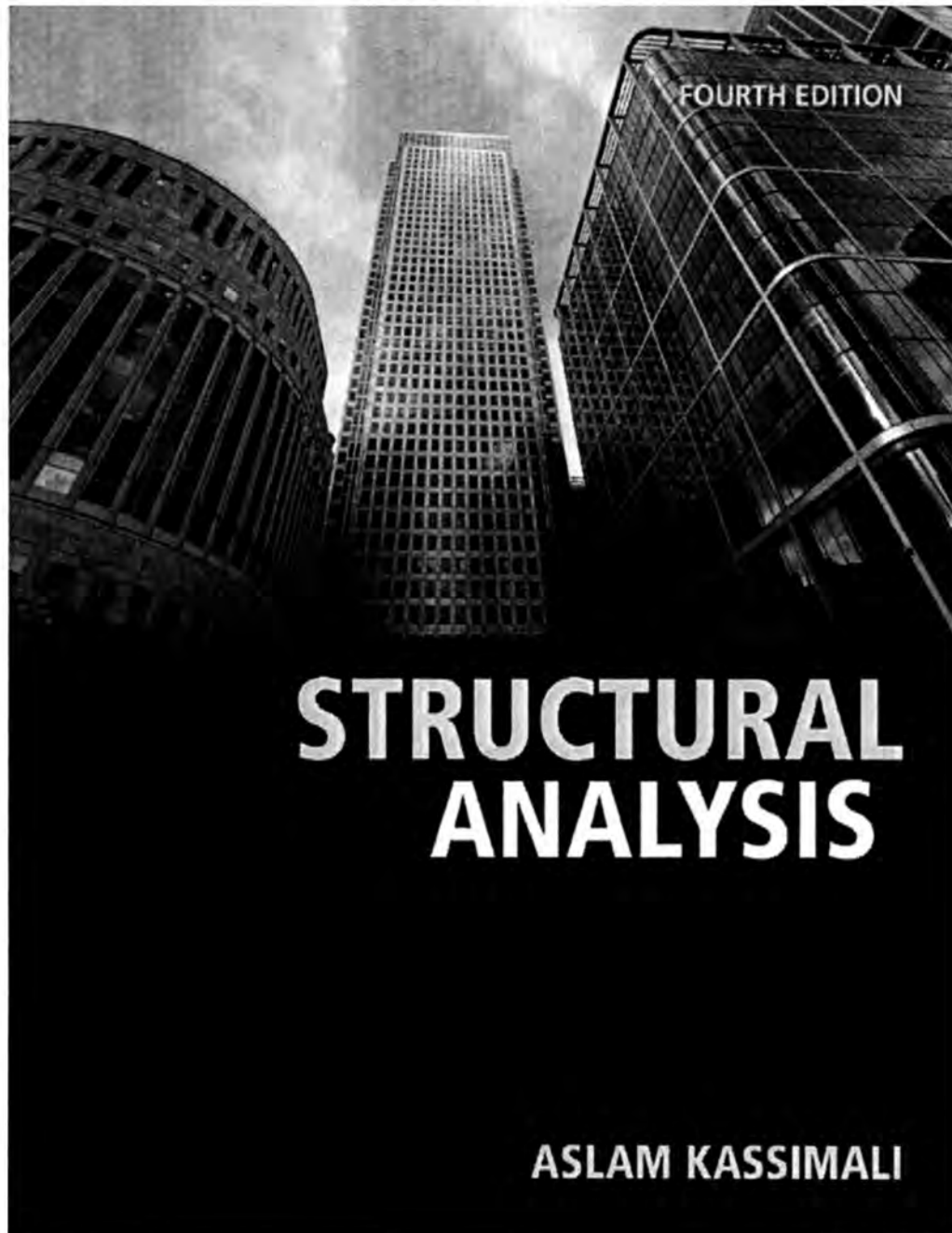


An Instructor's Solutions Manual to Accompany
Structural Analysis, 4th Edition
Aslam Kassimali



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An Instructor's Solutions Manual

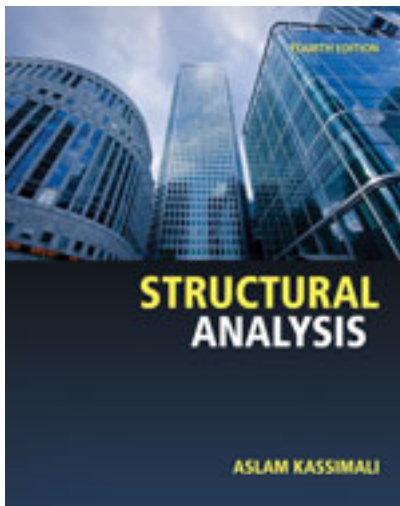
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Structural Analysis

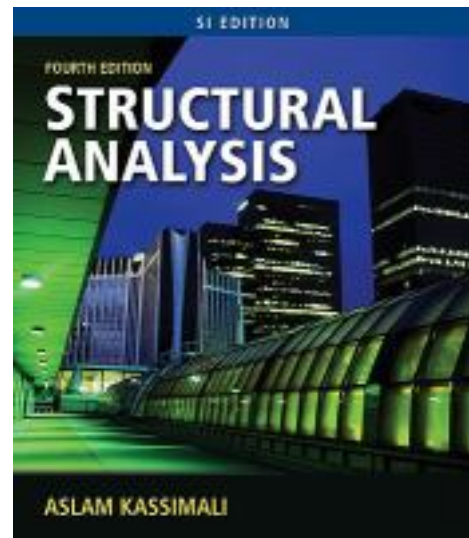
Fourth Edition

Aslam Kassimali

Southern Illinois University Carbondale



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SI



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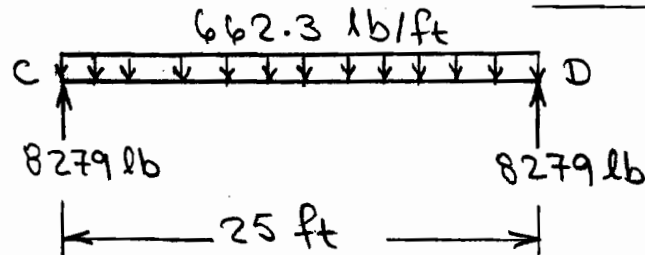
Chapter Two

Loads on Structures

CHAPTER 2

2.1 Beam CD

$$\begin{aligned}\text{Uniformly distributed load} &= 150(12)\left(\frac{4}{12}\right) + 490\left(\frac{18.3}{144}\right) \\ &= \underline{662.3 \text{ lb/ft}}\end{aligned}$$



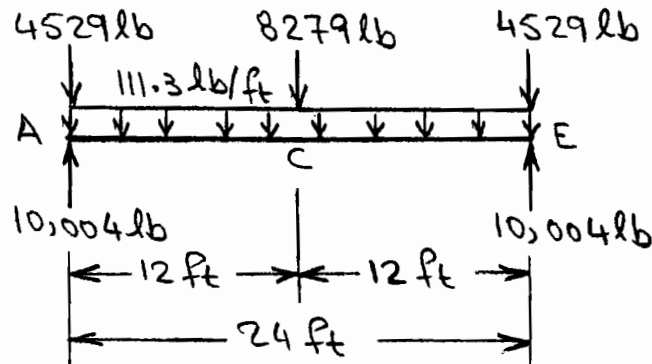
Girder AE

$$\text{Uniformly distributed load} = 490\left(\frac{32.7}{144}\right) = \underline{111.3 \text{ lb/ft}}$$

$$\text{Concentrated load at C} = \underline{8279 \text{ lb}}$$

Concentrated loads at A and E

$$= \left[150(6)\left(\frac{4}{12}\right) + 490\left(\frac{18.3}{144}\right)\right]\left(\frac{25}{2}\right) = \underline{4529 \text{ lb}}$$



2.2 See solution of Problem 2.1

Beam CD Uniformly distributed load

$$= 662.3 + 120 \left(\frac{6}{12} \right) (7) = 662.3 + 420 = \underline{1082.3 \text{ lb/ft}}$$

Girder AE Uniformly distributed load = $\underline{111.3 \text{ lb/ft}}$

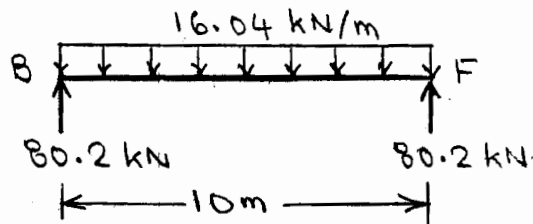
$$\text{Concentrated load at C} = 8279 + 420 \left(\frac{25}{2} \right) = \underline{13,529 \text{ lb}}$$

$$\text{Concentrated loads at A and E} = \underline{4529 \text{ lb}}$$

2.3 Beam BF

Uniformly distributed load

$$= 23.6 (5) \left(\frac{130}{1000} \right) + 77 \left(\frac{9100}{106} \right) = \underline{16.04 \text{ kN/m}}$$



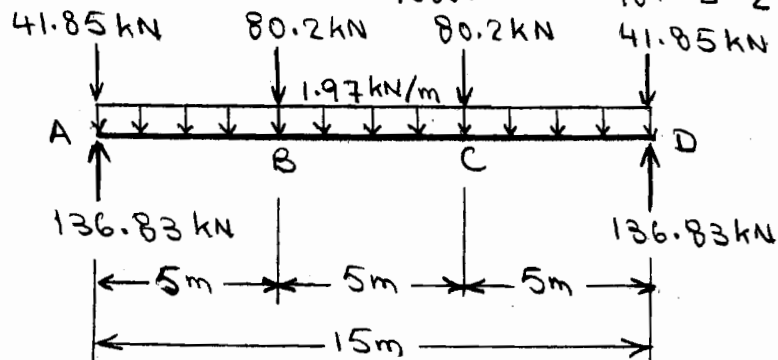
Girder AD

$$\text{Uniformly distributed load} = 77 \left(\frac{25600}{106} \right) = \underline{1.97 \text{ kN/m}}$$

$$\text{Concentrated loads at B and C} = \underline{80.2 \text{ kN}}$$

Concentrated loads at A and D

$$= \left[23.6 (2.5) \left(\frac{130}{1000} \right) + 77 \left(\frac{9100}{106} \right) \right] \frac{10}{2} = \underline{41.85 \text{ kN}}$$



2.4

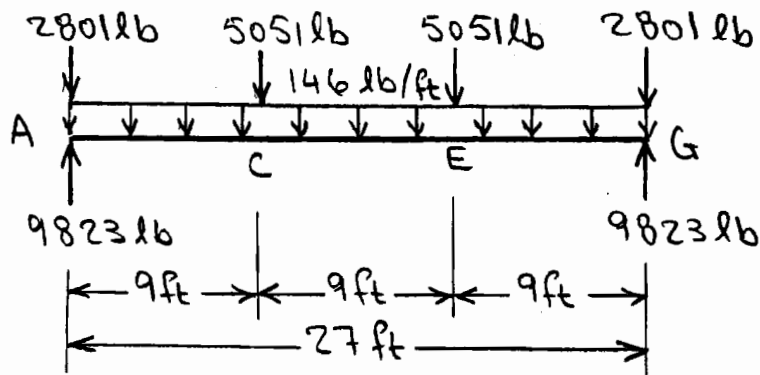
Uniformly distributed load = $490 \left(\frac{42.9}{144} \right) = \underline{146 \text{ lb/ft}}$

Concentrated loads at A and G

$$= \left[150(4.5) \left(\frac{4}{12} \right) + 490 \left(\frac{16.2}{144} \right) \right] \left(\frac{20}{2} \right) = \underline{2801 \text{ lb}}$$

Concentrated loads at C and E

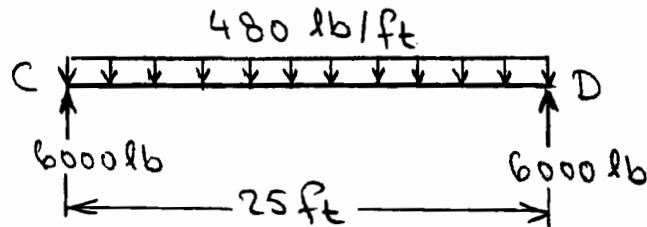
$$= \left[150(9) \left(\frac{4}{12} \right) + 490 \left(\frac{16.2}{144} \right) \right] \left(\frac{20}{2} \right) = \underline{5051 \text{ lb}}$$



2.5 Live load = 40 psf

Beam CD

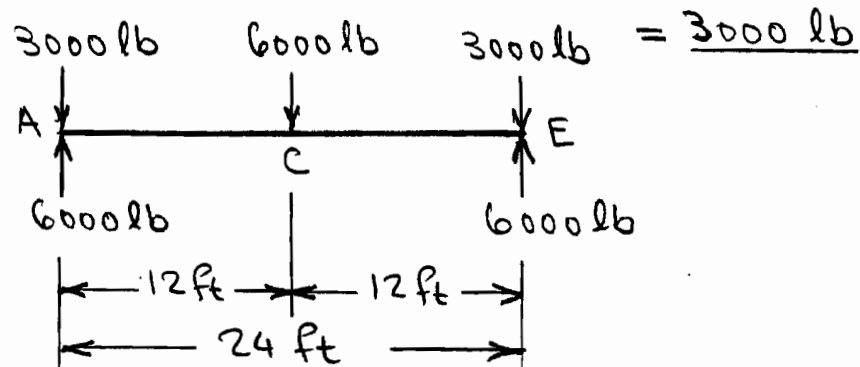
Uniformly distributed load = $40(12) = \underline{480 \text{ lb/ft}}$



Girder AE

Concentrated load at C = $\underline{6000 \text{ lb}}$

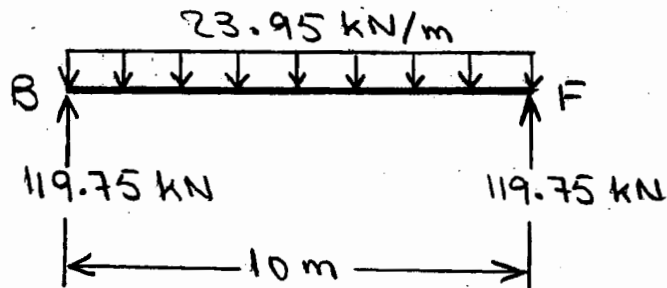
Concentrated loads at A and E = $[40(6)](\frac{25}{2})$



2.6 Live load = $4.79 \text{ kPa} = 4.79 \text{ kN/m}^2$

Beam BF

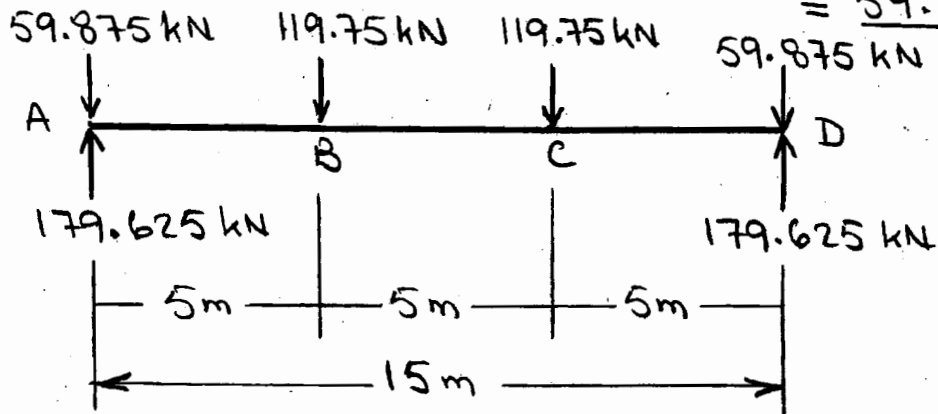
Uniformly distributed load = $4.79(5) = \underline{23.95 \text{ kN/m}}$



Girder AD

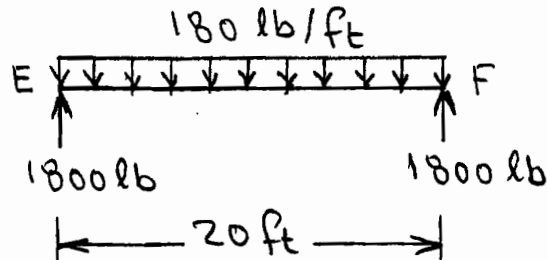
Concentrated loads at B and C = $\underline{119.75 \text{ kN}}$

Concentrated loads at A and D = $\left[4.79(2.5) \right] \frac{10}{2}$
 $= \underline{59.875 \text{ kN}}$



2.7 Beam EF

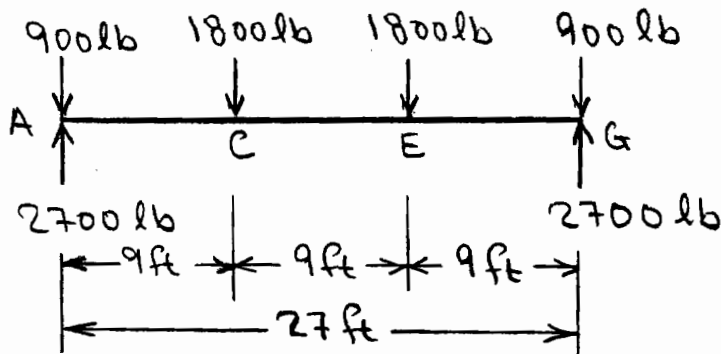
Uniformly distributed load = $20(9) = \underline{180 \text{ lb/ft}}$



Girder AG

Concentrated loads at C and E = 1800 lb

Concentrated loads at A and G = $1800/2 = \underline{900 \text{ lb}}$



Column A Concentrated load = 2700 lb

2.8 $V = 85 \text{ mph}$, $h = 40 + (15/2) = 47.5 \text{ ft}$,
 $I = 1.0$, $z_g = 1200 \text{ ft}$, $\alpha = 7.0$, $K_{zt} = 1$
 and $K_d = 1$
 $K_h = 2.01 \left(\frac{47.5}{1200} \right)^{2.7} = 0.8$
 $q_h = 0.00256 (0.8)(1)(1)(85)^2 (1) = 14.8 \text{ psf}$
 $G = 0.85$

For $\theta = 45^\circ$ and $h/L = 47.5/30 = 1.58$:

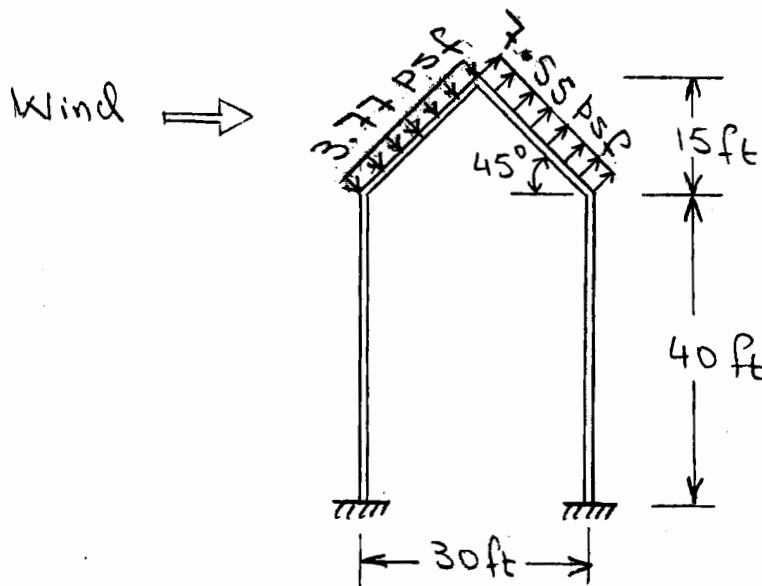
$C_p = 0.3$ for windward side

$C_p = -0.6$ for leeward side

Thus, the wind pressures are:

$p_h = 14.8 (0.85)(0.3) = \underline{3.77 \text{ psf}}$ for windward side

$p_h = 14.8 (0.85)(-0.6) = \underline{-7.55 \text{ psf}}$ for leeward side



2.9 $V = 40 \text{ m/s}$, $h = 12 + \frac{5}{2} = 14.5 \text{ m}$
 $I = 1.15$, $z_g = 366 \text{ m}$, $\alpha = 7.0$, $K_{zt} = 1$
 and $K_d = 1$
 $K_h = 2.01 \left(\frac{14.5}{366} \right)^{2/7} = 0.8$
 $q_h = 0.613 (0.8) (1) (1) (40)^2 (1.15) = 902.34 \text{ N/m}^2$
 $G = 0.85$

Roof slope: $\theta = \tan^{-1}(5/6) = 39.8^\circ$

$\frac{h}{L} = \frac{14.5}{12} = 1.21$

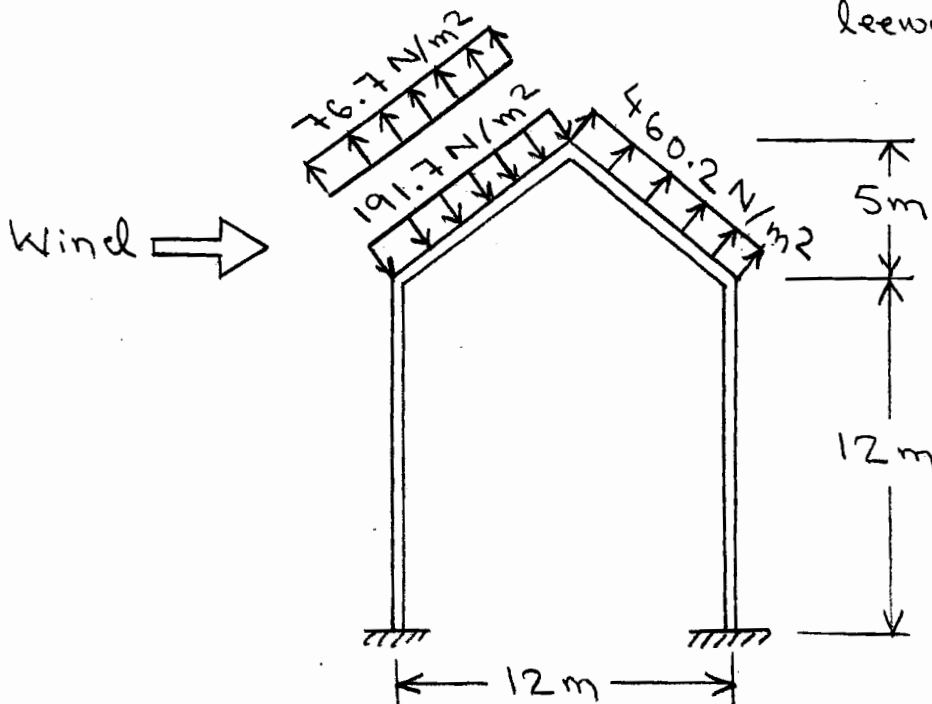
$C_p = -0.1$ and 0.25 for windward side

$C_p = -0.6$ for leeward side

Thus, the wind pressures are:

$$\begin{aligned} p_h &= (902.34)(0.85)(-0.1) = -76.7 \text{ N/m}^2 \\ p_h &= (902.34)(0.85)(0.25) = 191.7 \text{ N/m}^2 \\ p_h &= (902.34)(0.85)(-0.6) = -460.2 \text{ N/m}^2 \end{aligned} \quad \begin{array}{l} \text{for} \\ \text{windward} \\ \text{side} \end{array}$$

leeward side



2.10

$$V = 90 \text{ mph}, \quad h = 30 + \frac{11}{2} = 35.5 \text{ ft}$$

$$I = 1.15, \quad z_g = 900 \text{ ft}, \quad \alpha = 9.5, \quad k_{zt} = 1$$

$$\text{and } k_d = 1$$

$$K_h = 2.01 \left(\frac{35.5}{900} \right)^{2/9.5} = 1.02$$

$$q_h = 0.00256 (1.02)(1)(1)(90)^2 (1.15) = 24.32 \text{ psf}$$

$$G = 0.85$$

$$\text{Roof slope: } \theta = \tan^{-1}(11/20) = 28.8^\circ$$

$$\frac{h}{L} = \frac{35.5}{40} = 0.89$$

$$C_p = -0.3 \text{ and } 0.2 \text{ for windward side}$$

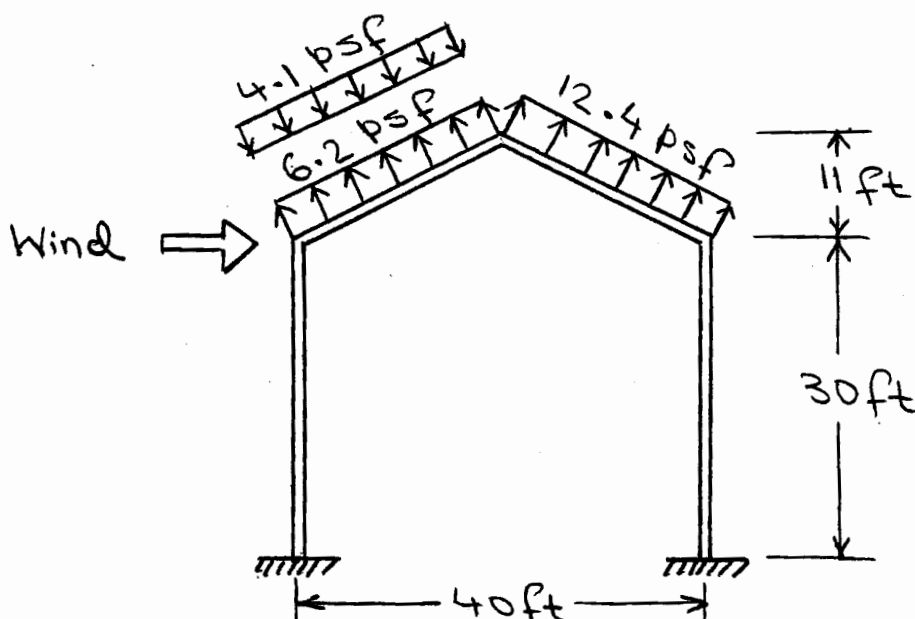
$$C_p = -0.6 \text{ for leeward side}$$

Thus, the wind pressures are:

$$p_h = 24.32 (0.85)(-0.3) = \underline{-6.2 \text{ psf}} \quad \text{for windward side}$$

$$p_h = 24.32 (0.85)(0.2) = \underline{4.1 \text{ psf}}$$

$$p_h = 24.32 (0.85)(-0.6) = \underline{-12.4 \text{ psf}} \quad \text{for leeward side}$$



2.11 $V = 90 \text{ mph}$, $I = 1.15$, $z_g = 900 \text{ ft}$, $\alpha = 9.5$

From the solution of Problem 2.10:

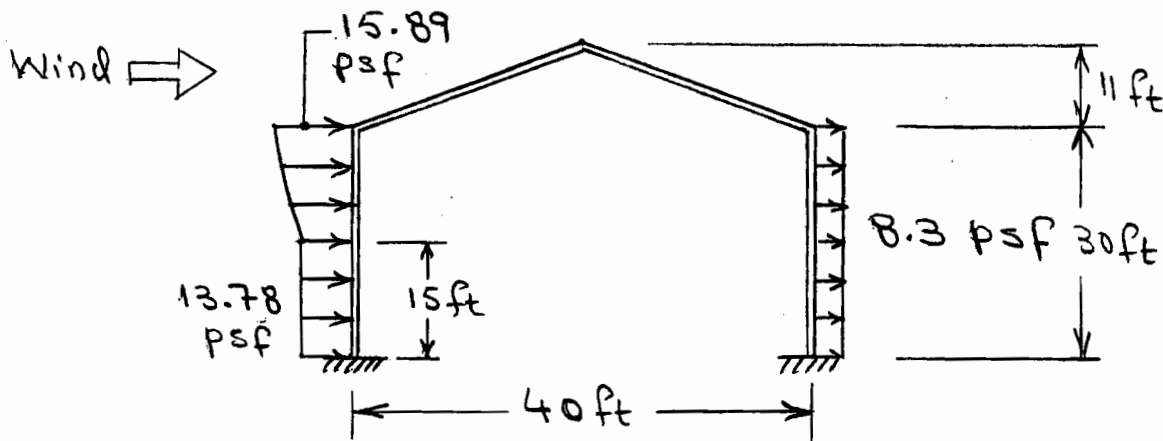
$$q_h = 24.32 \text{ psf} \quad \text{and} \quad G = 0.85$$

Leeward wall: For $L/B = 40/30 = 1.33$, $C_p = -0.4$

$$\begin{aligned} \text{Thus, the wind pressure, } p_h &= 24.32(0.85)(-0.4) \\ &= \underline{-8.3 \text{ psf}} \end{aligned}$$

Windward wall: $C_p = 0.8$

z (ft)	K_z	q_z (psf)	p_z (psf)
30	0.98	23.37	15.89
25	0.95	22.65	15.4
20	0.90	21.46	14.59
15	0.85	20.27	13.78



2.12 $p_g = 20 \text{ psf}$, $C_e = 1$, $C_t = 1$, $I = 1.2$

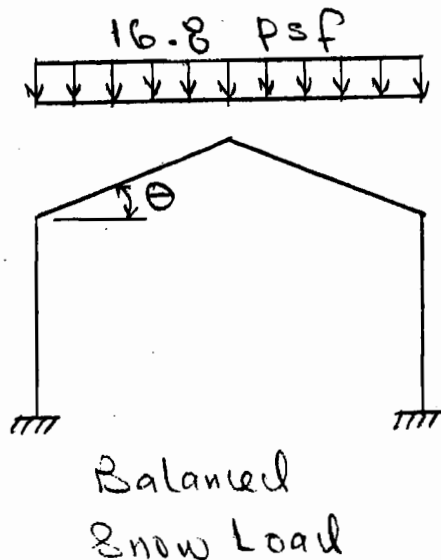
$$p_f = 0.7 C_e C_t I p_g = 0.7 (1)(1)(1.2)(20) = 16.8 \text{ psf}$$

$$\theta = \tan^{-1}(11/20) = 28.8^\circ, \quad \frac{70}{W} + 0.5 = \frac{70}{20} + 0.5 = 4^\circ$$

Therefore, the minimum values of p_f need not be considered.

$$C_s = 1$$

$$\text{Balanced load} = p_s = C_s p_f = 1(16.8) = \underline{16.8 \text{ psf}}$$



$$2.13 \quad p_g = 1.2 \text{ kN/m}^2, \quad C_e = 1, \quad C_t = 1, \quad I = 1.1$$

$$p_f = 0.7 C_e C_t I p_g = 0.7 (1) (1) (1.1) (1.2) = 0.92 \text{ kN/m}^2$$

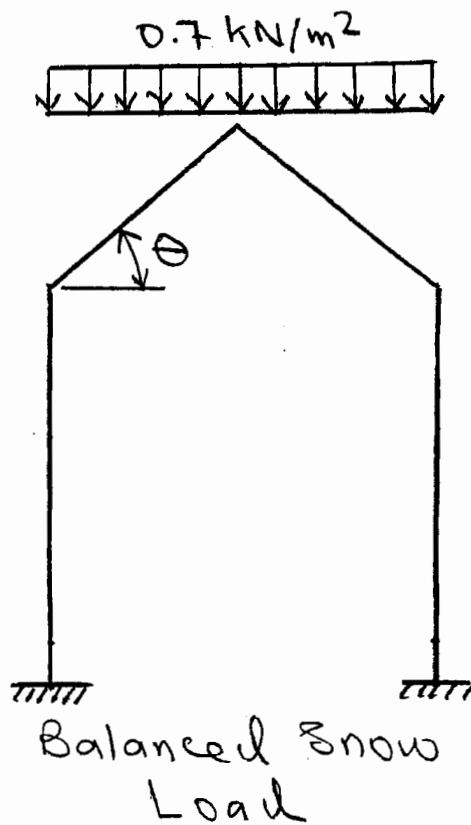
$$\theta = \tan^{-1}(5/6) = 39.8^\circ, \quad W = 6\text{m} = 19.7 \text{ ft}$$

$$\frac{70}{W} + 0.5 = \frac{70}{19.7} + 0.5 = 4.1^\circ$$

Therefore, the minimum values of p_f need not be considered.

$$C_s = 1 - \frac{\theta - 30^\circ}{40^\circ} = 0.76$$

$$\text{Balanced Load} = p_s = C_s p_f = 0.76 (0.92) = \underline{0.7 \text{ kN/m}^2}$$



Chapter Three

Equilibrium and Support Reactions

CHAPTER 3

- 3.1** (a) Internally stable, with $r=3$. It is statically determinate
- (b) Internally stable, with $r=5$. It is statically indeterminate. $i_e = 5-3 = \underline{2}$
- (c) Internally unstable, with $r=6$ and $e_c=2$.
As $r > 3+e_c$, the structure is statically indeterminate. $i_e = 6-(3+2) = \underline{1}$
- (d) Internally unstable with $r=3$ and $e_c=1$.
As $r < 3+e_c$, the beam is statically unstable.

- 3.2** (a) Internally stable, with $r=5$. It is statically indeterminate. $i_e = 5-3 = \underline{2}$.
- (b) Internally unstable, with $r=5$ and $e_c=2$.
As $r = 3+e_c$, the beam is statically determinate.
- (c) Internally stable, with $r=6$. It is statically indeterminate. $i_e = 6-3 = \underline{3}$
- (d) Internally unstable, with $r=6$ and $e_c=1$.
As $r > 3+e_c$, the arch is statically indeterminate. $i_e = 6-(3+1) = \underline{2}$.
- (e) Internally unstable, with $r=5$ and $e_c=2$.
As $r = 3+e_c$, the frame is statically determinate.

- 3.3 (a) Internally unstable, with $r=5$ and $e_c=3$.
As $r < 3 + e_c$, the beam is statically unstable.
- (b) Internally unstable, with $r=4$ and $e_c=2$.
As $r < 3 + e_c$, the beam is statically unstable.
- (c) Internally stable, with $r=4$. It is
statically indeterminate. $i_e = 4 - 3 = 1$.
- (d) Internally unstable, with $r=4$ and $e_c=2$.
As $r < 3 + e_c$, the frame is statically unstable.

3.4 (a) Internally stable, with $r = 4$.

It is statically indeterminate.

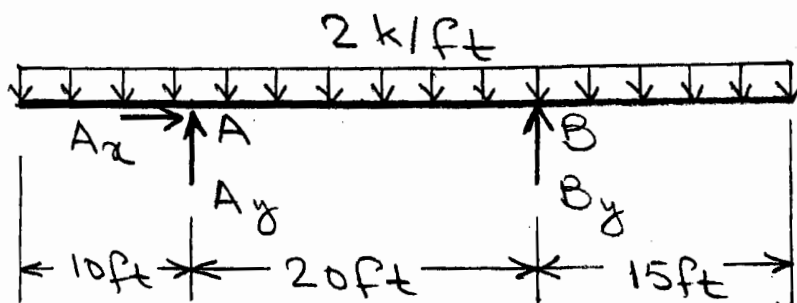
$$i_e = 4 - 3 = \underline{1}.$$

(b) Internally unstable, with $r = 4$ and $e_c = 2$. As $r < 3 + e_c$, the frame is statically unstable.

(c) Internally stable, with $r = 6$. It is statically indeterminate. $i_e = 6 - 3 = \underline{3}$.

(d) Internally unstable, with $r = 6$ and $e_c = 3$. As $r = 3 + e_c$, the frame is statically determinate.

3.5



$$\Sigma F_x = 0$$

$$A_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0$$

$$-A_y(20) + 2(45)(7.5) = 0$$

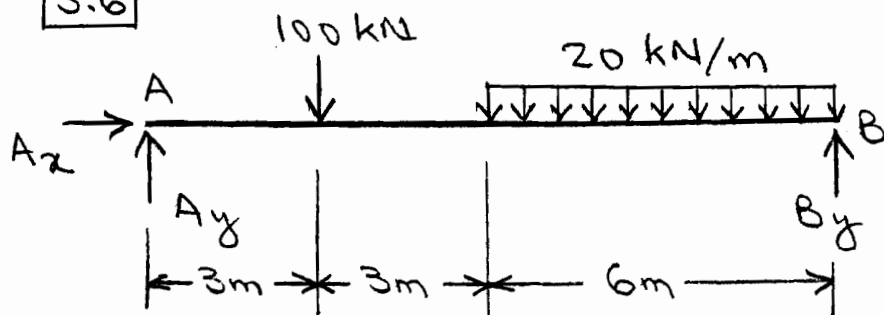
$$A_y = 33.75 \text{ k} \uparrow$$

$$+\uparrow \Sigma F_y = 0$$

$$33.75 - 2(45) + B_y = 0$$

$$B_y = 56.25 \text{ k} \uparrow$$

3.6



$$\Sigma F_x = 0$$

$$A_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0$$

$$-A_y(12) + 100(9) + 20(6)(3) = 0$$

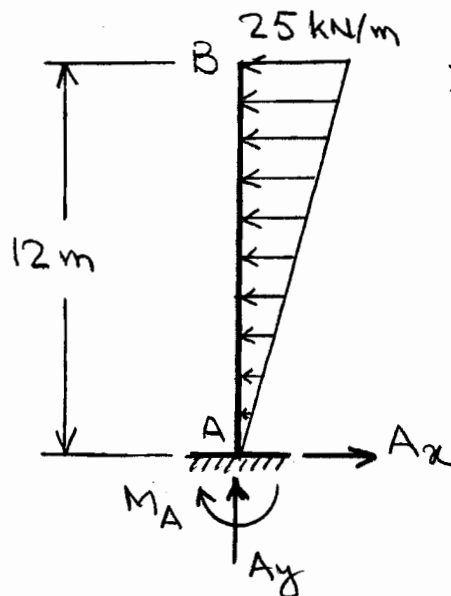
$$A_y = 105 \text{ kN} \uparrow$$

$$+\uparrow \Sigma F_y = 0$$

$$105 - 100 - 20(6) + B_y = 0$$

$$B_y = 115 \text{ kN} \uparrow$$

3.7



$$\rightarrow \sum F_x = 0$$

$$-\frac{1}{2}(25)(12) + A_x = 0$$

$$\underline{A_x = 150 \text{ kN} \rightarrow}$$

$$\sum F_y = 0$$

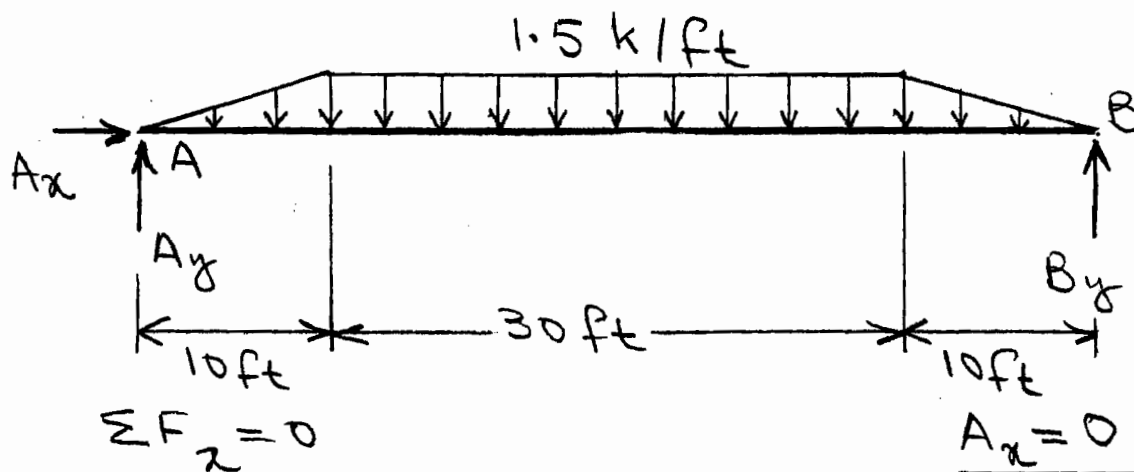
$$\underline{A_y = 0}$$

$$+\circlearrowleft \sum M_A = 0$$

$$-M_A + \frac{1}{2}(25)(12)(8) = 0$$

$$\underline{M_A = 1200 \text{ kN}\cdot\text{m} \curvearrowright}$$

3.8



$$+\circlearrowleft \Sigma M_B = 0$$

$$-A_y(50) + \frac{1}{2}(1.5)(10)\left(\frac{10}{3} + 40\right) + 1.5(30)(25) + \frac{1}{2}(1.5)(10)\left(\frac{2}{3}\right)(10) = 0$$

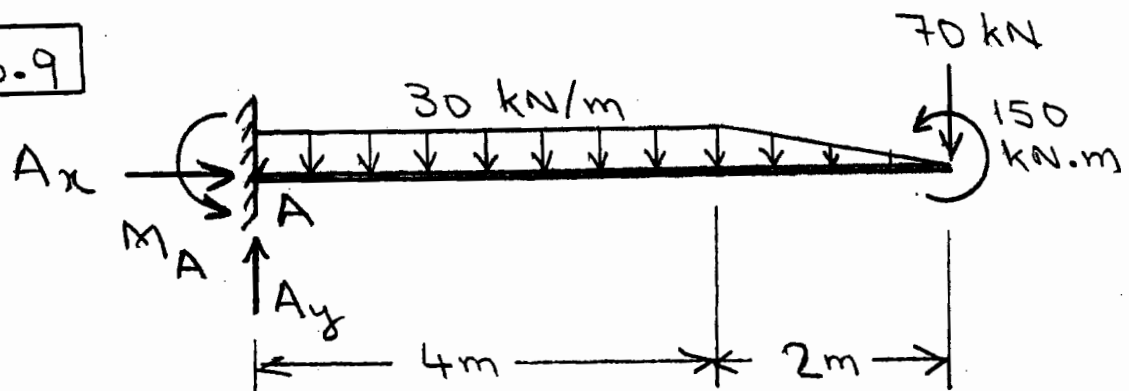
$$\underline{A_y = 30 \text{ k} \uparrow}$$

$$+\uparrow \Sigma F_y = 0$$

$$30 - \frac{1}{2}(1.5)(10)(2) - 1.5(30) + B_y = 0$$

$$\underline{B_y = 30 \text{ k} \uparrow}$$

3.9



$$\sum F_x = 0$$

$$A_x = 0$$

$$+\uparrow \sum F_y = 0$$

$$A_y - 30(4) - \frac{1}{2}(30)(2) - 70 = 0$$

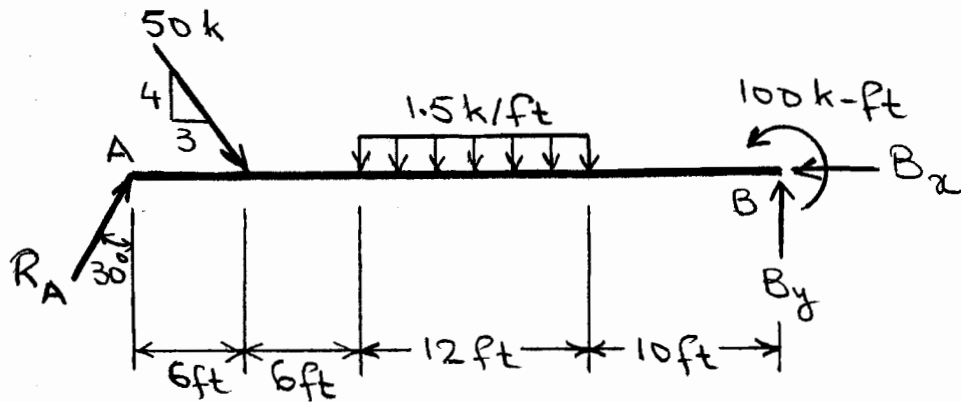
$$A_y = 220 \text{ kN} \uparrow$$

$$+\circlearrowleft \sum M_A = 0$$

$$M_A - 30(4)(2) - \frac{1}{2}(30)(2)\left(4 + \frac{2}{3}\right) - 70(6) + 150 = 0$$

$$M_A = 650 \text{ kN.m} \circlearrowleft$$

3.10



$$+\circlearrowleft \Sigma M_B = 0$$

$$-R_A (\cos 30^\circ)(34) + \frac{4}{5}(50)(28) + 1.5(12)(16) + 100 = 0$$

$$\underline{R_A = 51.2 \text{ k} \nearrow}$$

$$\rightarrow \Sigma F_x = 0$$

$$51.2(\sin 30^\circ) + \frac{3}{5}(50) - B_x = 0$$

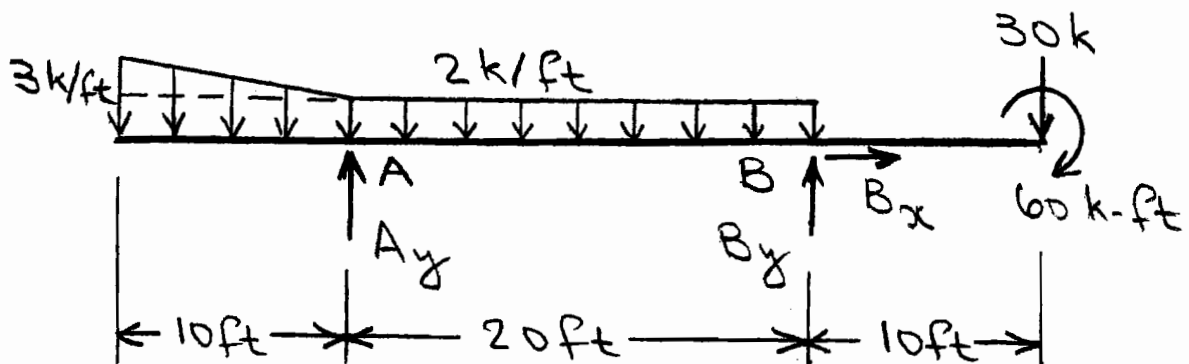
$$\underline{B_x = 55.6 \text{ k} \leftarrow}$$

$$+\uparrow \Sigma F_y = 0$$

$$51.2(\cos 30^\circ) - \frac{4}{5}(50) - 1.5(12) + B_y = 0$$

$$\underline{B_y = 13.65 \text{ k} \uparrow}$$

3.11



$$\sum F_x = 0$$

$$\underline{B_x = 0}$$

$$+\circlearrowleft \sum M_A = 0$$

$$\frac{1}{2}(1)(10)\left(\frac{20}{3}\right) - 2(30)(5) + B_y(20) - 30(30) - 60 = 0$$

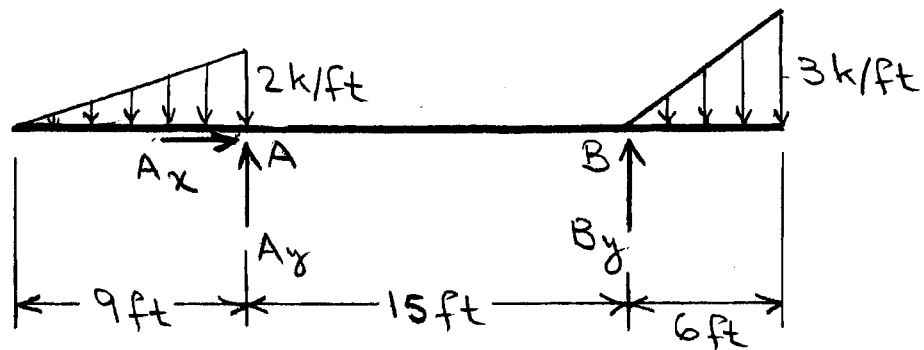
$$\underline{B_y = 61.33 \text{ k} \uparrow}$$

$$+\uparrow \sum F_y = 0$$

$$-\frac{1}{2}(1)(10) - 2(30) + A_y + 61.33 - 30 = 0$$

$$\underline{A_y = 33.67 \text{ k} \uparrow}$$

3.12



$$\sum F_x = 0$$

$$\underline{A_x = 0}$$

$$+\circlearrowleft \sum M_B = 0$$

$$\frac{1}{2}(2)(9)\left[\frac{1}{3}(9) + 15\right] - A_y(15) - \frac{1}{2}(3)(6)(4) = 0$$

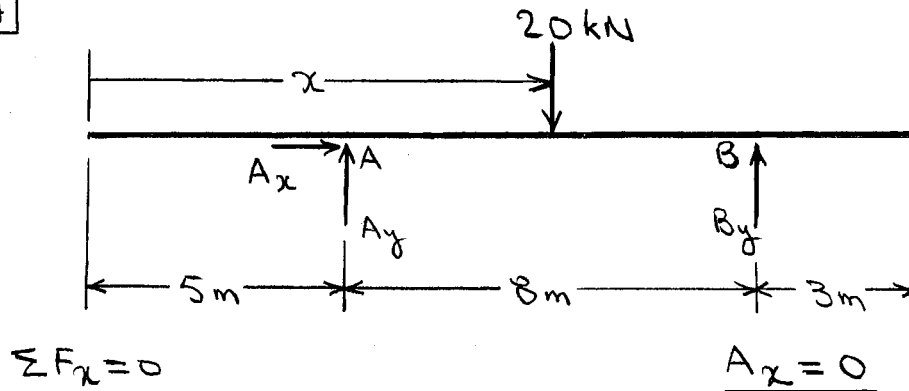
$$\underline{A_y = 8.4 \text{ k} \uparrow}$$

$$+\uparrow \sum F_y = 0$$

$$-\frac{1}{2}(2)(9) + 8.4 + B_y - \frac{1}{2}(3)(6) = 0$$

$$\underline{B_y = 9.6 \text{ k} \uparrow}$$

3.14



$+\circlearrowleft \sum M_B = 0$

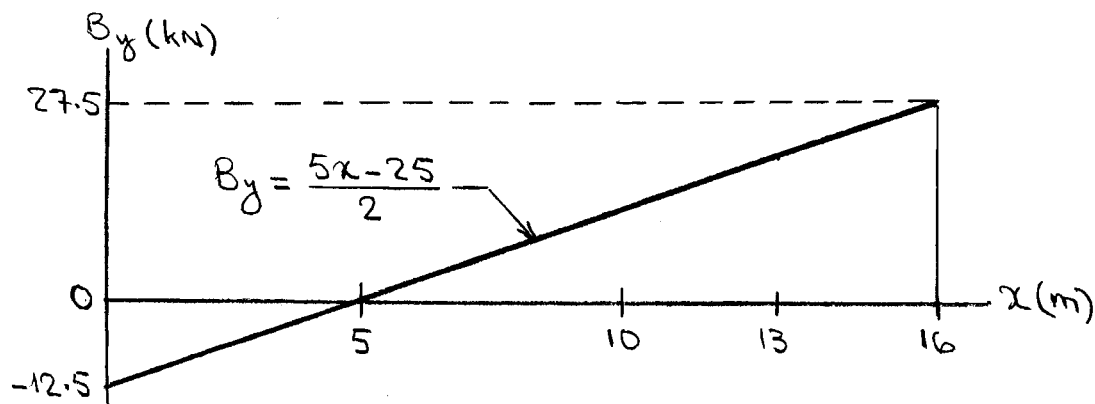
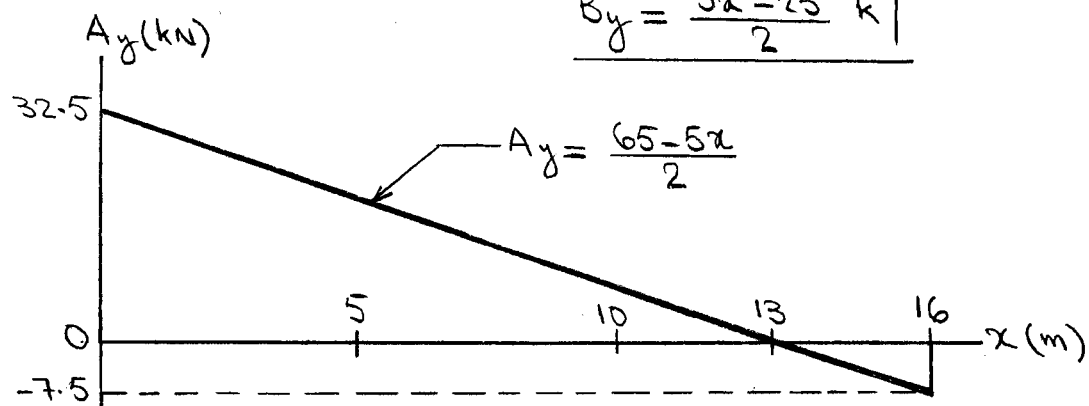
$-A_y(8) + 20(13-x) = 0$

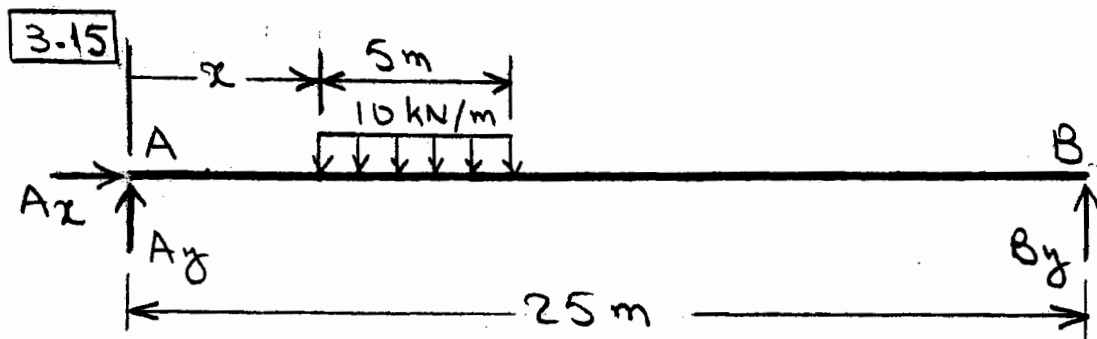
$A_y = \frac{65-5x}{2} \text{ k} \uparrow$

$+\uparrow \sum F_y = 0$

$\frac{65-5x}{2} - 20 + B_y = 0$

$B_y = \frac{5x-25}{2} \text{ k} \uparrow$





$$\sum F_x = 0$$

$$A_x = 0$$

$$0 \leq x \leq 20 \text{ m:}$$

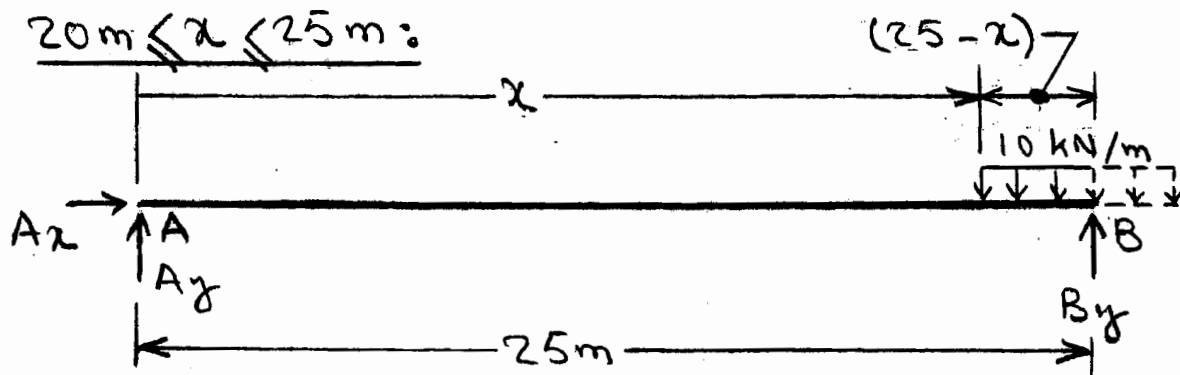
$$+\circlearrowleft \sum M_B = 0 \quad -A_y(25) + 10(5)[25 - (x + 25)] = 0$$

$$A_y = 45 - 2x \text{ kN} \uparrow$$

$$+\uparrow \sum F_y = 0 \quad (45 - 2x) - 10(5) + B_y = 0$$

$$B_y = 5 + 2x \text{ kN} \uparrow$$

$$20 \text{ m} \leq x \leq 25 \text{ m:}$$



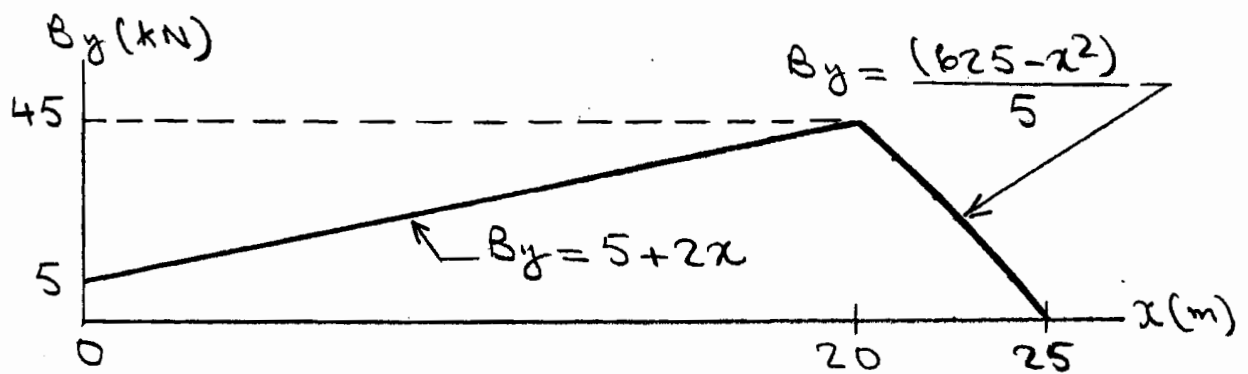
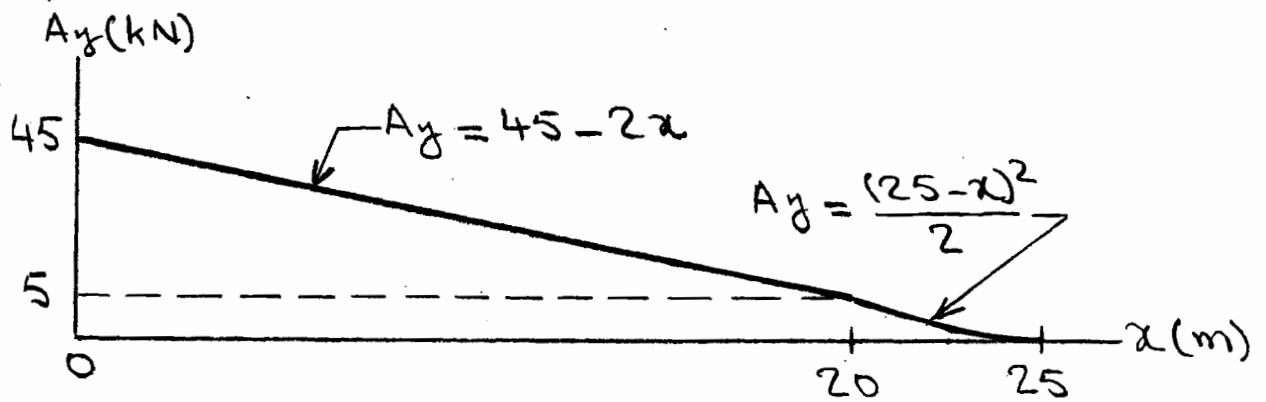
$$+\circlearrowleft \sum M_B = 0$$

$$-A_y(25) + 10(25 - x)\left(\frac{25 - x}{2}\right) = 0 \quad A_y = \frac{(25 - x)^2}{5} \text{ kN} \uparrow$$

$$+\uparrow \sum F_y = 0 \quad \frac{(25 - x)^2}{5} - 10(25 - x) + B_y = 0$$

$$B_y = \frac{(625 - x^2)}{5} \text{ kN} \uparrow$$

13.15 (contd.)

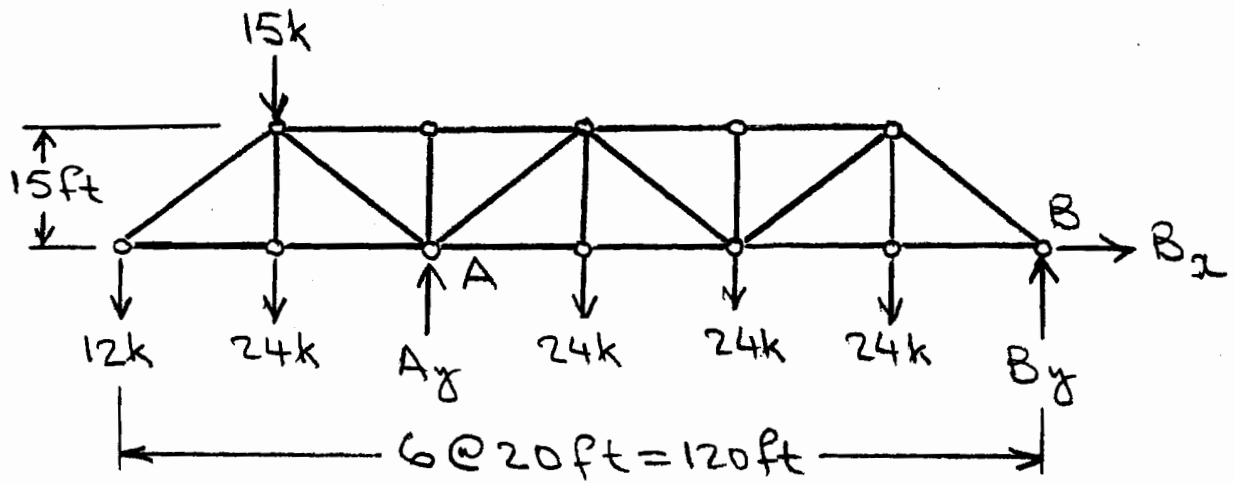


3.16

$$\underline{B_x = 0}$$

$$A_y = B_y = \frac{2(50) + 70}{2} = \underline{85 \text{ kN} \uparrow}$$

3.17



$$\Sigma F_x = 0$$

$$\underline{B_x = 0}$$

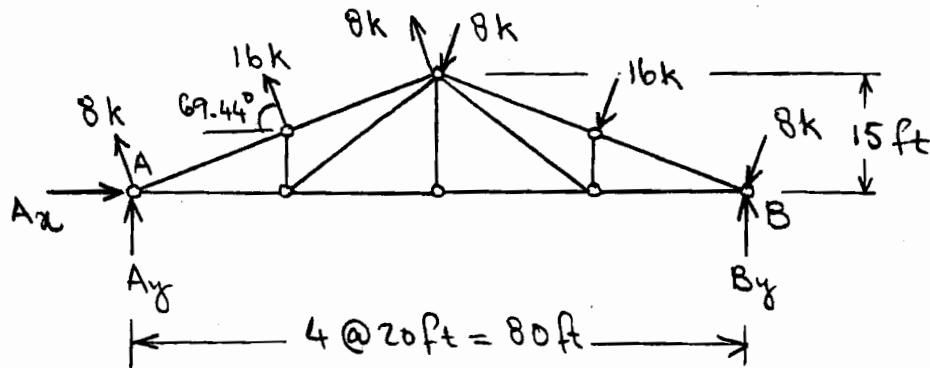
$$A_y = 12\left(\frac{6}{4}\right) + (24 + 15)\left(\frac{5}{4}\right) + 24\left(\frac{3}{4} + \frac{1}{2} + \frac{1}{4}\right)$$

$$\underline{A_y = 102.75 \text{ k} \uparrow}$$

$$B_y = 12 + 15 + 4(24) - 102.75$$

$$\underline{B_y = 20.25 \text{ k} \uparrow}$$

3.18



$$\rightarrow \Sigma F_x = 0 \quad A_x - (8 + 16 + 8 + 8 + 16 + 8) \cos 69.44^\circ = 0$$

$$\underline{A_x = 22.48 \text{ k} \rightarrow}$$

$$+\circlearrowleft \Sigma M_B = 0 \quad -A_y(80) + 2[16(7.5) + 8(15)] \cos 69.44^\circ$$

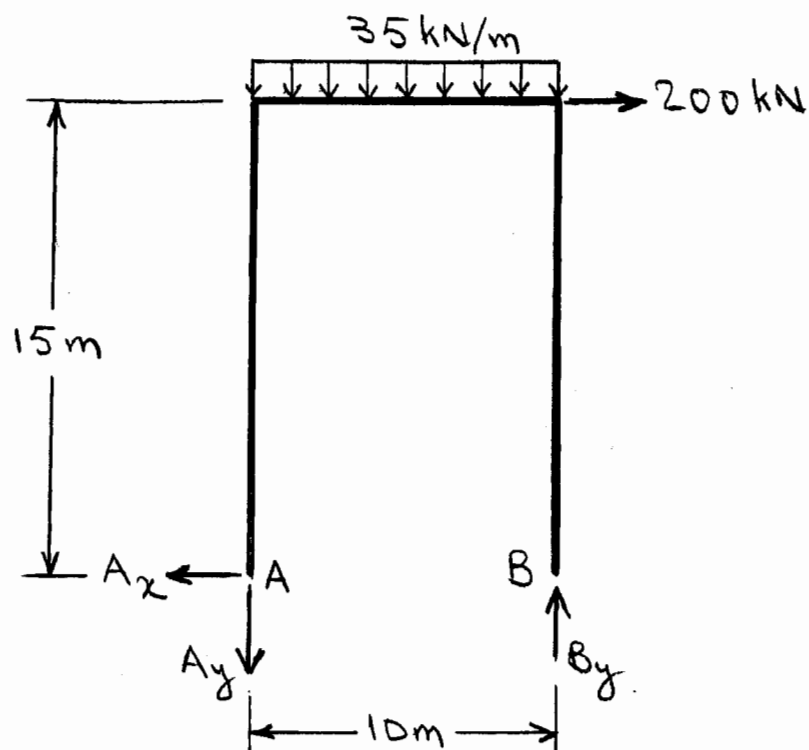
$$+ [-8(80) - 16(60) + 16(20)] \sin 69.44^\circ = 0$$

$$A_y = -12.87 \text{ k} \quad \underline{A_y = 12.87 \text{ k} \downarrow}$$

$$+\uparrow \Sigma F_y = 0 \quad -12.87 + (8 + 16 + 8 - 8 - 16 - 8) \sin 69.44^\circ + B_y = 0$$

$$\underline{B_y = 12.87 \text{ k} \uparrow}$$

3.19



$$\rightarrow \sum F_x = 0$$

$$\underline{A_x = 200 \text{ kN} \leftarrow}$$

$$+\curvearrowright \sum M_B = 0$$

$$A_y(10) + 35(10)(5) - 200(15) = 0$$

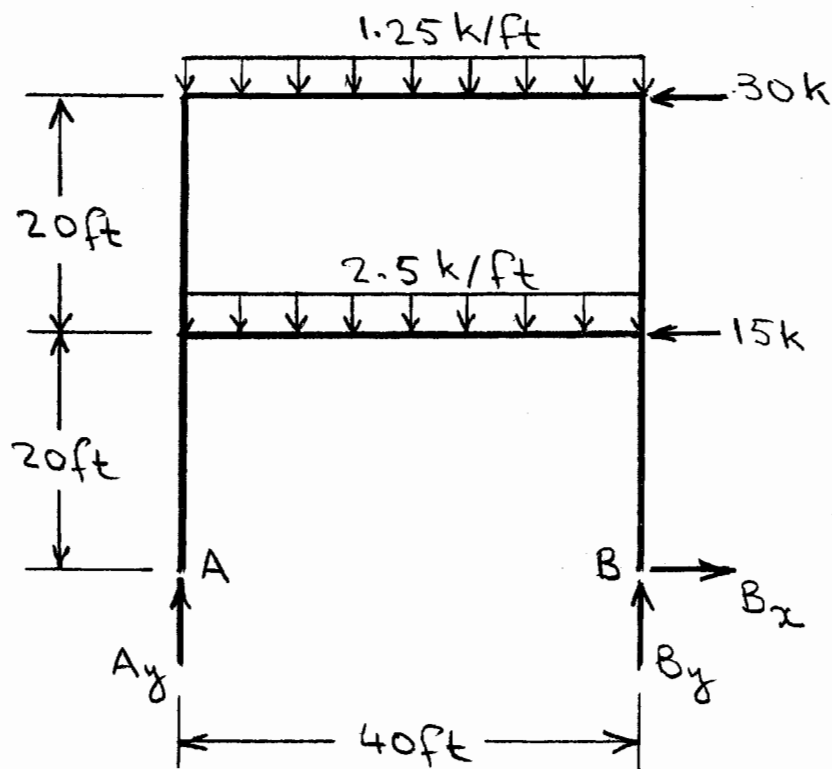
$$\underline{A_y = 125 \text{ kN} \downarrow}$$

$$+\uparrow \sum F_y = 0$$

$$-125 - 35(10) + B_y = 0$$

$$\underline{B_y = 475 \text{ kN} \uparrow}$$

3.20



$$\rightarrow \Sigma F_x = 0 \quad B_x - 15 - 30 = 0 \quad \underline{B_x = 45 \text{ k} \rightarrow}$$

$$+\circlearrowleft \Sigma M_B = 0$$

$$-A_y(40) + 2.5(40)(20) + 15(20) + 1.25(40)(20) + 30(40) = 0$$

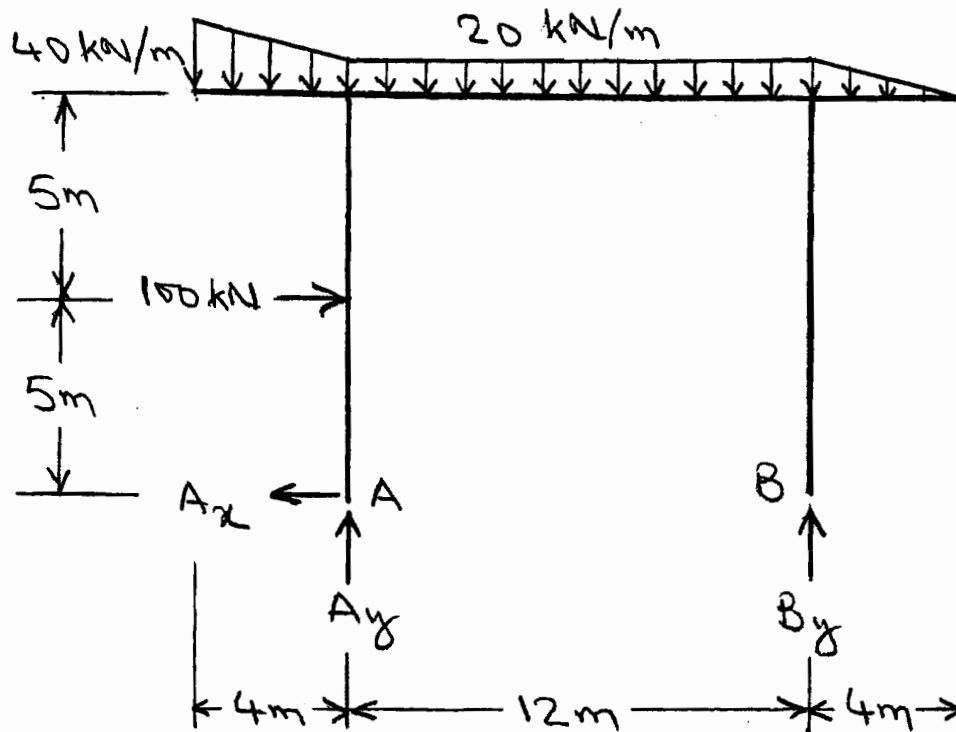
$$\underline{A_y = 112.5 \text{ k} \uparrow}$$

$$+\uparrow \Sigma F_y = 0$$

$$112.5 - 2.5(40) - 1.25(40) + B_y = 0$$

$$\underline{B_y = 37.5 \text{ k} \uparrow}$$

3.21

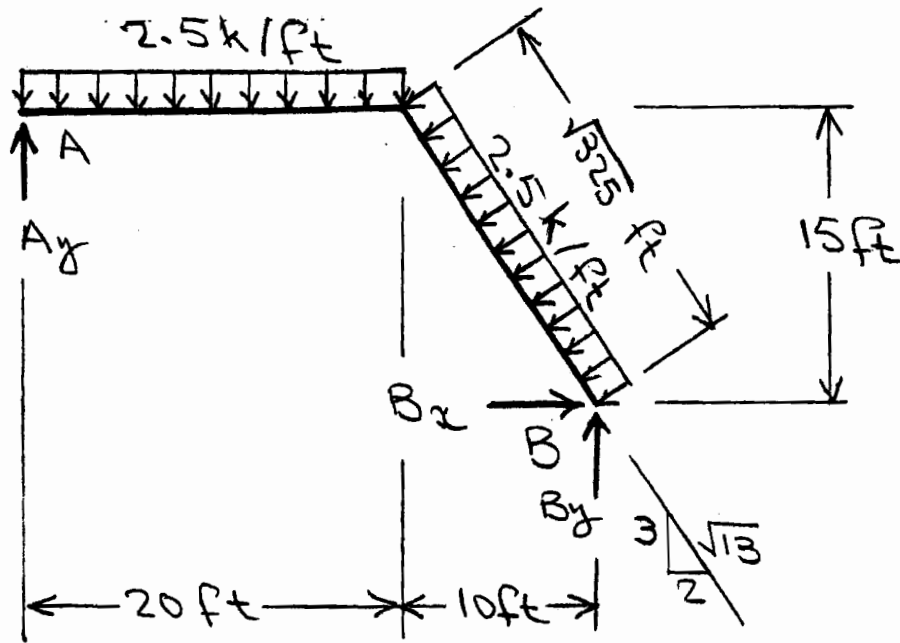


$$\rightarrow \Sigma F_x = 0 \quad 100 - A_x = 0 \quad \underline{A_x = 100 \text{ kN} \leftarrow}$$

$$\begin{aligned} + \curvearrowright \Sigma M_A = 0 \\ -100(5) + \frac{1}{2}(20)(4)\left(\frac{2}{3}\right)(4) - 20(16)(4) + B_y(12) \\ - \frac{1}{2}(20)(4)\left(12 + \frac{4}{3}\right) = 0 \\ \underline{B_y = 183.89 \text{ kN} \uparrow} \end{aligned}$$

$$\begin{aligned} + \uparrow \Sigma F_y = 0 \\ A_y - \frac{1}{2}(20)(4) - 20(16) - \frac{1}{2}(20)(4) + 183.89 = 0 \\ \underline{A_y = 216.11 \text{ kN} \uparrow} \end{aligned}$$

3.22



$$+\rightarrow \Sigma F_x = 0 \quad B_x - 2.5(\sqrt{325})\left(\frac{3}{\sqrt{13}}\right) = 0$$

$$\underline{B_x = 37.5 \text{ k} \rightarrow}$$

$$+\circlearrowleft \Sigma M_B = 0$$

$$-A_y(30) + 2.5(20)(20) + 2.5(\sqrt{325})\left(\frac{\sqrt{325}}{2}\right) = 0$$

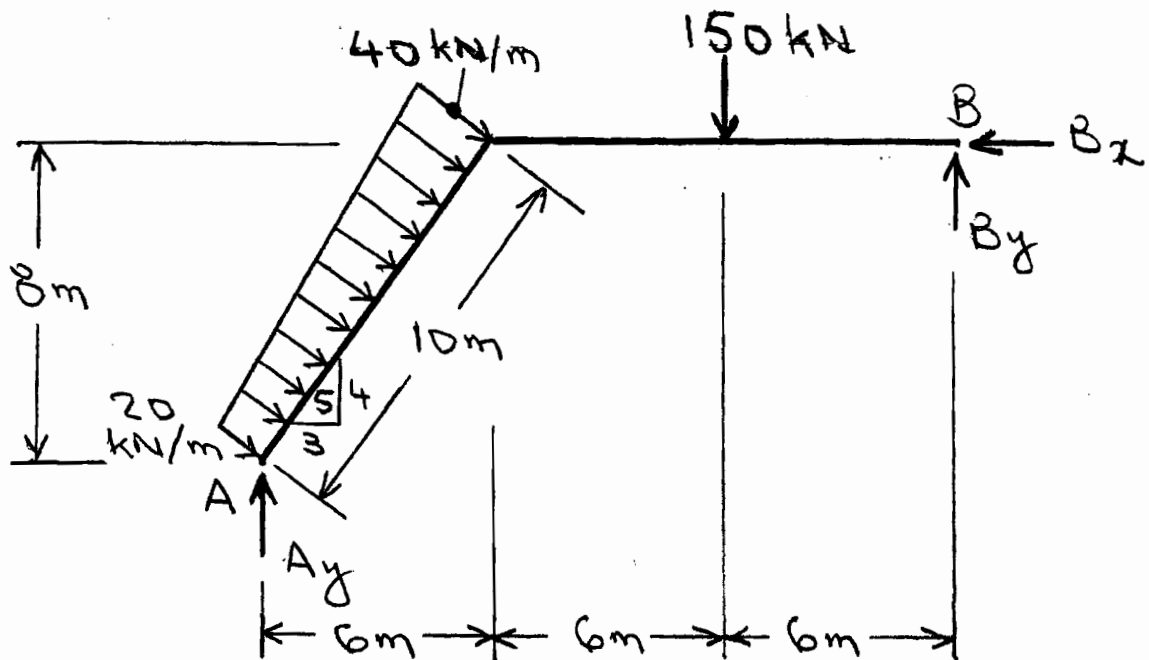
$$\underline{A_y = 46.875 \text{ k} \uparrow}$$

$$+\uparrow \Sigma F_y = 0$$

$$46.875 - 2.5(20) - 2.5(\sqrt{325})\left(\frac{2}{\sqrt{13}}\right) + B_y = 0$$

$$\underline{B_y = 28.125 \text{ k} \uparrow}$$

3.23



$$\rightarrow \Sigma F_x = 0 \quad \left(\frac{20+40}{2} \right) 10 \left(\frac{4}{5} \right) - B_x = 0$$

$$\underline{B_x = 240 \text{ kN} \leftarrow}$$

$$+\circlearrowleft \Sigma M_A = 0$$

$$-20(10)5 - \frac{1}{2}(20)10\left(\frac{20}{3}\right) - 150(12) + 240(8) + B_y(18) = 0$$

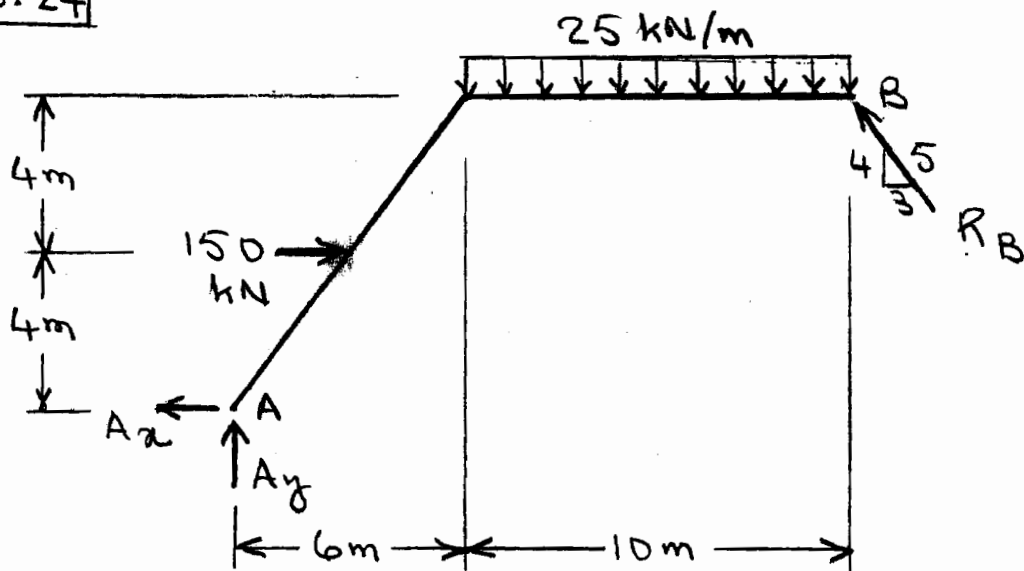
$$\underline{B_y = 85.93 \text{ kN} \uparrow}$$

$$+\uparrow \Sigma F_y = 0$$

$$A_y - \left(\frac{20+40}{2} \right) 10 \left(\frac{3}{5} \right) - 150 + 85.93 = 0$$

$$\underline{A_y = 244.07 \text{ kN} \uparrow}$$

3.24



$$+\circlearrowleft \sum M_A = 0$$

$$-150(4) - 25(10)(11) + \frac{3}{5}R_B(8) + \frac{4}{5}R_B(16) = 0$$

$$R_B = 190.3 \text{ kN} \nearrow$$

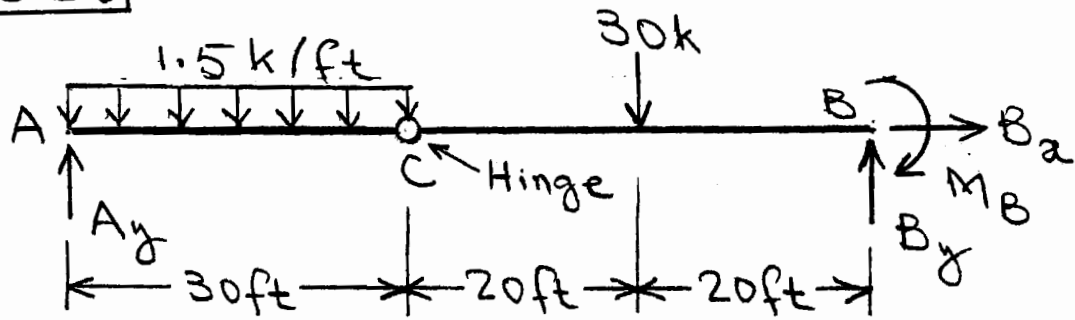
$$\rightarrow \sum F_x = 0 \quad -A_x + 150 - \frac{3}{5}(190.3) = 0$$

$$A_x = 35.8 \text{ kN} \leftarrow$$

$$+\uparrow \sum F_y = 0 \quad A_y - 25(10) + \frac{4}{5}(190.3) = 0$$

$$A_y = 97.7 \text{ kN} \uparrow$$

3.25



$$\sum F_x = 0$$

$$\underline{B_x = 0}$$

$$+\circlearrowleft \sum M_C^{AC} = 0$$

$$-A_y(30) + 1.5(30)(15) = 0$$

$$\underline{A_y = 22.5 \text{ k} \uparrow}$$

$$+\uparrow \sum F_y = 0$$

$$22.5 - 1.5(30) - 30 + B_y = 0$$

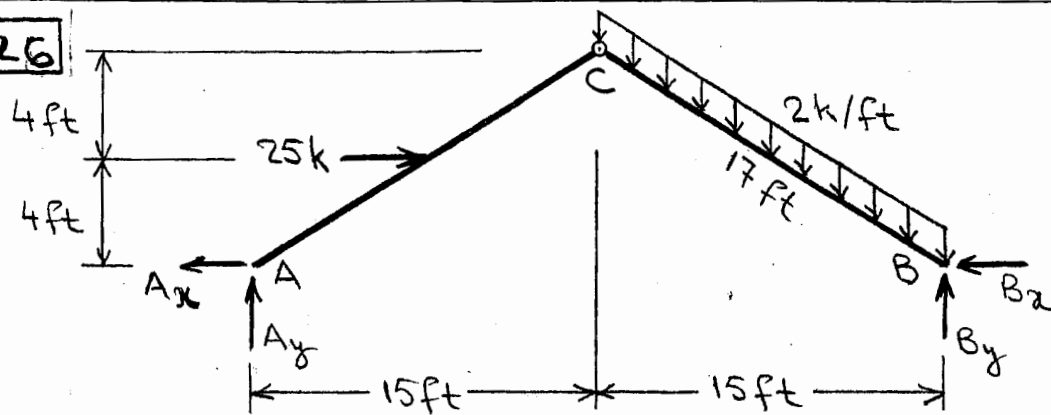
$$\underline{B_y = 52.5 \text{ k} \uparrow}$$

$$+\circlearrowleft \sum M_A = 0$$

$$-1.5(30)(15) - 30(50) + 52.5(70) - M_B = 0$$

$$\underline{M_B = 1500 \text{ k-ft} \curvearrowright}$$

3.26



$$+\circlearrowleft \Sigma M_B = 0$$

$$-A_y(30) - 25(4) + 2(17)\left(\frac{15}{2}\right) = 0$$

$$A_y = 5.17 \text{ k} \uparrow$$

$$+\circlearrowleft \Sigma M_C^{AC} = 0$$

$$-5.17(15) - A_x(8) + 25(4) = 0$$

$$A_x = 2.81 \text{ k} \leftarrow$$

$$\rightarrow \Sigma F_x = 0$$

$$-2.81 + 25 - B_x = 0$$

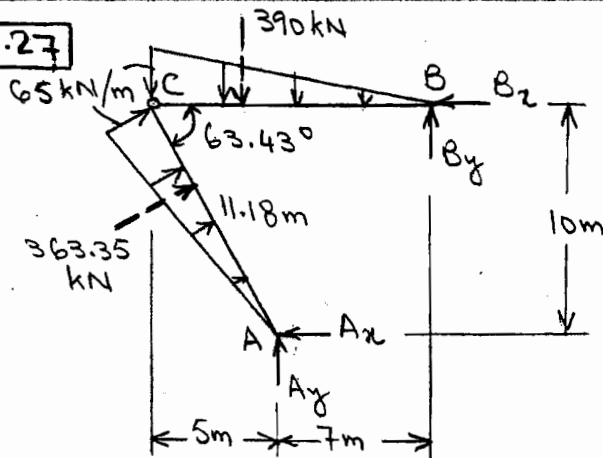
$$B_x = 22.19 \text{ k} \leftarrow$$

$$+\uparrow \Sigma F_y = 0$$

$$5.17 - 2(17) + B_y = 0$$

$$B_y = 28.83 \text{ k} \uparrow$$

3.27



$$+\circlearrowleft \Sigma M_C^{BC} = 0$$

$$-390\left(\frac{12}{3}\right) + B_y(12) = 0$$

$$B_y = 130 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0$$

$$B_x(10) + 130(7) + 390(1)$$

$$- 363.35\left(\frac{2}{3}\right)11.18 = 0$$

$$B_x = 140.82 \text{ kN} \leftarrow$$

$$\rightarrow \Sigma F_x = 0$$

$$363.35 \sin 63.43^\circ - A_x - 140.82 = 0$$

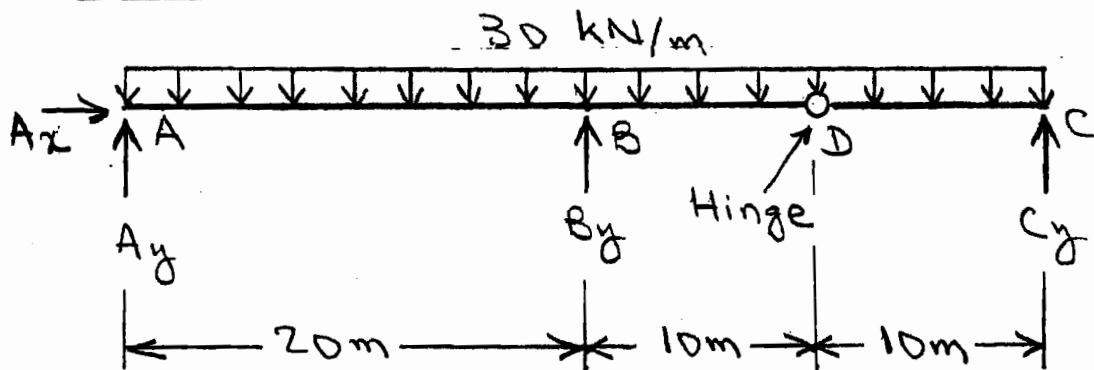
$$A_x = 184.16 \text{ kN} \leftarrow$$

$$+\uparrow \Sigma F_y = 0$$

$$A_y + 363.35 \cos 63.43^\circ - 390 + 130 = 0$$

$$A_y = 97.48 \text{ kN} \uparrow$$

3.28



$$\sum F_x = 0$$

$$\underline{A_x = 0}$$

$$+\circlearrowleft \sum M_D^{CD} = 0$$

$$C_y(10) - 30(10)(5) = 0$$

$$\underline{C_y = 150 \text{ kN} \uparrow}$$

$$+\circlearrowleft \sum M_A = 0$$

$$-30(40)(20) + B_y(20) + 150(40) = 0$$

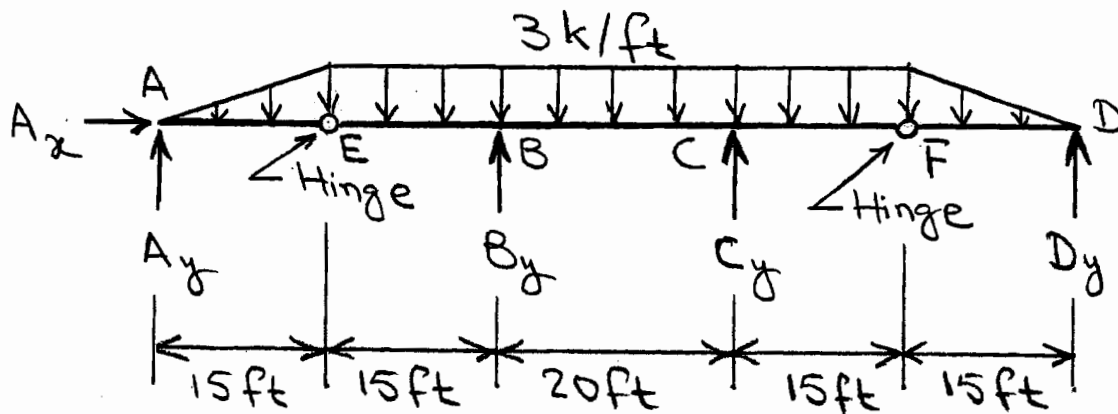
$$\underline{B_y = 900 \text{ kN} \uparrow}$$

$$+\uparrow \sum F_y = 0$$

$$A_y - 30(40) + 900 + 150 = 0$$

$$\underline{A_y = 150 \text{ kN} \uparrow}$$

3.29



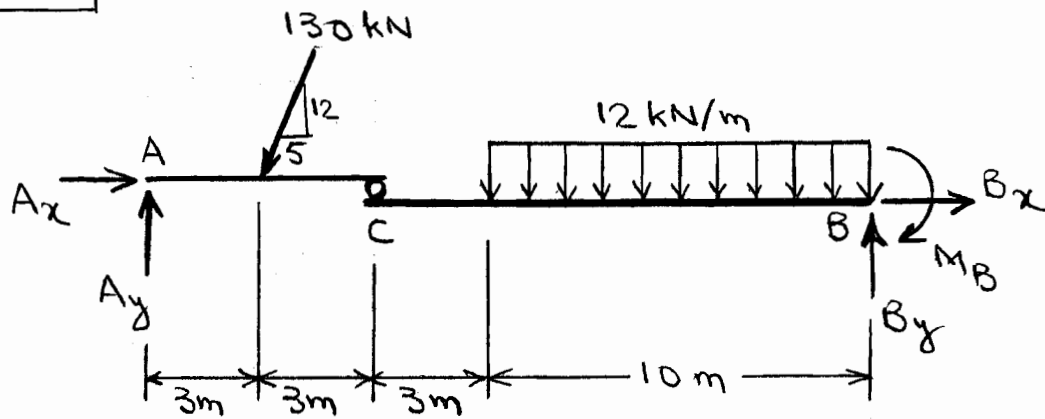
$$\begin{aligned} \sum \bar{F}_x &= 0 & \underline{A_x = 0} \\ +\circlearrowleft \sum M_E^{AE} &= 0 & -A_y(15) + \frac{1}{2}(3)(15)(5) = 0 \\ & & \underline{A_y = 7.5 \text{ k} \uparrow} \end{aligned}$$

$$\begin{aligned} +\circlearrowleft \sum M_F^{DF} &= 0 & D_y(15) - \frac{1}{2}(3)(15)(5) = 0 \\ & & \underline{D_y = 7.5 \text{ k} \uparrow} \end{aligned}$$

$$\begin{aligned} +\circlearrowleft \sum M_C &= 0 \\ -7.5(50) + \frac{1}{2}(3)(15)(40) + 3(50)(10) - B_y(20) \\ -\frac{1}{2}(3)(15)(20) + 7.5(30) &= 0 \\ \underline{B_y = 90 \text{ k} \uparrow} \end{aligned}$$

$$\begin{aligned} +\uparrow \sum F_y &= 0 \\ 7.5 - \frac{1}{2}(3)(15) - 3(50) - \frac{1}{2}(3)(15) + 90 + C_y + 7.5 &= 0 \\ \underline{C_y = 90 \text{ k} \uparrow} \end{aligned}$$

3.30



$$\begin{aligned} \rightarrow \sum F_x^{AC} = 0 \quad A_x - \frac{5}{13}(130) = 0 \quad \underline{A_x = 50 \text{ kN} \rightarrow} \end{aligned}$$

$$\begin{aligned} +\curvearrowright \sum M_C^{AC} = 0 \quad -A_y(6) + \frac{12}{13}(130)(3) = 0 \quad \underline{A_y = 60 \text{ kN} \uparrow} \end{aligned}$$

$$\begin{aligned} \rightarrow \sum F_x = 0 \quad 50 - \frac{5}{13}(130) + B_x = 0 \quad \underline{B_x = 0} \end{aligned}$$

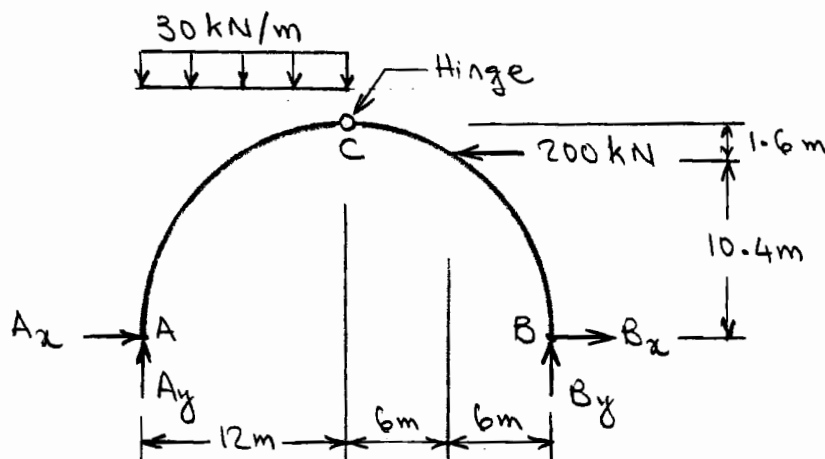
$$\begin{aligned} \uparrow \sum F_y = 0 \quad 60 - \frac{12}{13}(130) - 12(10) + B_y = 0 \quad \underline{B_y = 180 \text{ kN} \uparrow} \end{aligned}$$

$$+\curvearrowright \sum M_A = 0$$

$$-\frac{12}{13}(130)(3) - 12(10)(9+5) + 180(19) - M_B = 0$$

$$\underline{M_B = 1380 \text{ kN} \cdot \text{m} \curvearrowright}$$

3.31



$$+\circlearrowleft \sum M_B = 0 \quad -A_y(24) + 30(12)(18) + 200(10.4) = 0$$

$$A_y = 356.67 \text{ kN} \uparrow$$

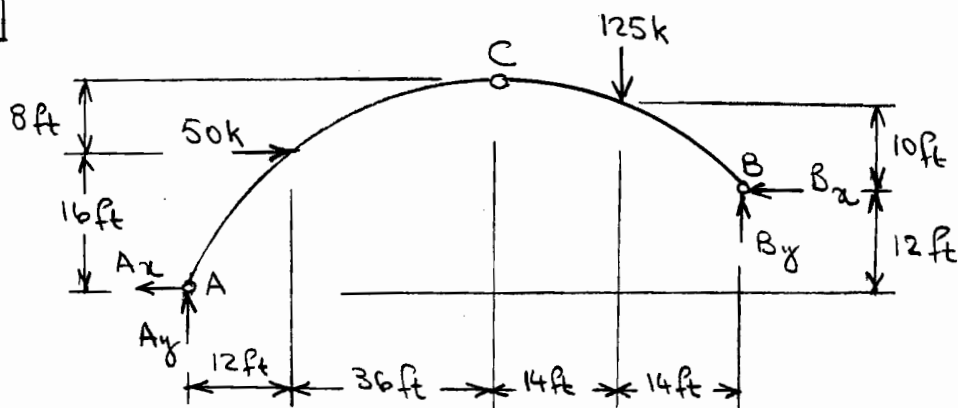
$$+\circlearrowleft \sum M_C^{AC} = 0 \quad A_x(12) - 356.67(12) + 30(12)(6) = 0$$

$$A_x = 176.67 \text{ kN} \rightarrow$$

$$\sum F_x = 0 \quad 176.67 - 200 + B_x = 0 \quad B_x = 23.33 \text{ kN} \rightarrow$$

$$\sum F_y = 0 \quad 356.67 - 30(12) + B_y = 0 \quad B_y = 3.33 \text{ kN} \uparrow$$

3.32



$$+\circlearrowleft \sum M_C^{AC} = 0 \quad -A_x(24) - A_y(48) + 50(8) = 0$$

$$3A_x + 6A_y = 50 \quad (1)$$

$$+\circlearrowleft \sum M_B = 0 \quad -A_x(12) - A_y(76) - 50(4) + 125(14) = 0$$

$$6A_x + 38A_y = 775 \quad (2)$$

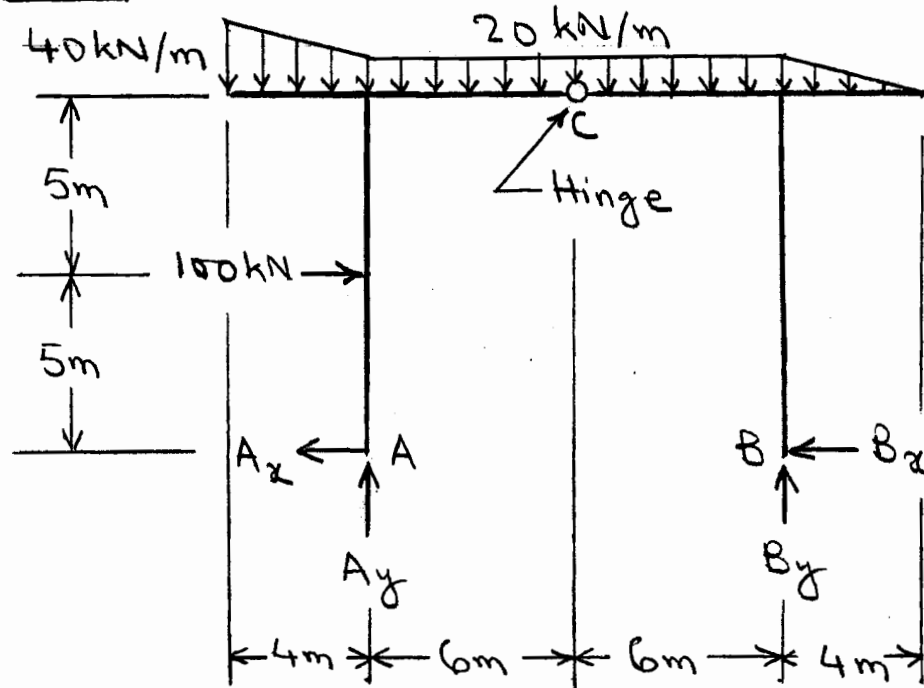
Solving (1) and (2) simultaneously, we obtain

$$A_x = -35.25 \text{ k} = 35.25 \text{ k} \rightarrow \quad A_y = 25.96 \text{ k} \uparrow$$

$$\rightarrow \sum F_x = 0 \quad 35.25 + 50 - B_x = 0 \quad B_x = 85.25 \text{ k} \leftarrow$$

$$\uparrow \sum F_y = 0 \quad 25.96 - 125 + B_y = 0 \quad B_y = 99.04 \text{ k} \uparrow$$

3.33



$$+\circlearrowleft \sum M_A = 0$$

$$-100(5) + \frac{1}{2}(20)(4)\left(\frac{8}{3}\right) - 20(16)4 - \frac{1}{2}(20)4\left(12 + \frac{4}{3}\right) + B_y(12) = 0$$

$$B_y = 183.89 \text{ kN} \uparrow$$

$$+\circlearrowleft \sum M_C^{BC} = 0$$

$$-20(6)3 - \frac{1}{2}(20)4\left(6 + \frac{4}{3}\right) + 183.89(6) - B_x(10) = 0$$

$$B_x = 45 \text{ kN} \leftarrow$$

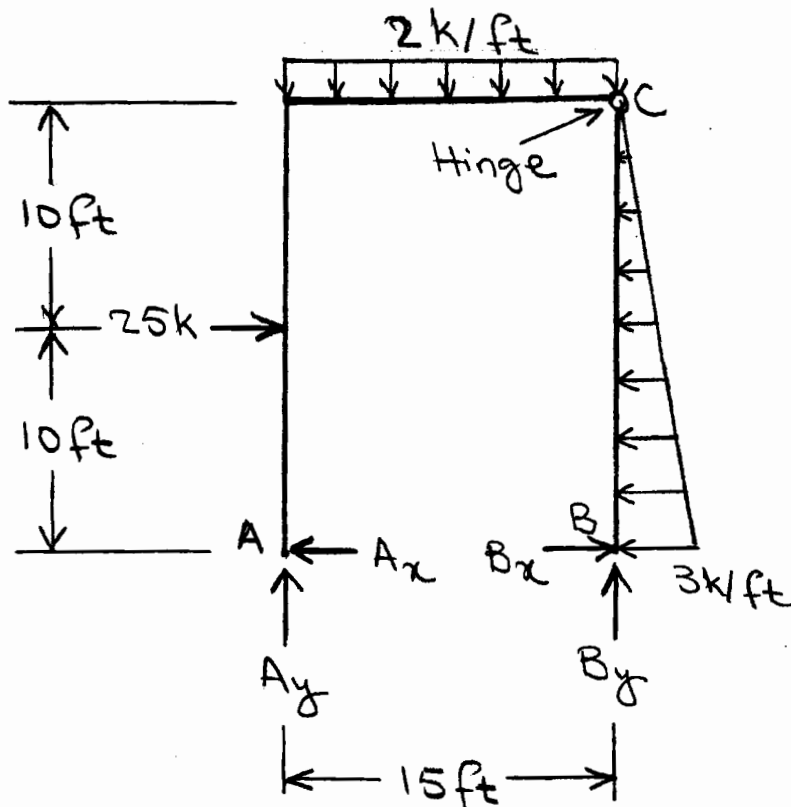
$$\rightarrow \sum F_x = 0 \quad 100 - A_x - 45 = 0$$

$$A_x = 55 \text{ kN} \leftarrow$$

$$+\uparrow \sum F_y = 0 \quad A_y - \frac{1}{2}(20)4 - 20(16) - \frac{1}{2}(20)4 + 183.89 = 0$$

$$A_y = 216.11 \text{ kN} \uparrow$$

3.34



$$+\circlearrowleft \Sigma M_A = 0$$

$$-25(10) - 2(15)7.5 + \frac{1}{2}(3)20\left(\frac{20}{3}\right) + B_y(15) = 0$$

$$\underline{B_y = 18.33 \text{ k} \uparrow}$$

$$+\Sigma M_C^{BC} = 0$$

$$-\frac{1}{2}(3)20\left(\frac{40}{3}\right) + B_x(20) = 0$$

$$\underline{B_x = 20 \text{ k} \rightarrow}$$

$$+\rightarrow \Sigma F_x = 0$$

$$-A_x + 25 - \frac{1}{2}(3)20 + 20 = 0$$

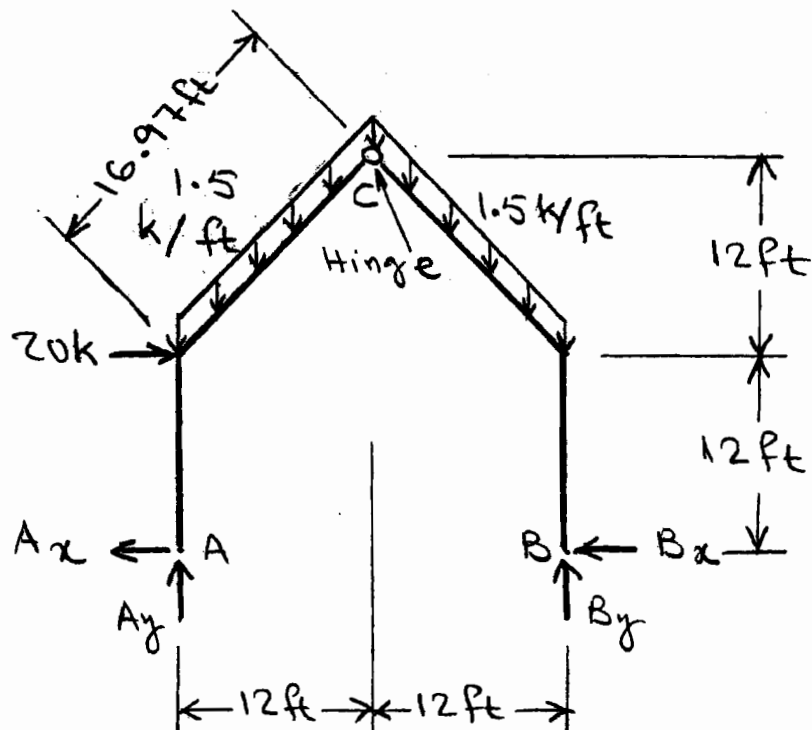
$$\underline{A_x = 15 \text{ k} \leftarrow}$$

$$+\uparrow \Sigma F_y = 0$$

$$A_y - 2(15) + 18.33 = 0$$

$$\underline{A_y = 11.67 \text{ k} \uparrow}$$

3.35



$$+\circlearrowleft \sum M_B = 0$$

$$-A_y(24) - 20(12) + 1.5(16.97)(18) + 1.5(16.97)(6) = 0$$

$$\underline{A_y = 15.46 \text{ k} \uparrow}$$

$$+\circlearrowleft \sum M_C^{AC} = 0$$

$$-A_x(24) - 15.46(12) + 20(12) + 1.5(16.97)(6) = 0$$

$$\underline{A_x = 8.63 \text{ k} \leftarrow}$$

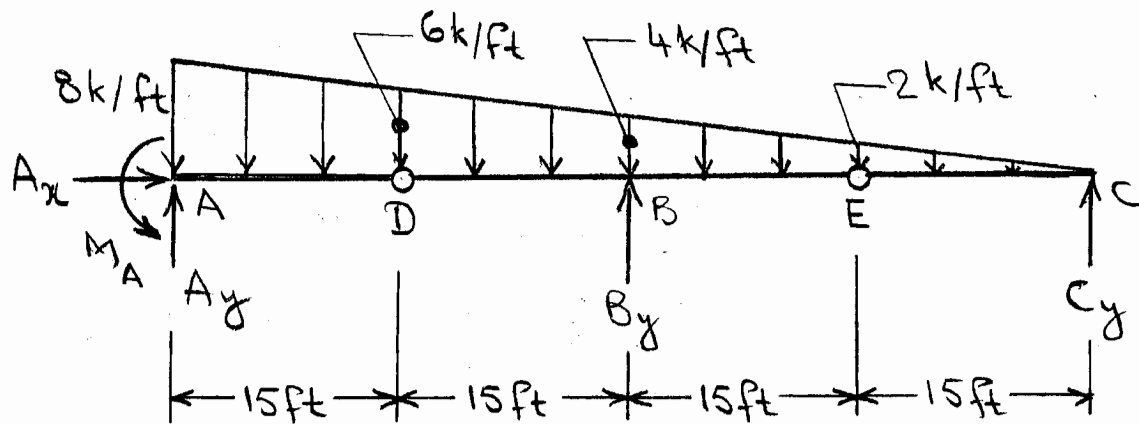
$$+\rightarrow \sum F_x = 0 \quad -8.63 + 20 - B_x = 0$$

$$\underline{B_x = 11.37 \text{ k} \leftarrow}$$

$$+\uparrow \sum F_y = 0 \quad 15.46 - 2(1.5)(16.97) + B_y = 0$$

$$\underline{B_y = 35.45 \text{ k} \uparrow}$$

3.36



$$\sum F_x = 0$$

$$\underline{A_x = 0}$$

$$+\circlearrowleft \sum M_E = 0$$

$$C_y(15) - \frac{1}{2}(2)(15)(5) = 0$$

$$\underline{C_y = 5 \text{ k} \uparrow}$$

$$+\circlearrowleft \sum M_D = 0$$

$$5(45) - \frac{1}{2}(6)(45)(15) + B_y(15) = 0$$

$$\underline{B_y = 120 \text{ k} \uparrow}$$

$$+\circlearrowleft \sum M_A = 0$$

$$M_A - \frac{1}{2}(8)(60)(20) + 120(30) + 5(60) = 0$$

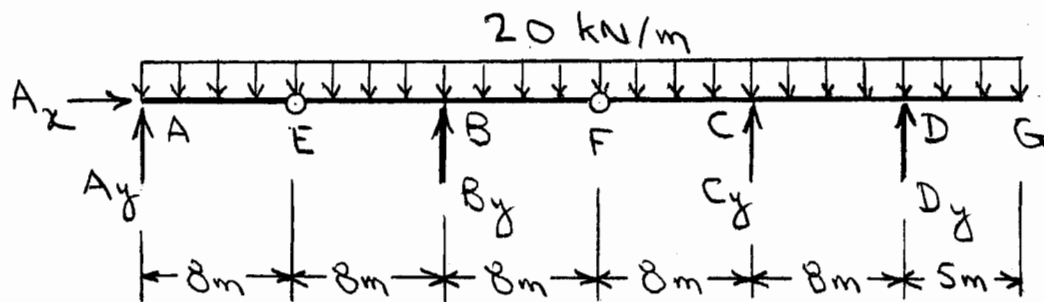
$$\underline{M_A = 900 \text{ k-ft} \curvearrowleft}$$

$$+\uparrow \sum F_y = 0$$

$$A_y - \frac{1}{2}(8)(60) + 120 + 5 = 0$$

$$\underline{A_y = 115 \text{ k} \uparrow}$$

3.37



$$\sum F_x = 0$$

$$\underline{A_x = 0}$$

$$+\circlearrowleft \sum M_E^{AE} = 0$$

$$-A_y(8) + 20(8)(4) = 0$$

$$\underline{A_y = 80 \text{ kN} \uparrow}$$

$$+\circlearrowleft \sum M_F^{AF} = 0$$

$$-80(24) + 20(24)(12) - B_y(8) = 0$$

$$\underline{B_y = 480 \text{ kN} \uparrow}$$

$$+\circlearrowleft \sum M_D = 0$$

$$-80(40) - 480(24) - C_y(8) + 20(45)(17.5) = 0$$

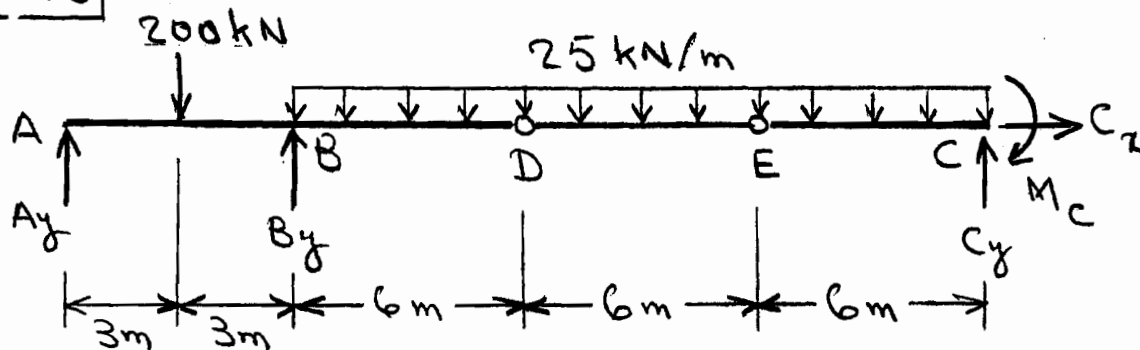
$$\underline{C_y = 128.75 \text{ kN} \uparrow}$$

$$+\uparrow \sum F_y = 0$$

$$80 + 480 + 128.75 + D_y - 20(45) = 0$$

$$\underline{D_y = 211.25 \text{ kN} \uparrow}$$

3.38



$$\sum F_x = 0$$

$$C_x = 0$$

$$+\circlearrowleft \sum M_D^{AD} = 0 \quad -A_y(12) + 200(9) - B_y(6) + 25(6)(3) = 0$$

$$2A_y + B_y = 375 \quad (1)$$

$$+\circlearrowleft \sum M_E^{AE} = 0 \quad -A_y(18) + 200(15) - B_y(12) + 25(12)(6) = 0$$

$$1.5A_y + B_y = 400 \quad (2)$$

Solving Eqs. (1) and (2) simultaneously, we obtain

$$A_y = -50 \text{ kN} = \underline{50 \text{ kN} \downarrow}$$

$$B_y = \underline{475 \text{ kN} \uparrow}$$

$$+\uparrow \sum F_y = 0 \quad -50 - 200 + 475 - 25(18) + C_y = 0$$

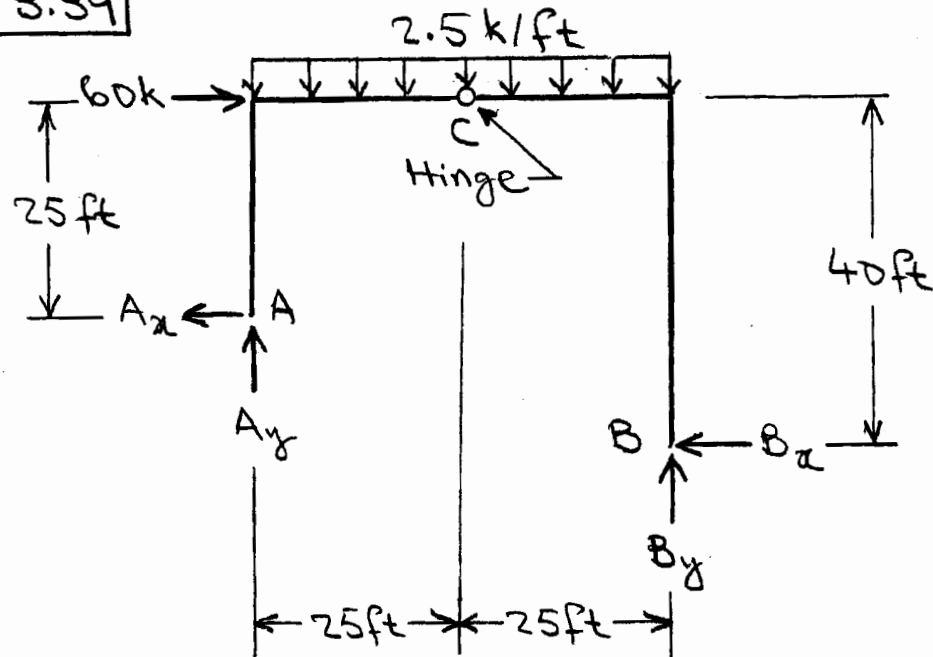
$$C_y = \underline{225 \text{ kN} \uparrow}$$

$$+\circlearrowleft \sum M_c = 0$$

$$50(24) + 200(21) - 475(18) + 25(18)(9) - M_c = 0$$

$$M_c = \underline{900 \text{ kN}\cdot\text{m} \curvearrowright}$$

3.39



$$+\circlearrowleft \sum M_A = 0$$

$$-60(25) - 2.5(50)(25) - B_x(15) + B_y(50) = 0$$

$$3B_x - 10B_y = -925 \quad (1)$$

$$+\circlearrowleft \sum M_C^{BC} = 0$$

$$-2.5(25)(12.5) - B_x(40) + B_y(25) = 0$$

$$8B_x - 5B_y = -156.25 \quad (2)$$

Solving Eqs. (1) and (2) simultaneously, we obtain

$$B_x = 47.12 \text{ k} \leftarrow$$

$$B_y = 106.63 \text{ k} \uparrow$$

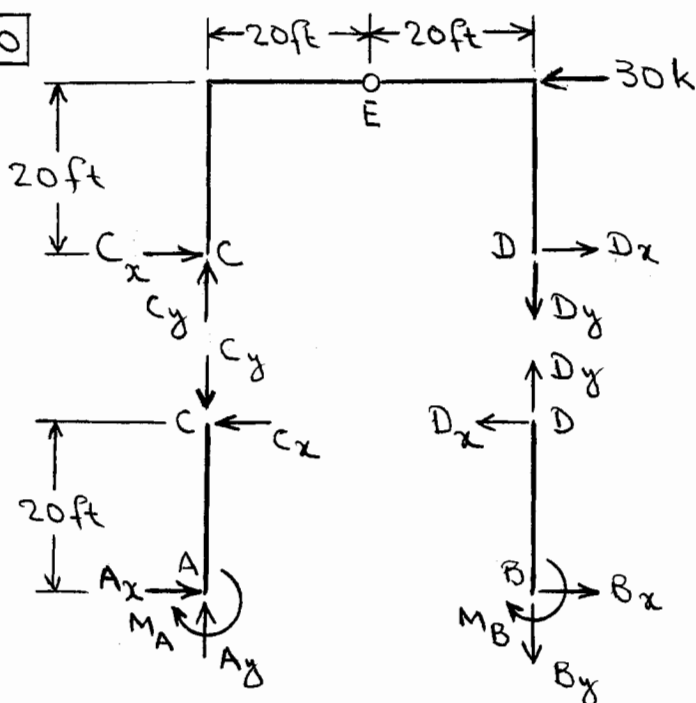
$$+\rightarrow \sum F_x = 0 \quad -A_x + 60 - 47.12 = 0$$

$$A_x = 12.88 \text{ k} \leftarrow$$

$$+\uparrow \sum F_y = 0 \quad A_y - 2.5(50) + 106.63 = 0$$

$$A_y = 18.37 \text{ k} \uparrow$$

3.40



Considering the free body of portion CED, we determine the forces at the internal hinges C and D to be

$$\begin{aligned}
 +\curvearrowright \sum M_D = 0 & \quad -C_y(40) + 30(20) = 0 & C_y = 15 \text{ k} \\
 +\curvearrowright \sum M_E^{CE} = 0 & \quad C_x(20) - 15(20) = 0 & C_x = 15 \text{ k} \\
 +\rightarrow \sum F_x = 0 & \quad 15 - 30 + D_x = 0 & D_x = 15 \text{ k} \\
 +\uparrow \sum F_y = 0 & \quad 15 - D_y = 0 & D_y = 15 \text{ k}
 \end{aligned}$$

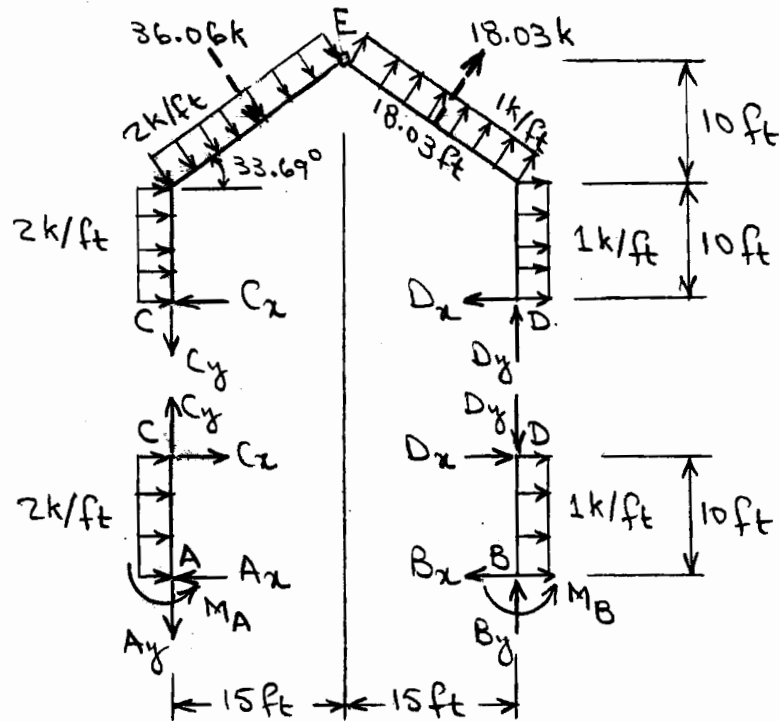
Considering the equilibrium of portion AC, we compute the reactions at support A to be

$$\begin{aligned}
 +\rightarrow \sum F_x = 0 & \quad A_x - 15 = 0 & A_x = 15 \text{ k} \rightarrow \\
 +\uparrow \sum F_y = 0 & \quad A_y - 15 = 0 & A_y = 15 \text{ k} \uparrow \\
 +\curvearrowright \sum M_A = 0 & \quad 15(20) - M_A = 0 & M_A = 300 \text{ k-ft} \curvearrowright
 \end{aligned}$$

Similarly, considering the equilibrium of portion BD:

$$\begin{aligned}
 +\rightarrow \sum F_x = 0 & \quad B_x - 15 = 0 & B_x = 15 \text{ k} \rightarrow \\
 +\uparrow \sum F_y = 0 & \quad -B_y + 15 = 0 & B_y = 15 \text{ k} \downarrow \\
 +\curvearrowright \sum M_B = 0 & \quad 15(20) - M_B = 0 & M_B = 300 \text{ k-ft} \curvearrowright
 \end{aligned}$$

3.41



Considering the free body of portion CED:

$$+\circlearrowleft \sum M_D = 0 \quad C_y(30) - (2+1)(10)5 - (36.06 + 18.03) \sin 33.69^\circ (15) + 36.06 \cos 33.69^\circ (22.5) - 18.03 \cos 33.69^\circ (7.5) = 0$$

$$C_y = 1.25 \text{ k}$$

$$+\circlearrowleft \sum M_E^{CE} = 0 \quad -C_x(20) + 1.25(15) + 2(10)(15) + 36.06 \left(\frac{18.03}{2} \right) = 0$$

$$C_x = 32.19 \text{ k}$$

$$\rightarrow \sum F_x = 0 \quad -32.19 + (2+1)10 + (36.06 + 18.03) \sin 33.69^\circ - D_x = 0$$

$$D_x = 27.81 \text{ k}$$

$$+\uparrow \sum F_y = 0 \quad -1.25 - 36.06 \cos 33.69^\circ + 18.03 \cos 33.69^\circ + D_y = 0$$

$$D_y = 16.25 \text{ k}$$

Free body of portion AC:

$$\rightarrow \sum F_x = 0 \quad 32.19 + 2(10) - A_x = 0$$

$$A_x = 52.19 \text{ k} \leftarrow$$

$$+\uparrow \sum F_y = 0 \quad -A_y + 1.25 = 0$$

$$A_y = 1.25 \text{ k} \downarrow$$

$$+\circlearrowleft \sum M_A = 0 \quad M_A - 32.19(10) - 2(10)5 = 0$$

$$M_A = 421.9 \text{ k-ft} \curvearrowright$$

Free body of portion BD:

$$\rightarrow \sum F_x = 0 \quad 27.81 + 1(10) - B_x = 0$$

$$B_x = 37.81 \text{ k} \leftarrow$$

$$+\uparrow \sum F_y = 0 \quad B_y - 16.25 = 0$$

$$B_y = 16.25 \text{ k} \uparrow$$

$$+\circlearrowleft \sum M_B = 0 \quad M_B - 27.81(10) - 1(10)5 = 0$$

$$M_B = 328.1 \text{ k-ft} \curvearrowright$$

Chapter Four

Plane and Space Trusses

CHAPTER 4

4.1 (a) $m=2, r=3, j=3; m+r < 2j$; Unstable

(b) $m=3, r=3, j=3; m+r=2j$
Statically determinate

(c) $m=2, r=4, j=3; m+r=2j$
Statically determinate

(d) $m=3, r=4, j=4; m+r < 2j$; Unstable

4.2 (a) $m=6, r=3, j=4; m+r > 2j$
Statically indeterminate; $i = (6+3) - 2(4) = \underline{1}$

(b) $m=9, r=3, j=6; m+r=2j$
Statically determinate

(c) $m=9, r=3, j=6; m+r=2j$
Unstable, because two rigid portions are connected by an internal hinge and three reactions are not sufficient to prevent relative rotation of one rigid part with respect to the other.

(d) $m=7, r=4, j=5; m+r > 2j$
Statically indeterminate; $i = (7+4) - 2(5) = \underline{1}$

4.3 (a) $m=17, r=3, j=9; m+r > 2j$
Statically indeterminate; $i = (17+3) - 2(9) = \underline{2}$

(b) $m=12, r=3, j=7; m+r > 2j$
Statically indeterminate; $i = (12+3) - 2(7) = \underline{1}$

(c) $m=25, r=4, j=14; m+r > 2j$
Statically indeterminate; $i = (25+4) - 2(14) = \underline{1}$

(d) $m=24, r=6, j=15; m+r = 2j$
Statically determinate

4.4 (a) $m=22, r=4, j=13; m+r = 2j$
Statically determinate

(b) $m=14, r=3, j=9; m+r < 2j; \underline{\text{Unstable}}$

(c) $m=11, r=3, j=7; m+r = 2j$
Statically determinate

(d) $m=23, r=4, j=13; m+r > 2j$
Statically indeterminate; $i = (23+4) - 2(13) = \underline{1}$

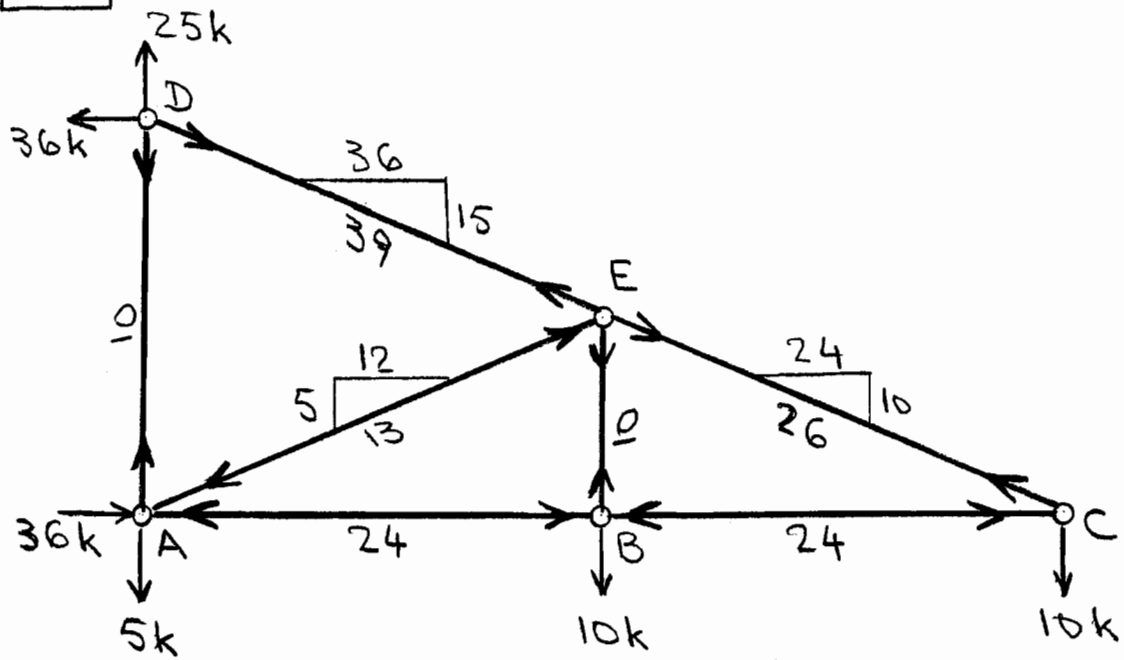
4.5 (a) $m=14, r=4, j=9; m+r = 2j$
Unstable, because two rigid portions are connected by a hinge.

(b) $m=24, r=3, j=14; m+r < 2j; \underline{\text{Unstable}}$

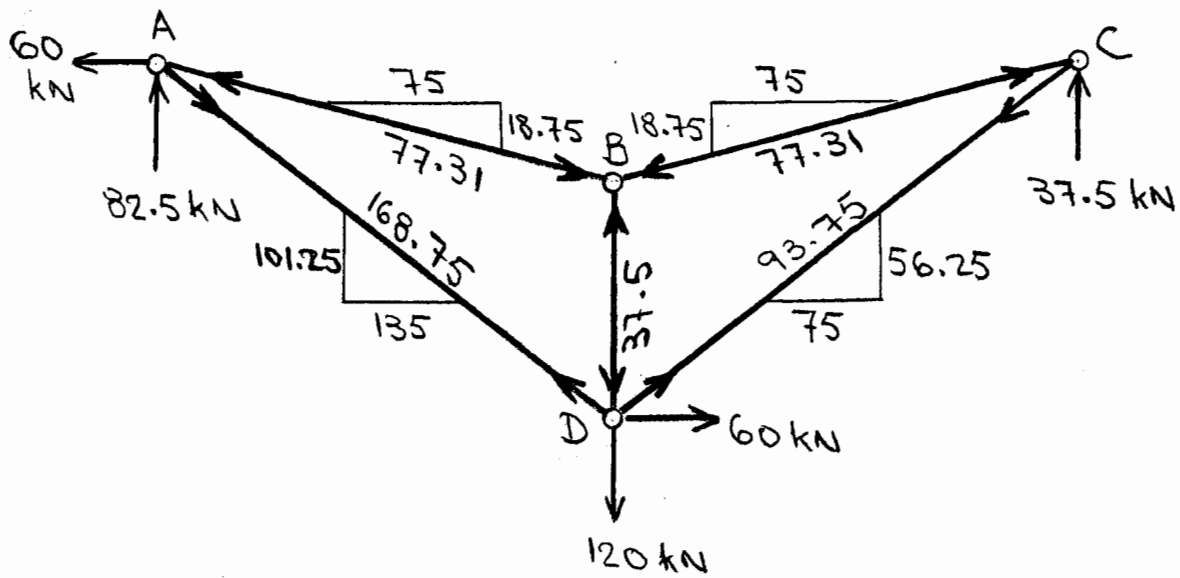
(c) $m=9, r=3, j=6; m+r = 2j$
Statically determinate

(d) $m=25, r=3, j=14; m+r = 2j$
Unstable, because two rigid portions are connected by three parallel members

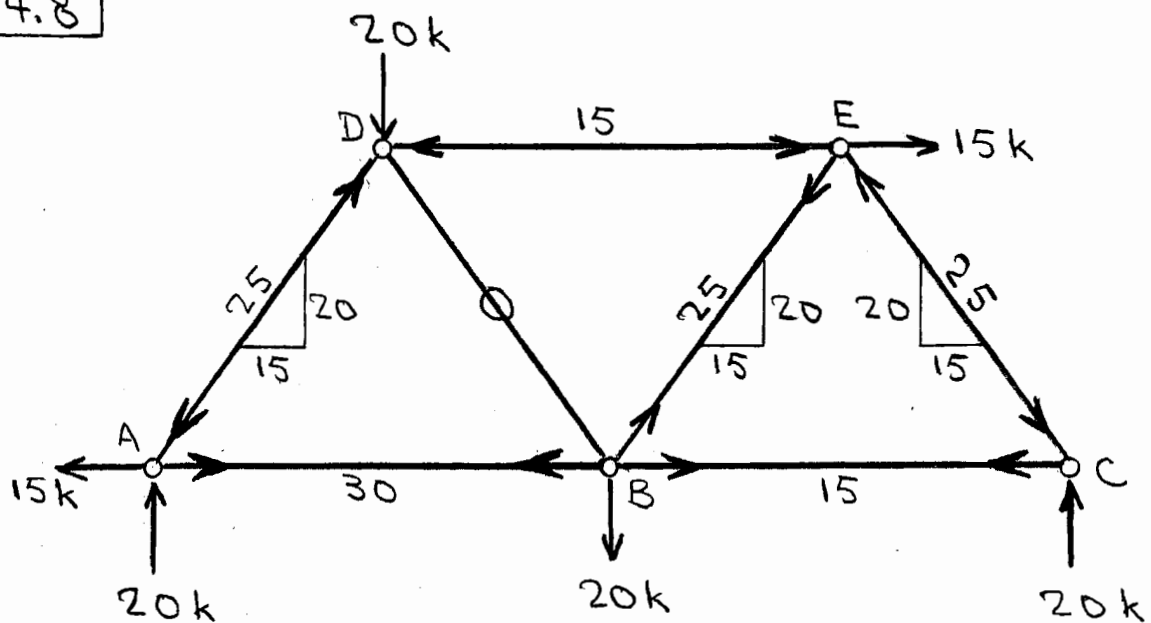
4.6



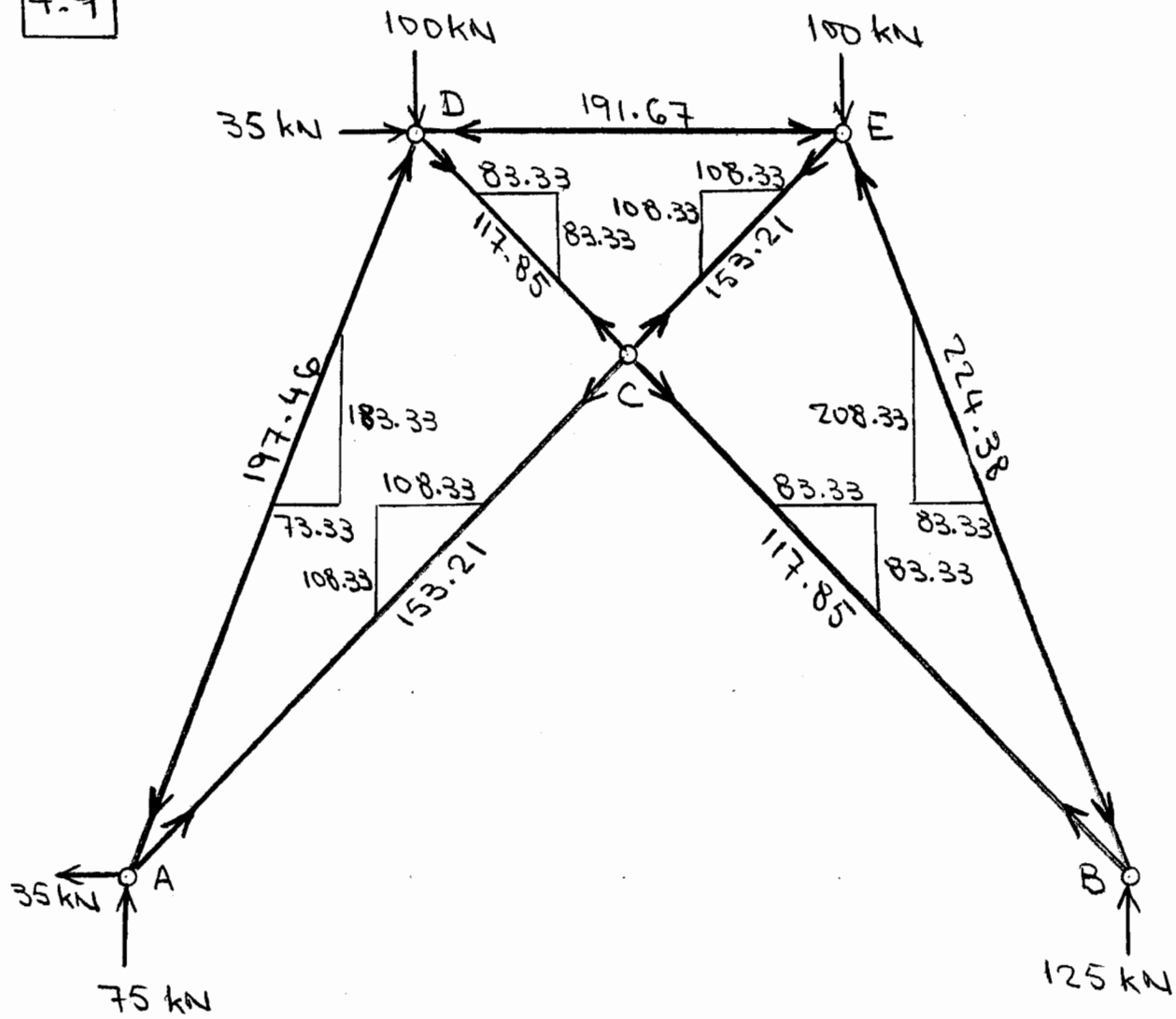
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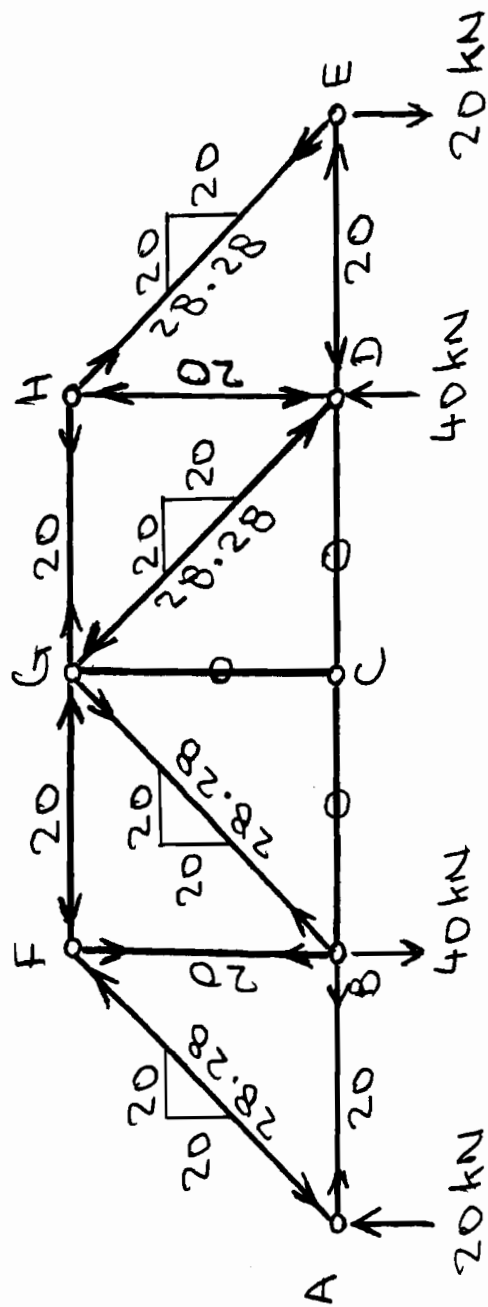
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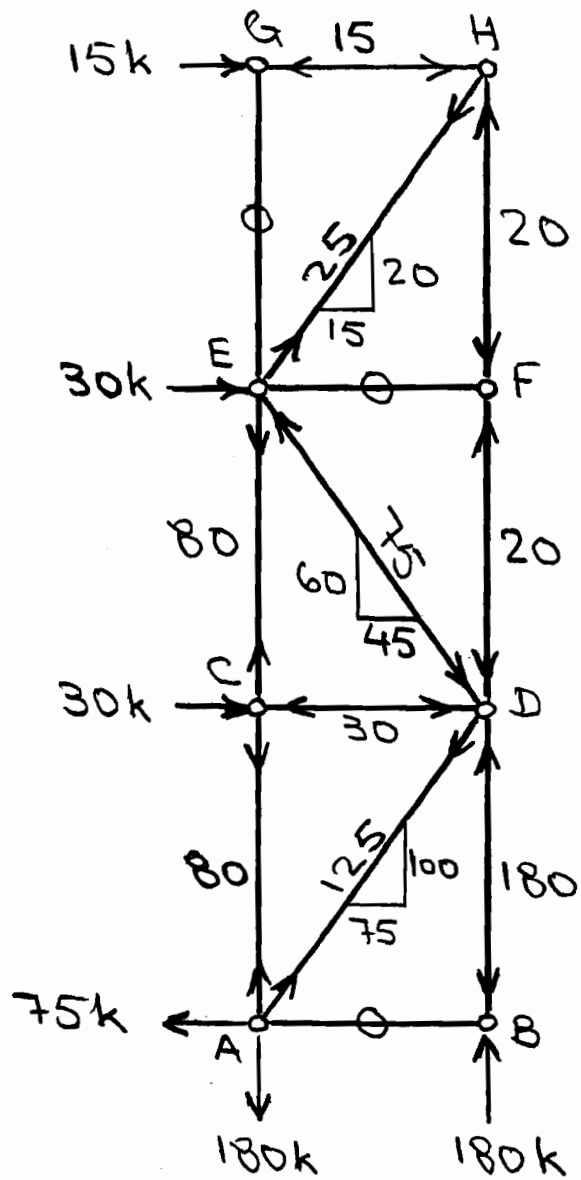
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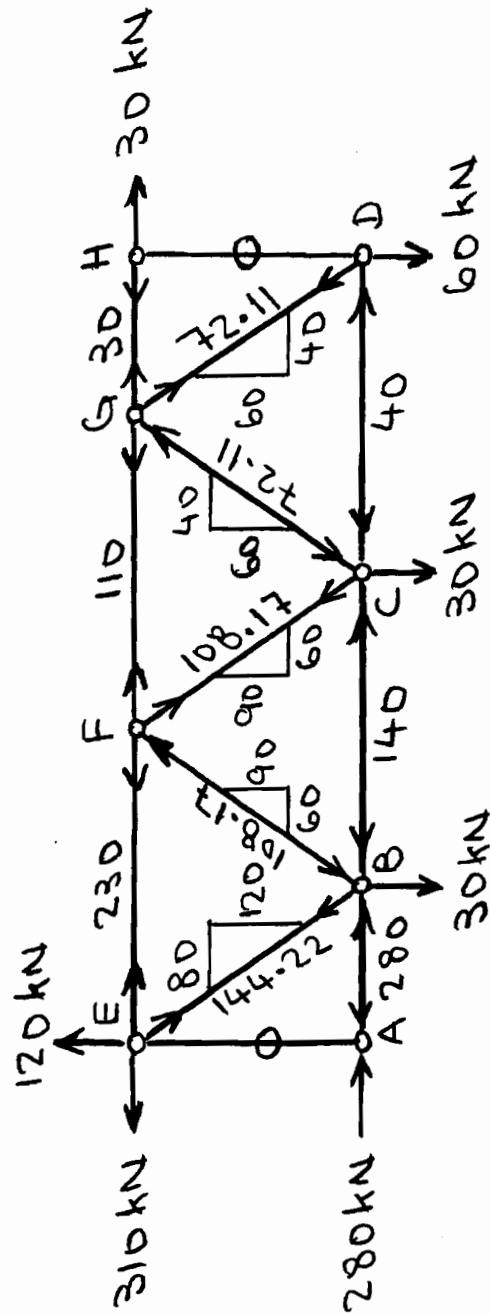
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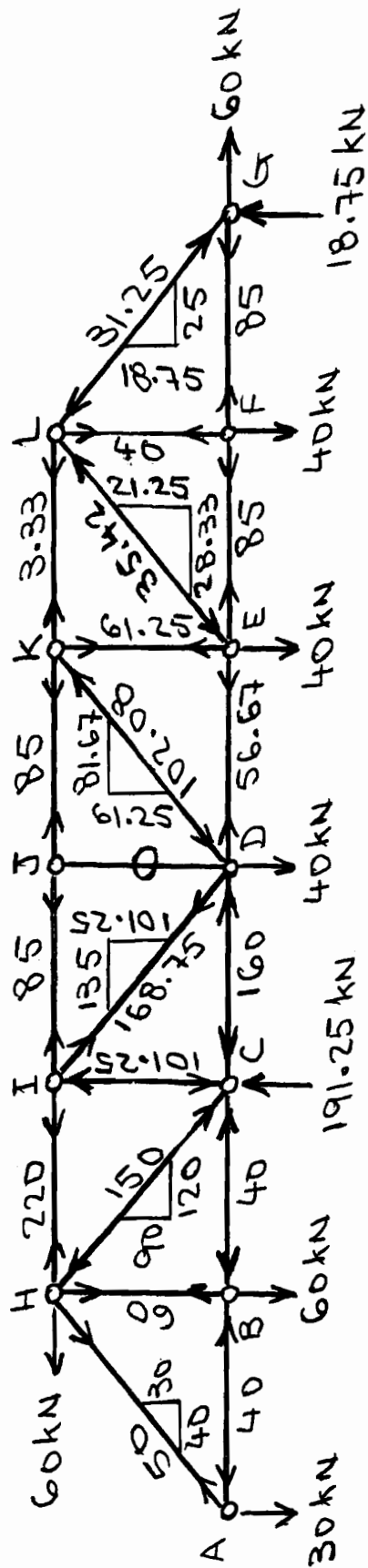
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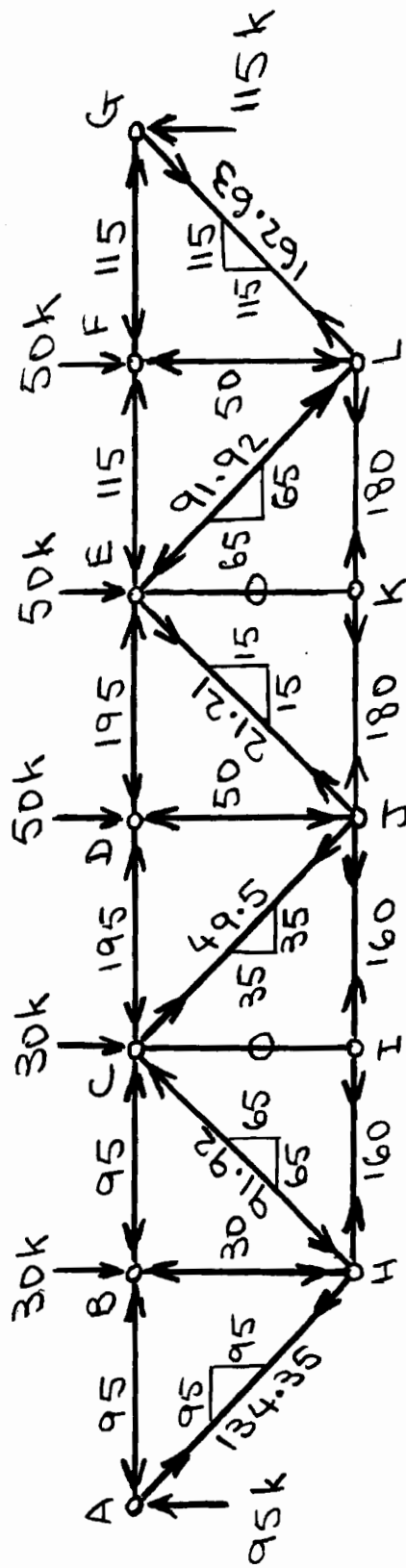
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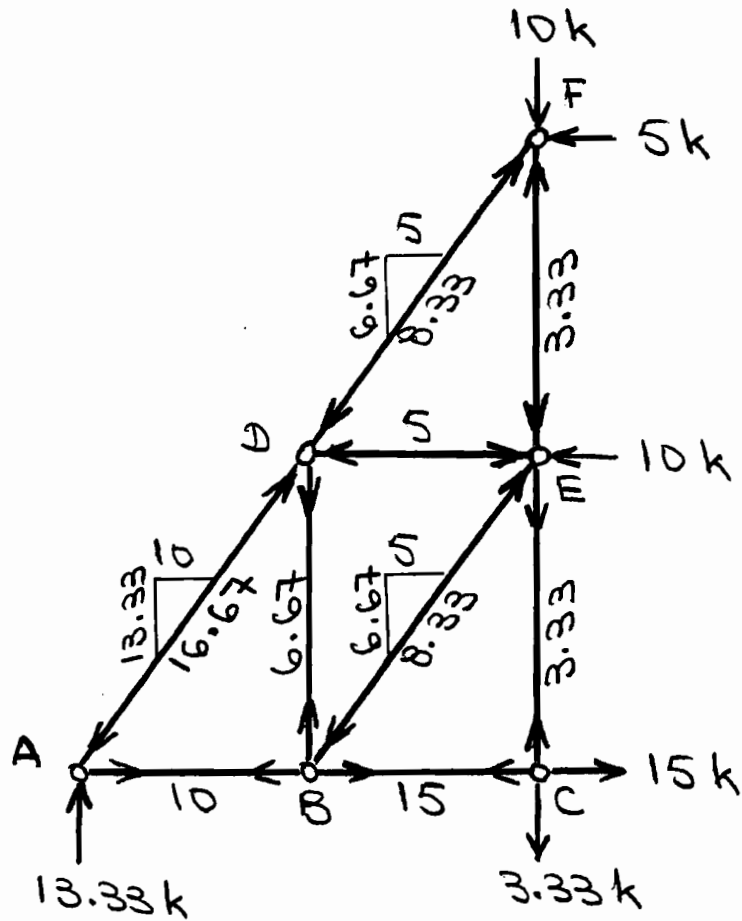
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4.15



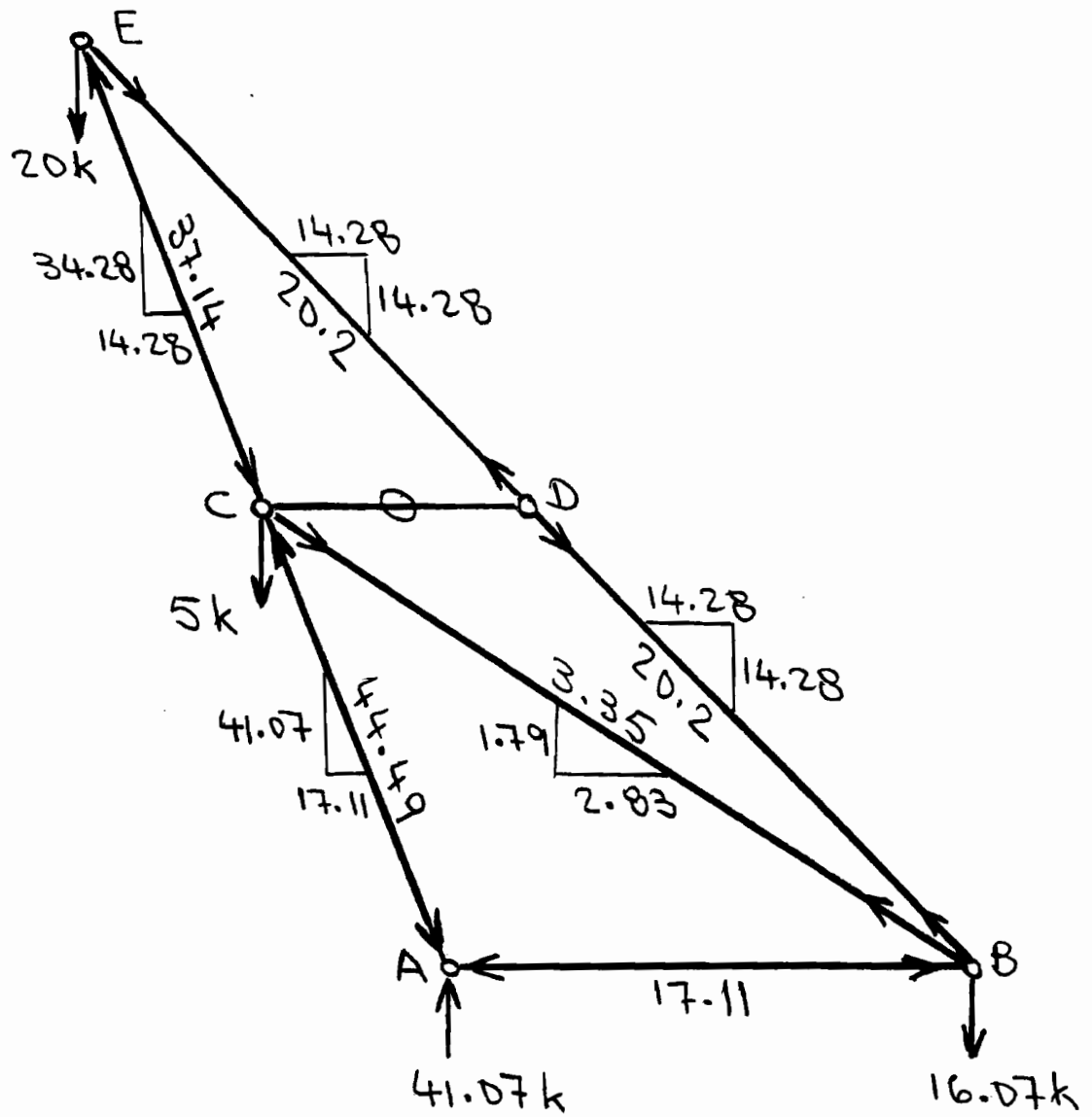
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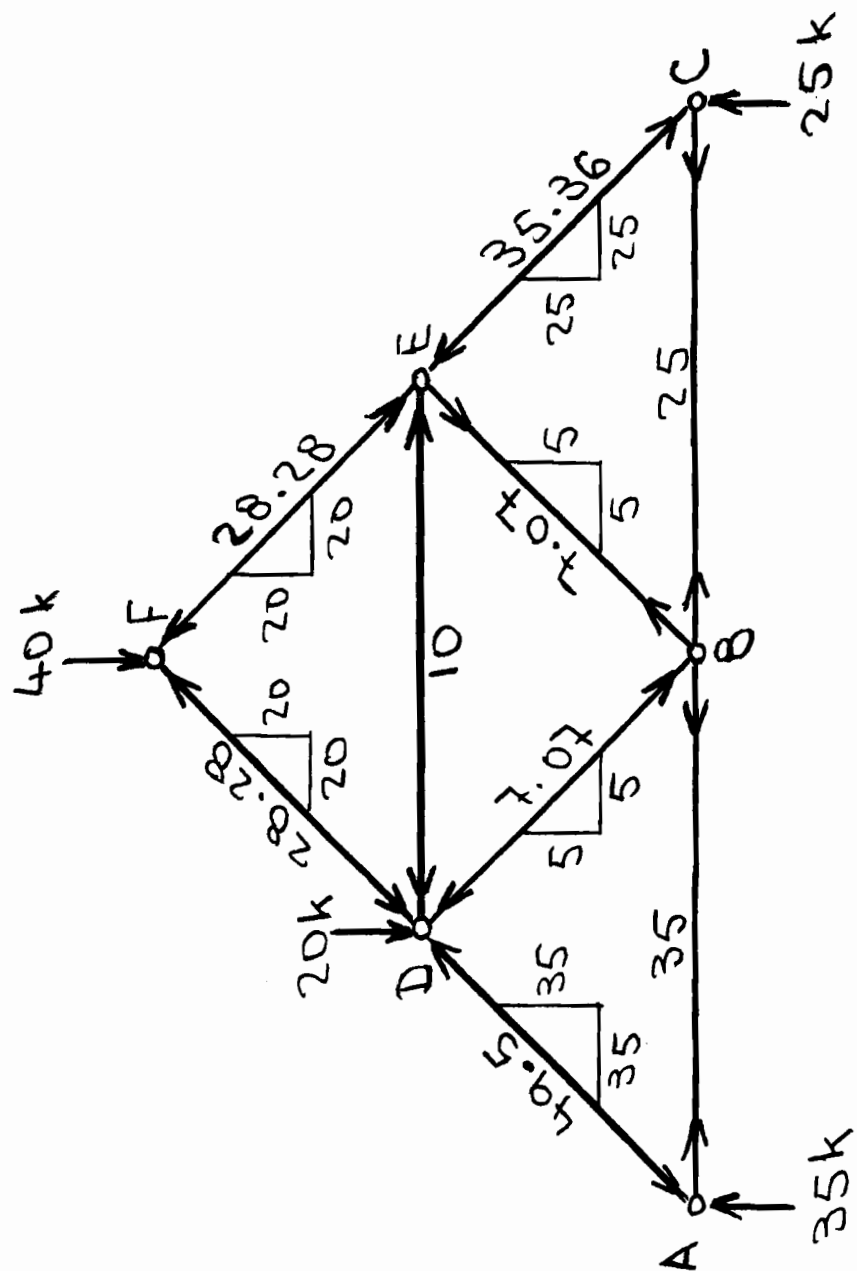
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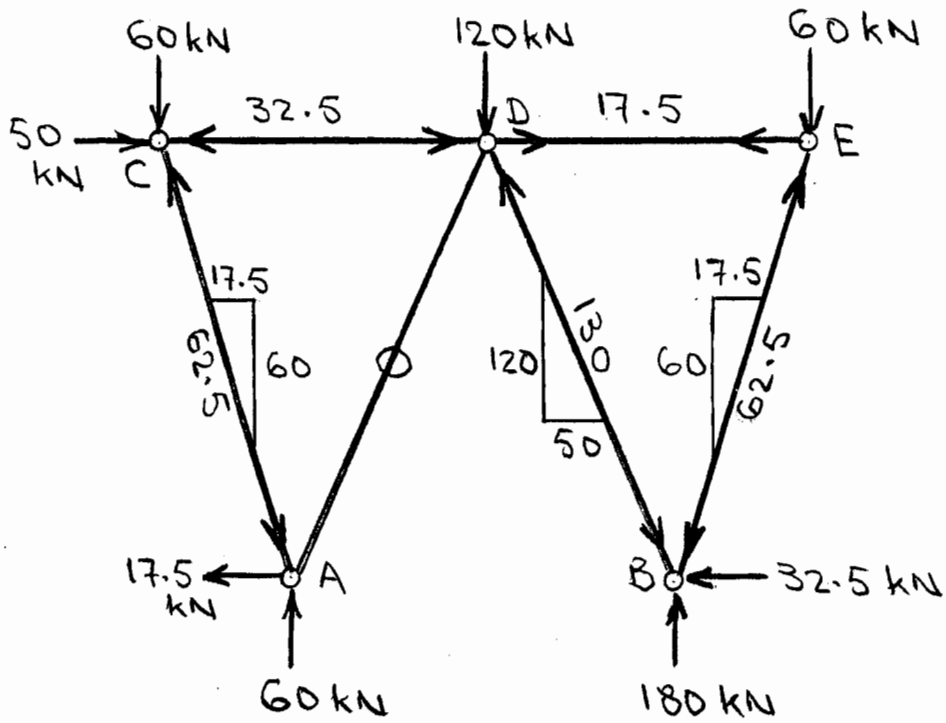
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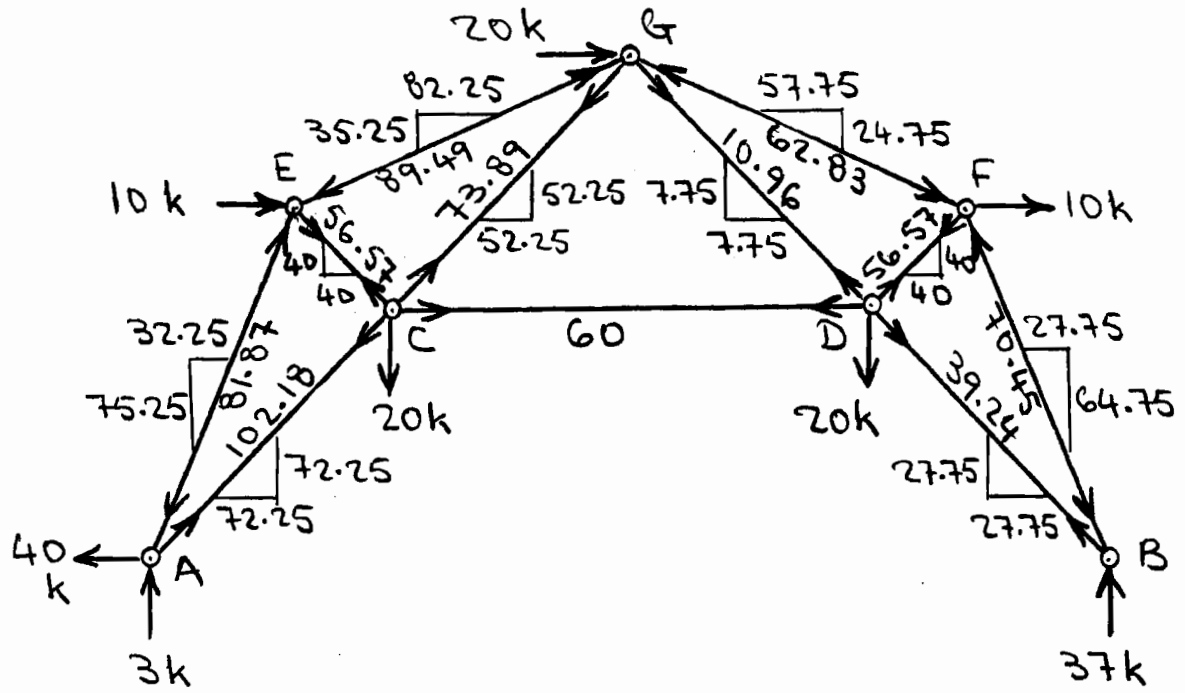
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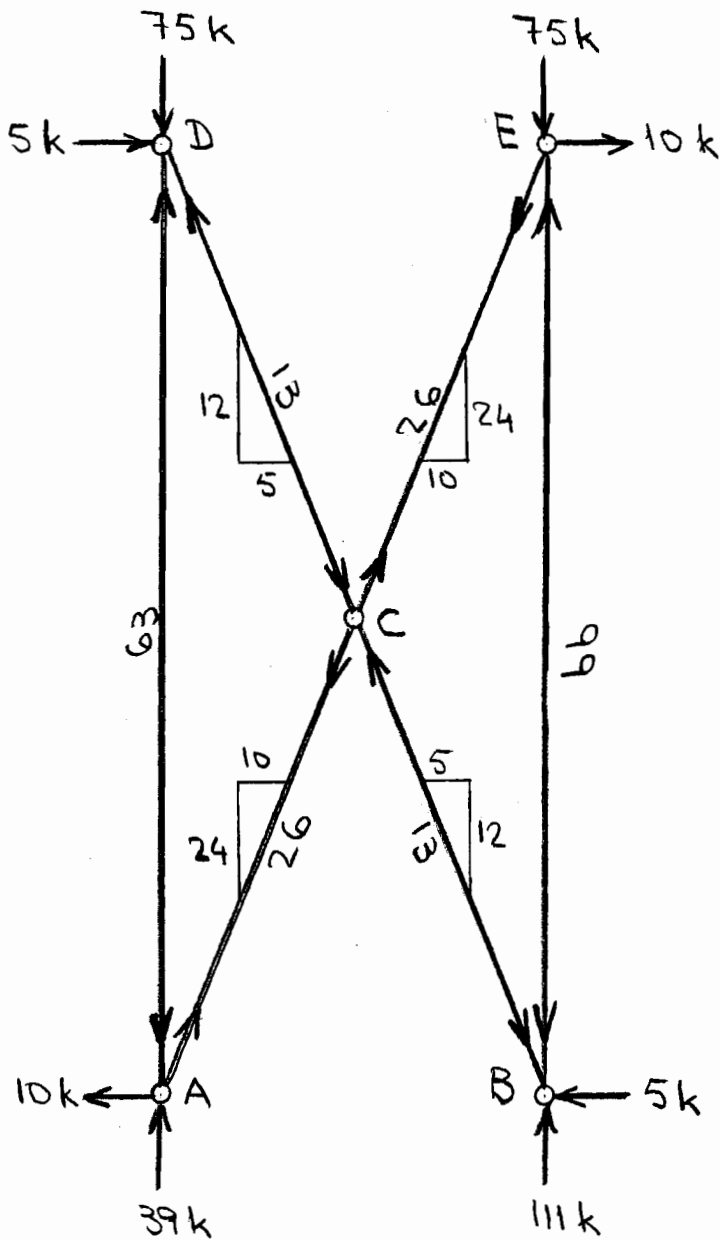
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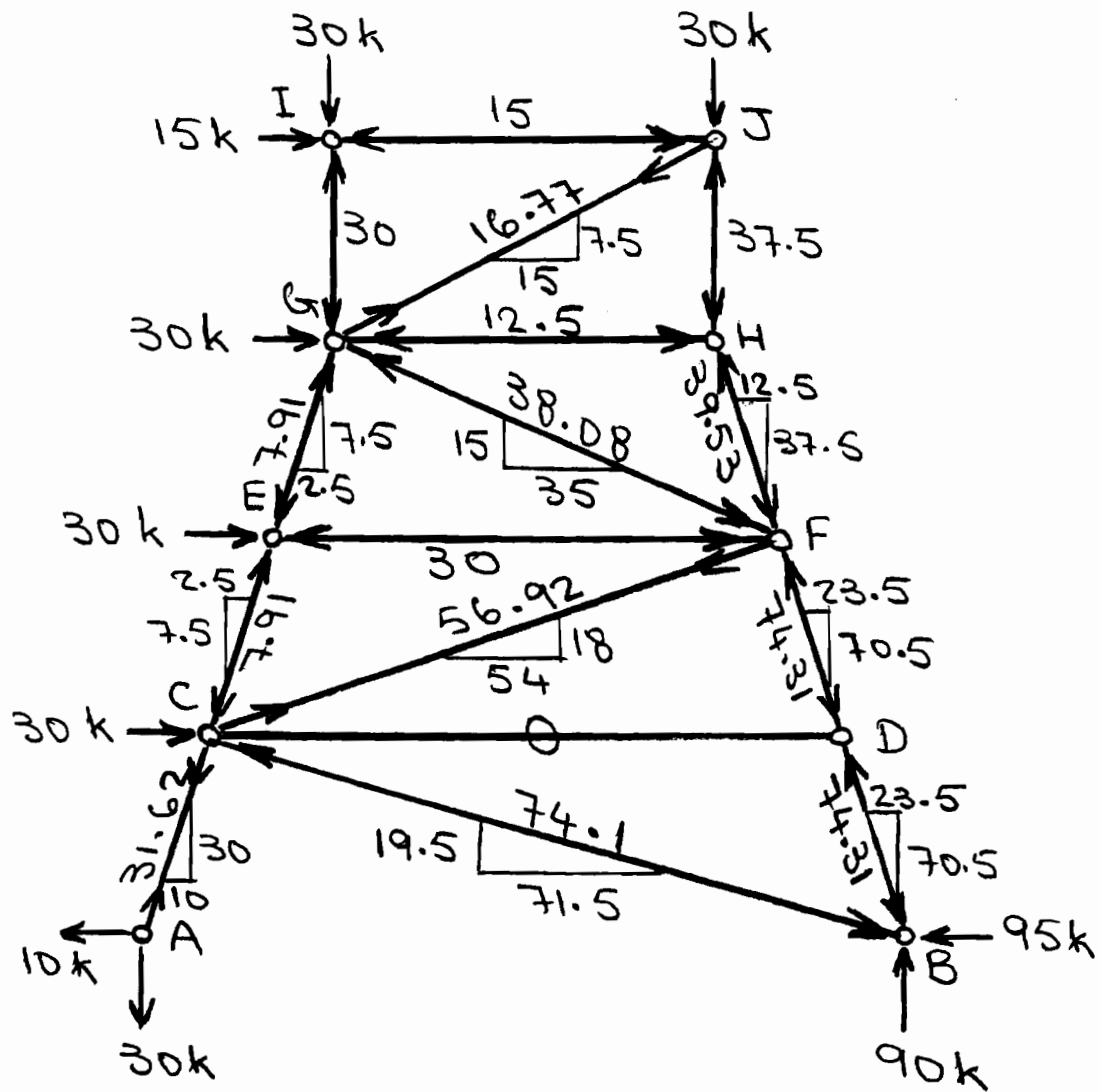
4.22



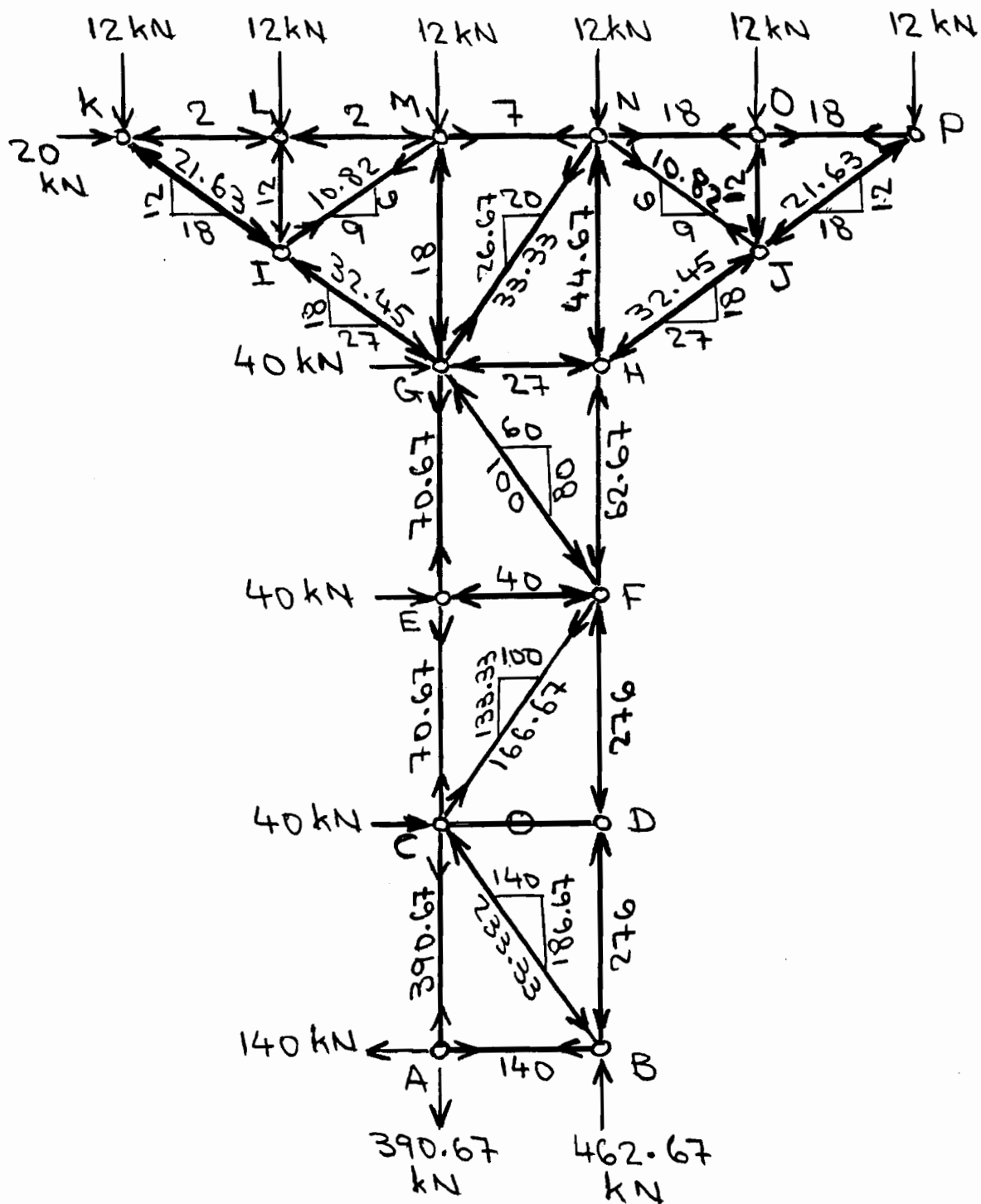
4.23



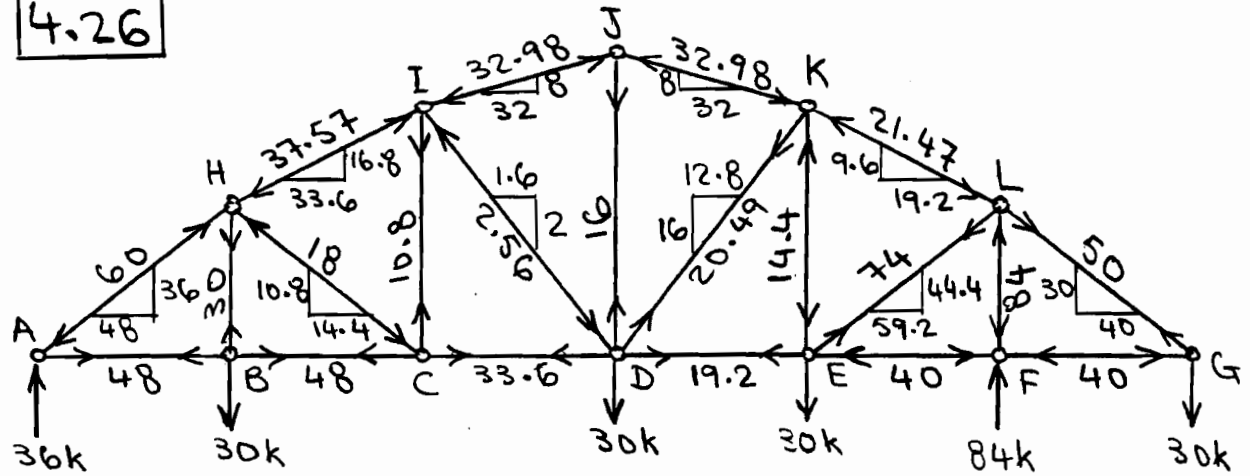
4.24



4.25



4.26



4.27 Reactions:

$$\pm \rightarrow \sum F_x = 0 \quad A_x + 50 = 0 \quad A_x = -50 \quad \underline{A_x = 50 \text{ kN} \leftarrow}$$

$$+\curvearrowleft \sum M_C^{AC} = 0 \quad -A_y(8) + 120(4) = 0 \quad \underline{A_y = 60 \text{ kN} \uparrow}$$

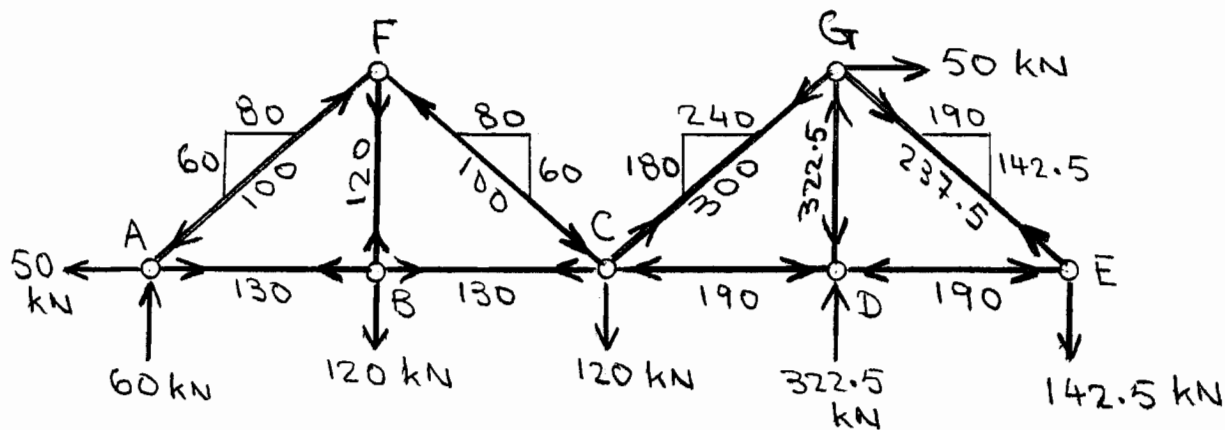
$$+\curvearrowleft \sum M_E = 0 \quad -60(16) + 120(12) + 120(8) - 50(3) - D_y(4) = 0$$

$$\underline{D_y = 322.5 \text{ kN} \uparrow}$$

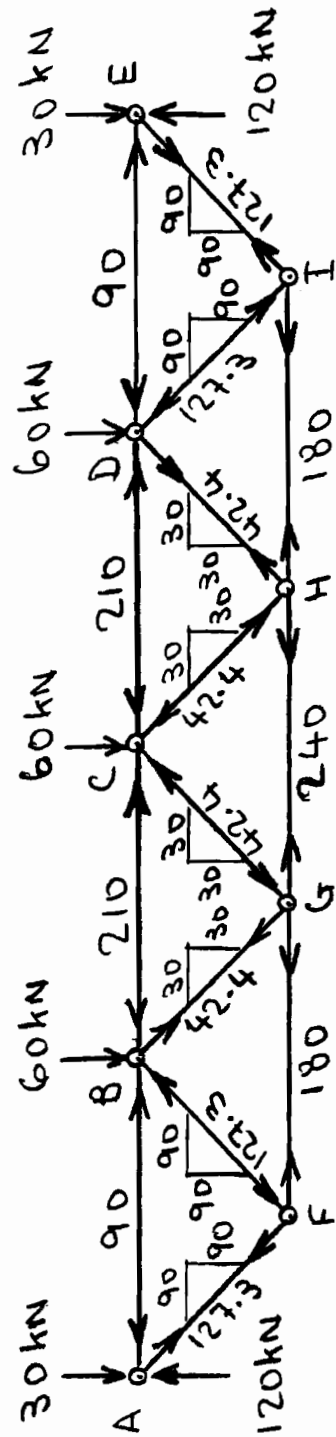
$$+\uparrow \sum F_y = 0 \quad 60 - 120 - 120 + 322.5 + E_y = 0$$

$$E_y = -142.5 \text{ kN} \quad \underline{E_y = 142.5 \text{ kN} \downarrow}$$

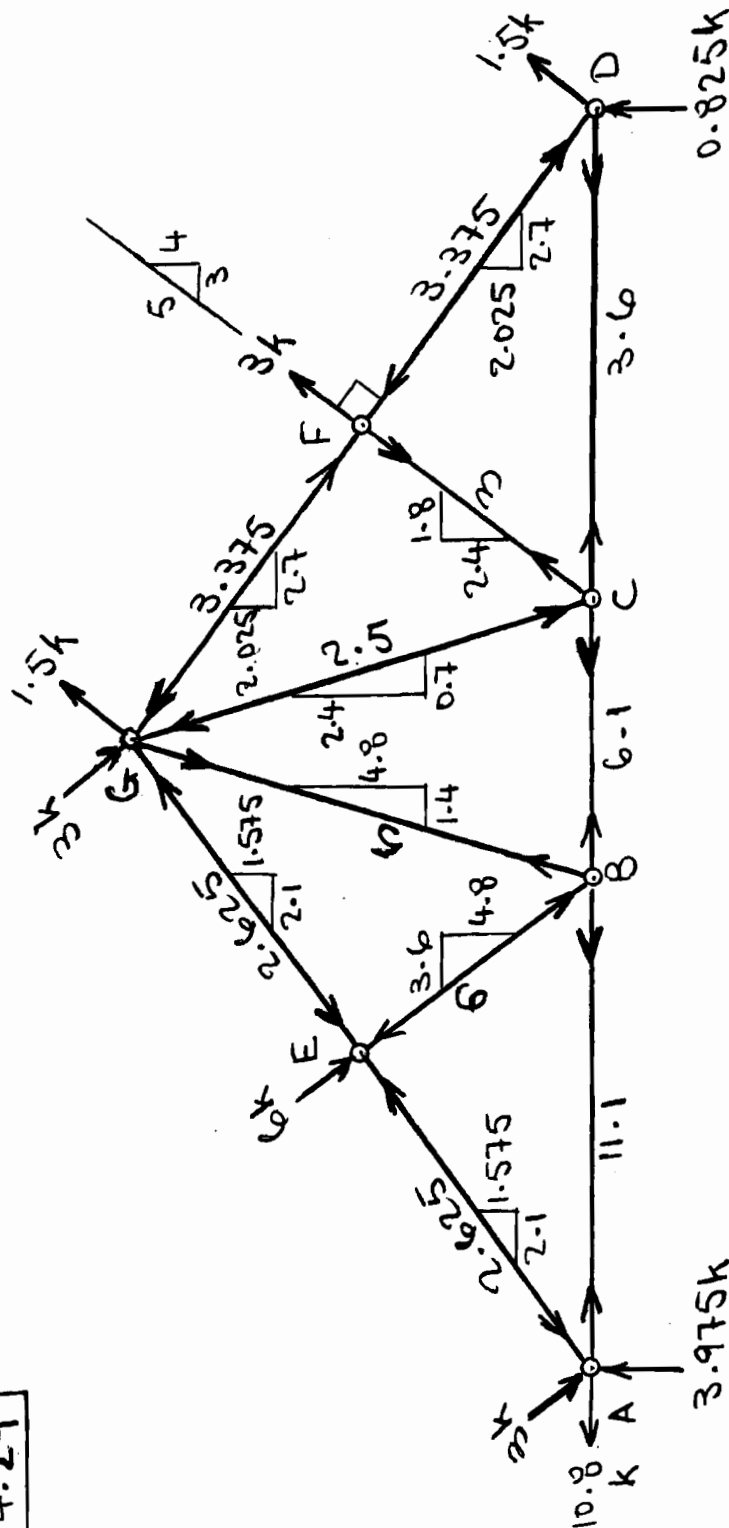
Member Forces:



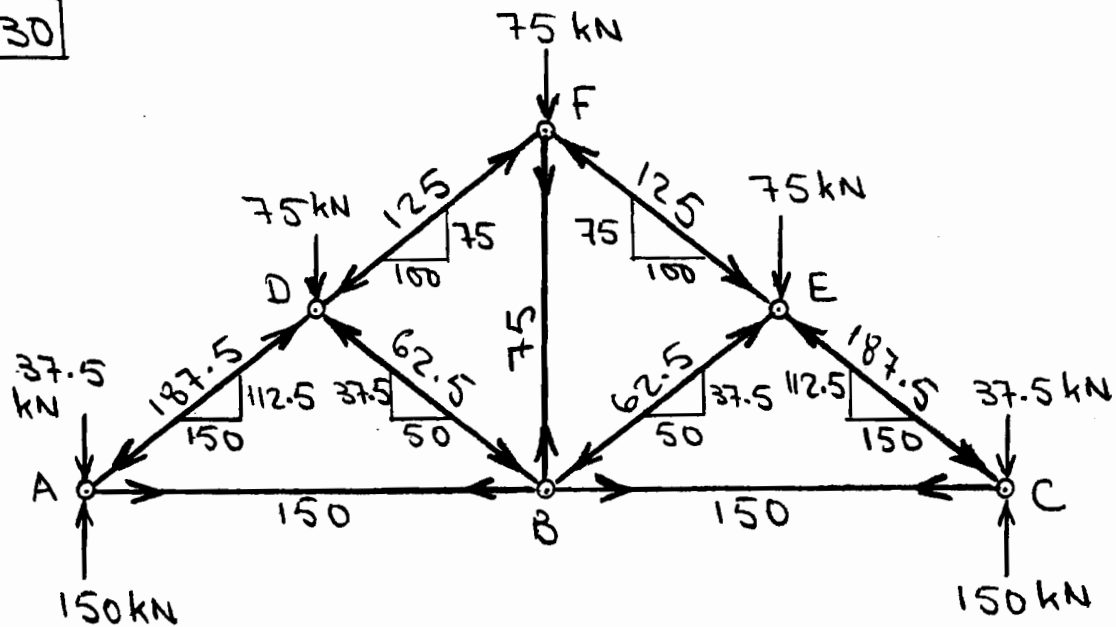
4.28



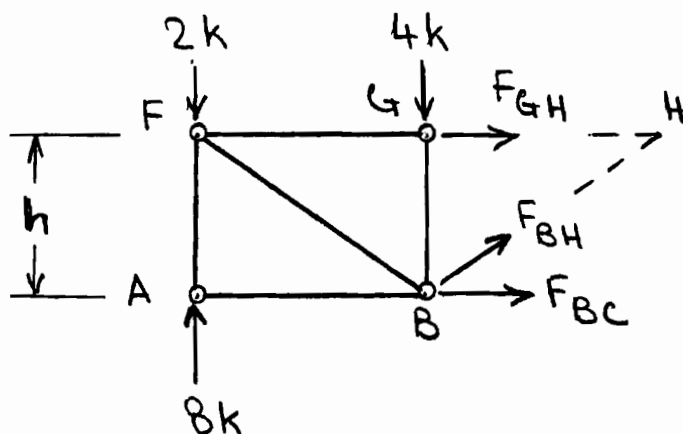
4.29



4.30



4.31 Section through members BC, BH and GH:



$$+\circlearrowleft \sum M_B = 0 \quad -8(6) + 2(6) - F_{GH}(h) = 0$$

$$F_{GH} = -\frac{36}{h} \quad (1)$$

$$+\circlearrowleft \sum M_H = 0 \quad -8(12) + 2(12) + 4(6) + F_{BC}(h) = 0$$

$$F_{BC} = \frac{48}{h} \quad (2)$$

Equations (1) and (2) indicate that the magnitudes of F_{GH} and F_{BC} are inversely proportional to the truss height h .

For $h = 3$ ft: $F_{GH} = -\frac{36}{3} = -12k = \underline{12k (C)}$

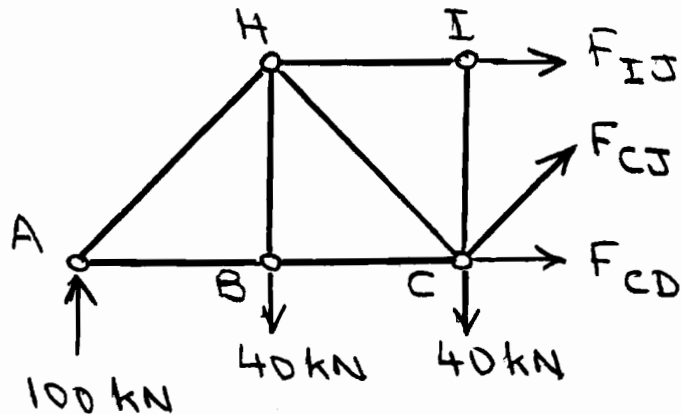
$$F_{BC} = \frac{48}{3} = \underline{16k (T)}$$

For $h = 6$ ft: $F_{GH} = -\frac{36}{6} = -6k = \underline{6k (C)}$

$$F_{BC} = \frac{48}{6} = \underline{8k (T)}$$

4.32

Section through members CD, CJ and IJ:



$$+\uparrow \Sigma F_y = 0$$

$$100 - 2(40) + \left(\frac{1}{\sqrt{2}}\right) F_{CJ} = 0$$

$$F_{CJ} = -28.28 \text{ kN}$$

$$\underline{F_{CJ} = 28.28 \text{ kN (C)}}$$

$$+\curvearrowright \Sigma M_C = 0$$

$$-100(10) + 40(5) - F_{IJ}(5) = 0$$

$$F_{IJ} = -160 \text{ kN}$$

$$\underline{F_{IJ} = 160 \text{ kN (C)}}$$

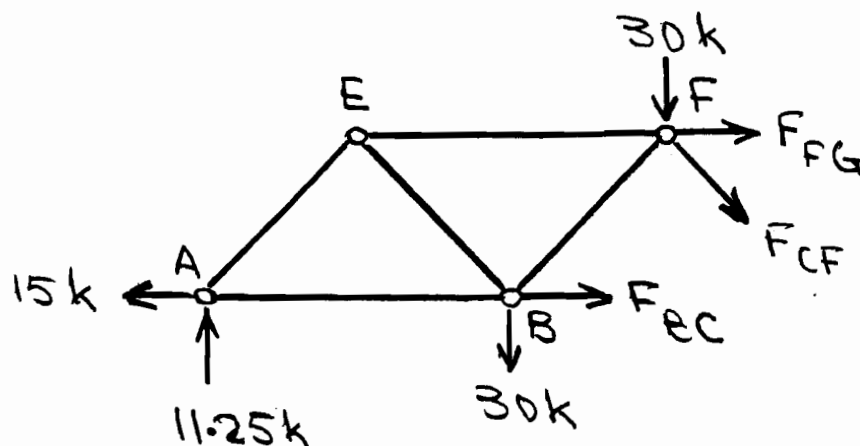
$$+\rightarrow \Sigma F_x = 0$$

$$F_{CD} - \left(\frac{1}{\sqrt{2}}\right) 28.28 - 160 = 0$$

$$\underline{F_{CD} = 180 \text{ kN (T)}}$$

4.33

Section through members BC, CF and FG:



$$+\uparrow \Sigma F_y = 0$$

$$11.25 - 30 - 30 - \left(\frac{1}{\sqrt{2}}\right) F_{CF} = 0$$

$$F_{CF} = -68.94 \text{ k}$$

$$\underline{F_{CF} = 68.94 \text{ k (C)}}$$

$$+\circlearrowleft \Sigma M_F = 0$$

$$-15(10) - 11.25(30) + 30(10) + F_{BC}(10) = 0$$

$$\underline{F_{BC} = 18.75 \text{ k (T)}}$$

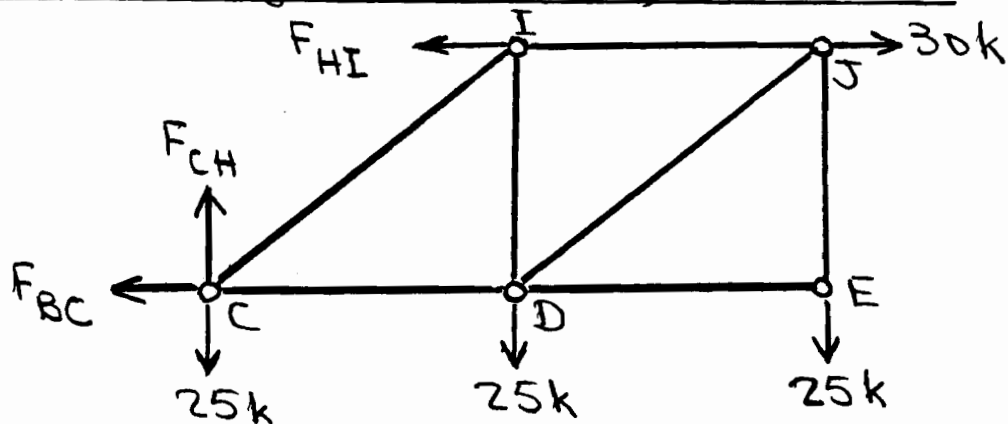
$$+\rightarrow \Sigma F_x = 0$$

$$-15 + 18.75 - \left(\frac{1}{\sqrt{2}}\right) 68.94 + F_{FG} = 0$$

$$\underline{F_{FG} = 45 \text{ k (T)}}$$

4.34

Section through members BC, CH and HI:



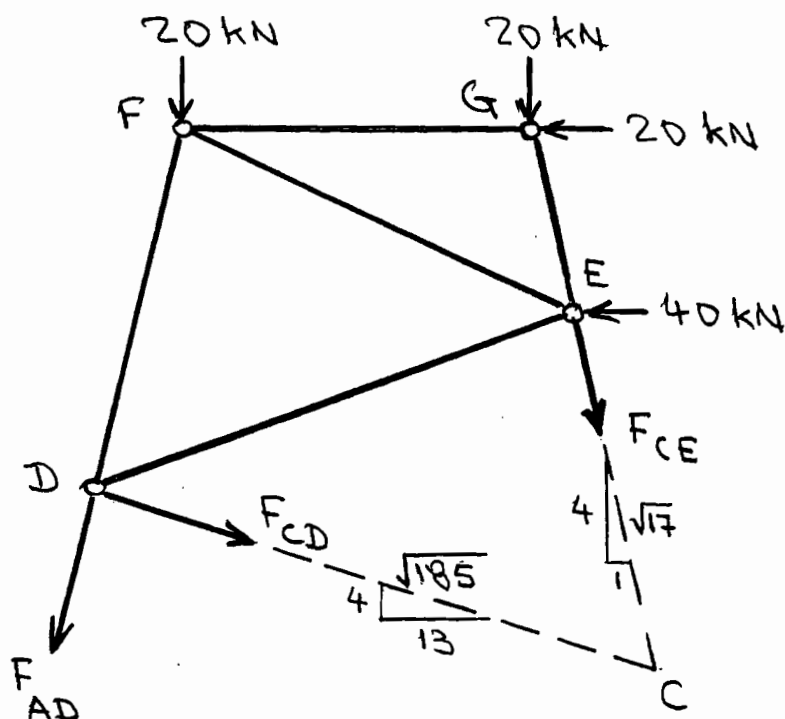
$$\begin{aligned}
 +\uparrow \Sigma F_y &= 0 & F_{CH} - 3(25) &= 0 \\
 & & F_{CH} &= 75 \text{ k (T)}
 \end{aligned}$$

$$\begin{aligned}
 +\curvearrowright \Sigma M_C &= 0 \\
 F_{HI}(15) - 30(15) - 25(20) - 25(40) &= 0 \\
 F_{HI} &= 130 \text{ k (T)}
 \end{aligned}$$

$$\begin{aligned}
 \pm \rightarrow \Sigma F_x &= 0 \\
 -F_{BC} - 130 + 30 &= 0 \\
 F_{BC} &= -100 \text{ k} \\
 F_{BC} &= 100 \text{ k (C)}
 \end{aligned}$$

4.35

Section through members AD, CD and CE:



$$+\circlearrowleft \sum M_D = 0$$

$$-20(1.5) - 20(7.5) + 20(6) + 40(3)$$

$$- \frac{1}{\sqrt{17}} F_{CE}(3) - \frac{4}{\sqrt{17}} F_{CE}(8.25) = 0$$

$$F_{CE} = 6.87 \text{ kN (T)}$$

$$+\circlearrowleft \sum M_C = 0$$

$$20(8.25) + 20(2.25) + 20(9) + 40(6)$$

$$+ \frac{1}{\sqrt{17}} F_{AD}(3) + \frac{4}{\sqrt{17}} F_{AD}(9.75) = 0$$

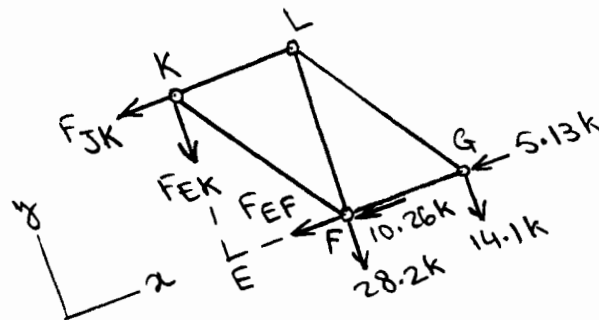
$$F_{AD} = -61.85 \text{ kN} \quad F_{AD} = 61.85 \text{ kN (C)}$$

$$+\uparrow \sum F_y = 0$$

$$\frac{4}{\sqrt{17}} (61.85) - 20 - 20 - \frac{4}{\sqrt{17}} (6.87) - \frac{4}{\sqrt{185}} F_{CD} = 0$$

$$F_{CD} = 45.34 \text{ kN (T)}$$

4.36 Section through members EF, EK and JK:



$$\sum F_y = 0 \quad -F_{EK} - 28.2 - 14.1 = 0 \quad F_{EK} = -42.3 \text{ k} = \underline{42.3 \text{ k (C)}}$$

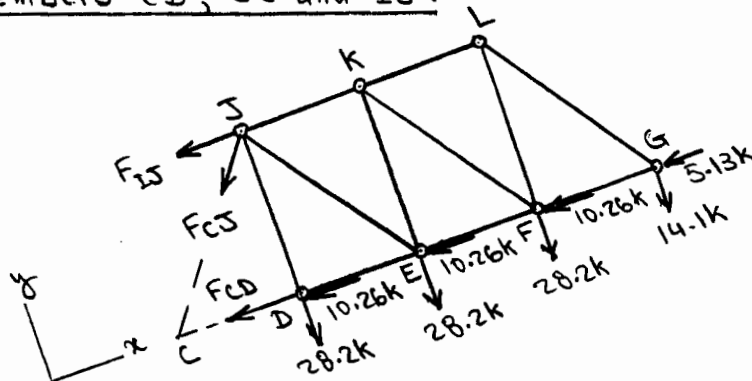
$$+\circlearrowleft \sum M_K = 0 \quad -F_{EF}(20) - 28.2(15) - 14.1(30) - (5.13 + 10.26)(20) = 0$$

$$F_{EF} = -57.69 \text{ k} = \underline{57.69 \text{ k (C)}}$$

$$+\circlearrowleft \sum M_E = 0 \quad F_{JK}(20) - 28.2(15) - 14.1(30) = 0$$

$$F_{JK} = 42.3 \text{ k (T)}$$

Section through members CD, CJ and IJ:



$$\sum F_y = 0 \quad -\left(\frac{4}{5}\right)F_{CJ} - 3(28.2) - 14.1 = 0 \quad F_{CJ} = -123.38 \text{ k} = \underline{123.38 \text{ k (C)}}$$

$$+\circlearrowleft \sum M_J = 0 \quad -F_{CD}(20) - [3(10.26) + 5.13](20) - 28.2(15 + 30) - 14.1(45) = 0$$

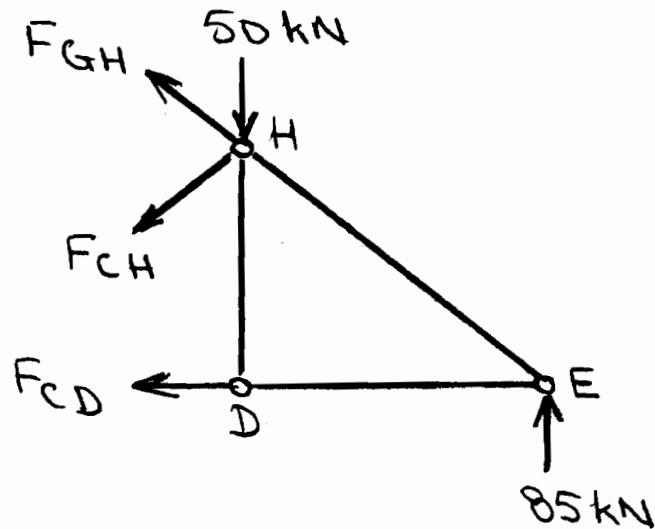
$$F_{CD} = -131.09 \text{ k} = \underline{131.09 \text{ k (C)}}$$

$$+\circlearrowleft \sum M_C = 0 \quad F_{IJ}(20) - 28.2(15 + 30 + 45) - 14.1(60) = 0$$

$$F_{IJ} = 169.2 \text{ k (T)}$$

4.37

Section through members CD, CH and GH:



$$+\circlearrowleft \Sigma M_E = 0$$

$$50(4) + \frac{4}{5} F_{CH}(3) + \frac{3}{5} F_{CH}(4) = 0$$

$$F_{CH} = -41.67 \text{ kN}$$

$$\underline{F_{CH} = 41.67 \text{ kN (C)}}$$

$$+\uparrow \Sigma F_y = 0$$

$$\frac{3}{5} F_{GH} - 50 + \frac{3}{5} (41.67) + 85 = 0$$

$$F_{GH} = -100 \text{ kN}$$

$$\underline{F_{GH} = 100 \text{ kN (C)}}$$

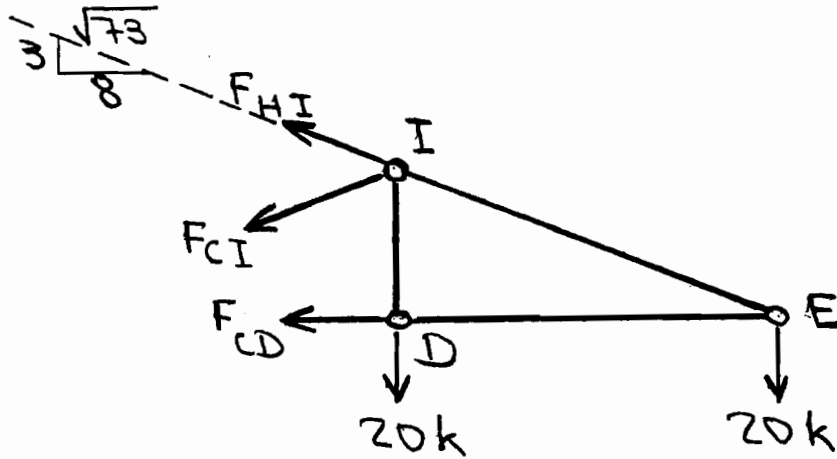
$$+\circlearrowleft \Sigma M_H = 0$$

$$-F_{CD}(3) + 85(4) = 0$$

$$\underline{F_{CD} = 113.33 \text{ kN (T)}}$$

4.38

Section through members CD, CI and HI:



$$+\circlearrowleft \sum M_I = 0 \quad -F_{CD}(3.75) - 20(10) = 0$$

$$F_{CD} = -53.33 \text{ k} \quad \underline{F_{CD} = 53.33 \text{ k (C)}}$$

$$+\circlearrowleft \sum M_E = 0$$

$$20(10) + \frac{8}{\sqrt{73}} F_{CI}(3.75) + \frac{3}{\sqrt{73}} F_{CI}(10) = 0$$

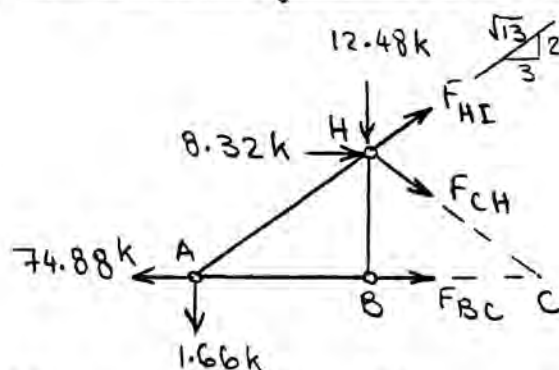
$$F_{CI} = -28.48 \text{ k} \quad \underline{F_{CI} = 28.48 \text{ k (C)}}$$

$$+\uparrow \sum F_y = 0$$

$$\frac{3}{\sqrt{73}} F_{HI} + \frac{3}{\sqrt{73}} (28.48) - 20 - 20 = 0$$

$$\underline{F_{HI} = 85.44 \text{ k (T)}}$$

4.39 Section through members BC, CH and HI:



$$+\circlearrowleft \sum M_H = 0 \quad -74.88 \left(\frac{40}{3} \right) + 1.66(20) + F_{BC} \left(\frac{40}{3} \right) = 0$$

$$F_{BC} = 72.39 \text{ k (T)}$$

$$+\circlearrowleft \sum M_C = 0 \quad 1.66(40) + 12.48(20) - 8.32 \left(\frac{40}{3} \right) - \frac{3}{\sqrt{13}} F_{HI} \left(\frac{40}{3} \right) - \frac{2}{\sqrt{13}} F_{HI}(20) = 0$$

$$F_{HI} = 9.24 \text{ k (T)}$$

$$+\circlearrowleft \sum M_A = 0 \quad -8.32 \left(\frac{40}{3} \right) - 12.48(20) - \frac{3}{\sqrt{13}} F_{CH} \left(\frac{40}{3} \right) - \frac{2}{\sqrt{13}} F_{CH}(20) = 0$$

$$F_{CH} = -16.25 \text{ k} = 16.25 \text{ k (C)}$$

Section through members DE, EK and KL:

$$+\circlearrowleft \sum M_K = 0$$

$$-F_{DE} \left(\frac{80}{3} \right) + (24.96 - 35.78)20$$

$$+ 12.48(40) + 16.64 \left(\frac{40}{3} \right) + 8.32 \left(\frac{80}{3} \right) = 0$$

$$F_{DE} = 27.25 \text{ k (T)}$$

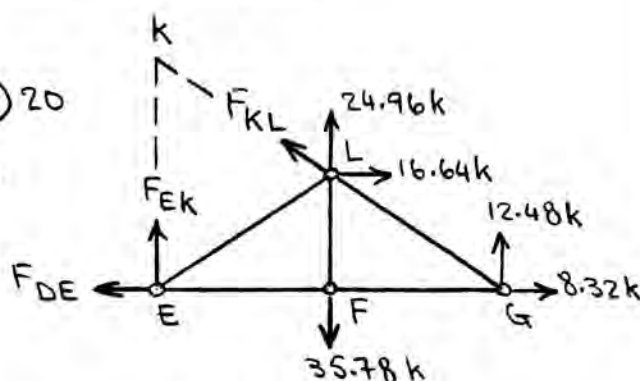
$$+\circlearrowleft \sum M_G = 0$$

$$-F_{EK}(40) + (35.78 - 24.96)20 - 16.64 \left(\frac{40}{3} \right) = 0$$

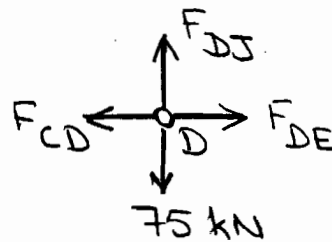
$$F_{EK} = -0.14 \text{ k} = 0.14 \text{ k (C)}$$

$$+\circlearrowleft \sum M_E = 0 \quad (24.96 - 35.78)20 + 12.48(40) - 16.64 \left(\frac{40}{3} \right) + \frac{3}{\sqrt{13}} F_{KL} \left(\frac{40}{3} \right) + \frac{2}{\sqrt{13}} F_{KL}(20) = 0$$

$$F_{KL} = -2.75 \text{ k} = 2.75 \text{ k (C)}$$

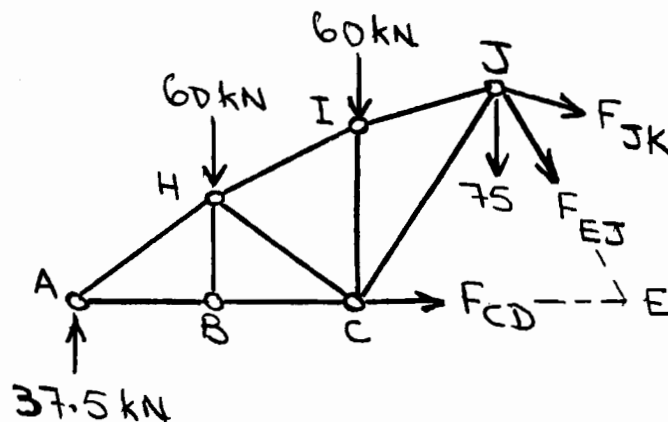


4.40 Joint D:



$$\sum F_y = 0 \quad F_{DJ} = 75 \text{ kN (T)}$$

Section through members CD, DJ, EJ and JK:



$$+\circlearrowleft \sum M_J = 0$$

$$-37.5(12) + 60(8) + 60(4) + F_{CD}(6) = 0$$

$$F_{CD} = -45 \text{ kN}$$

$$\underline{F_{CD} = 45 \text{ kN (C)}}$$

$$+\circlearrowleft \sum M_E = 0$$

$$-37.5(16) + 60(12) + 60(8) + 75(4)$$

$$- \frac{4}{\sqrt{17}} F_{JK}(6) + \frac{1}{\sqrt{17}} F_{JK}(4) = 0$$

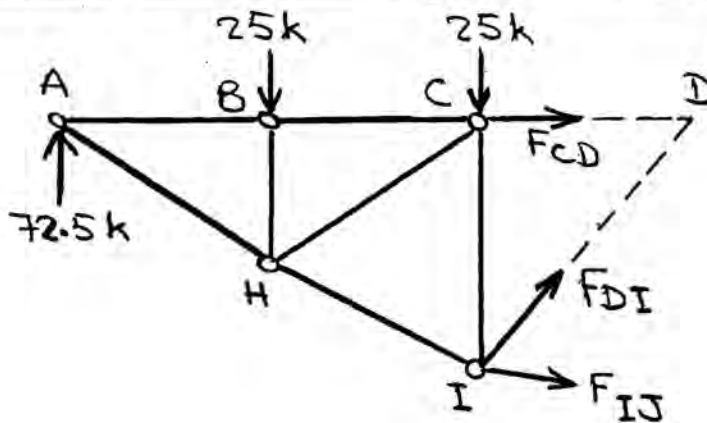
$$\underline{F_{JK} = 185.54 \text{ kN (T)}}$$

$$+\uparrow \sum F_y = 0$$

$$37.5 - 60 - 60 - 75 - \frac{3}{\sqrt{13}} F_{EJ} - \frac{1}{\sqrt{17}} (185.54) = 0$$

$$F_{EJ} = -243.37 \text{ kN} \quad F_{EJ} = 243.37 \text{ kN (C)}$$

4.41 Section through members CD, DI and IJ:



$$+\circlearrowleft \sum M_I = 0 \quad -72.5(60) + 25(30) - F_{CD}(35) = 0$$

$$F_{CD} = -102.86 \text{ k} \quad \underline{F_{CD} = 102.86 \text{ k (C)}}$$

$$+\circlearrowleft \sum M_D = 0$$

$$-72.5(90) + 25(60) + 25(30) + \frac{6}{\sqrt{37}} F_{IJ}(35) + \frac{1}{\sqrt{37}} F_{DI}(30) = 0$$

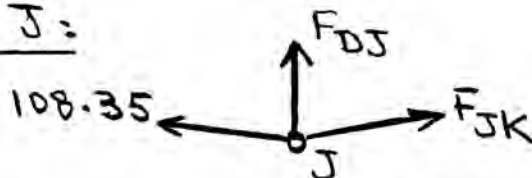
$$F_{IJ} = 108.35 \text{ k (T)}$$

$$+\uparrow \sum F_y = 0$$

$$72.5 - 25 - 25 - \frac{1}{\sqrt{37}} (108.35) + \frac{7}{\sqrt{85}} F_{DI} = 0$$

$$F_{DI} = -6.17 \text{ k} \quad \underline{F_{DI} = 6.17 \text{ k (C)}}$$

Joint J:



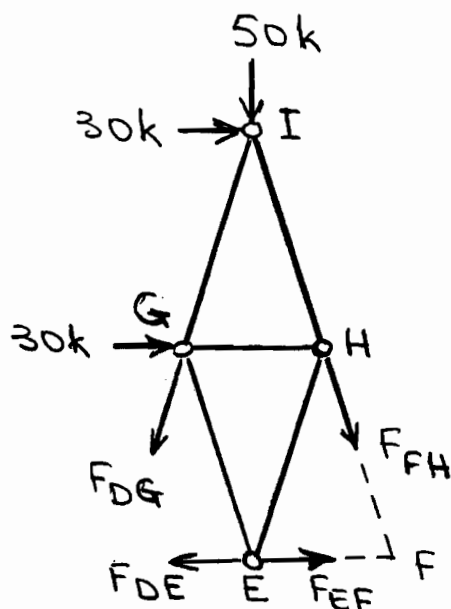
$$+\rightarrow \sum F_x = 0 \quad -\frac{6}{\sqrt{37}} (108.35) + \frac{6}{\sqrt{37}} F_{JK} = 0$$

$$F_{JK} = 108.35 \text{ k (T)}$$

$$+\uparrow \sum F_y = 0 \quad 2\left(\frac{1}{\sqrt{37}}\right) 108.35 + F_{DJ} = 0$$

$$F_{DJ} = -35.63 \text{ k} \quad \underline{F_{DJ} = 35.63 \text{ k (C)}}$$

4.42 Section through members DG, DE, EF and FH:



$$+\circlearrowleft \sum M_F = 0$$

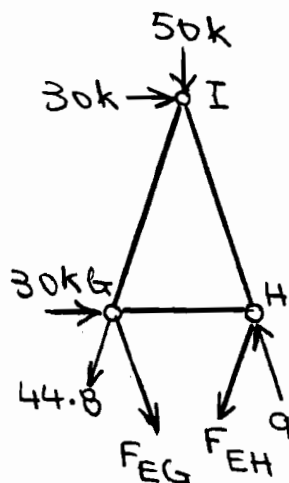
$$50(6) - 30(18) - 30(9) + \frac{1}{\sqrt{10}} F_{DG}(9) + \frac{3}{\sqrt{10}} F_{DG}(9) = 0$$

$$F_{DG} = 44.8 \text{ k (T)}$$

$$+\uparrow \sum F_y = 0 \quad -\frac{3}{\sqrt{10}}(44.8) - 50 - \frac{3}{\sqrt{10}} F_{FH} = 0$$

$$F_{FH} = -97.5 \text{ k} = 97.5 \text{ k (C)}$$

Section through members DG, EG, EH and FH:



$$+\circlearrowleft \sum M_G = 0$$

$$-30(9) - 50(3) + \frac{3}{\sqrt{10}}(97.5)(6) - \frac{3}{\sqrt{10}} F_{EH}(6) = 0$$

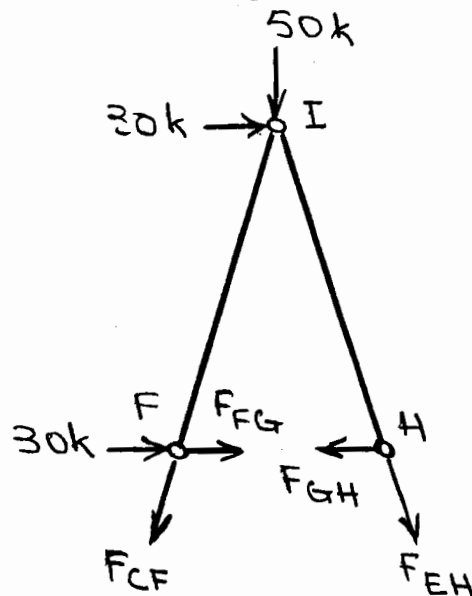
$$F_{EH} = 23.72 \text{ k (T)}$$

$$+\circlearrowleft \sum M_H = 0$$

$$50(3) - 30(9) + \frac{3}{\sqrt{10}}(44.8)(6) + \frac{3}{\sqrt{10}} F_{EG}(6) = 0$$

$$F_{EG} = -23.72 \text{ k} \quad F_{EG} = 23.72 \text{ k (C)}$$

4.43 Section through members CF, FG, GH and EH:



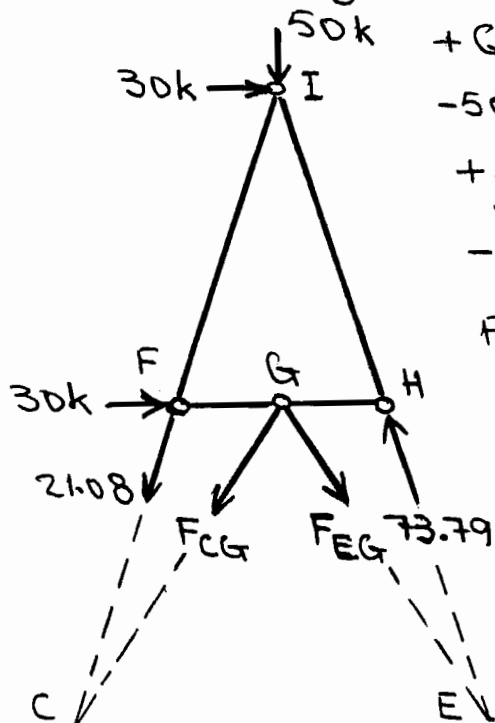
$$+\circlearrowleft \sum M_H = 0 \quad 50(3) - 30(9) + \frac{3}{\sqrt{10}} F_{CF}(6) = 0$$

$$\underline{F_{CF} = 21.08 \text{ k (T)}}$$

$$+\circlearrowleft \sum M_F = 0 \quad -50(3) - 30(9) - \frac{3}{\sqrt{10}} F_{EH}(6) = 0$$

$$F_{EH} = -73.79 \text{ k} = 73.79 \text{ k (C)}$$

Section through members CF, CG, EG and EH:



$$+\circlearrowleft \sum M_C = 0$$

$$-50(6) - 30(18) - 30(9) + \frac{1}{\sqrt{10}} (73.79)(9)$$

$$+ \frac{3}{\sqrt{10}} (73.79)(9) - \frac{2}{\sqrt{13}} F_{EG}(9)$$

$$- \frac{3}{\sqrt{13}} F_{EG}(6) = 0$$

$$F_{EG} = -27.04 \text{ k}$$

$$\underline{F_{EG} = 27.04 \text{ k (C)}}$$

$$+\circlearrowleft \sum M_E = 0$$

$$50(6) - 30(18) - 30(9)$$

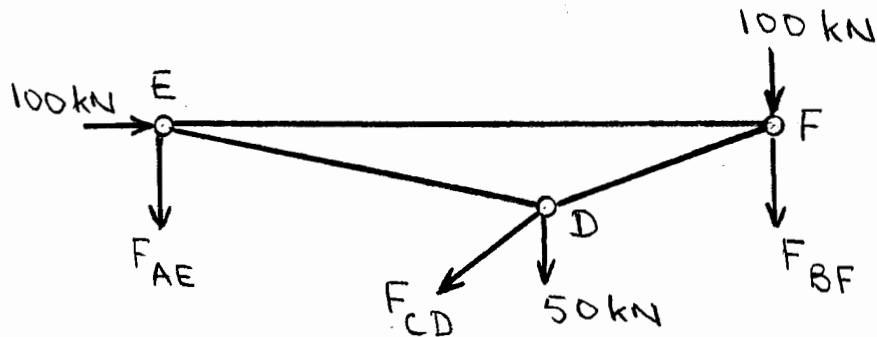
$$+ \frac{1}{\sqrt{10}} (21.08)(9) + \frac{3}{\sqrt{10}} (21.08)(9)$$

$$+ \frac{2}{\sqrt{13}} F_{CG}(9) + \frac{3}{\sqrt{13}} F_{CG}(6) = 0$$

$$\underline{F_{CG} = 27.04 \text{ k (T)}}$$

4.44

Section through members AE, CD and BF:



$$\begin{aligned} \rightarrow \Sigma F_x = 0 \quad 100 - \left(\frac{4}{5}\right) F_{CD} = 0 \quad \underline{F_{CD} = 125 \text{ kN (T)}} \end{aligned}$$

$$\begin{aligned} + \curvearrowright \Sigma M_F = 0 \quad F_{AE}(16) - \frac{4}{5}(125)(2) + \frac{3}{5}(125)(6) + 50(6) = 0 \\ F_{AE} = -34.38 \text{ kN} = \underline{34.38 \text{ kN (C)}} \end{aligned}$$

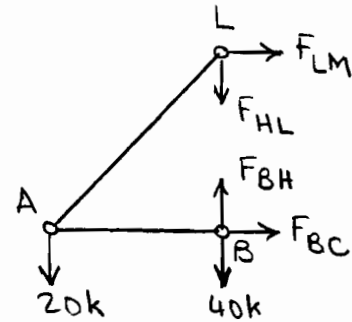
$$\begin{aligned} + \uparrow \Sigma F_y = 0 \quad 34.38 - \frac{3}{5}(125) - 50 - 100 - F_{BF} = 0 \\ F_{BF} = -190.62 \text{ kN} = \underline{190.62 \text{ kN (C)}} \end{aligned}$$

4.45 Section through members BC, BH, HL and LM:

$$+\circlearrowleft \sum M_B = 0$$

$$20(30) - F_{LM}(30) = 0$$

$$\underline{F_{LM} = 20 \text{ k (T)}}$$



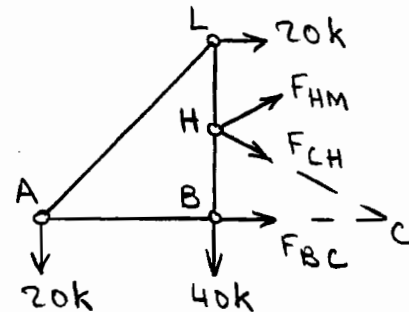
Section through members BC, CH, HM and LM:

$$+\circlearrowleft \sum M_C = 0$$

$$20(60) + 40(30) - 20(30)$$

$$- \frac{2}{\sqrt{5}} F_{HM}(15) - \frac{1}{\sqrt{5}} F_{HM}(30) = 0$$

$$\underline{F_{HM} = 67.08 \text{ k (T)}}$$



$$+\uparrow \sum F_y = 0 \quad -20 - 40 + \frac{1}{\sqrt{5}}(67.08) - \frac{1}{\sqrt{5}} F_{CH} = 0$$

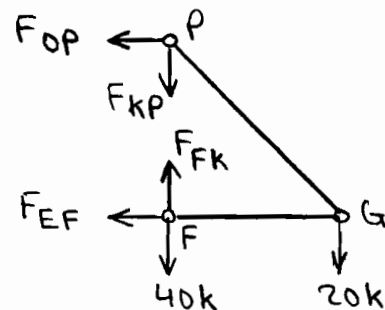
$$F_{CH} = -67.08 \text{ k} = \underline{67.08 \text{ k (C)}}$$

Section through members EF, FK, KP, and OP:

$$+\circlearrowleft \sum M_P = 0$$

$$- F_{EF}(30) - 20(30) = 0$$

$$F_{EF} = -20 \text{ k} = \underline{20 \text{ k (C)}}$$



Section through members EF, EK, KO and OP:

$$+\circlearrowleft \sum M_O = 0$$

$$-20(60) - 40(30) + 20(30)$$

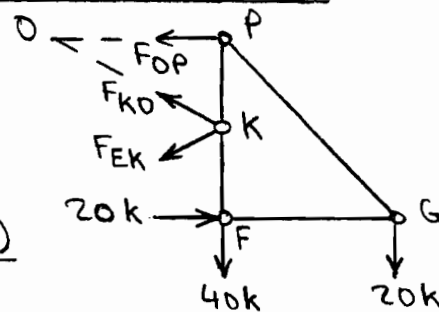
$$- \frac{2}{\sqrt{5}} F_{EK}(15) - \frac{1}{\sqrt{5}} F_{EK}(30) = 0$$

$$F_{EK} = -67.08 \text{ k} = \underline{67.08 \text{ k (C)}}$$

$$+\uparrow \sum F_y = 0$$

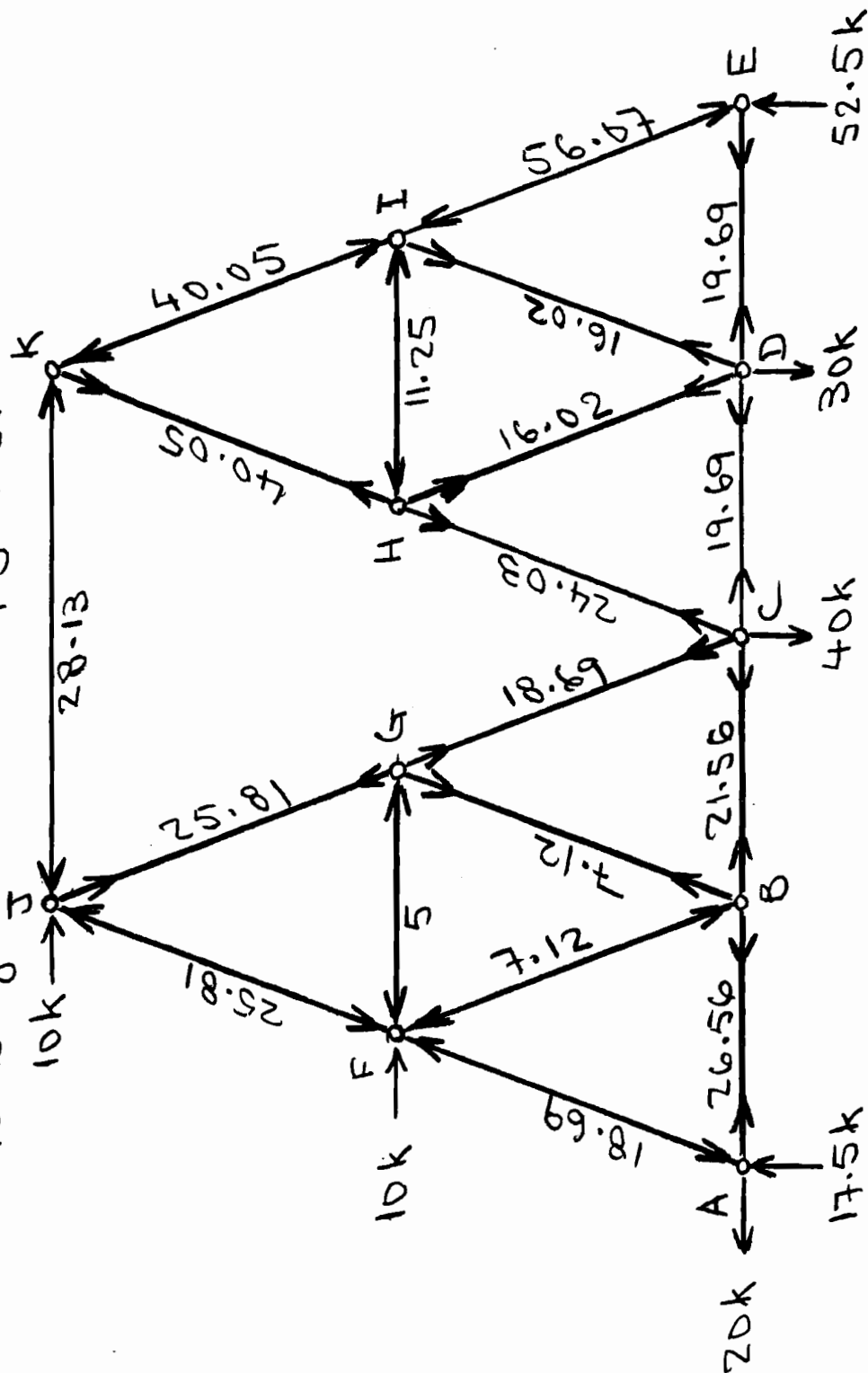
$$\frac{1}{\sqrt{5}}(67.08) + \frac{1}{\sqrt{5}} F_{KO} - 40 - 20 = 0$$

$$\underline{F_{KO} = 67.08 \text{ k (T)}}$$

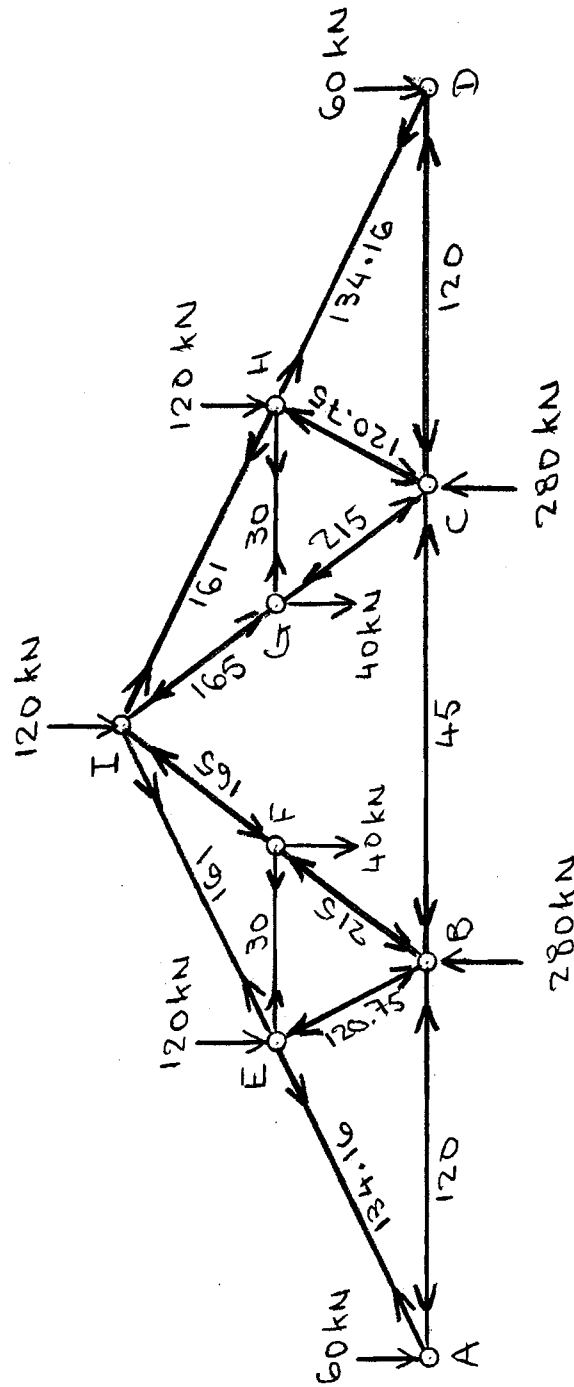


4.46

Considering a section through members BC, CG and JK, compute the force in member JK: $F_{JK} = 28.13 \text{ k}(C)$. Then, determine the remaining member forces by the method of joints.

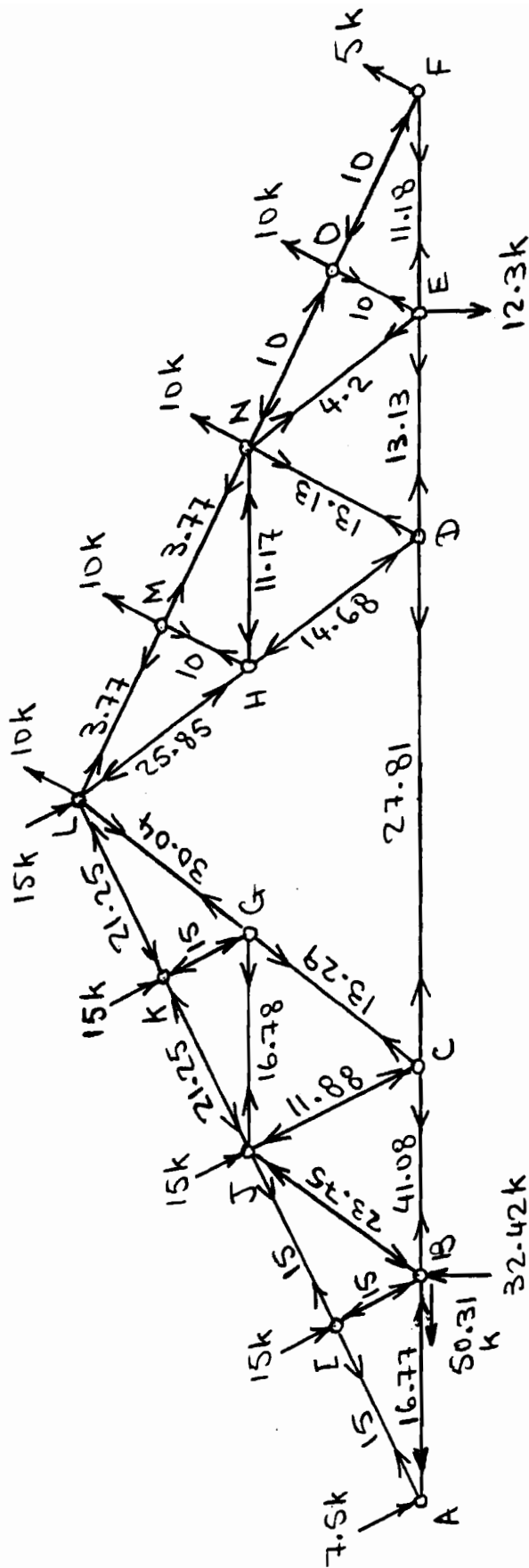


4.47 Considering a section through members BC, FI and EI, compute the force in member BC: $F_{BC} = 45 \text{ kN (C)}$. Then, determine the remaining member forces by the method of joints.



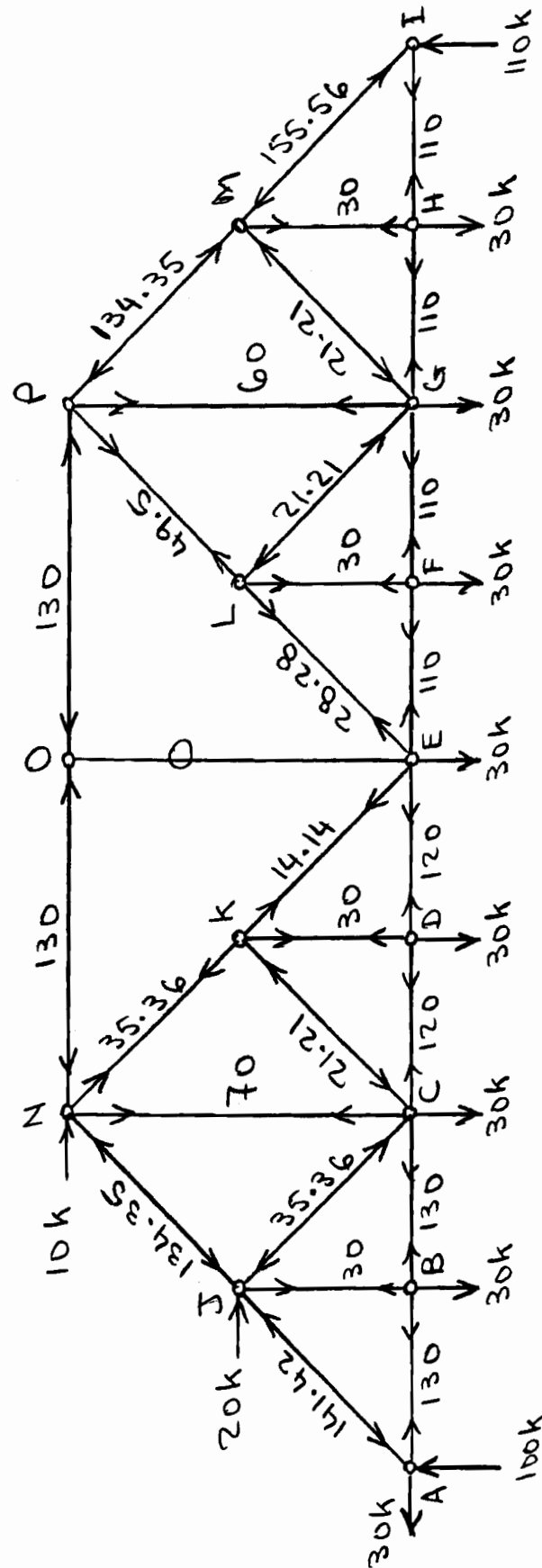
4.48

Considering a section through members CD, GL and KL, compute the force in member CD: $F_{CD} = 27.81 \text{ k (T)}$. Then, determine the remaining member forces by the method of joints.

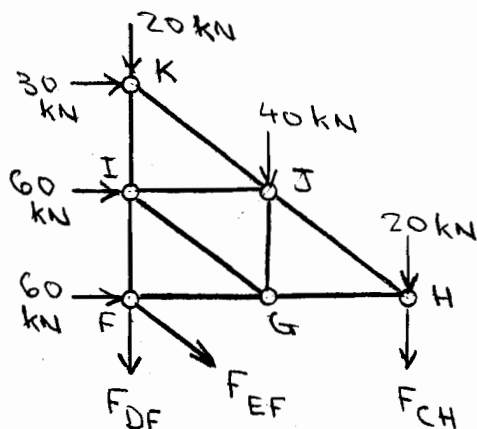


4.49 Considering a section through members DE, EK and NO, compute the force in member NO: $F_{NO} = 130 \text{ k (C)}$.

Then, determine the remaining member forces by the method of joints.



4.50 Section through members DF, EF and CH:



$$+\circlearrowleft \Sigma M_F = 0 \quad -30(6) - 60(3) - 40(4) - 20(8) - F_{CH}(8) = 0$$

$$F_{CH} = -85 \text{ kN} = \underline{85 \text{ kN (C)}}$$

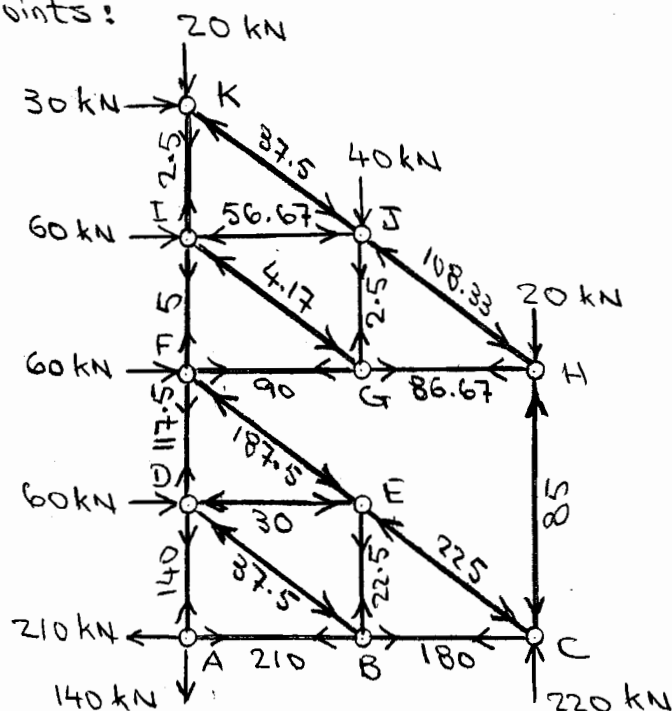
$$+\rightarrow \Sigma F_x = 0 \quad 30 + 60 + 60 + \left(\frac{4}{5}\right)F_{EF} = 0$$

$$F_{EF} = -187.5 \text{ kN} = \underline{187.5 \text{ kN (C)}}$$

$$+\uparrow \Sigma F_y = 0 \quad -20 - 40 - 20 - F_{DF} + 85 + \left(\frac{3}{5}\right)(187.5) = 0$$

$$F_{DF} = 117.5 \text{ (T)}$$

The remaining member forces can now be determined by the method of joints:



4.51

Member	Projection			Length (ft)
	x (ft)	y (ft)	z (ft)	
AD	3	10	5	11.58
BD	8	10	7	14.59
CD	2	10	4	10.95

Joint D:

$$\Sigma F_x = 0 \quad - \left(\frac{3}{11.58} \right) F_{AD} + \left(\frac{8}{14.59} \right) F_{BD} + \left(\frac{2}{10.95} \right) F_{CD} + 6 = 0$$

$$\Sigma F_y = 0 \quad - \left(\frac{10}{11.58} \right) F_{AD} - \left(\frac{10}{14.59} \right) F_{BD} - \left(\frac{10}{10.95} \right) F_{CD} - 12 = 0$$

$$\Sigma F_z = 0 \quad \left(\frac{5}{11.58} \right) F_{AD} + \left(\frac{7}{14.59} \right) F_{BD} - \left(\frac{4}{10.95} \right) F_{CD} = 0$$

Solving these simultaneous equations, we obtain:

$$\underline{F_{AD} = 1.12 \text{ k (T)}}$$

$$F_{BD} = -7.56 \text{ k} = \underline{7.56 \text{ k (C)}}$$

$$F_{CD} = -8.54 \text{ k} = \underline{8.54 \text{ k (C)}}$$

4.52

Member	Projection			Length (m)
	x (m)	y (m)	z (m)	
AB	0	0	3	3
AC	4	0	3	5
BC	4	0	0	4
AD	0	7	3	7.62
BD	0	7	0	7
CD	4	7	0	8.06

Joint D:

$$\Sigma F_x = 0 \quad \left(\frac{4}{8.06}\right) F_{CD} = 0 \quad \underline{F_{CD} = 0}$$

$$\Sigma F_y = 0 \quad \left(\frac{3}{7.62}\right) F_{AD} + 40 = 0 \quad F_{AD} = -101.6 \text{ kN}$$

$$\underline{F_{AD} = 101.6 \text{ kN (C)}}$$

$$\Sigma F_z = 0 \quad \left(\frac{7}{7.62}\right) 101.6 - F_{BD} - 60 = 0$$

$$\underline{F_{BD} = 33.33 \text{ kN (T)}}$$

Joint B:

$$\Sigma F_x = 0 \quad \underline{F_{BC} = 0}$$

$$\Sigma F_z = 0 \quad \underline{F_{AB} = 0}$$

Joint C:

$$\Sigma F_y = 0 \quad \left(\frac{3}{5}\right) F_{AC} = 0 \quad \underline{F_{AC} = 0}$$

4.53

Member	Projection			Length (ft)
	x (ft)	y (ft)	z (ft)	
AB	20	0	0	20
AC	14	0	10	17.2
BC	6	0	10	11.66
AD	10	20	5	22.91
BD	10	20	5	22.91
CD	4	20	5	21

Joint D:

$$\Sigma F_x = 0 \quad -\left(\frac{10}{22.91}\right)F_{AD} + \left(\frac{10}{22.91}\right)F_{BD} + \left(\frac{4}{21}\right)F_{CD} + 25 = 0$$

$$\Sigma F_y = 0 \quad -\left(\frac{20}{22.91}\right)F_{AD} - \left(\frac{20}{22.91}\right)F_{BD} - \left(\frac{20}{21}\right)F_{CD} - 30 = 0$$

$$\Sigma F_z = 0 \quad \left(\frac{5}{22.91}\right)F_{AD} + \left(\frac{5}{22.91}\right)F_{BD} - \left(\frac{5}{21}\right)F_{CD} = 0$$

Solving these equations, we obtain:

$$\underline{F_{AD} = 16.63 \text{ k (T)}}$$

$$F_{BD} = -33.84 \text{ k} = \underline{33.84 \text{ k (C)}}$$

$$F_{CD} = -15.74 \text{ k} = \underline{15.74 \text{ k (C)}}$$

Joint A:

$$\Sigma F_z = 0 \quad -\left(\frac{5}{22.91}\right)16.63 - \left(\frac{10}{17.2}\right)F_{AC} = 0$$

$$F_{AC} = -6.24 \text{ k} = \underline{6.24 \text{ k (C)}}$$

$$\Sigma F_x = 0 \quad \left(\frac{10}{22.91}\right)16.63 - \left(\frac{14}{17.2}\right)6.24 + F_{AB} = 0$$

$$F_{AB} = -2.18 \text{ k} = \underline{2.18 \text{ k (C)}}$$

Joint B:

$$\Sigma F_z = 0 \quad \left(\frac{5}{22.91}\right)33.84 - \left(\frac{10}{11.66}\right)F_{BC} = 0$$

$$\underline{F_{BC} = 8.61 \text{ k (T)}}$$

4.54

Member	Projection			Length (m)
	x(m)	y(m)	z(m)	
AB	8	0	0	8
CD	8	0	0	8
BC	0	0	4	4
AD	0	0	4	4
BD	8	0	4	8.94
AE	4	8	2	9.17
BE	4	8	2	9.17
CE	4	8	2	9.17
DE	4	8	2	9.17

Joint C: $\sum F_y = 0 \quad \left(\frac{8}{9.17}\right) F_{CE} = 0 \quad \underline{F_{CE} = 0}$
 $\sum F_z = 0 \quad \underline{F_{BC} = 0}$

Joint E: $\sum F_x = 0 \quad \frac{4}{9.17}(-F_{AE} + F_{BE} - F_{DE}) + 30 = 0$

$\sum F_y = 0 \quad -\frac{8}{9.17}(F_{AE} + F_{BE} + F_{DE}) - 60 = 0$

$\sum F_z = 0 \quad \frac{2}{9.17}(F_{AE} + F_{BE} - F_{DE}) + 40 = 0$

Solving these equations, we obtain:

$F_{AE} = -57.31 \text{ kN} = \underline{57.31 \text{ kN (C)}}$;

$F_{BE} = -68.78 \text{ kN} = \underline{68.78 \text{ kN (C)}}$ and $\underline{F_{DE} = 57.31 \text{ kN (T)}}$

Joint A:

$\sum F_x = 0 \quad -\left(\frac{4}{9.17}\right) 57.31 + F_{AB} = 0 \quad \underline{F_{AB} = 25 \text{ kN (T)}}$

$\sum F_z = 0 \quad \left(\frac{2}{9.17}\right) 57.31 - F_{AD} = 0 \quad \underline{F_{AD} = 12.5 \text{ kN (T)}}$

Joint D:

$\sum F_y = 0 \quad \left(\frac{2}{9.17}\right) 57.31 + 12.5 + \left(\frac{4}{8.94}\right) F_{BD} = 0$
 $F_{BD} = -55.87 \text{ kN} = \underline{55.87 \text{ kN (C)}}$

$\sum F_x = 0 \quad \left(\frac{4}{9.17}\right) 57.31 - \left(\frac{8}{8.94}\right) 55.87 + F_{CD} = 0$
 $\underline{F_{CD} = 25 \text{ kN (T)}}$

4.55

Member	Force (k)
AB	29.17 (T)
CD	15.83 (C)
BC	7.22 (T)
AD	0
AC	0.5 (T)
EF	28.33 (T)
GH	10.0 (C)
FG	5.55 (C)
EH	5.55 (C)
FH	2.0 (T)
AE	4.12 (T)
BF	8.25 (C)
CG	80.41 (C)
DH	39.17 (C)
BE	52.25 (C)
CF	8.83 (T)
DG	45.12 (T)
AH	2.21 (C)

Chapter Five

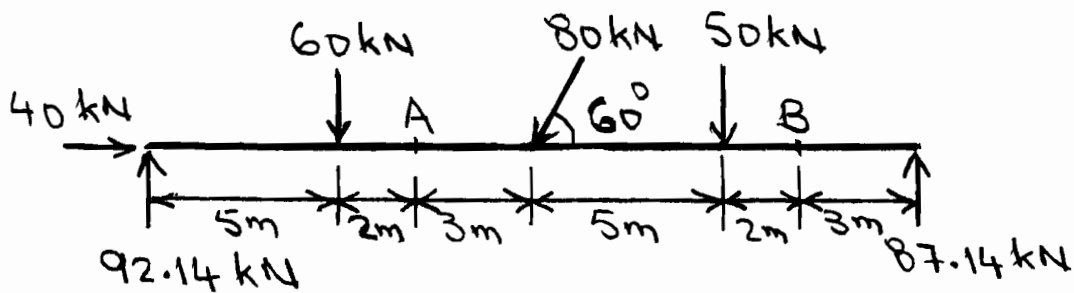
Beams and Frames:

Shear and Bending

Movement

CHAPTER 5

5.1



$$Q_A = -40 \text{ kN}$$

$$S_A = 92.14 - 60 = 32.14 \text{ kN}$$

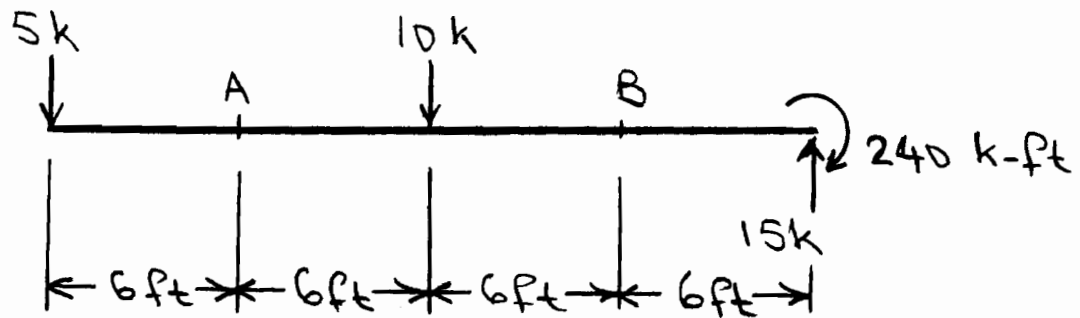
$$M_A = 92.14(7) - 60(2) = 524.98 \text{ kN.m}$$

$$Q_B = 0$$

$$S_B = -87.14 \text{ kN}$$

$$M_B = 87.14(3) = 261.42 \text{ kN.m}$$

5.2



$$Q_A = \underline{0}$$

$$S_A = \underline{-5 k}$$

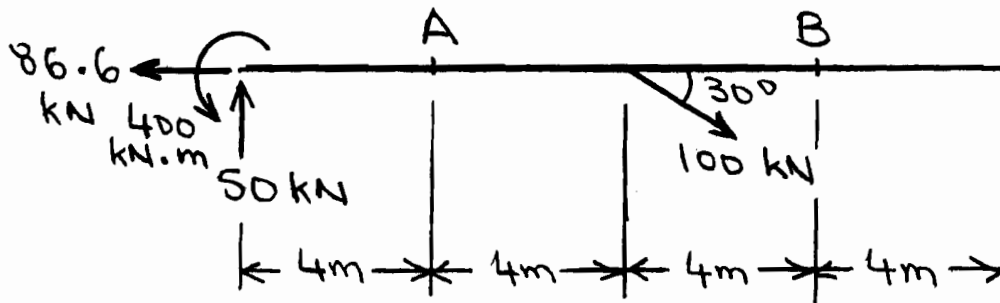
$$M_A = -5(6) = \underline{-30 k-ft}$$

$$Q_B = \underline{0}$$

$$S_B = -5 - 10 = \underline{-15 k}$$

$$M_B = -5(18) - 10(6) = \underline{-150 k-ft}$$

5.3



$$Q_A = 100 \cos 30^\circ = \underline{86.6 \text{ kN}}$$

$$S_A = 100 \sin 30^\circ = \underline{50 \text{ kN}}$$

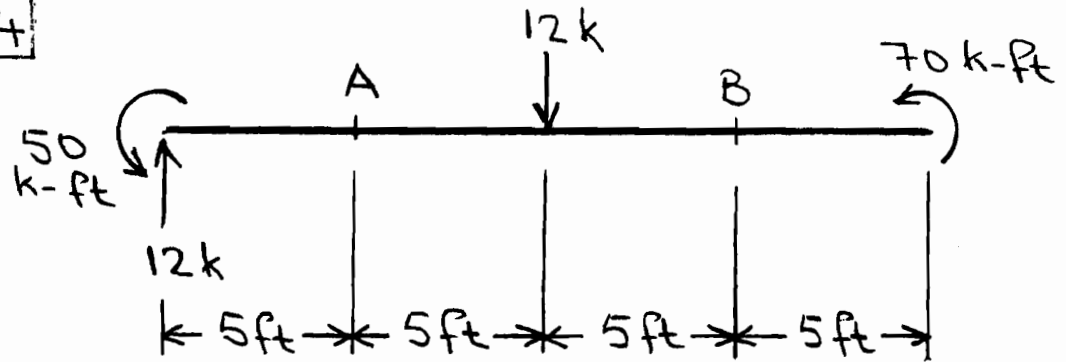
$$M_A = -100 \sin 30^\circ (4) = \underline{-200 \text{ kN.m}}$$

$$Q_B = \underline{0}$$

$$S_B = \underline{0}$$

$$M_B = \underline{0}$$

5.4



$$Q_A = \underline{0}$$

$$S_A = \underline{12 \text{ k}}$$

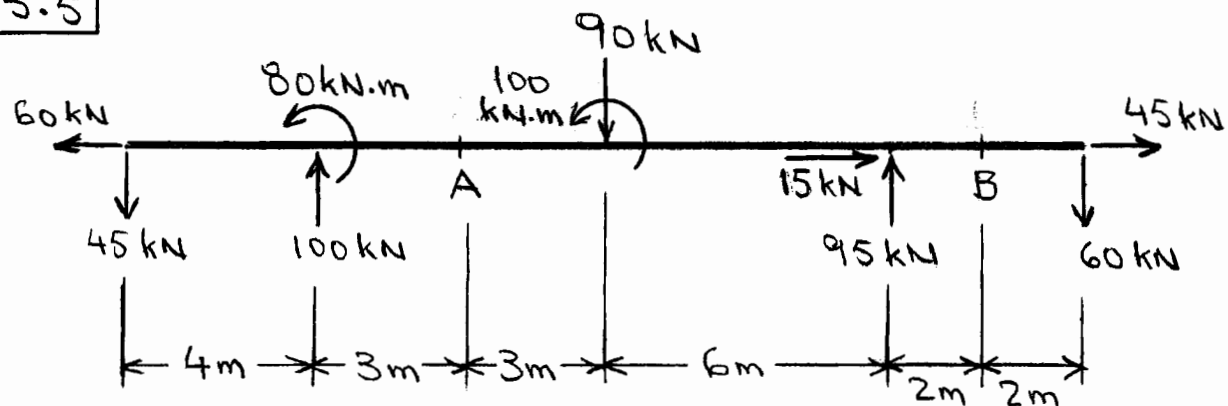
$$M_A = 12(5) - 50 = \underline{10 \text{ k-ft}}$$

$$Q_B = \underline{0}$$

$$S_B = \underline{0}$$

$$M_B = \underline{70 \text{ k-ft}}$$

5.5



$$Q_A = \underline{60 \text{ kN}}$$

$$S_A = -45 + 100 = \underline{55 \text{ kN}}$$

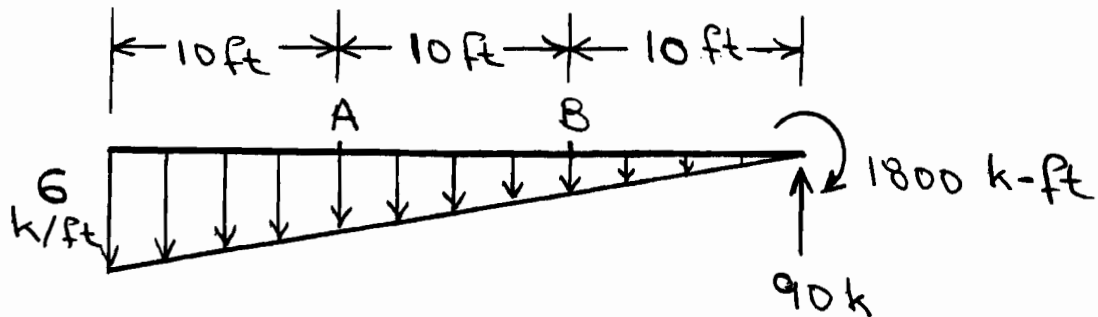
$$M_A = -45(7) - 80 + 100(3) = \underline{-95 \text{ kN.m}}$$

$$Q_B = \underline{45 \text{ kN}}$$

$$S_B = \underline{60 \text{ kN}}$$

$$M_B = -60(2) = \underline{-120 \text{ kN.m}}$$

5.6



$$Q_A = \underline{0}$$

$$S_A = -\frac{(6+4)}{2} (10) = \underline{-50 \text{ k}}$$

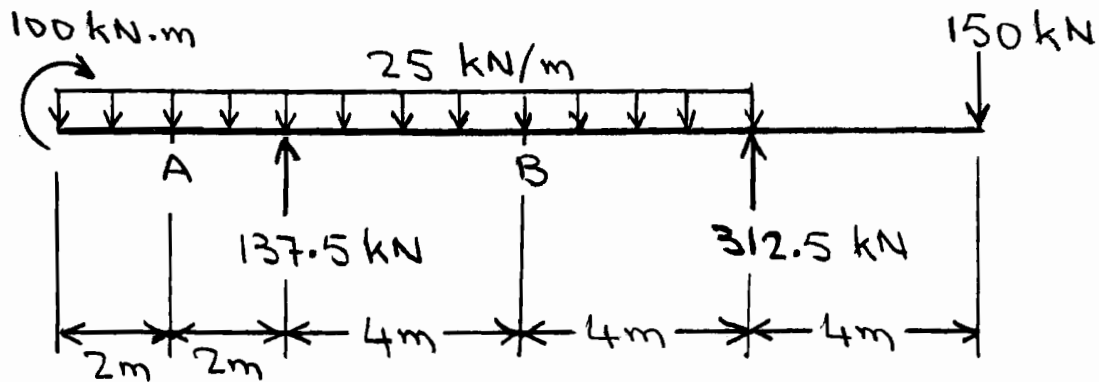
$$M_A = -4(10)5 - \frac{1}{2}(2)(10)\left(\frac{20}{3}\right) = \underline{-266.67 \text{ k-ft}}$$

$$Q_B = \underline{0}$$

$$S_B = -\frac{(6+2)}{2} (20) = \underline{-80 \text{ k}}$$

$$M_B = -2(20)10 - \frac{1}{2}(4)(20)\left(\frac{40}{3}\right) = \underline{-933.33 \text{ k-ft}}$$

5.7



$$Q_A = 0$$

$$S_A = -25(2) = \underline{-50 \text{ kN}}$$

$$M_A = 100 - 25(2)(1) = \underline{50 \text{ kN.m}}$$

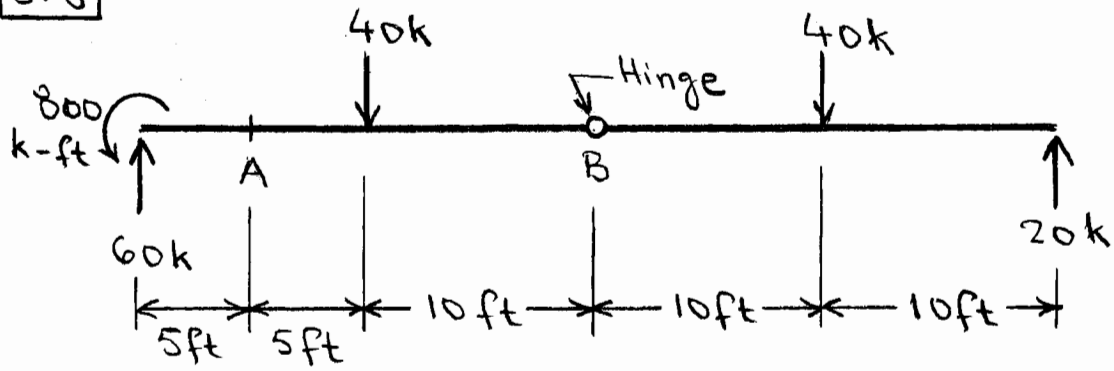
$$Q_B = 0$$

$$S_B = 150 - 312.5 + 25(4) = \underline{-62.5 \text{ kN}}$$

$$M_B = -150(8) + 312.5(4) - 25(4)(2)$$

$$M_B = \underline{-150 \text{ kN.m}}$$

5.8



$$Q_A = \underline{0}$$

$$S_A = \underline{60 \text{ k}}$$

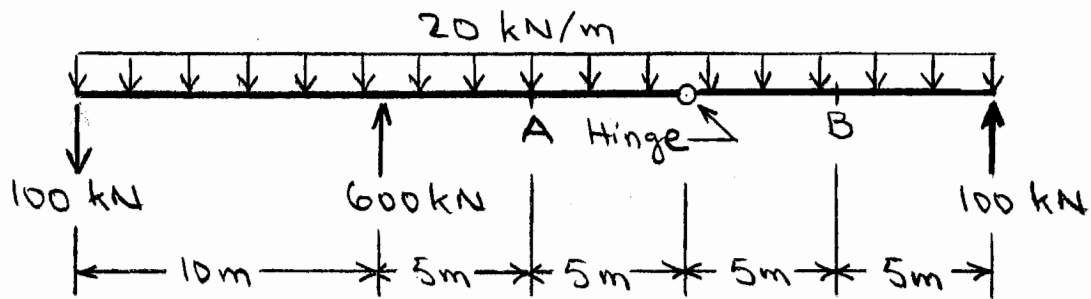
$$M_A = -800 + 60(5) = \underline{-500 \text{ k-ft}}$$

$$Q_B = \underline{0}$$

$$S_B = -20 + 40 = \underline{20 \text{ k}}$$

$$M_B = 20(20) - 40(10) = \underline{0}$$

5.9



$$Q_A = \underline{0}$$

$$S_A = -100 - 20(15) + 600 = \underline{200 \text{ kN}}$$

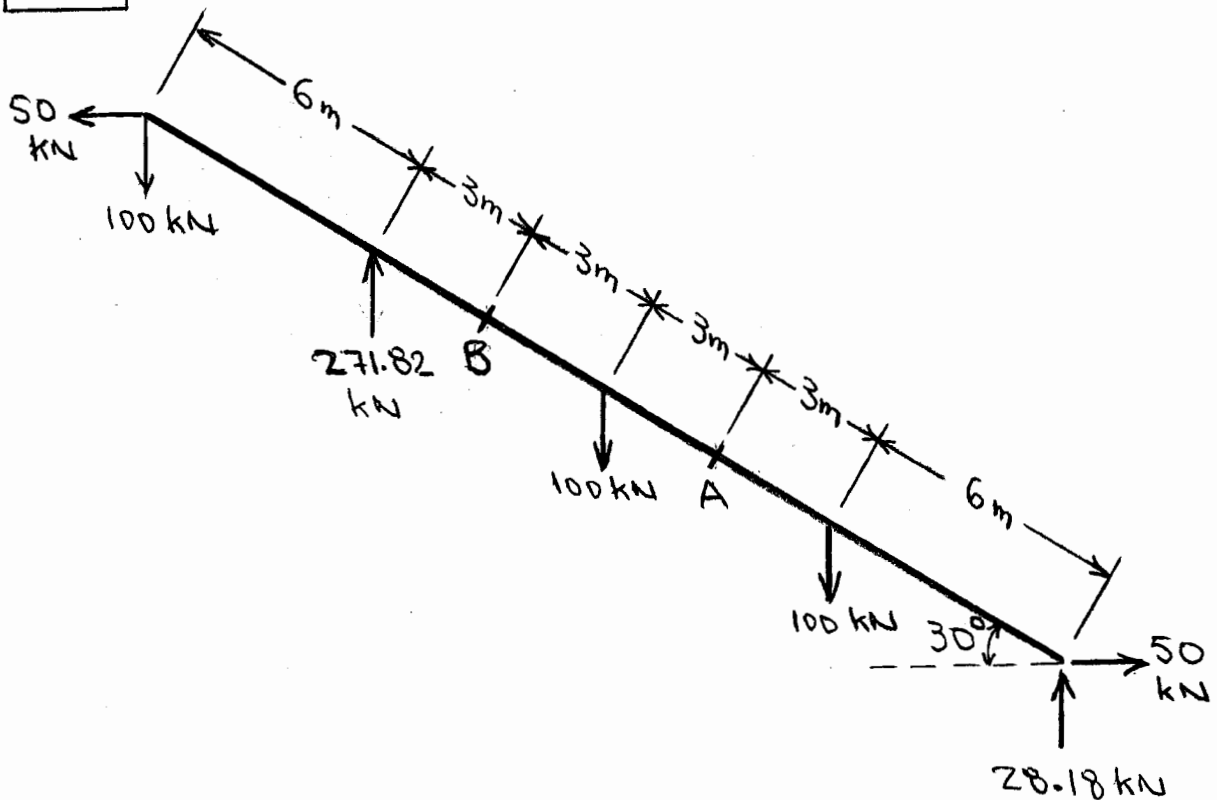
$$M_A = -100(15) - 20(15)(7.5) + 600(5) = \underline{-750 \text{ kN}\cdot\text{m}}$$

$$Q_B = \underline{0}$$

$$S_B = -100 + 20(5) = \underline{0}$$

$$M_B = 100(5) - 20(5)(2.5) = \underline{250 \text{ kN}\cdot\text{m}}$$

5.10



$$Q_A = 50(\cos 30^\circ) - 28.18(\sin 30^\circ) + 100(\sin 30^\circ) = \underline{79.2 \text{ kN}}$$

$$S_A = -50(\sin 30^\circ) - 28.18(\cos 30^\circ) + 100(\cos 30^\circ) = \underline{37.2 \text{ kN}}$$

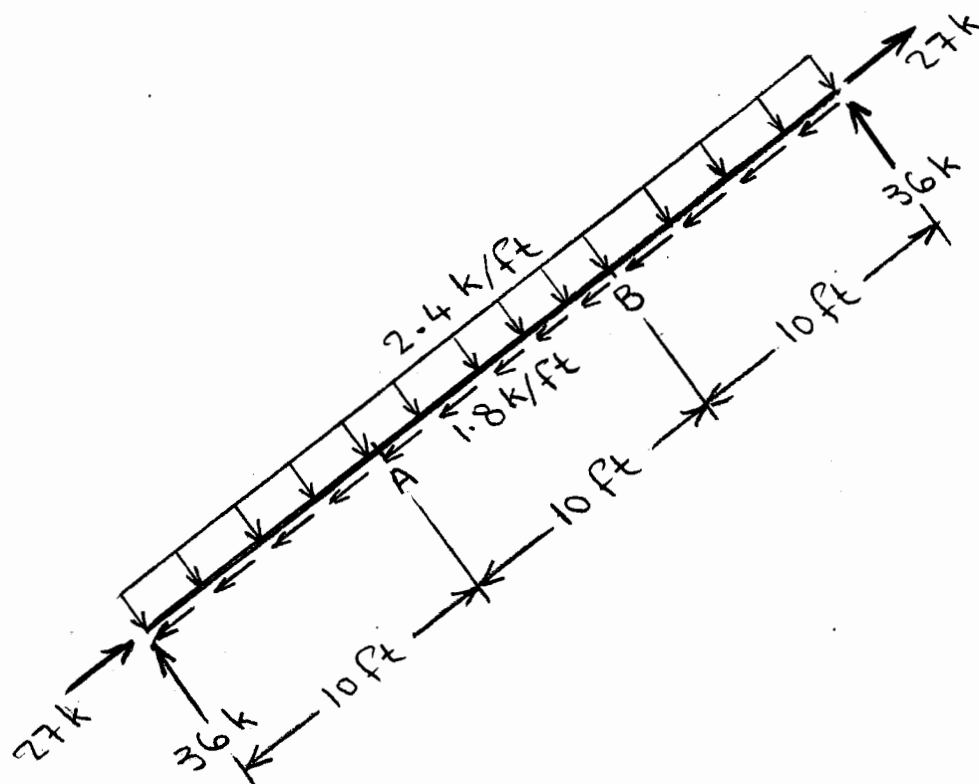
$$M_A = 50(\sin 30^\circ)(9) + 28.18(\cos 30^\circ)(9) - 100(\cos 30^\circ)(3) \\ = \underline{184.8 \text{ kN.m}}$$

$$Q_B = 50(\cos 30^\circ) - 100(\sin 30^\circ) + 271.82(\sin 30^\circ) = \underline{129.2 \text{ kN}}$$

$$S_B = -50(\sin 30^\circ) - 100(\cos 30^\circ) + 271.82(\cos 30^\circ) = \underline{123.8 \text{ kN}}$$

$$M_B = -50(\sin 30^\circ)(9) - 100(\cos 30^\circ)(9) + 271.82(\cos 30^\circ)(3) \\ = \underline{-298.2 \text{ kN.m}}$$

5.11



$$Q_A = -27 + 1.8(10) = \underline{-9 \text{ k}}$$

$$S_A = 36 - 2.4(10) = \underline{12 \text{ k}}$$

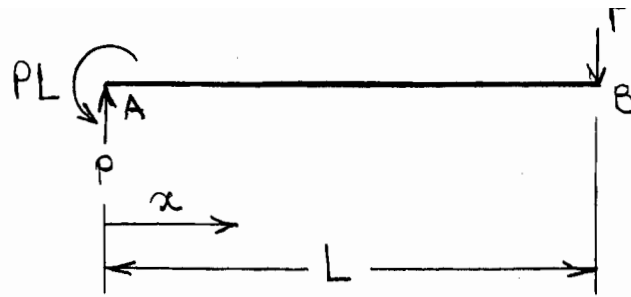
$$M_A = 36(10) - 2.4(10)(5) = \underline{240 \text{ k-ft}}$$

$$Q_B = 27 - 1.8(10) = \underline{9 \text{ k}}$$

$$S_B = -36 + 2.4(10) = \underline{-12 \text{ k}}$$

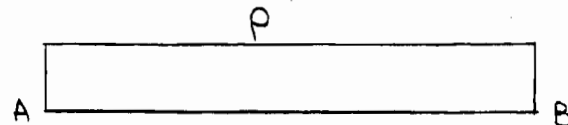
$$M_B = 36(10) - 2.4(10)(5) = \underline{240 \text{ k-ft}}$$

5.12

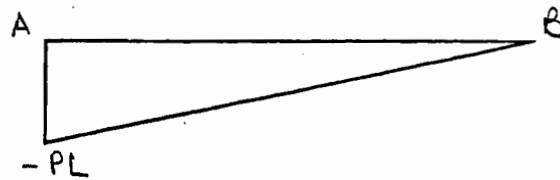


$$S = -P \quad \text{for } 0 < x < L$$

$$M = -PL + Px = P(x - L) \quad \text{for } 0 < x < L$$

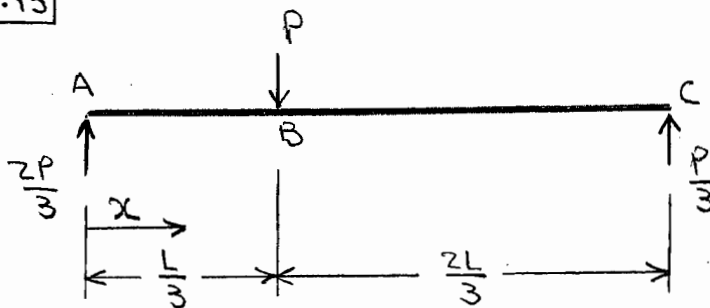


Shear Diagram



Bending Moment Diagram

5.13



$$0 < x < \frac{L}{3} :$$

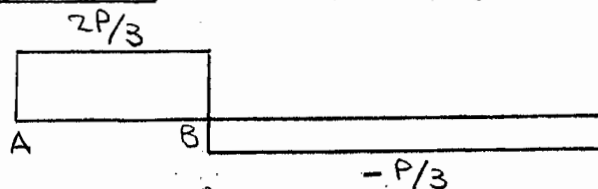
$$S = \frac{2P}{3}$$

$$M = \frac{2P}{3}x$$

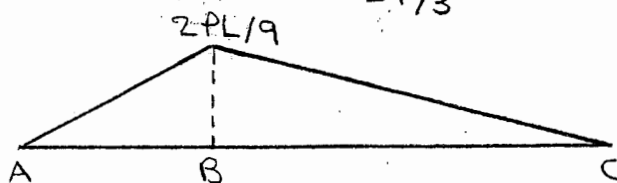
$$\frac{L}{3} < x < L :$$

$$S = \frac{2P}{3} - P = -\frac{P}{3}$$

$$M = \frac{2P}{3}x - P(x - \frac{L}{3}) = \frac{P}{3}(L - x)$$

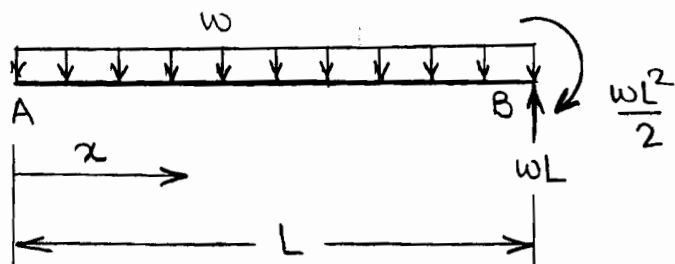


Shear Diagram



Bending Moment Diagram

5.14

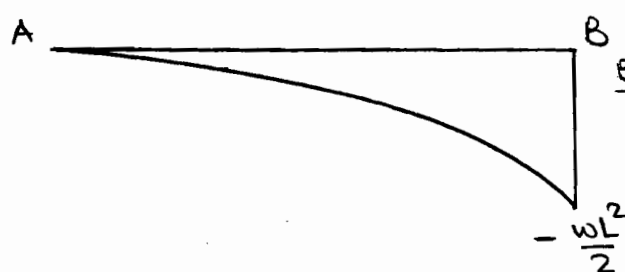


$$S = -wx \quad \text{for } 0 \leq x < L$$

$$M = -\frac{wx^2}{2} \quad \text{for } 0 \leq x < L$$

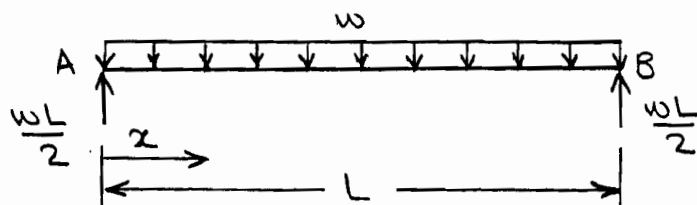


Shear Diagram



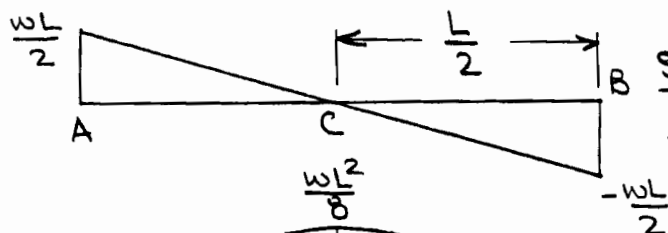
Bending Moment Diagram

5.15

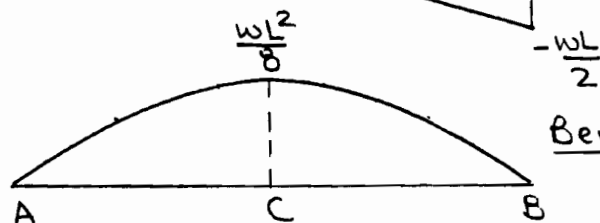


$$S = \frac{wL}{2} - wx = w\left(\frac{L}{2} - x\right) \quad \text{for } 0 < x < L$$

$$M = \frac{wL}{2}x - \frac{wx^2}{2} = \frac{wx}{2}(L - x) \quad \text{for } 0 \leq x \leq L$$

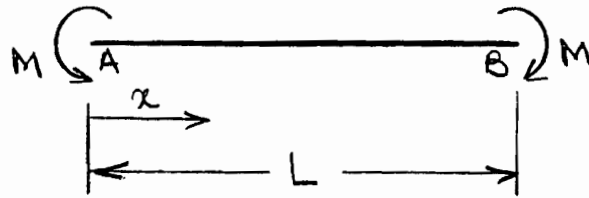


Shear Diagram



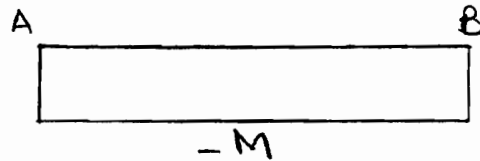
Bending Moment Diagram

5.16



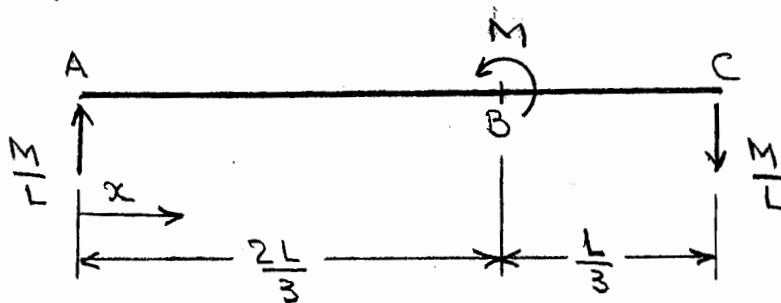
$$\delta = 0$$

$$M_x = -M \text{ for } 0 < x < L$$



Bending Moment Diagram

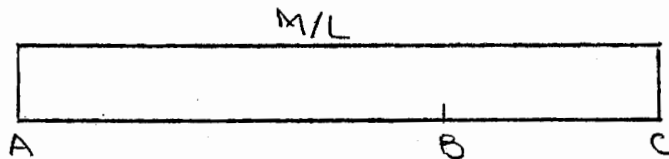
5.17



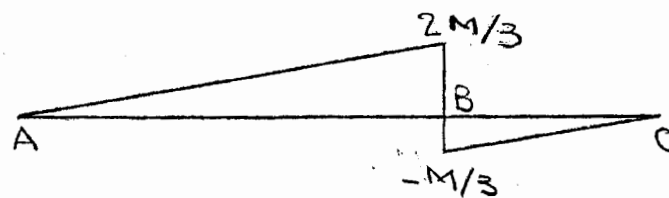
$$S = \frac{M}{L} \text{ for } 0 < x < L$$

$$\text{B.M.} = \frac{Mx}{L} \text{ for } 0 \leq x < \frac{2L}{3}$$

$$\text{B.M.} = \frac{Mx}{L} - M = \frac{M}{L}(x - L) \text{ for } \frac{2L}{3} < x \leq L$$

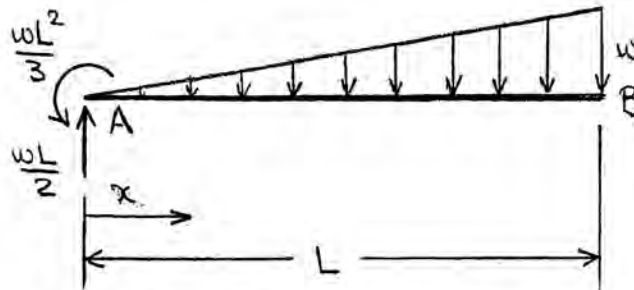


Shear Diagram



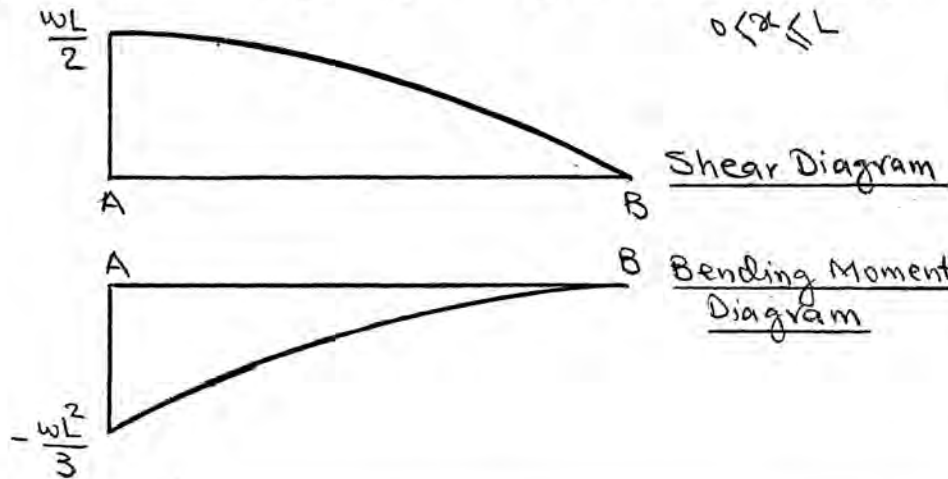
Bending Moment Diagram

5.18

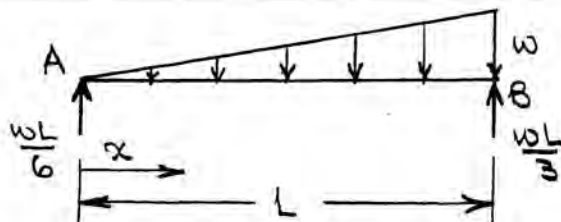


$$S = \frac{wL}{2} - \frac{1}{2}(x)\left(\frac{wx}{L}\right) = \frac{wL}{2} \left[1 - \left(\frac{x}{L}\right)^2\right] \quad 0 < x \leq L$$

$$M = -\frac{wL^2}{3} + \left(\frac{wL}{2}\right)x - \left(\frac{wx^2}{2L}\right)\left(\frac{x}{3}\right) = -\frac{wL^2}{3} \left[1 - \frac{3}{2}\left(\frac{x}{L}\right) + \frac{1}{2}\left(\frac{x}{L}\right)^3\right] \quad 0 < x \leq L$$

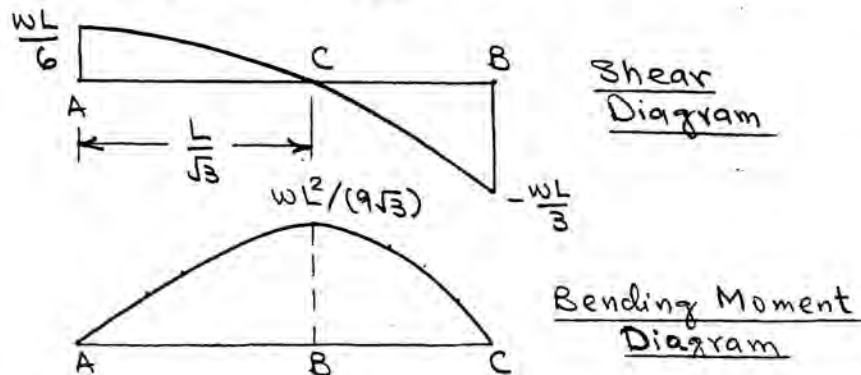


5.19

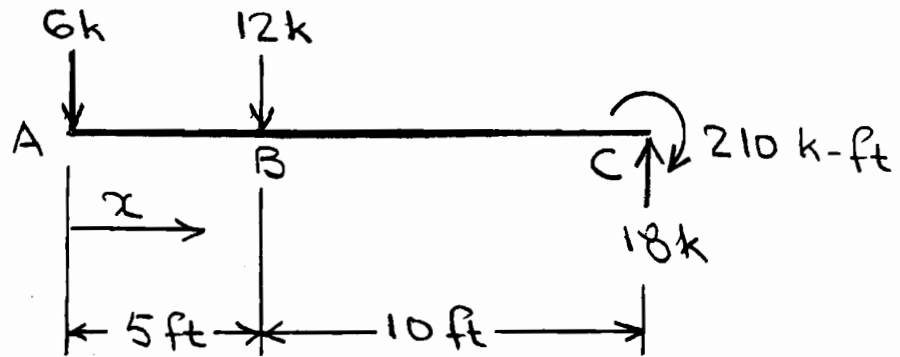


$$S = \frac{wL}{6} - \frac{1}{2}\left(\frac{wx}{L}\right)x = \frac{w}{6L}(L^2 - 3x^2) \quad \text{for } 0 < x < L$$

$$M = \frac{wL}{6}x - \frac{wx^2}{2L}\left(\frac{x}{3}\right) = \frac{wx}{6L}(L^2 - x^2) \quad \text{for } 0 \leq x \leq L$$

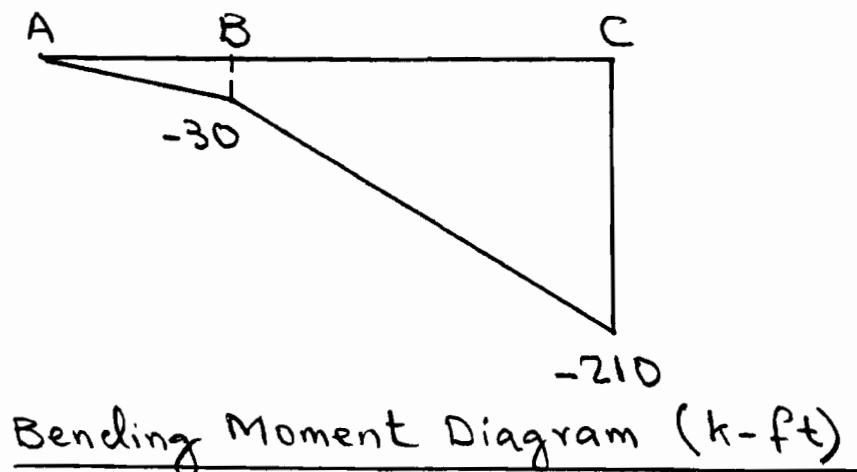
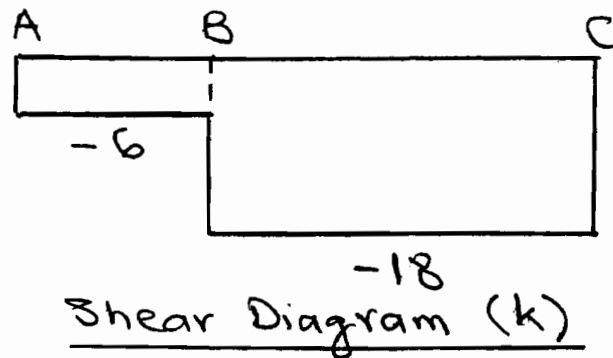


5.20

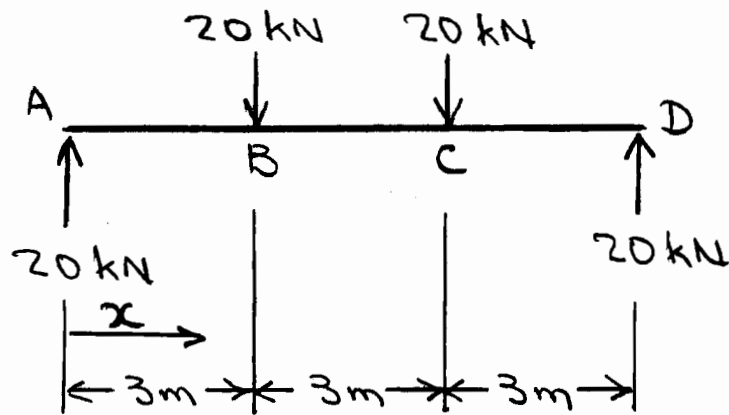


$0 < x < 5'$: $S = -6$
 $M = -6x$

$5' < x < 15'$: $S = -6 - 12 = -18$
 $M = -6x - 12(x - 5) = 60 - 18x$



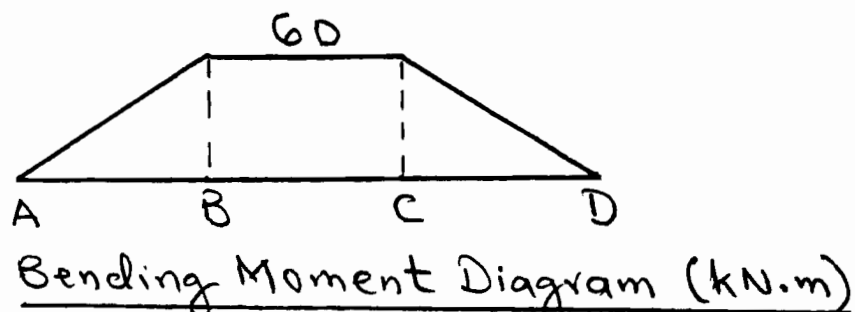
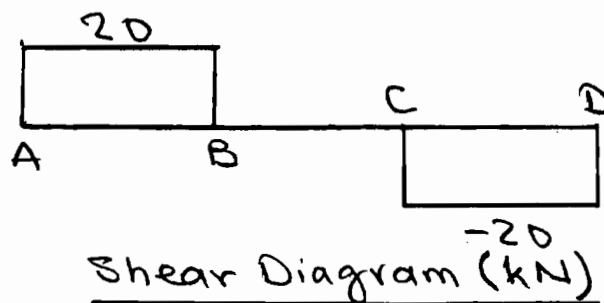
5.21



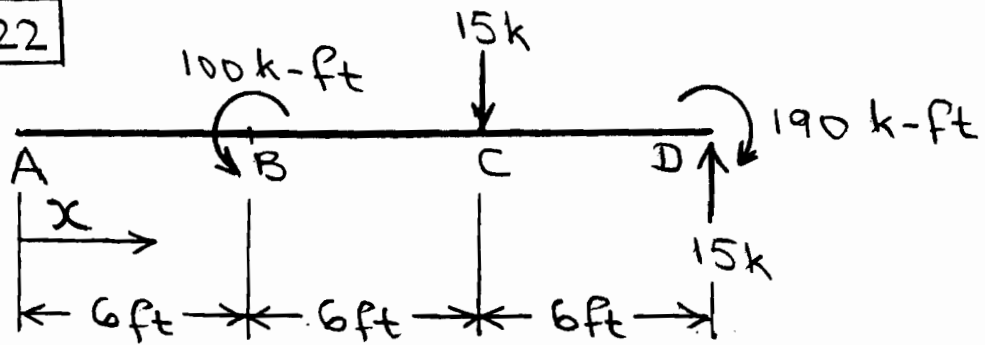
$0 < x < 3\text{m}:$ $S = 20$
 $M = 20x$

$3\text{m} < x < 6\text{m}:$ $S = 20 - 20 = 0$
 $M = 20x - 20(x - 3) = 60$

$6\text{m} < x < 9\text{m}:$ $S = 20 - 20 - 20 = -20$
 $M = 20x - 20(x - 3) - 20(x - 6)$
 $= 180 - 20x$



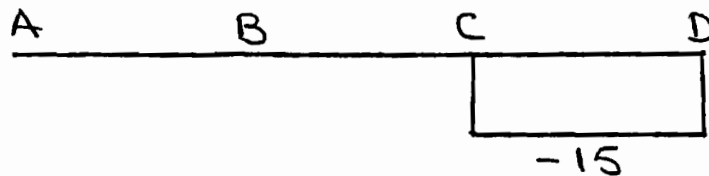
5.22



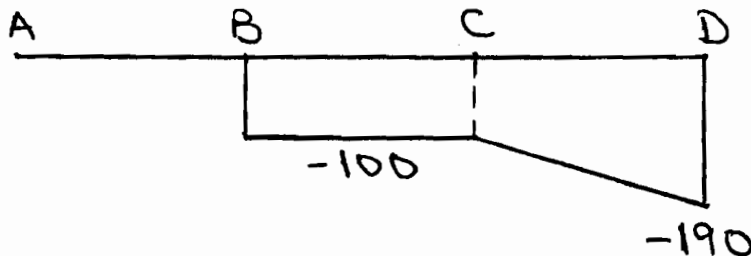
$0 \leq x < 6'$: $S = 0$
 $M = 0$

$6' < x < 12'$: $S = 0$
 $M = -100$

$12' < x < 18'$: $S = -15$
 $M = -100 - 15(x - 12) = 80 - 15x$

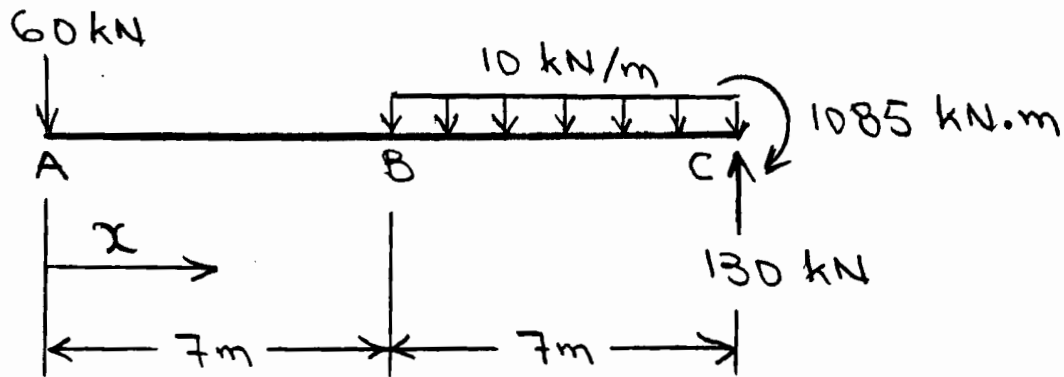


Shear Diagram (k)



Bending Moment Diagram (k-ft)

5.23



$0 < x \leq 7\text{m}$:

$$S = -60$$

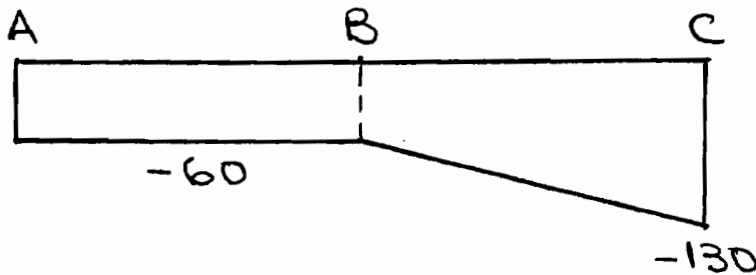
$$M = -60x$$

$7\text{m} \leq x < 14\text{m}$:

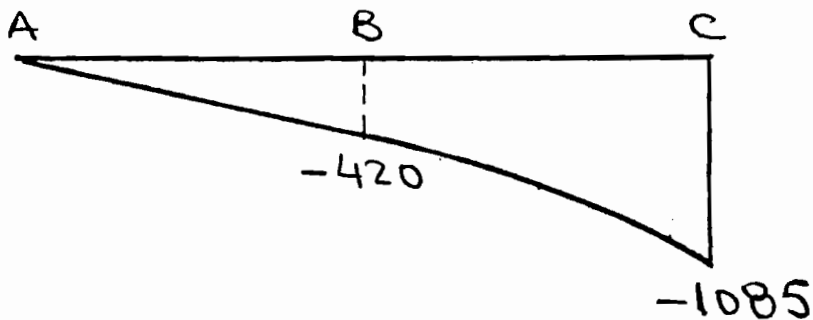
$$S = -60 - 10(x - 7) = 10(1 - x)$$

$$M = -60x - \frac{10(x - 7)^2}{2}$$

$$= -5x^2 + 10x - 245$$

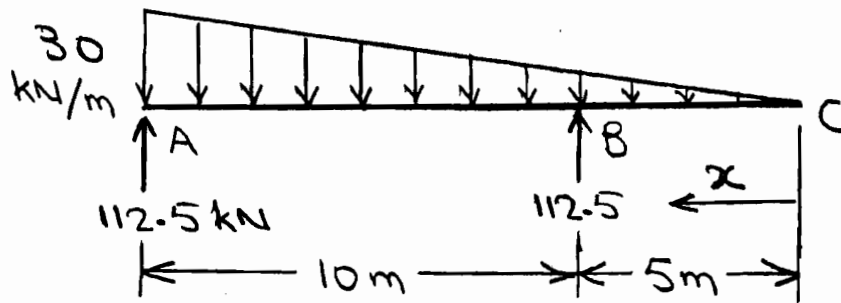


Shear Diagram (kN)



Bending Moment Diagram (kN.m)

5.24



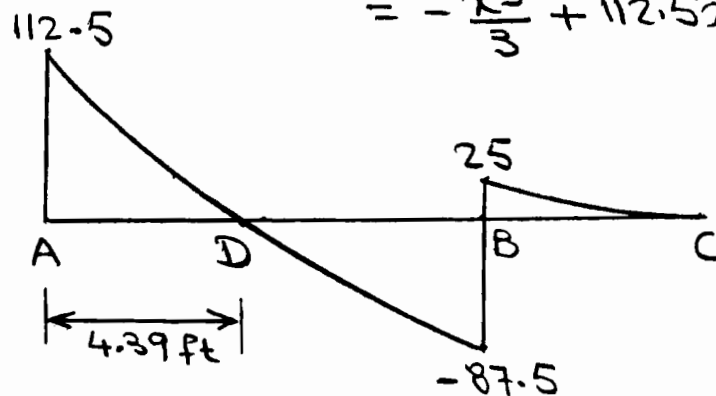
$$0 \leq x < 5\text{m}: \quad S = \frac{1}{2}(2x)x = x^2$$

$$M = -\frac{1}{2}(2x)x\left(\frac{x}{3}\right) = -\frac{x^3}{3}$$

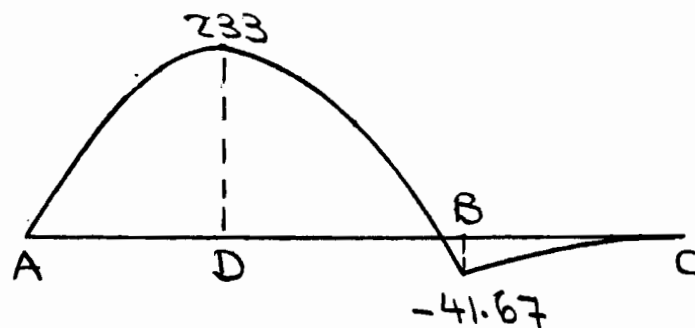
$$5\text{m} < x < 10\text{m}: \quad S = x^2 - 112.5$$

$$M = -\frac{x^3}{3} + 112.5(x - 5)$$

$$= -\frac{x^3}{3} + 112.5x - 562.5$$

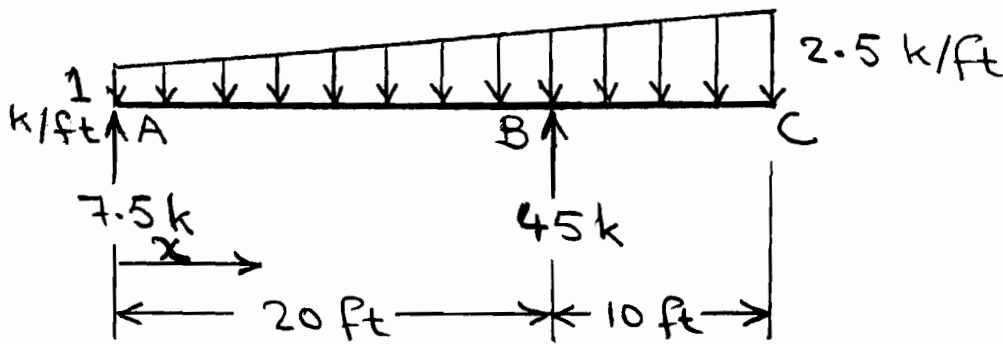


Shear Diagram (kN)



Bending Moment Diagram (kN.m)

5.25



$$\underline{0 < x < 20'}: \quad S = 7.5 - 1(x) - \frac{1}{2} \left(\frac{x}{20} \right) x$$

$$= -\frac{x^2}{40} - x + 7.5$$

$$M = 7.5x - 1 \left(\frac{x^2}{2} \right) - \frac{1}{2} \left(\frac{x}{20} \right) x \left(\frac{x}{3} \right)$$

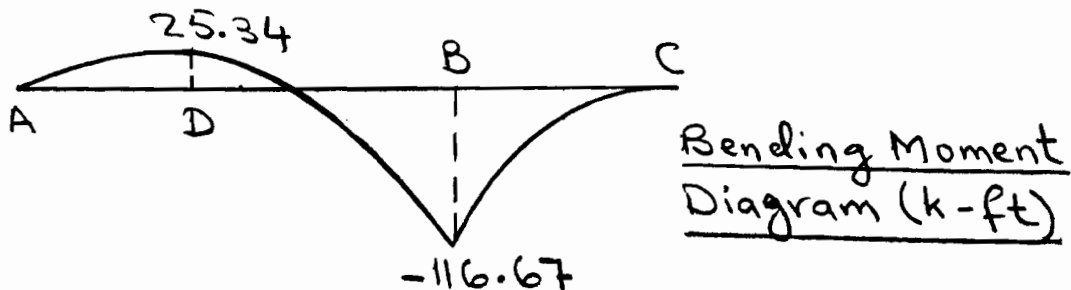
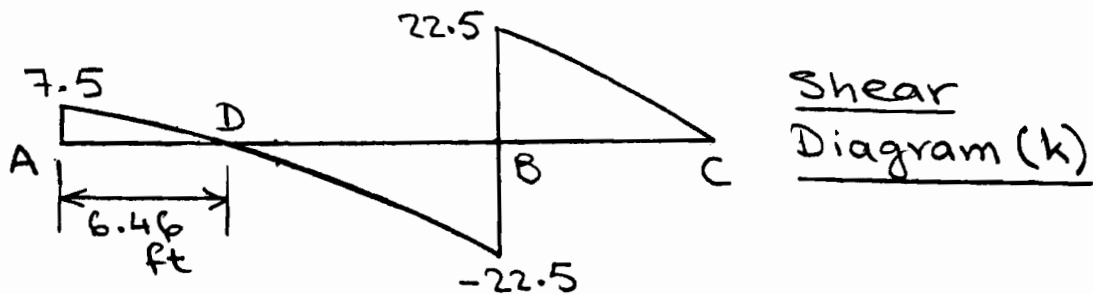
$$= -\frac{x^3}{120} - \frac{x^2}{2} + 7.5x$$

$$\underline{20' < x < 30'}: \quad S = 7.5 - 1(x) - \frac{1}{2} \left(\frac{x}{20} \right) x + 45$$

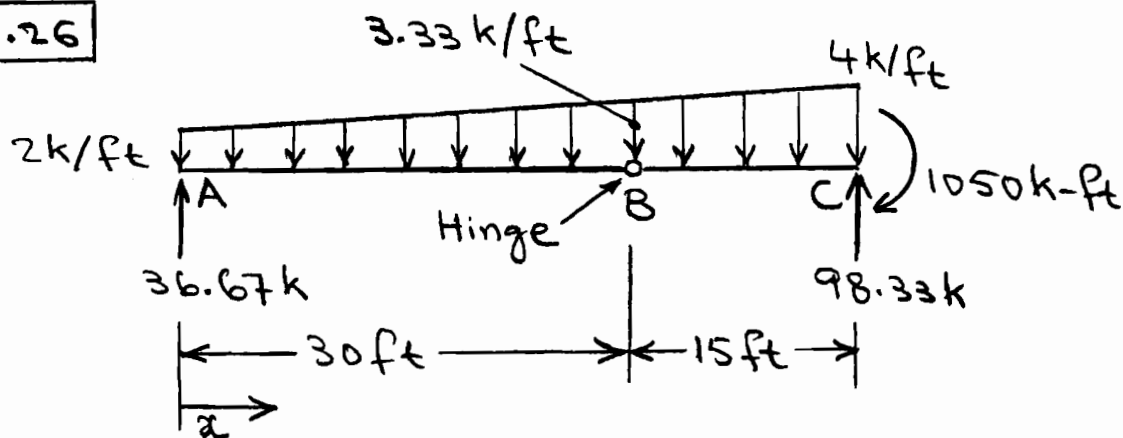
$$= -\frac{x^2}{40} - x + 52.5$$

$$M = 7.5x - 1 \left(\frac{x^2}{2} \right) - \frac{x^3}{120} + 45(x-20)$$

$$= -\frac{x^3}{120} - \frac{x^2}{2} + 52.5x - 900$$



5.26



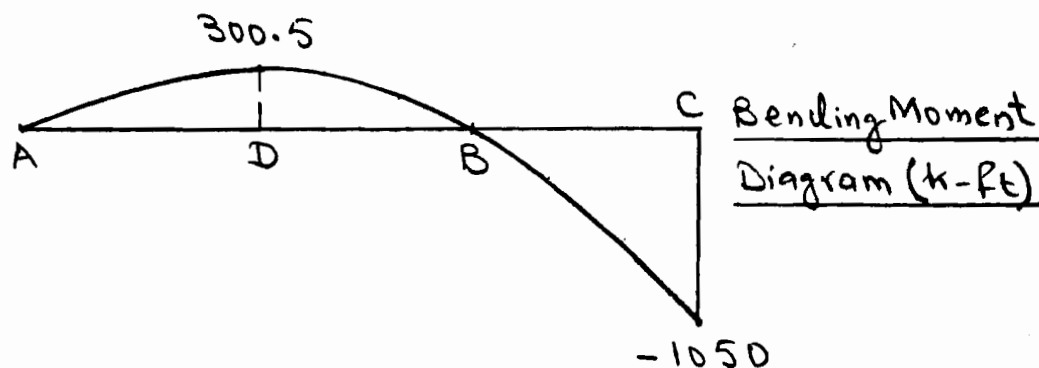
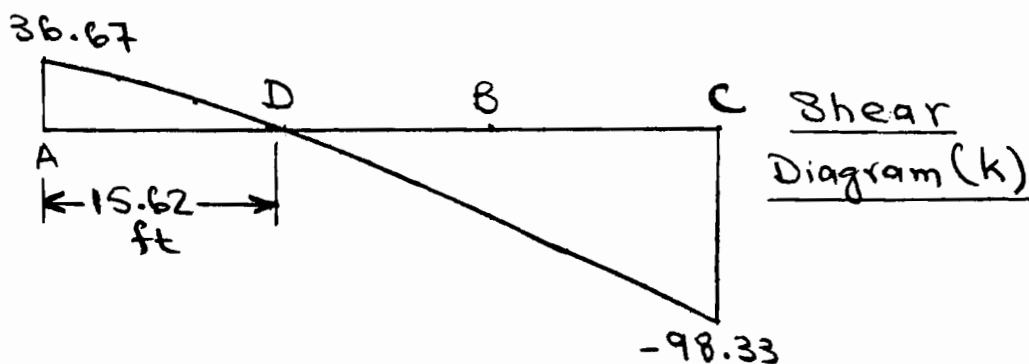
$0 \leq x \leq 45'$:

$$S = 36.67 - 2x - \frac{1}{2} \left(\frac{2}{45} x \right) x$$

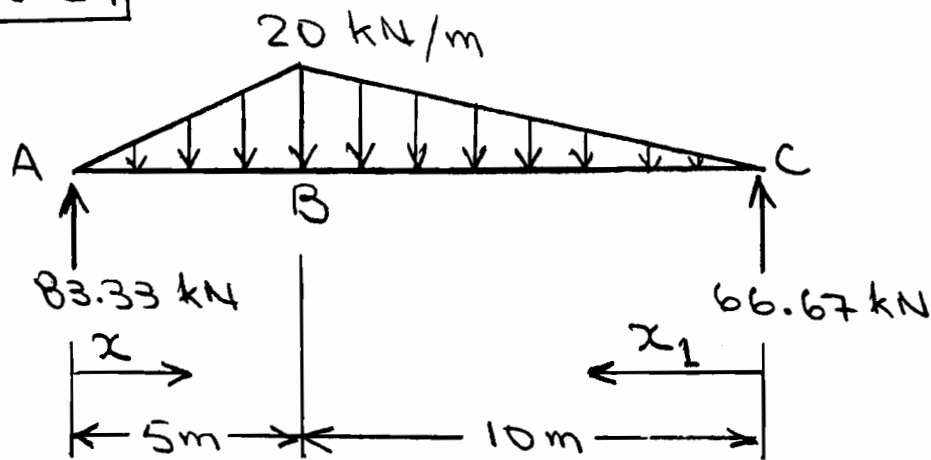
$$= 36.67 - 2x - \frac{x^2}{45}$$

$$M = 36.67x - 2(x) \frac{x}{2} - \frac{1}{2} \left(\frac{2}{45} x \right) x \left(\frac{x}{3} \right)$$

$$= 36.67x - x^2 - \frac{x^3}{135}$$



5.27



$0 < x \leq 5\text{m}$:

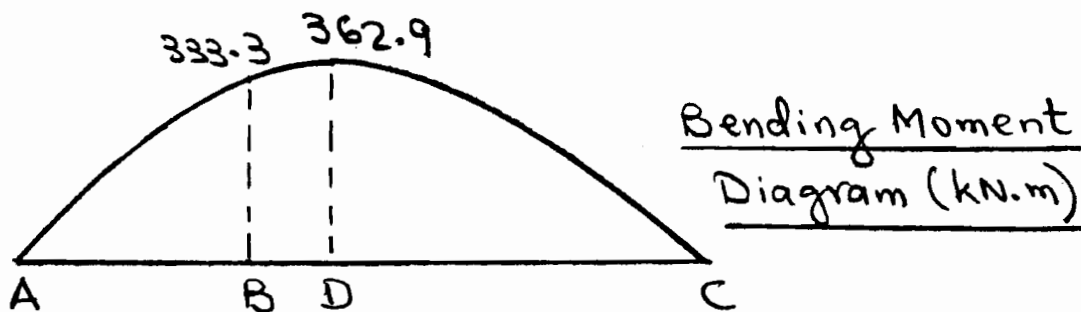
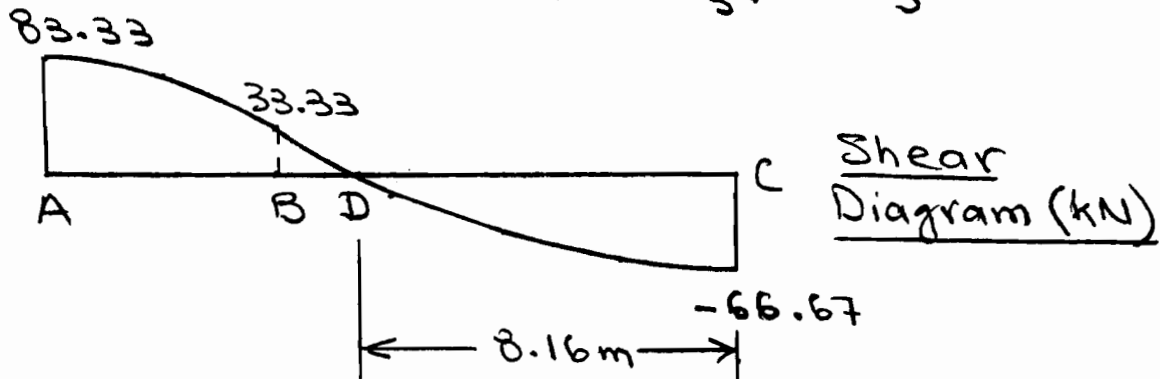
$$S = 83.33 - \frac{1}{2} \left(\frac{20x}{5} \right) x = -2x^2 + 83.33$$

$$M = 83.33x - \frac{1}{2} \left(\frac{20x}{5} \right) x \left(\frac{x}{3} \right) = -\frac{2x^3}{3} + 83.33x$$

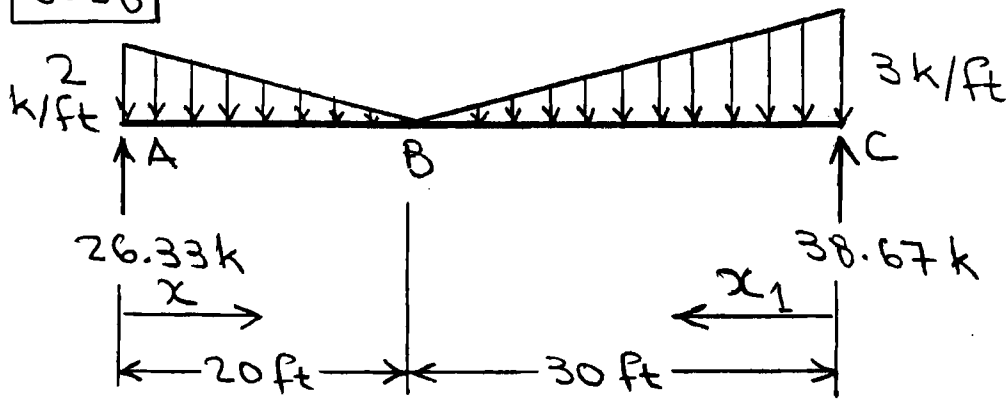
$0 < x_1 \leq 10\text{m}$:

$$S = -66.67 + \frac{1}{2} (2x_1) x_1 = x_1^2 - 66.67$$

$$M = 66.67x_1 - \frac{1}{2} (2x_1) x_1 \left(\frac{x_1}{3} \right) = -\frac{x_1^3}{3} + 66.67x_1$$



5.28



$0 < x \leq 20'$:

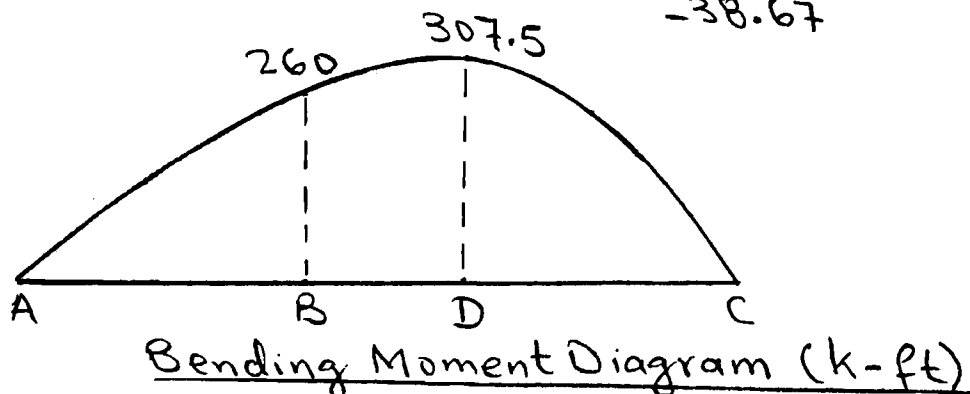
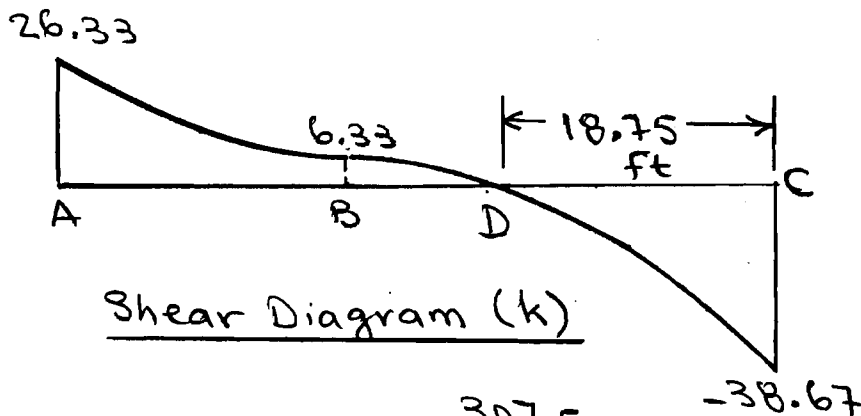
$$S = 26.33 - 2x + \frac{1}{2} \left(\frac{x}{10} \right) x = \frac{x^2}{20} - 2x + 26.33$$

$$M = 26.33x - x^2 + \frac{1}{2} \left(\frac{x}{10} \right) x \left(\frac{x}{3} \right) = \frac{x^3}{60} - x^2 + 26.33x$$

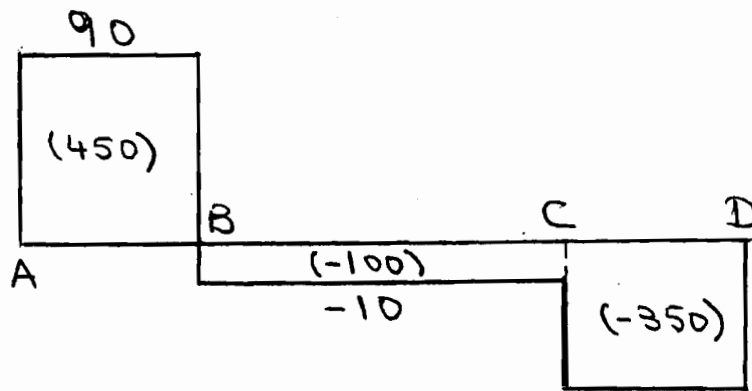
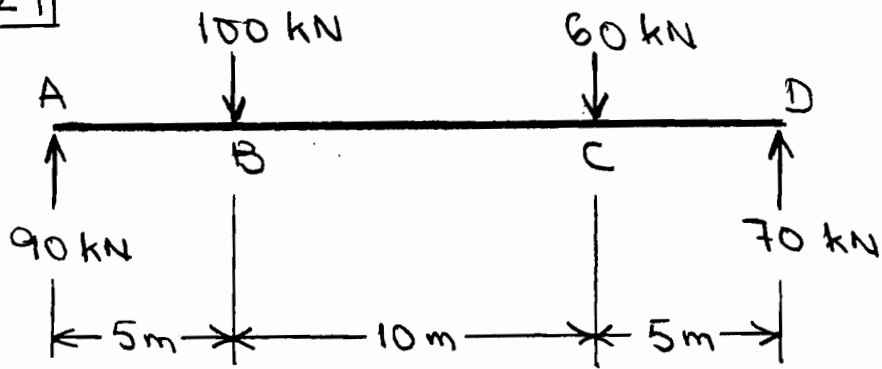
$0 < x_1 \leq 30'$:

$$S = -38.67 + 3x_1 - \frac{1}{2} \left(\frac{x_1}{10} \right) x_1 = -\frac{x_1^2}{20} + 3x_1 - 38.67$$

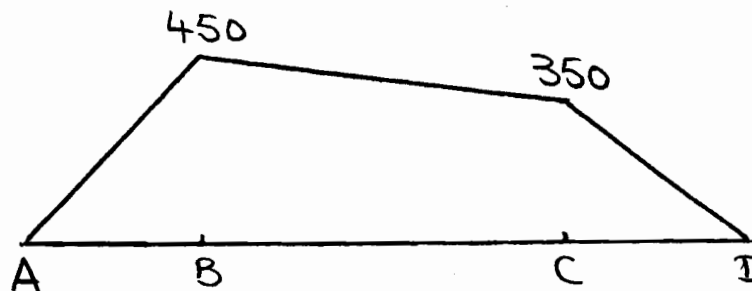
$$M = 38.67x_1 - 3 \frac{x_1^2}{2} + \frac{1}{2} \left(\frac{x_1}{10} \right) x_1 \left(\frac{x_1}{3} \right) = \frac{x_1^3}{60} - \frac{3x_1^2}{2} + 38.67x_1$$



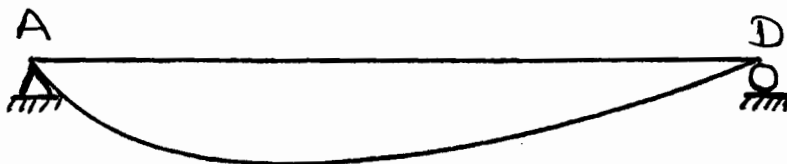
5.29



Shear Diagram (kN)

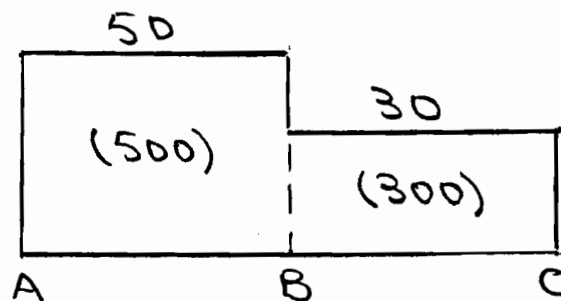
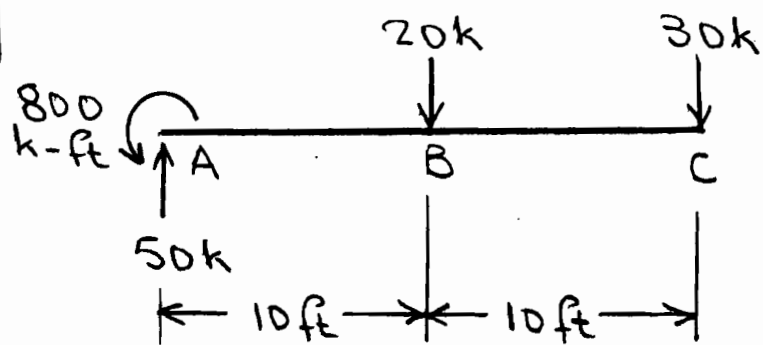


Bending Moment Diagram (kN.m)

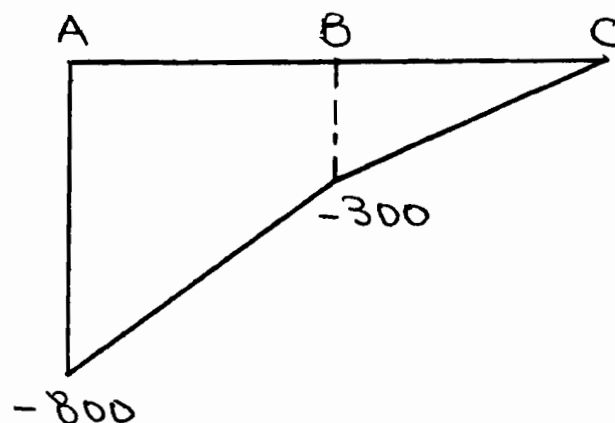


Qualitative Deflected Shape

5.30



Shear Diagram (k)

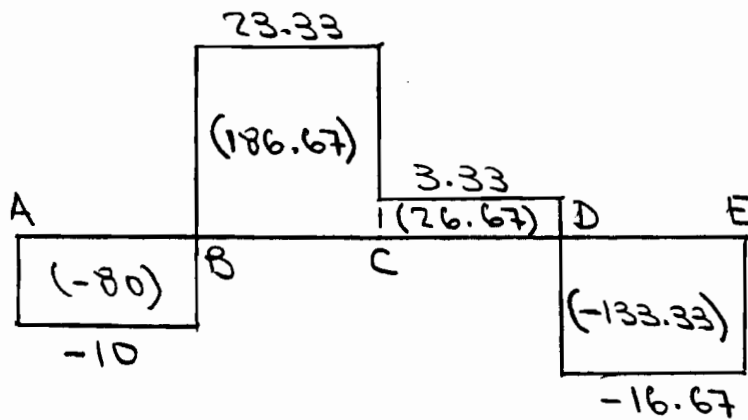
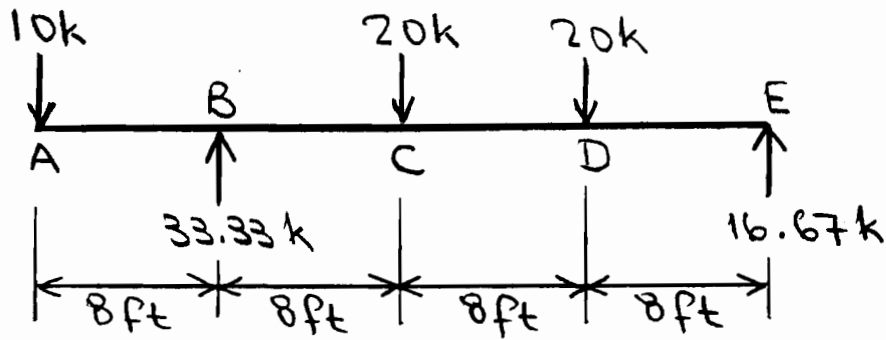


Bending Moment Diagram (k-ft)

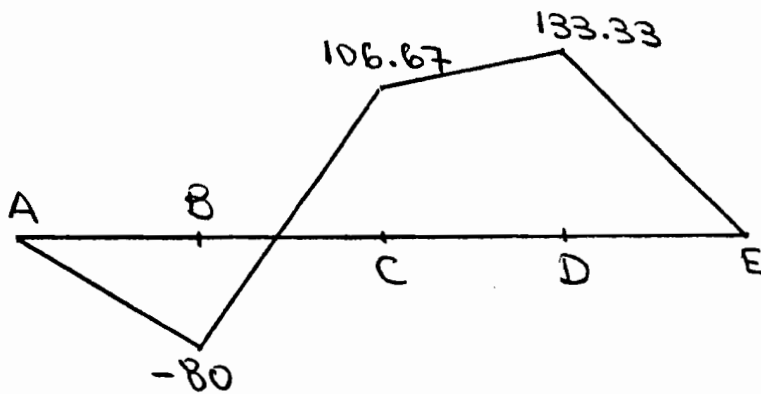


Qualitative Deflected Shape

5.31



Shear Diagram (k)

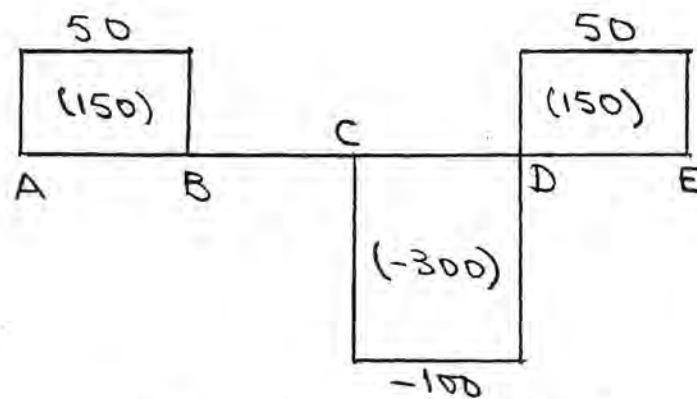
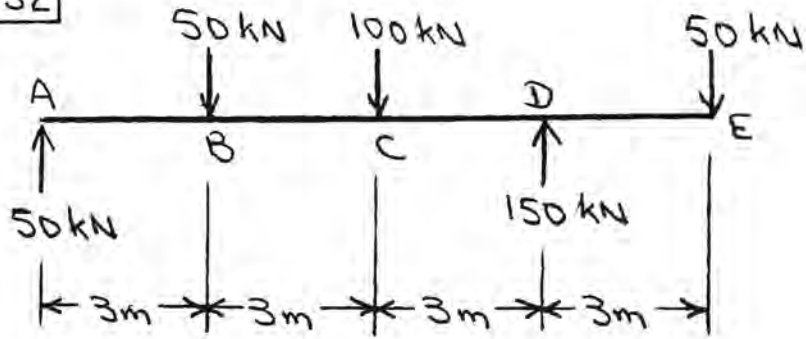


Bending Moment Diagram (k-ft)

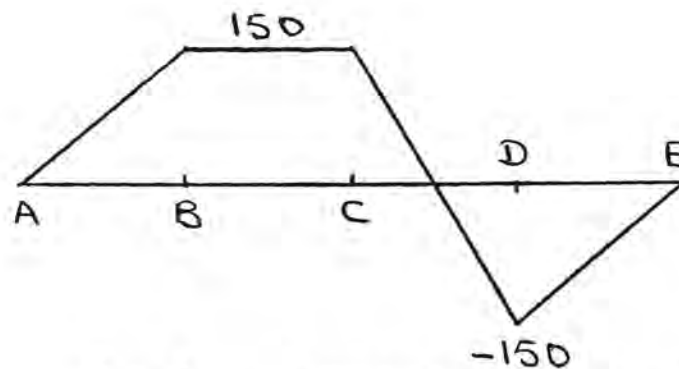


Qualitative Deflected Shape

5.32



Shear Diagram (kN)

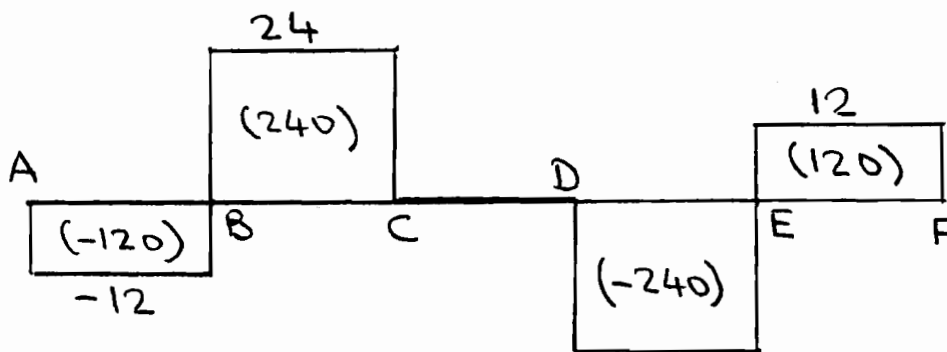
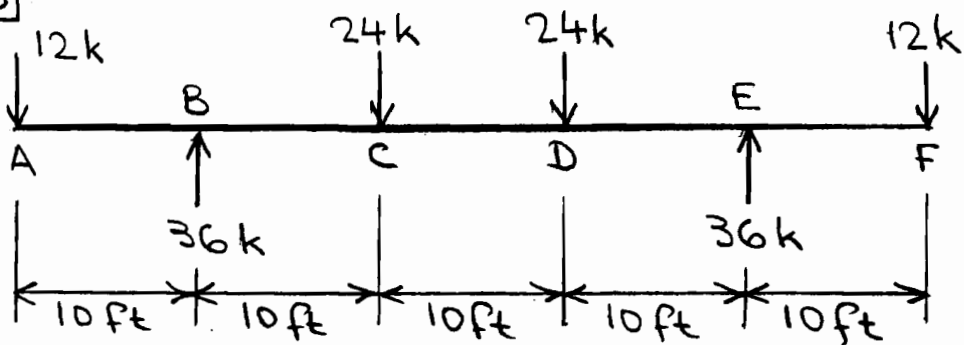


Bending Moment Diagram (kN.m)

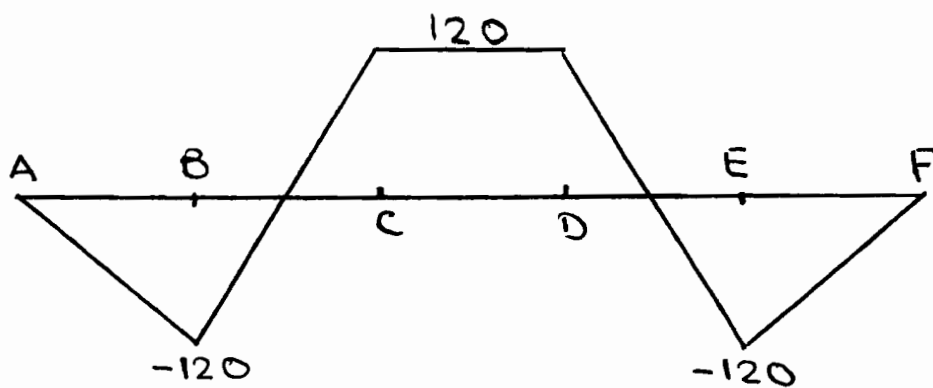


Qualitative Deflected Shape

5.33



Shear Diagram (k)

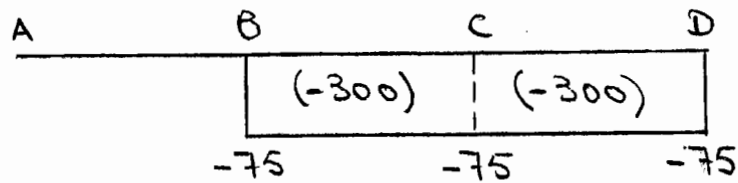
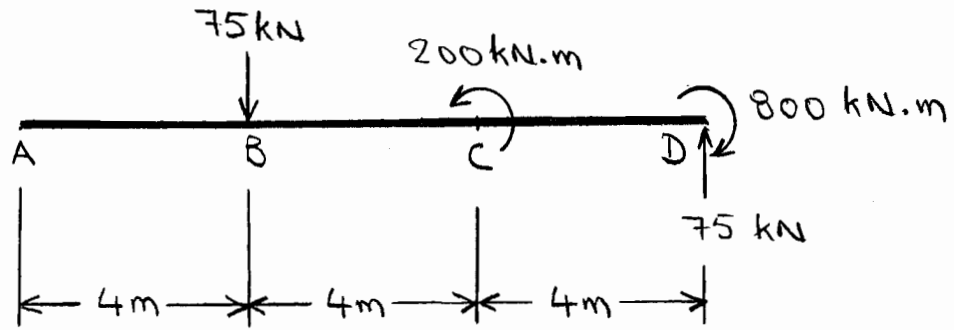


Bending Moment Diagram (k-ft)

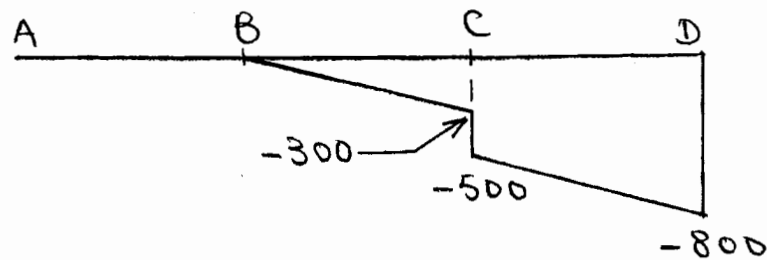


Qualitative Deflected Shape

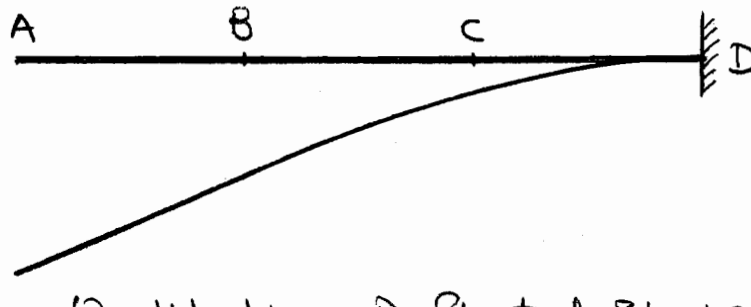
5.34



Shear Diagram (kN)

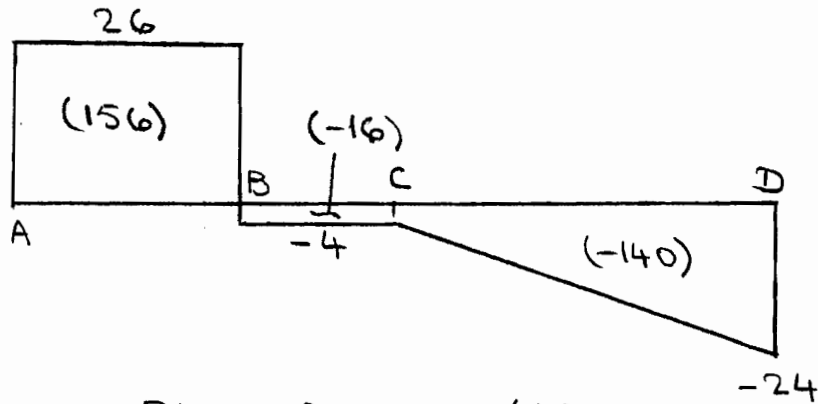
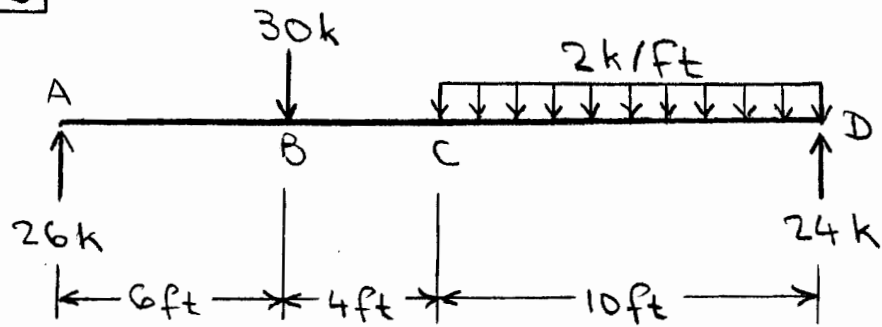


Bending Moment Diagram (kN.m)

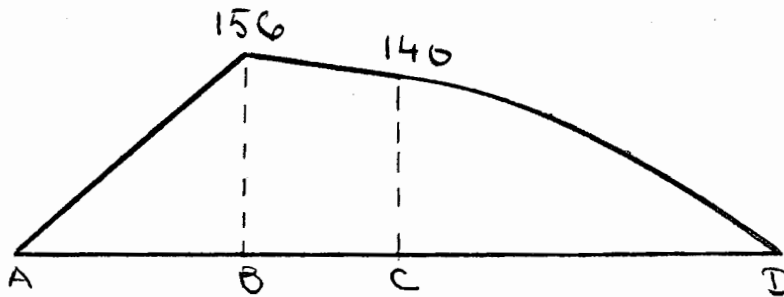


Qualitative Deflected Shape

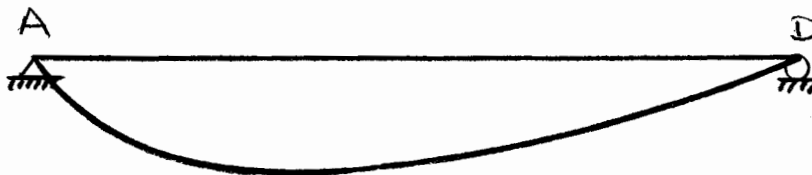
5.35



Shear Diagram (k)

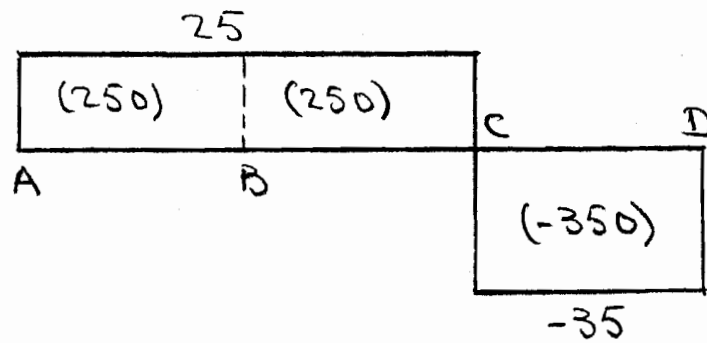
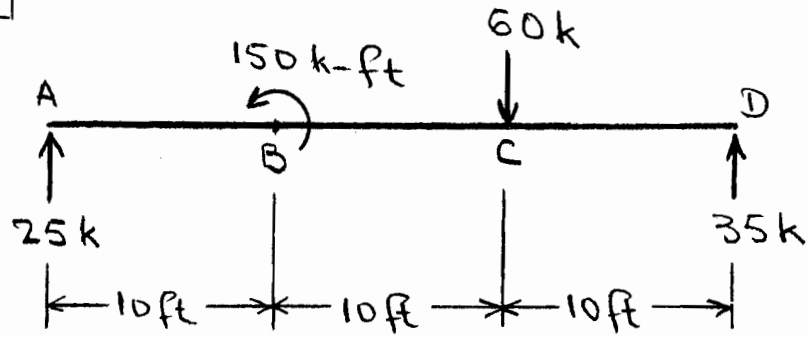


Bending Moment Diagram (k-ft)

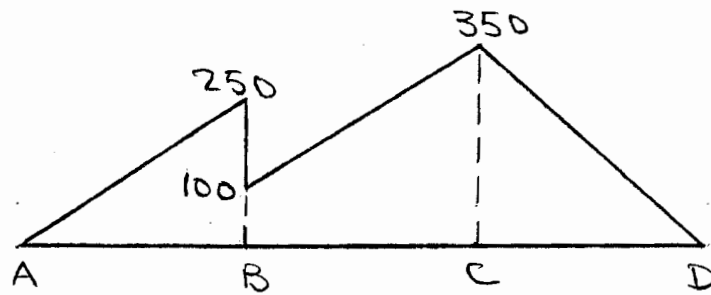


Qualitative Deflected Shape

5.36



Shear Diagram (k)

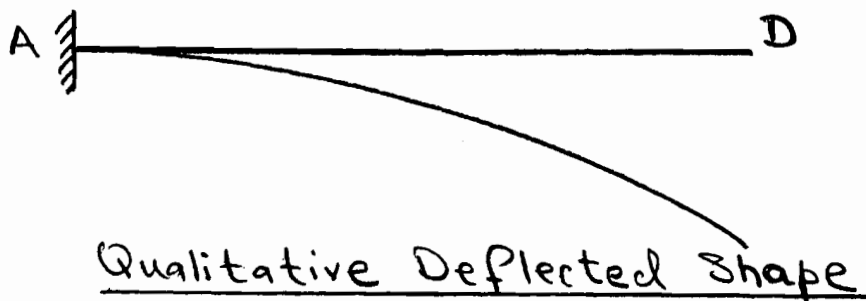
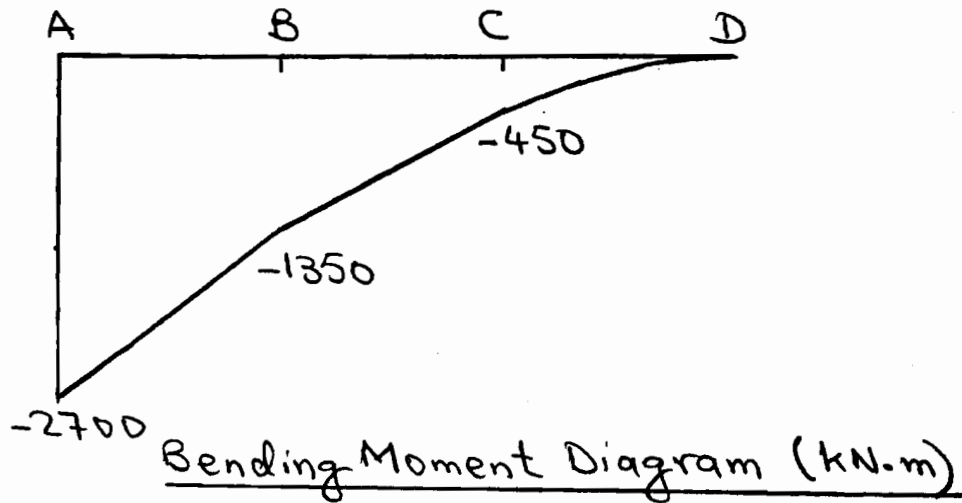
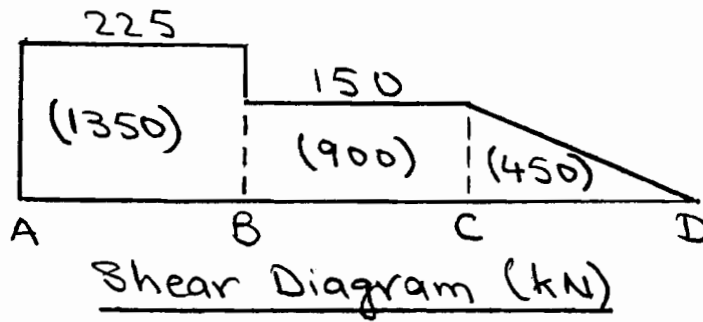
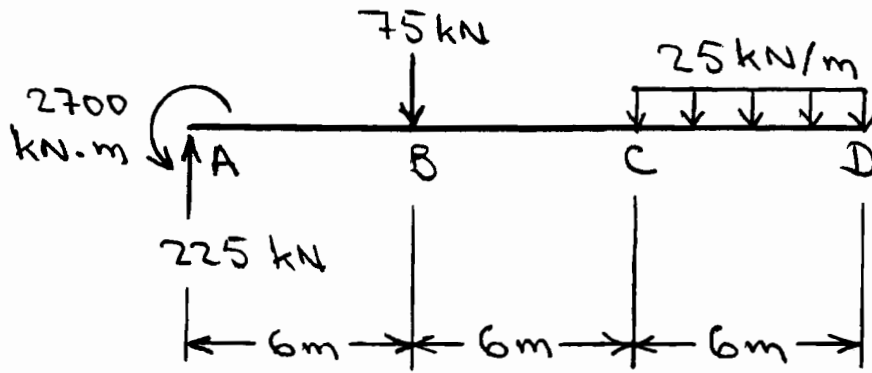


Bending Moment Diagram (k-ft)

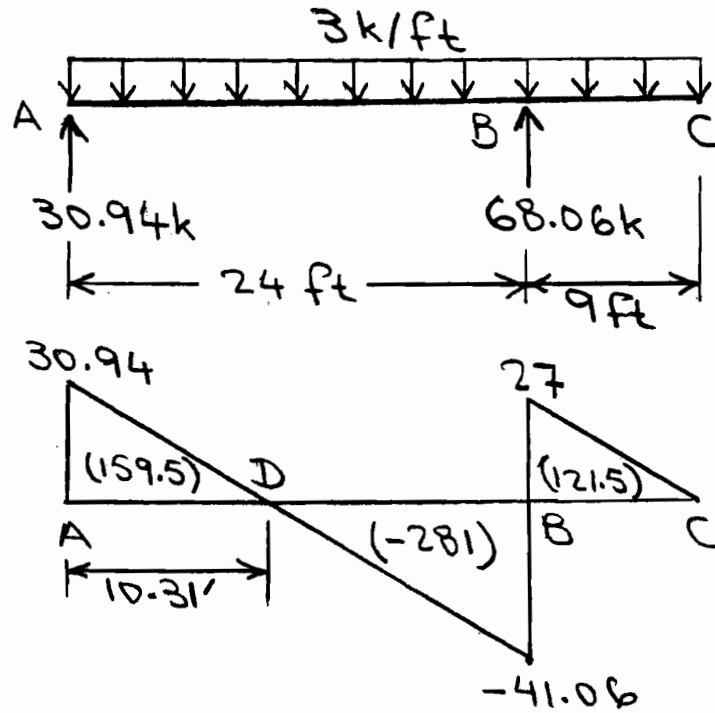


Qualitative Deflected Shape

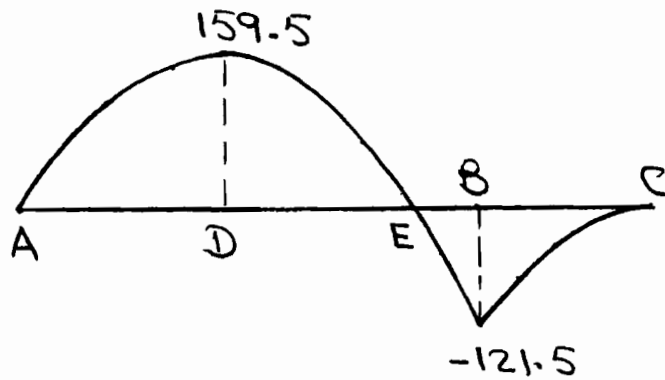
5.37



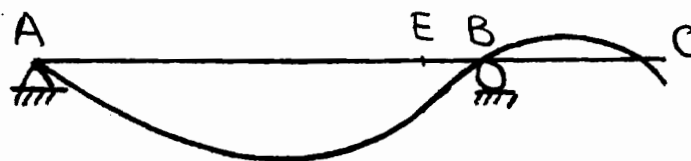
5.38



Shear Diagram (k)

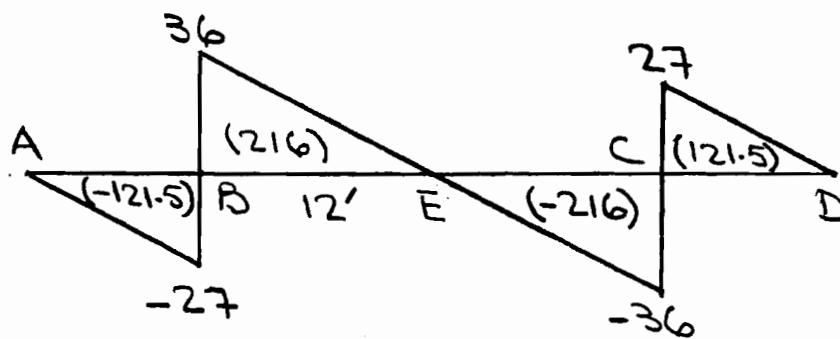
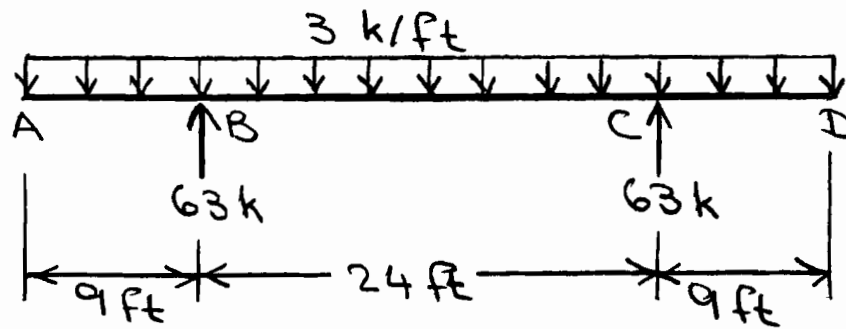


Bending Moment Diagram (k-ft)

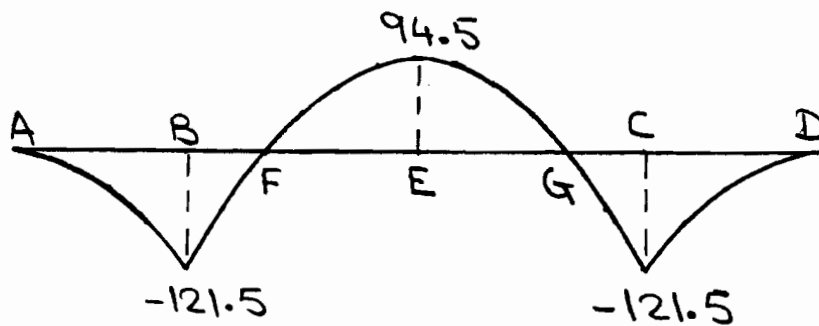


Qualitative Deflected Shape

5.39



Shear Diagram (k)

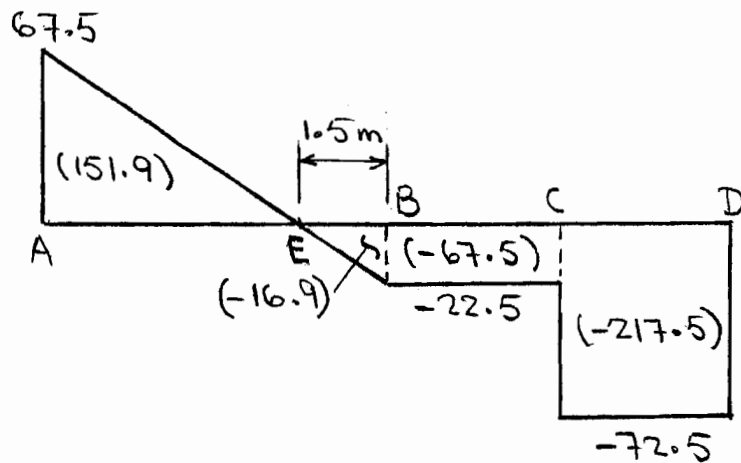
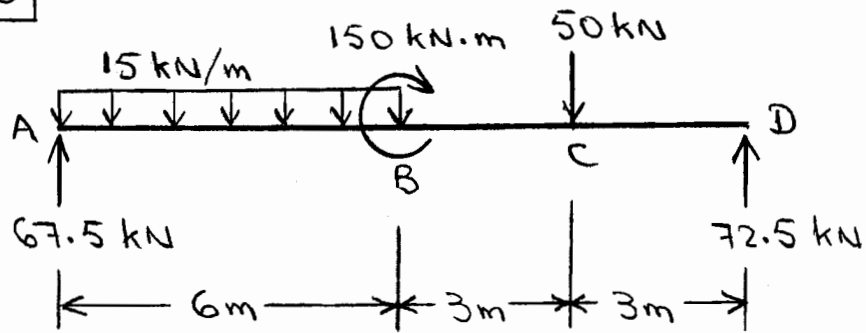


Bending Moment Diagram (k-ft)

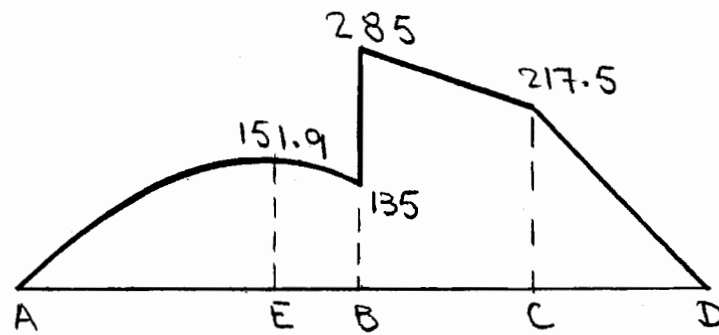


Qualitative Deflected Shape

5.40



Shear Diagram (kN)

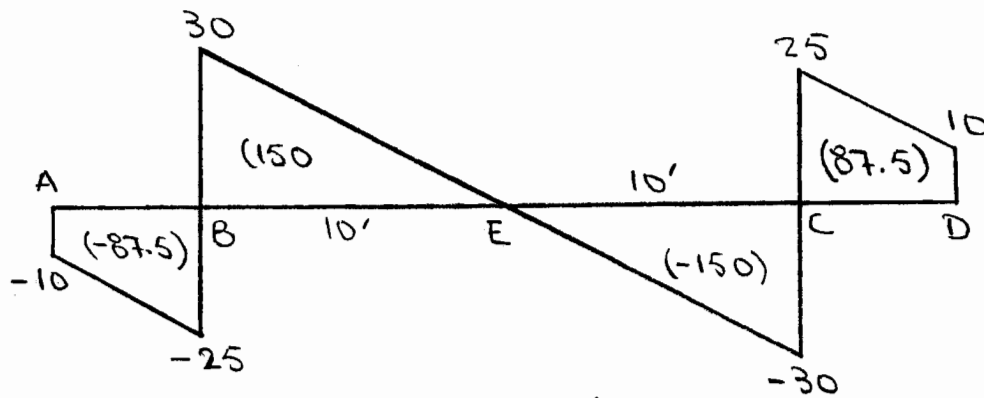
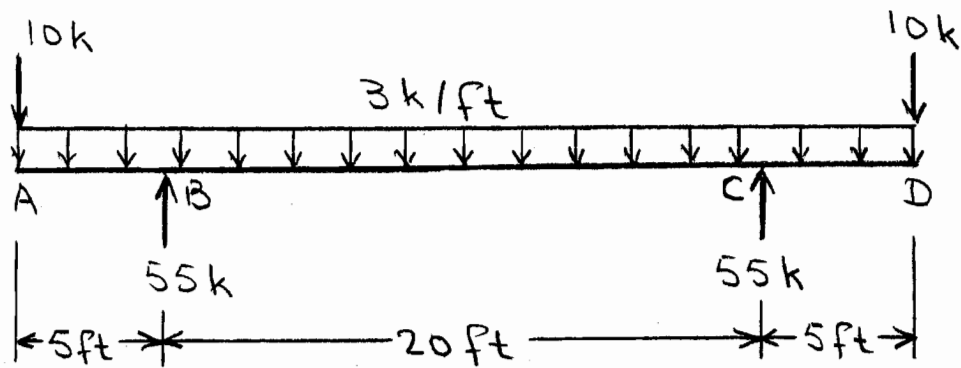


Bending Moment Diagram (kN.m)

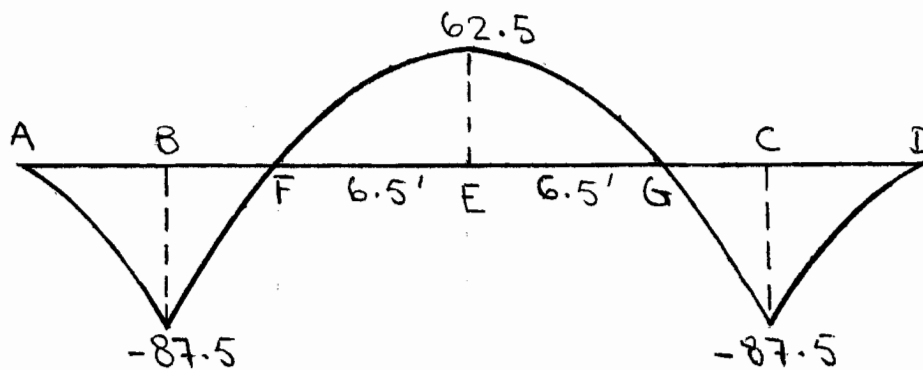


Qualitative Deflected Shape

5.41



Shear Diagram (k)

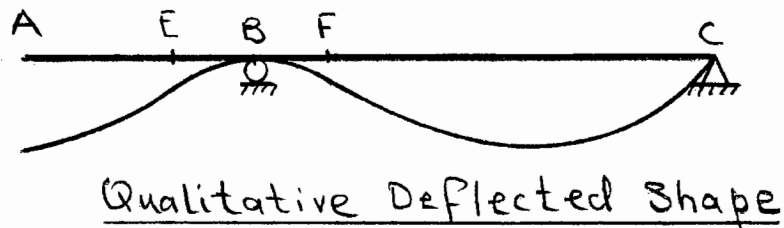
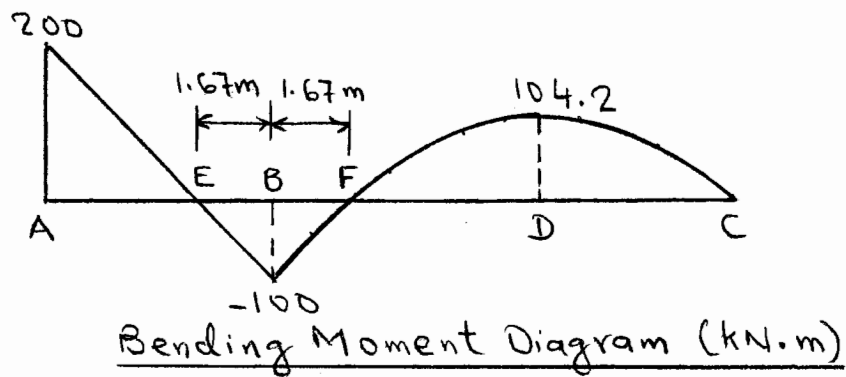
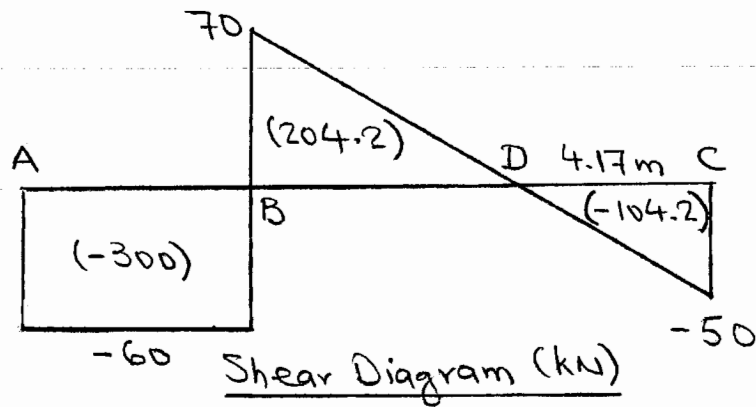
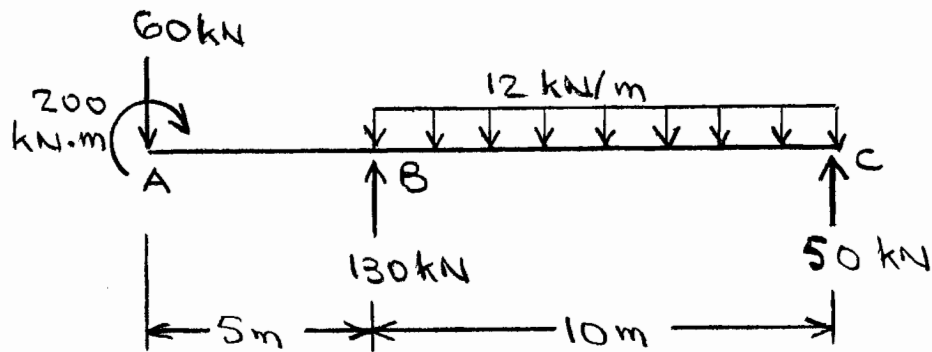


Bending Moment Diagram (k-ft)

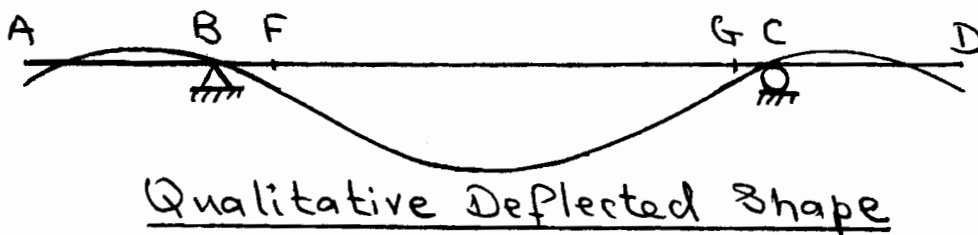
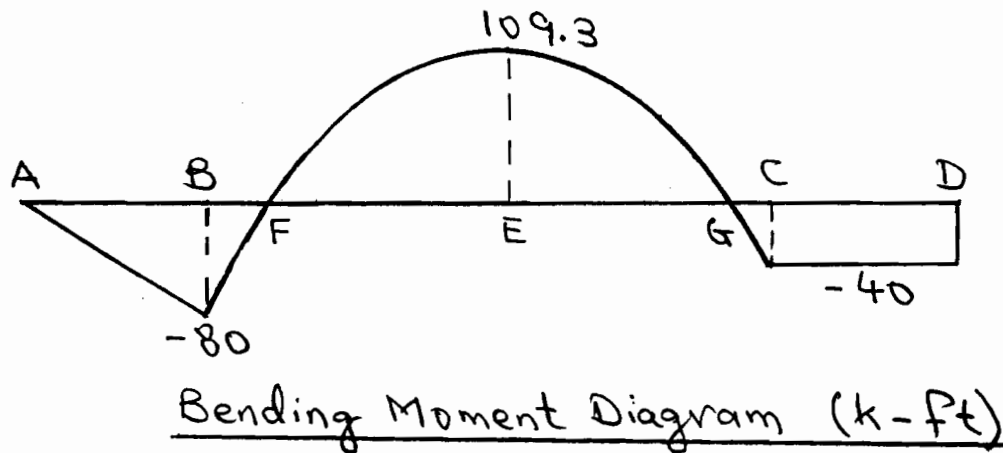
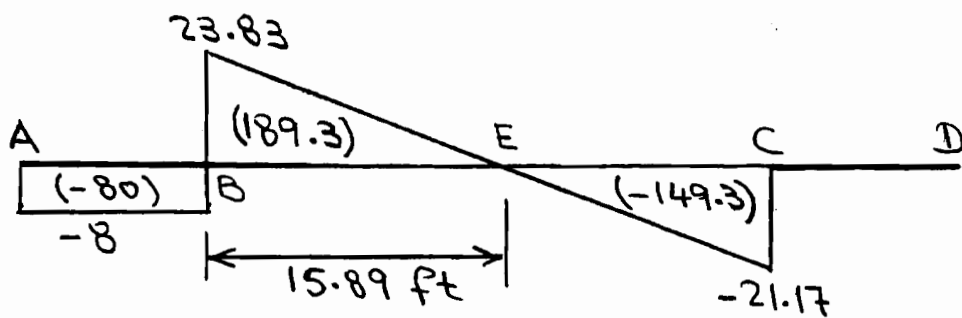
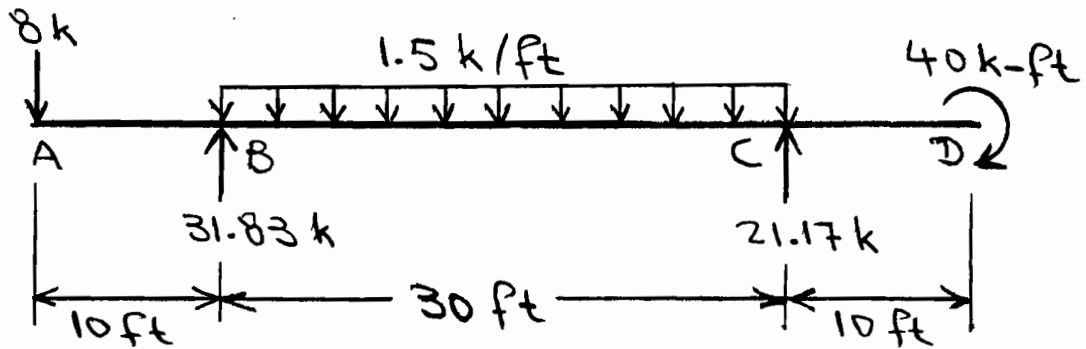


Qualitative Deflected Shape

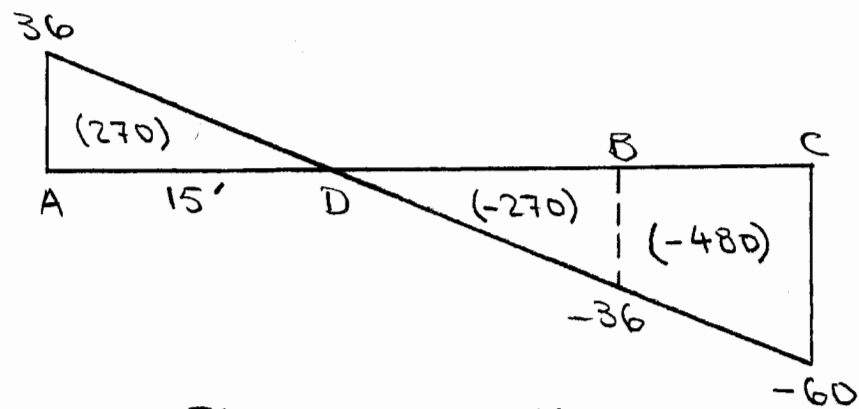
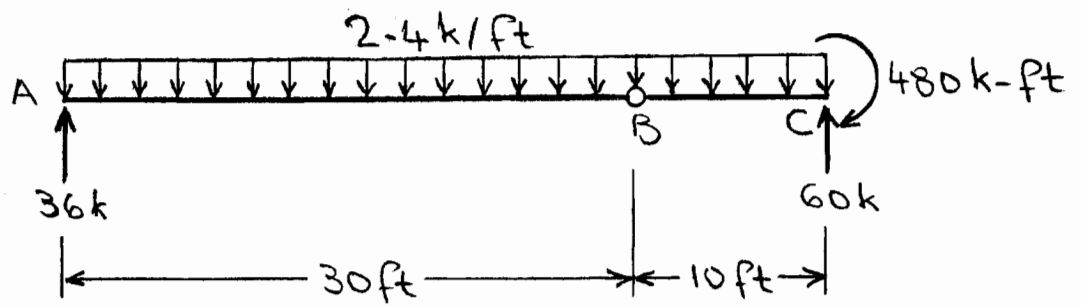
5.42



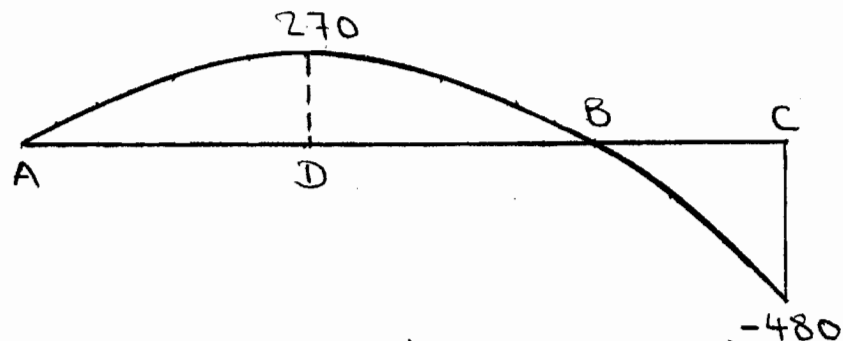
5.43



5.44



Shear Diagram (k)

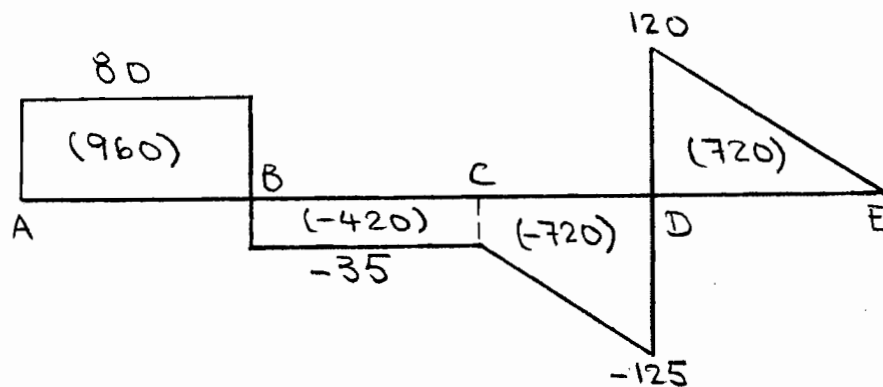
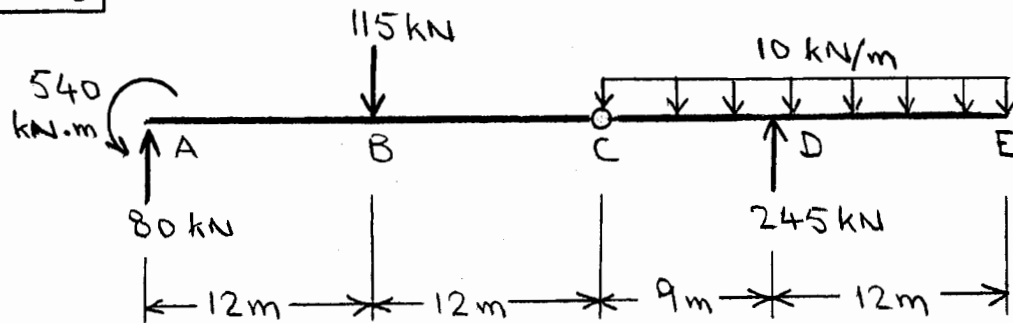


Bending Moment Diagram (k-ft)

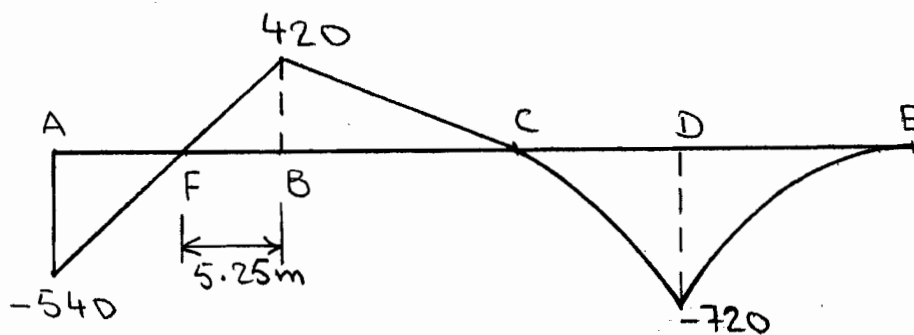


Qualitative Deflected Shape

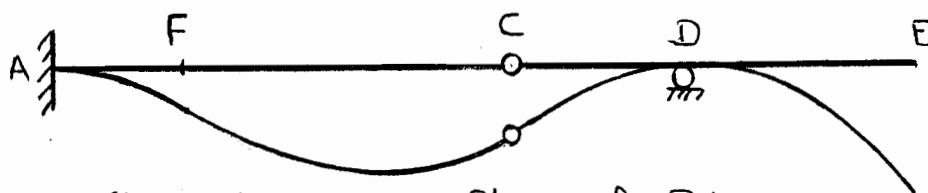
5.45



Shear Diagram (kN)

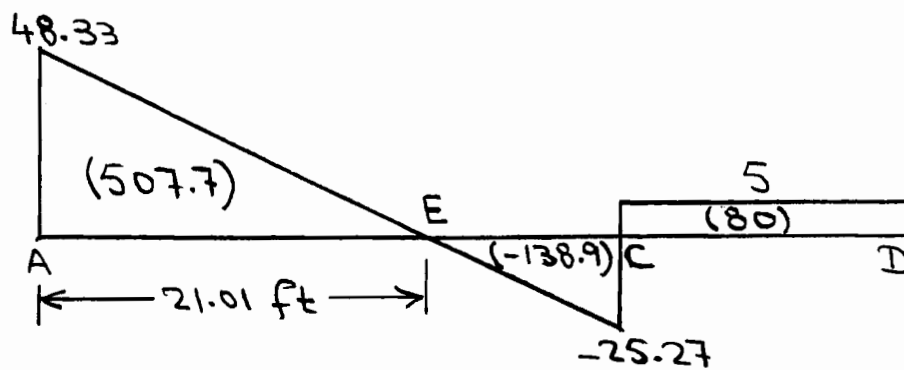
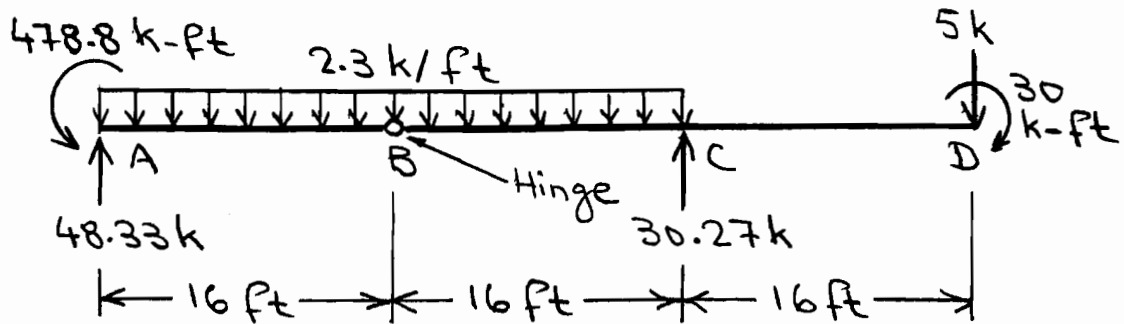


Bending Moment Diagram (kN.m)

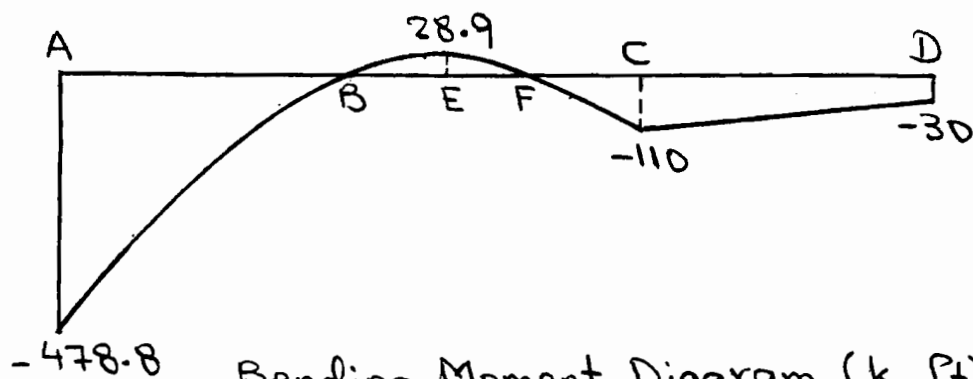


Qualitative Deflected Shape

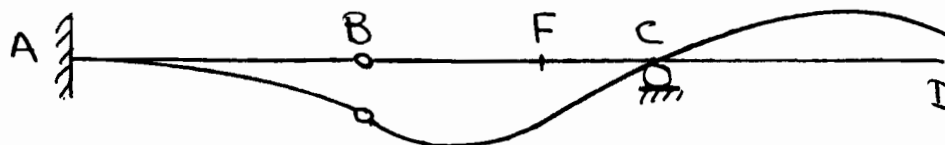
5.46



Shear Diagram (k)

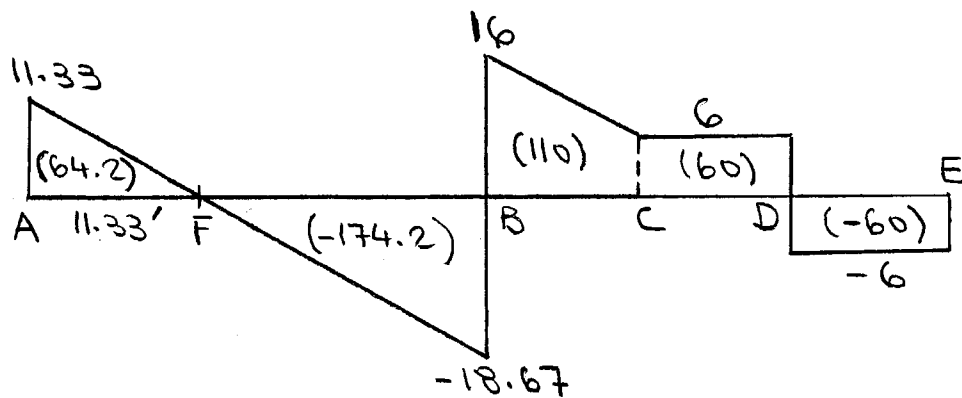
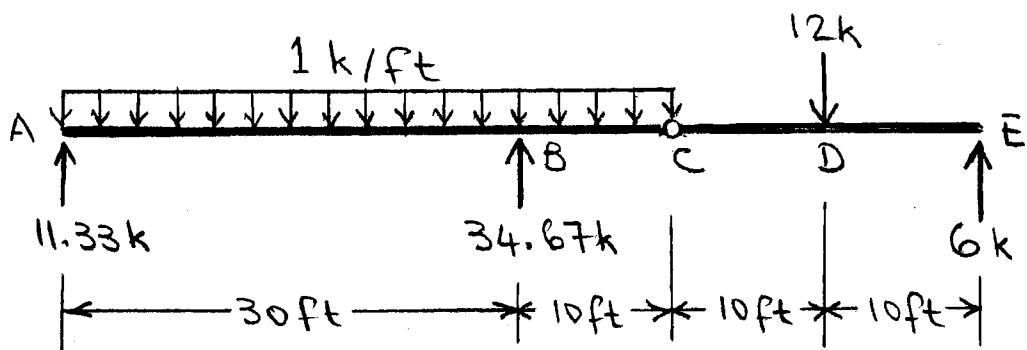


Bending Moment Diagram (k-ft)

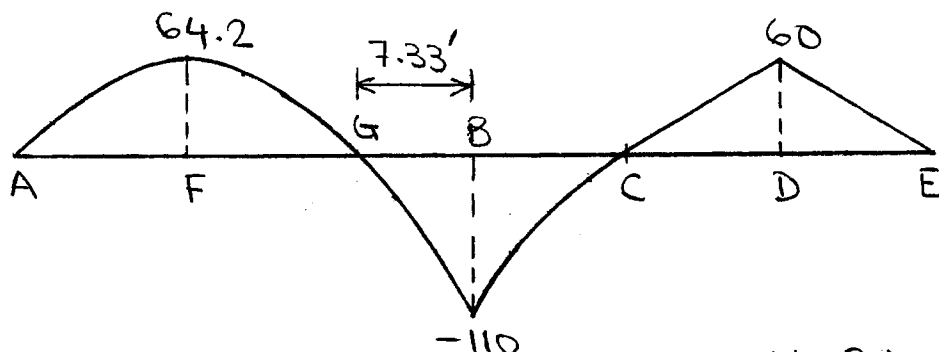


Qualitative Deflected Shape

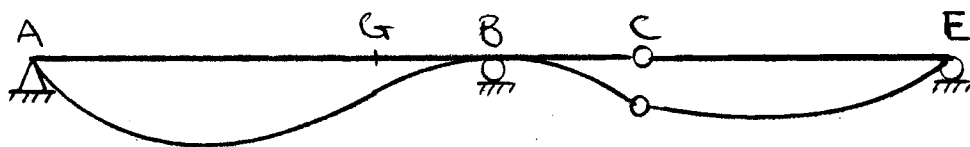
5.47



Shear Diagram (k)

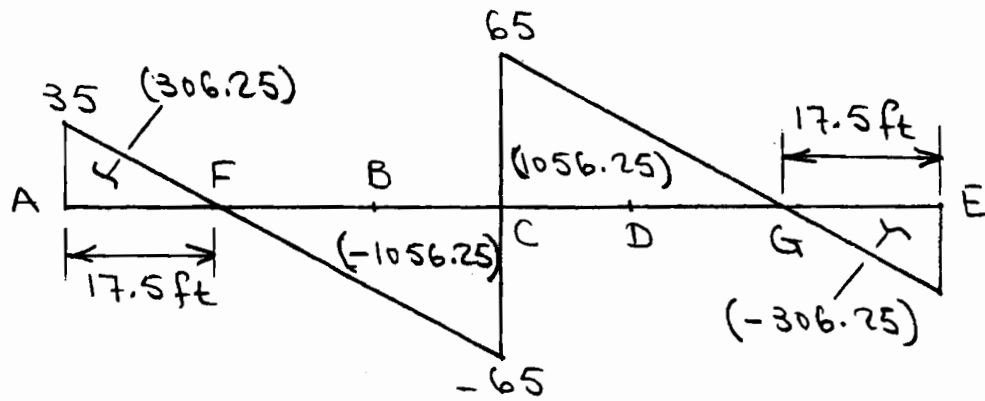
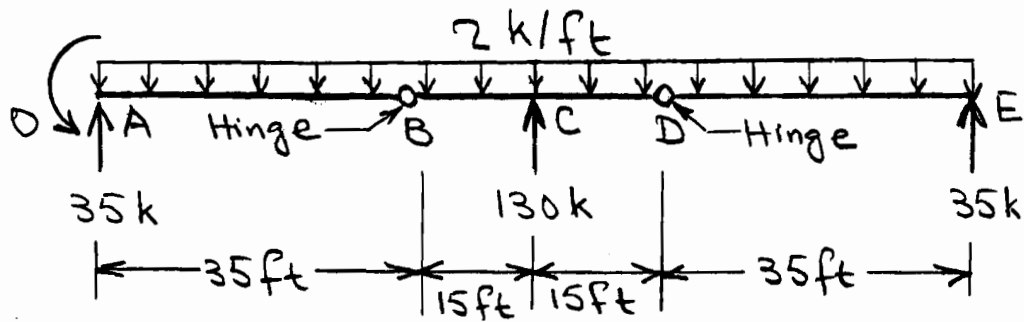


Bending Moment Diagram (k-ft)

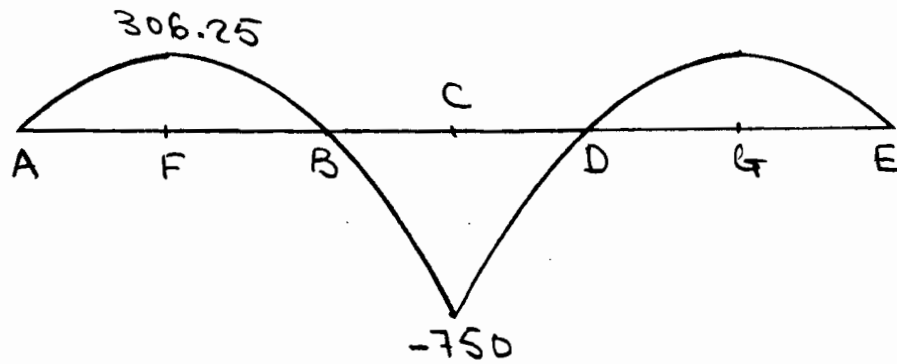


Qualitative Deflected Shape

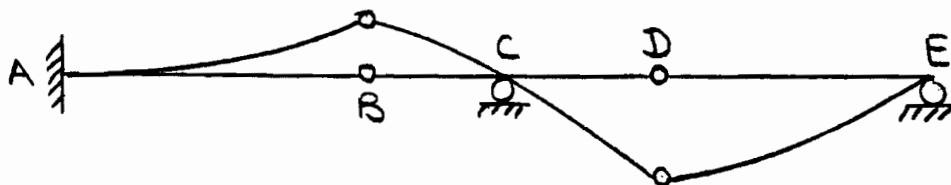
5.48



Shear Diagram (k)

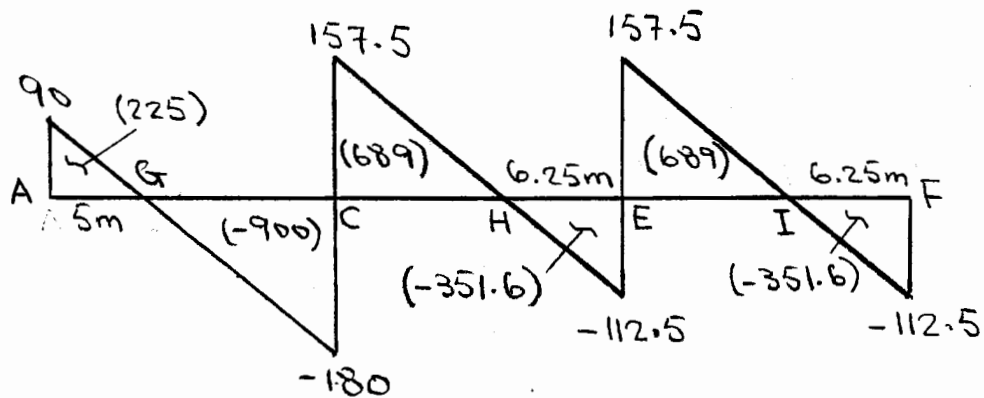
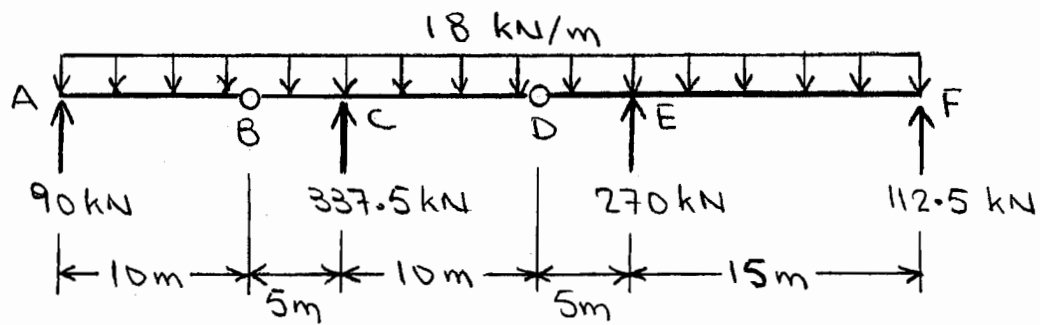


Bending Moment Diagram (k-ft)

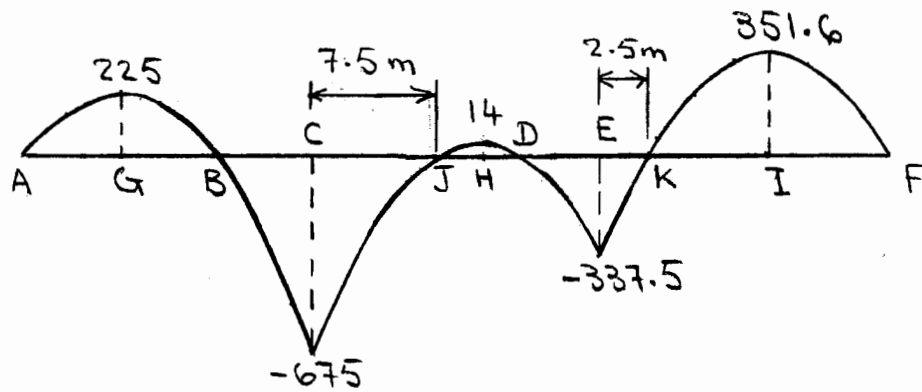


Qualitative Deflected Shape

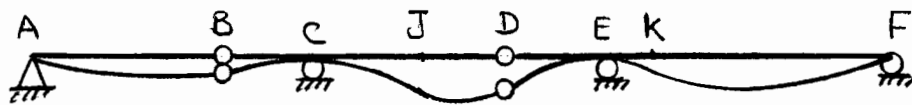
5.49



Shear Diagram (kN)

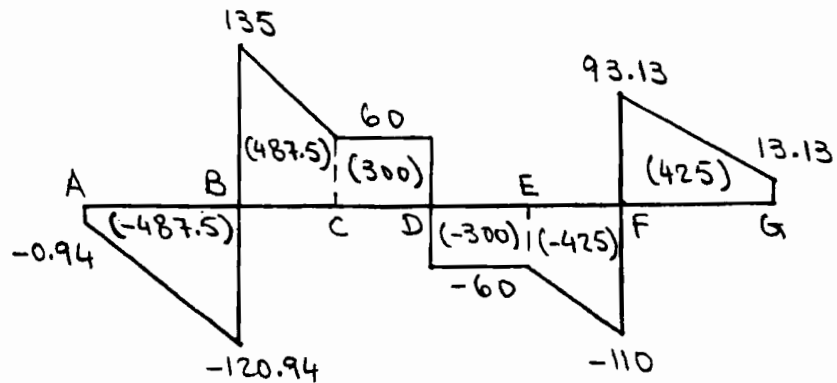
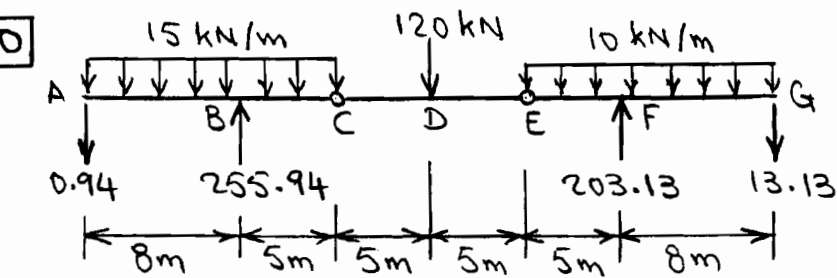


Bending Moment Diagram (kN.m)

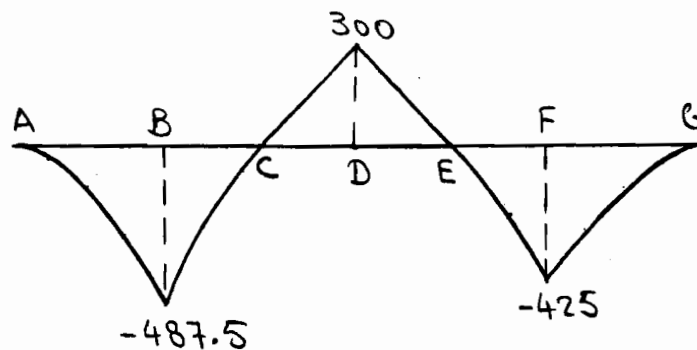


Qualitative Deflected Shape

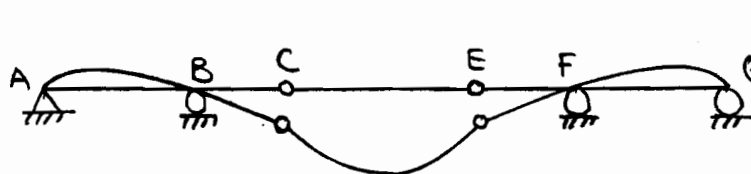
5.50



Shear
Diagram (kN)

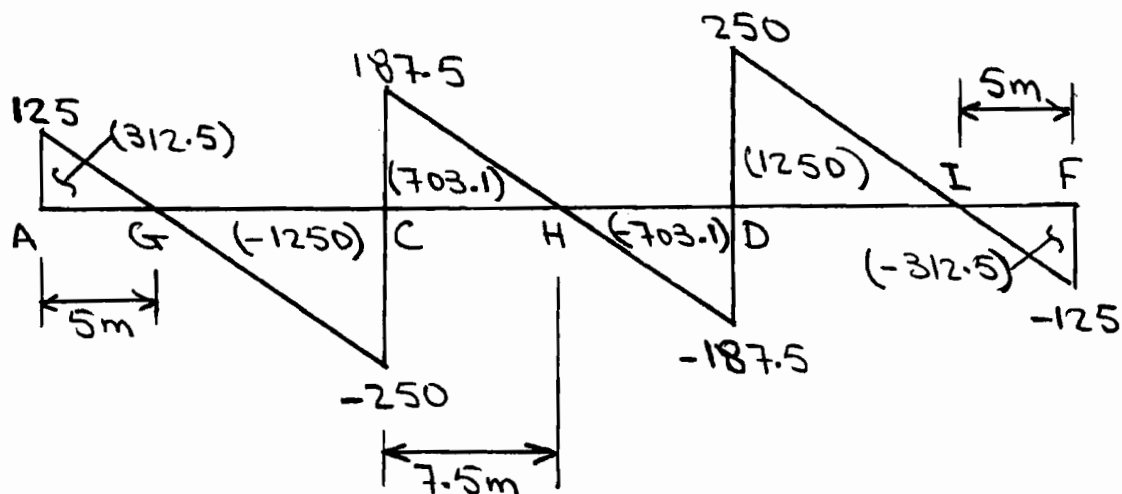
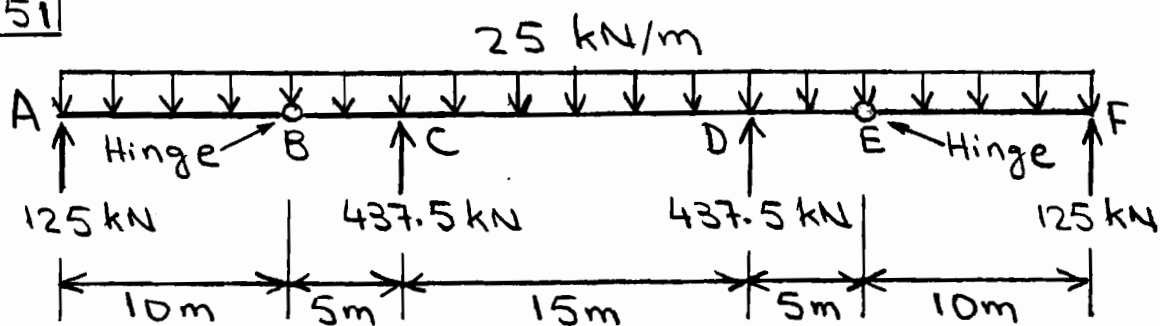


Bending Moment
Diagram (kN-m)

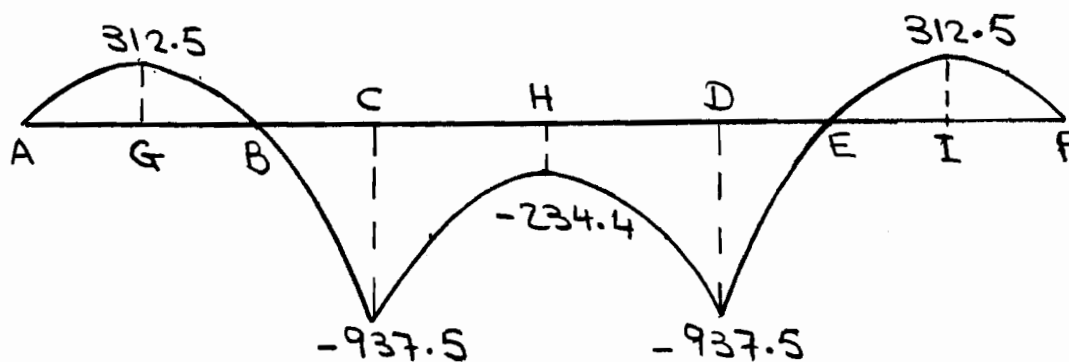


Qualitative
Deflected Shape

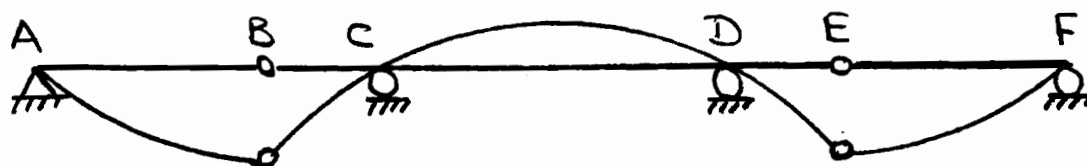
5.51



Shear Diagram (kN)

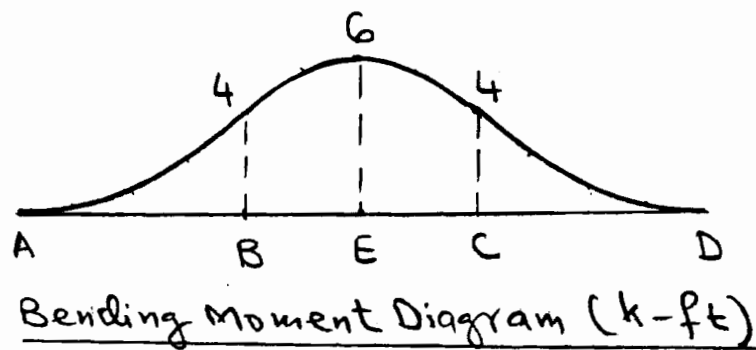
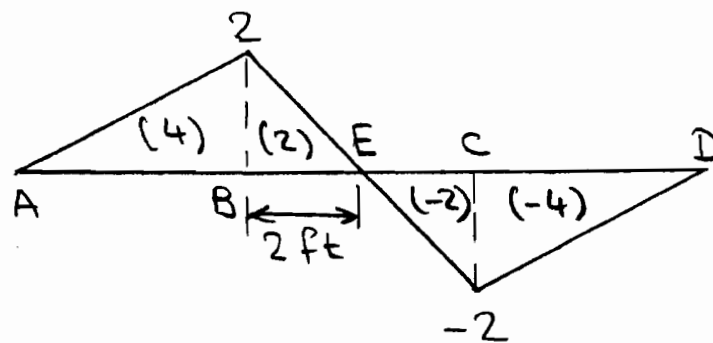
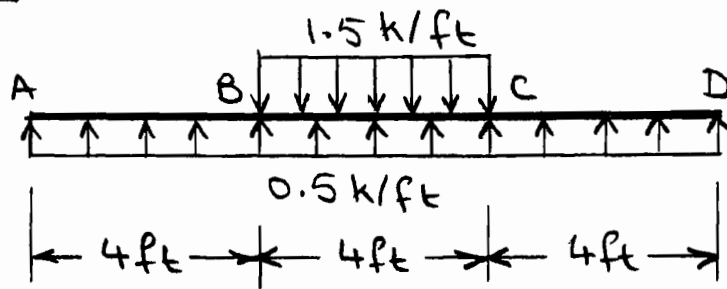


Bending Moment Diagram (kN.m)

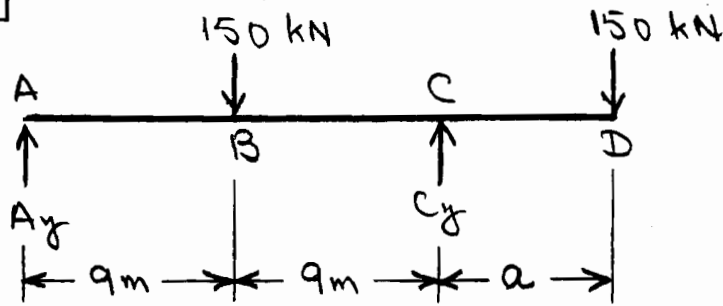


Qualitative Deflected Shape

5.52



5.53



$$+\circlearrowleft \sum M_C = 0 \quad -A_y(18) + 150(9) - 150a = 0$$

$$A_y = 75 - \frac{25}{3}a$$

Maximum positive bending moment occurs at B:

$$M_B = A_y(9) = 675 - 75a \quad (1)$$

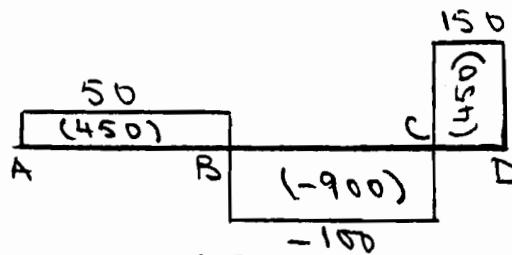
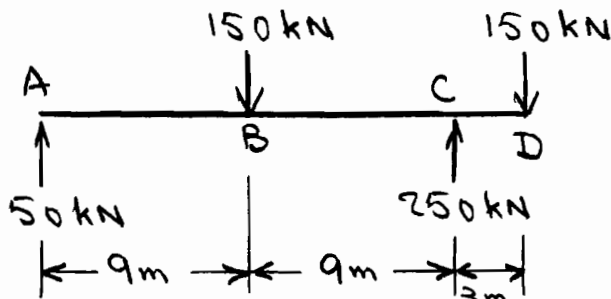
Maximum negative bending moment occurs at C:

$$M_C = 150a \quad (2)$$

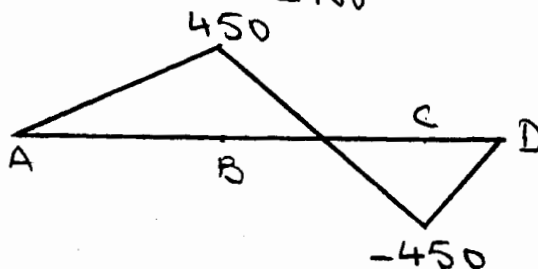
Equating Eqs. (1) and (2):

$$675 - 75a = 150a$$

$$\underline{a = 3\text{ m}}$$

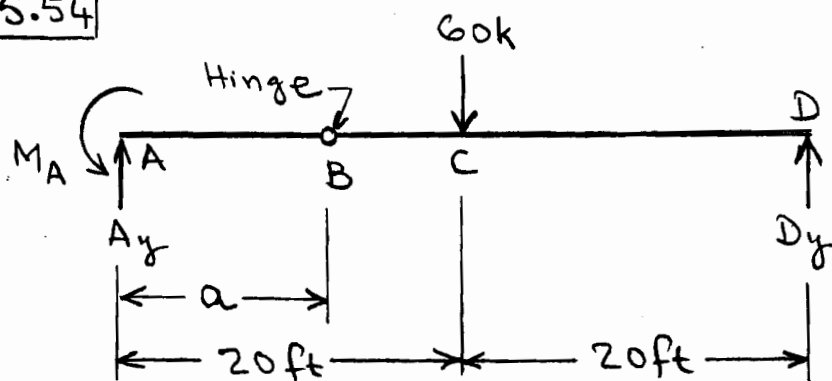


Shear Diagram (kN)



Bending Moment Diagram (kN.m)

5.54



$$+\circlearrowleft \sum M_B^{BD} = 0 \quad -60(20-a) + D_y(40-a) = 0$$

$$D_y = \frac{60(20-a)}{40-a}$$

Maximum positive bending moment occurs at C:

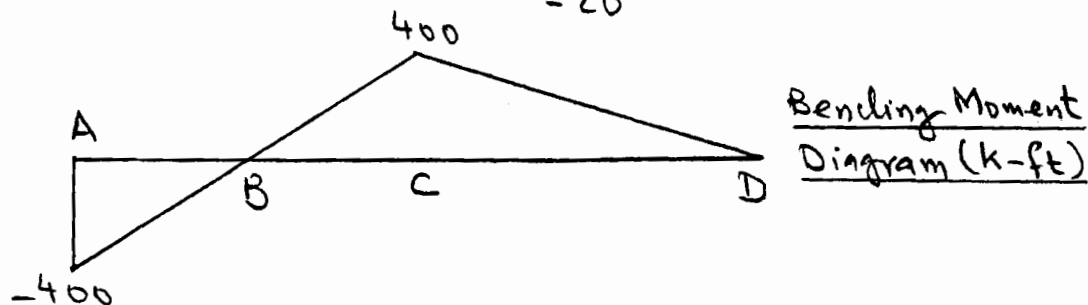
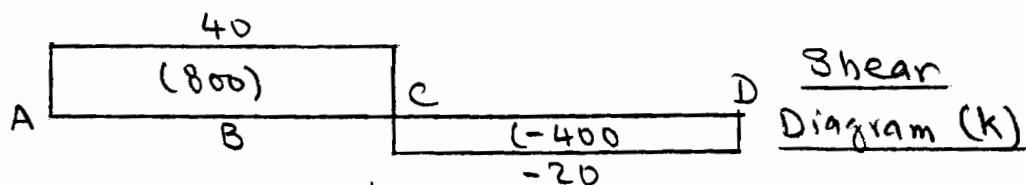
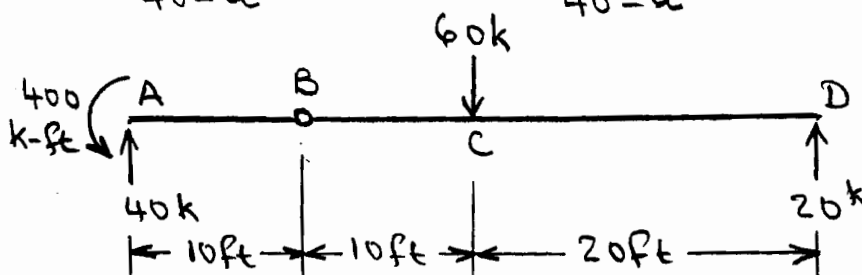
$$M_C = D_y(20) = \frac{1200(20-a)}{40-a} \quad (1)$$

Maximum negative bending moment occurs at A:

$$M_A = 60(20) - D_y(40) = 1200 - \frac{2400(20-a)}{40-a} \quad (2)$$

Equating Eqs. (1) and (2):

$$\frac{1200(20-a)}{40-a} = 1200 - \frac{2400(20-a)}{40-a} \quad \underline{a = 10 \text{ ft}}$$



5.55 (a) $m=4, j=5, r=3, e_c=0$

$$3m+r = 3j+e_c; \text{ Statically determinate}$$

(b) $m=6, j=7, r=3, e_c=1$

$$3m+r < 3j+e_c; \text{ Unstable}$$

(c) $m=7, j=6, r=3, e_c=0$

$$3m+r > 3j+e_c; \text{ Statically indeterminate}$$

$$i = (21+3) - 18 = \underline{6}$$

(d) $m=7, j=7, r=6, e_c=1$

$$3m+r > 3j+e_c; \text{ Statically indeterminate}$$

$$i = (21+6) - (21+1) = \underline{5}$$

5.56 (a) $m=6, j=6, r=4, e_c=2$

$$3m+r > 3j+e_c; \text{ Statically indeterminate}$$

$$i = (18+4) - (18+2) = \underline{2}$$

(b) $m=6, j=7, r=6, e_c=3$

$$3m+r = 3j+e_c; \text{ Statically determinate}$$

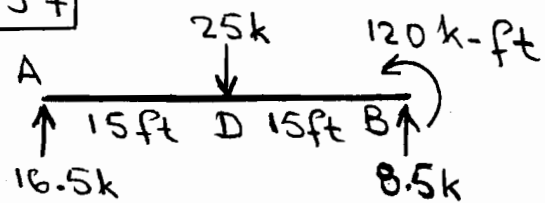
(c) $m=5, j=6, r=4, e_c=2$

$$3m+r < 3j+e_c; \text{ Unstable}$$

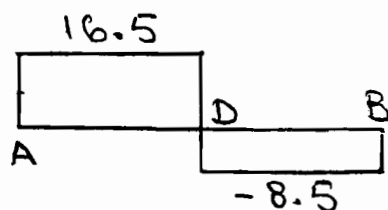
(d) Statically indeterminate

$$i = 3(8) = \underline{24}$$

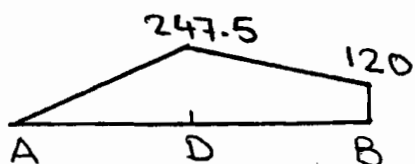
5.57



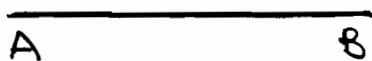
Member End Forces



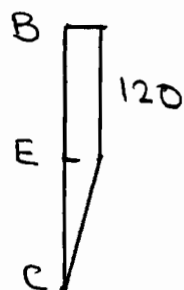
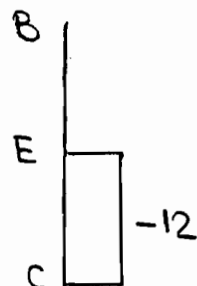
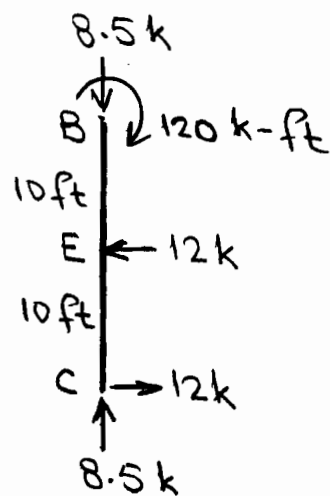
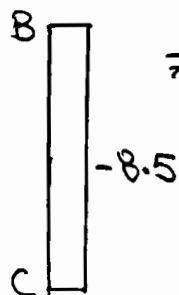
Shear Diagrams (k)



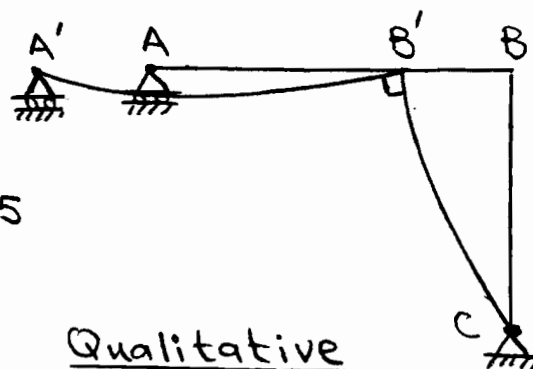
Bending Moment Diagrams (k-ft)



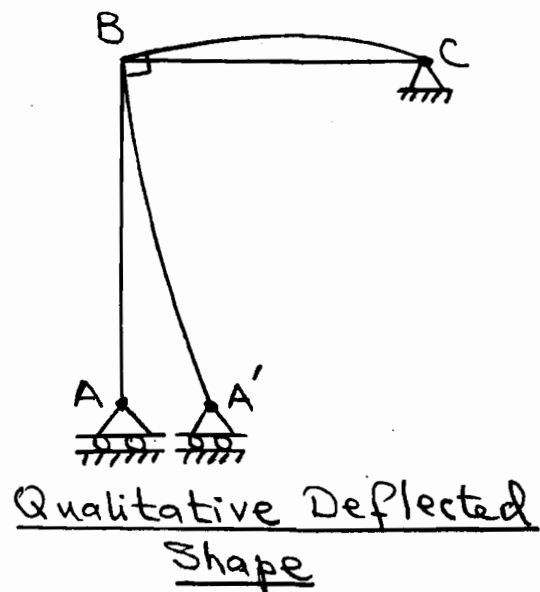
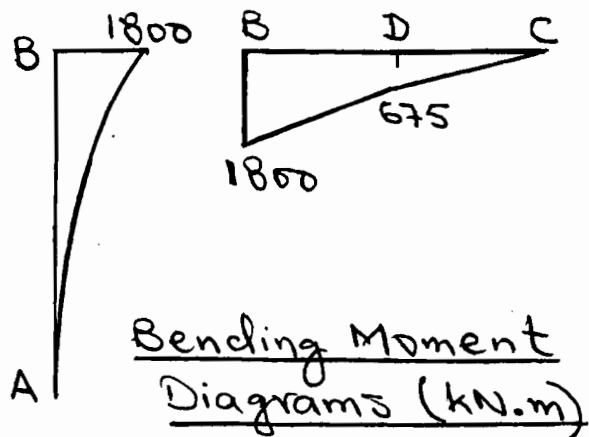
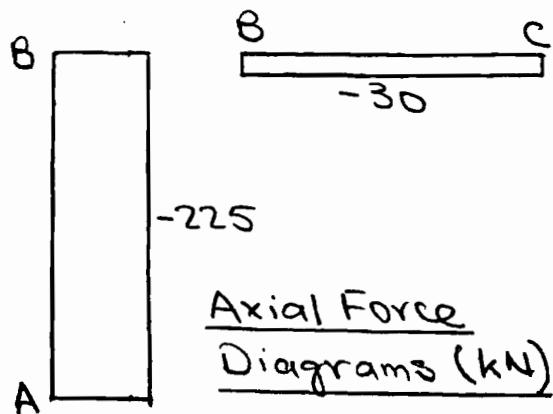
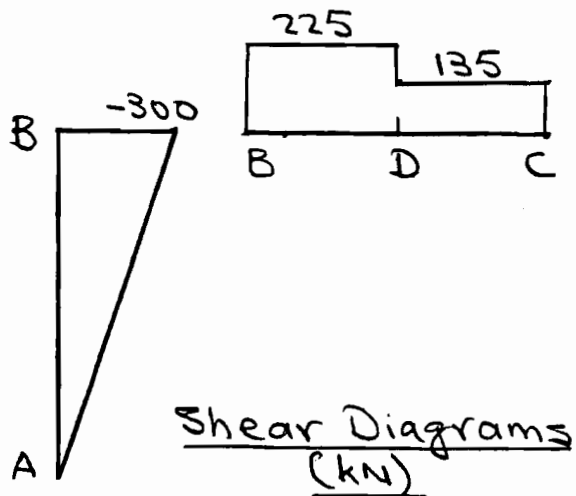
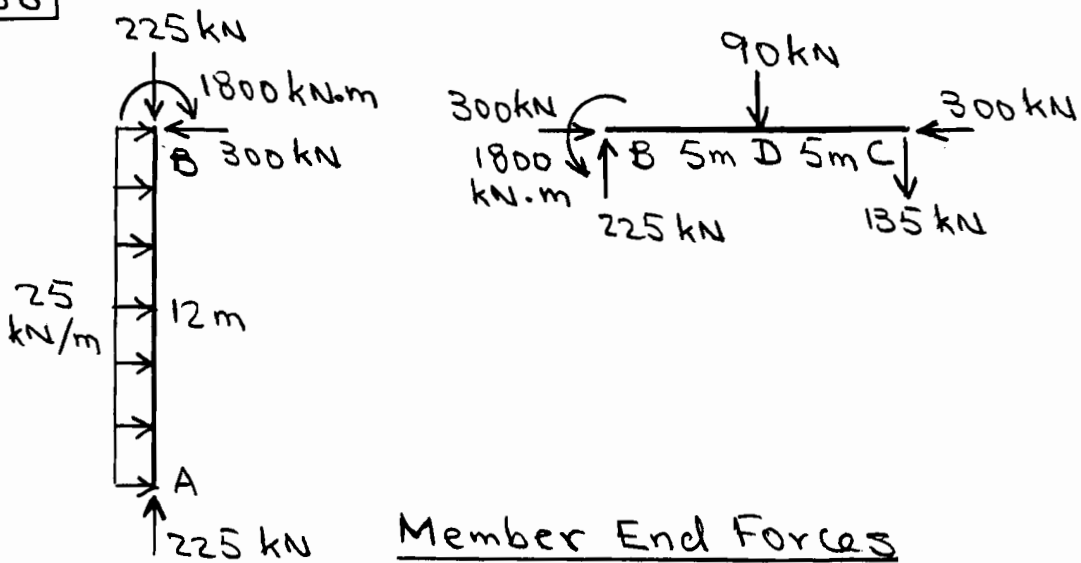
Axial Force Diagrams (k)



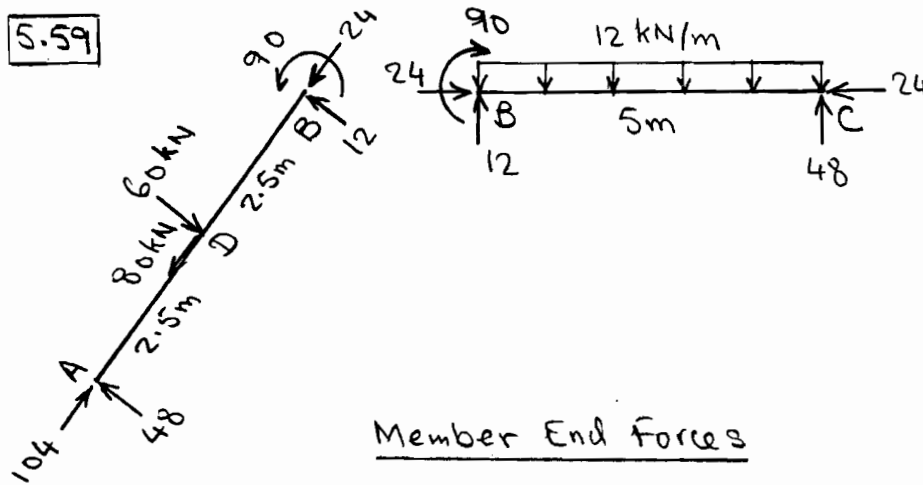
Qualitative Deflected Shape



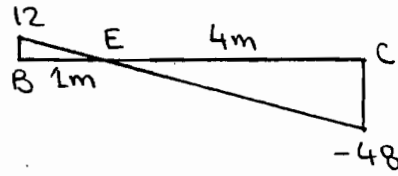
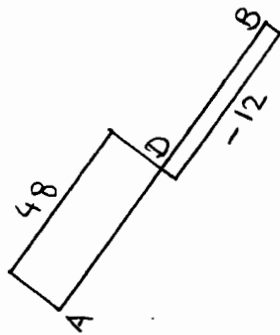
5.58



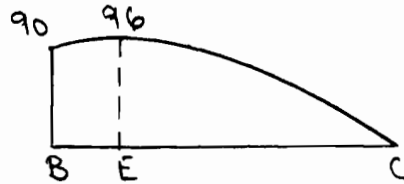
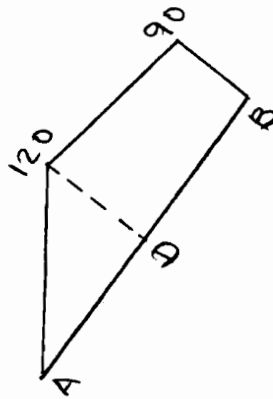
5.59



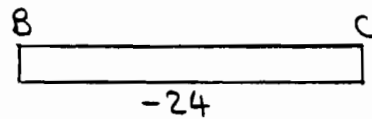
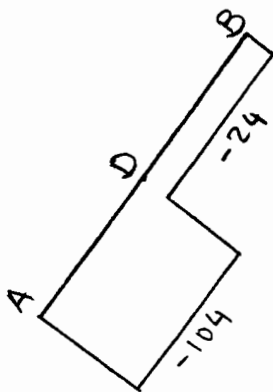
Member End Forces



Shear Diagrams (kN)

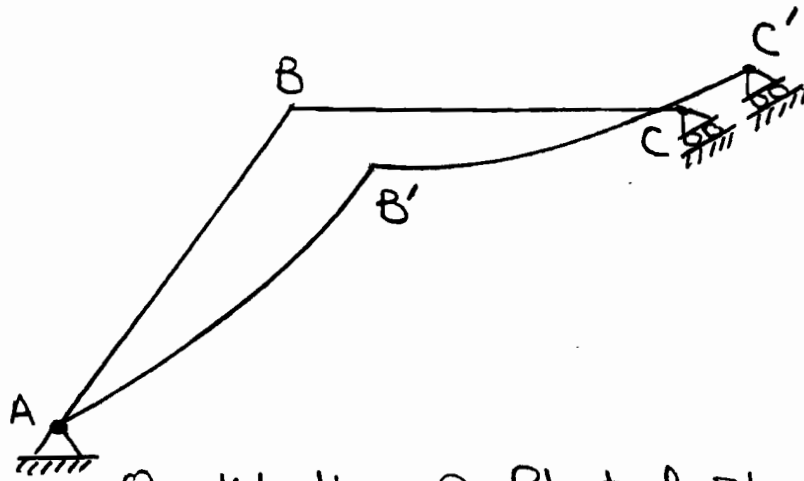


Bending Moment Diagrams (kN-m)



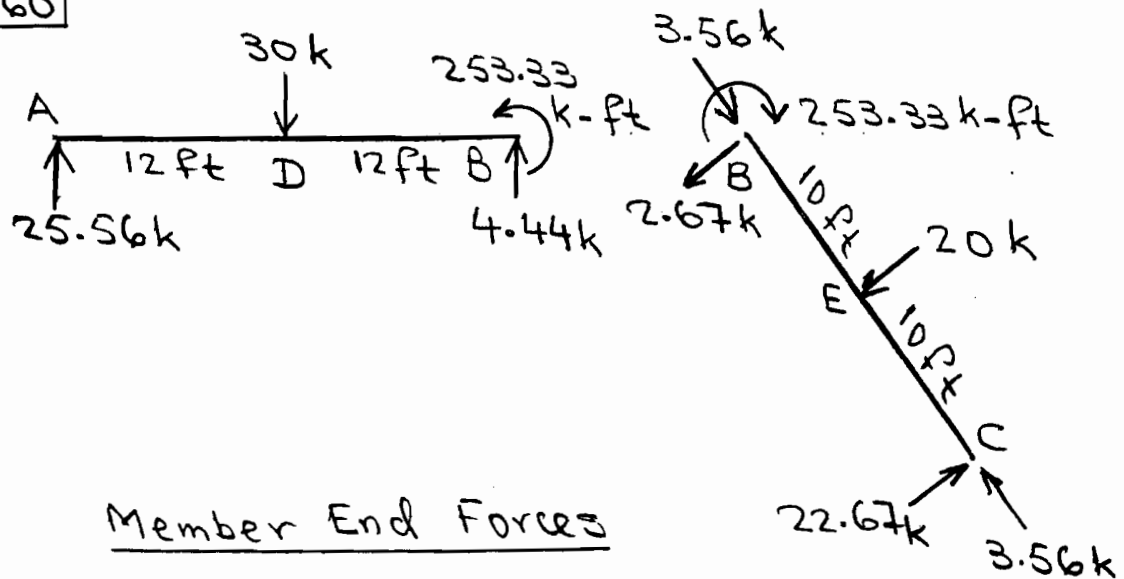
Axial Force Diagrams (kN)

5.59 (contd.)

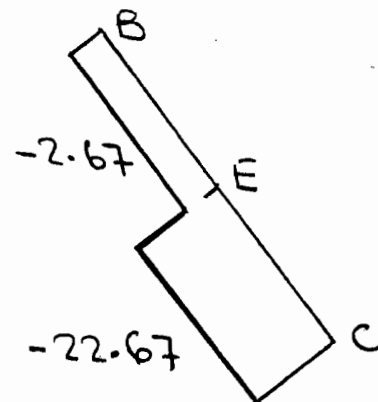
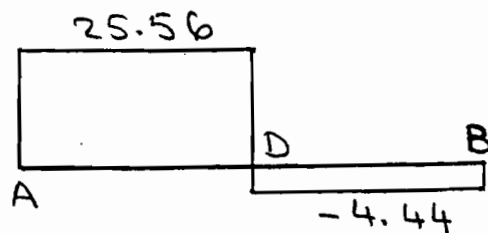


Qualitative Deflected Shape

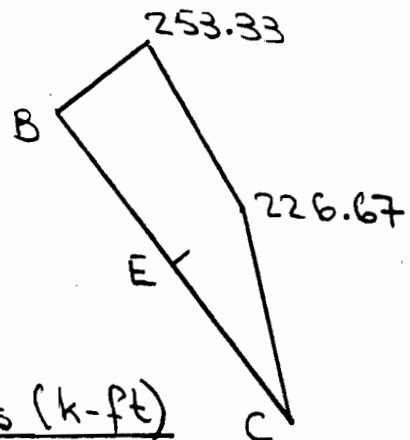
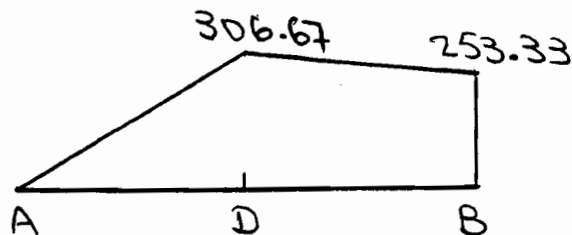
5.60



Member End Forces

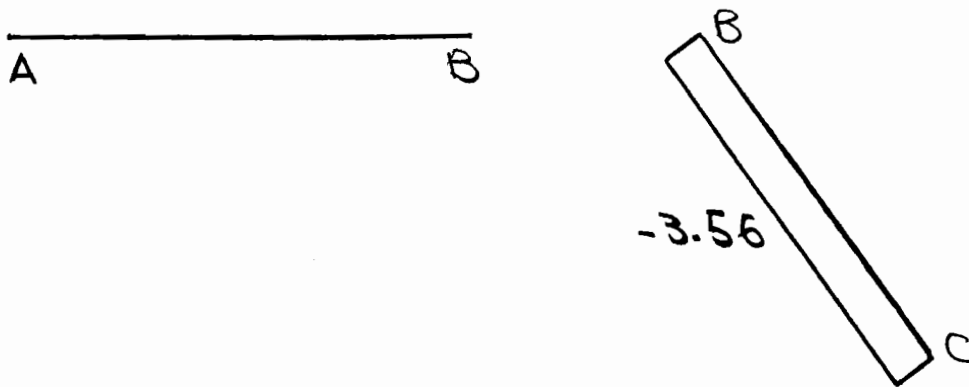


Shear Diagrams (k)

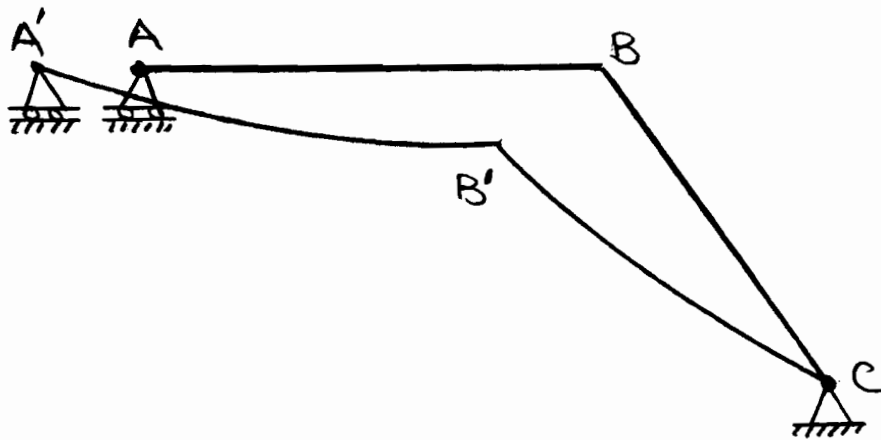


Bending Moment Diagrams (k-ft)

5.60 (contd.)

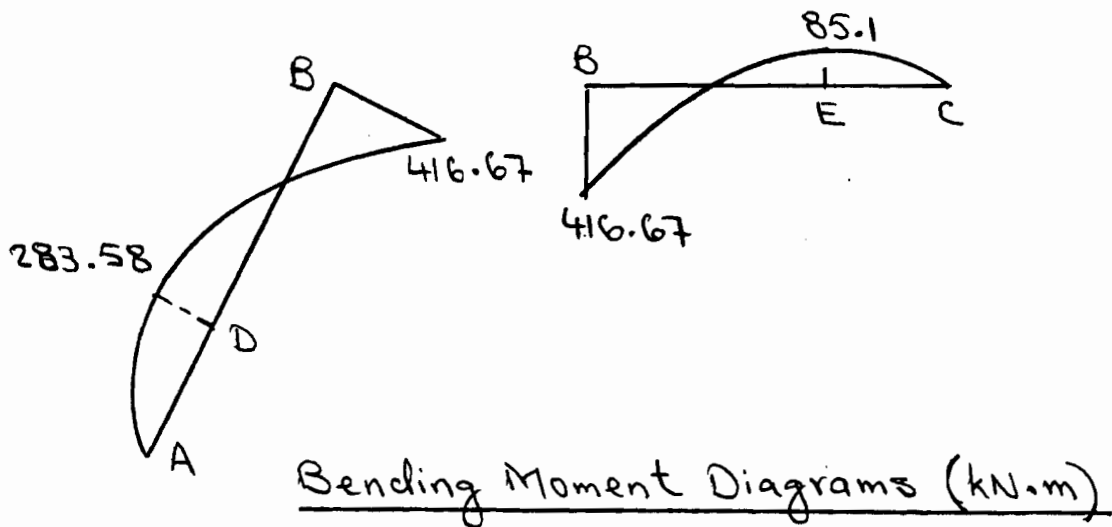
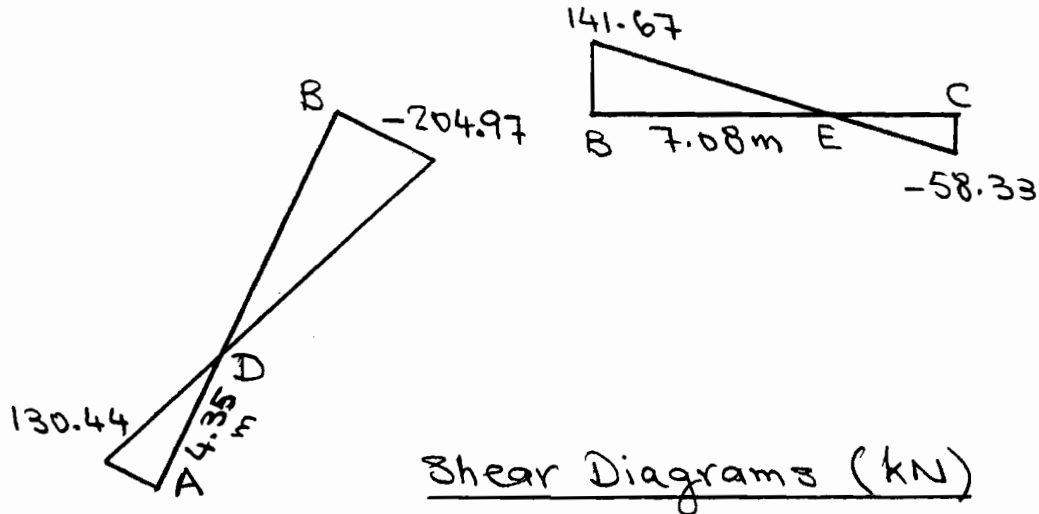
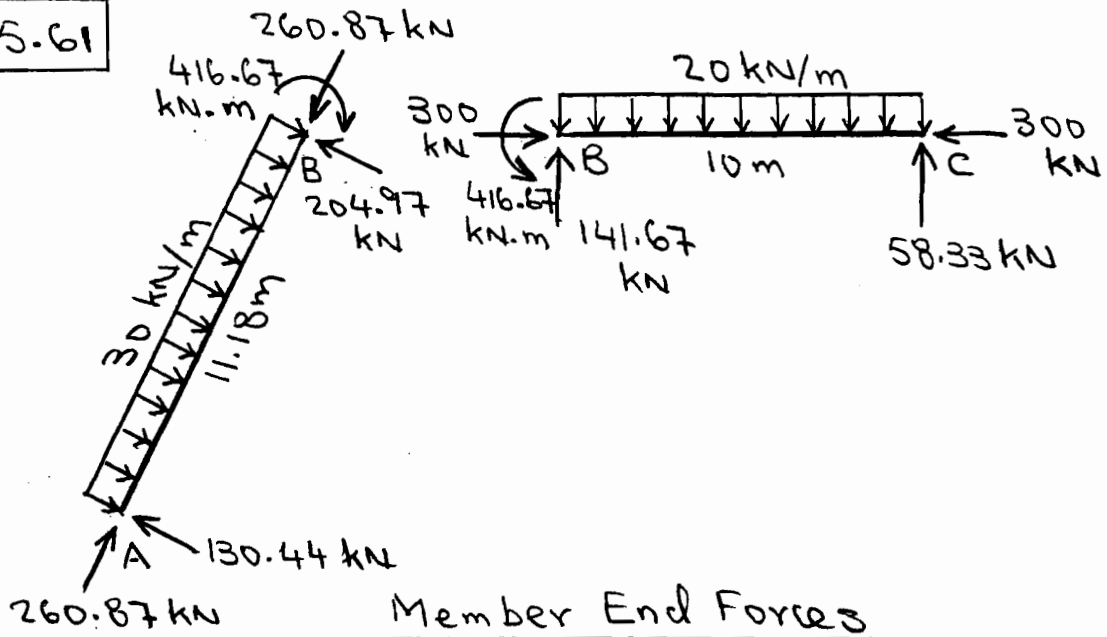


Axial Force Diagrams (k)

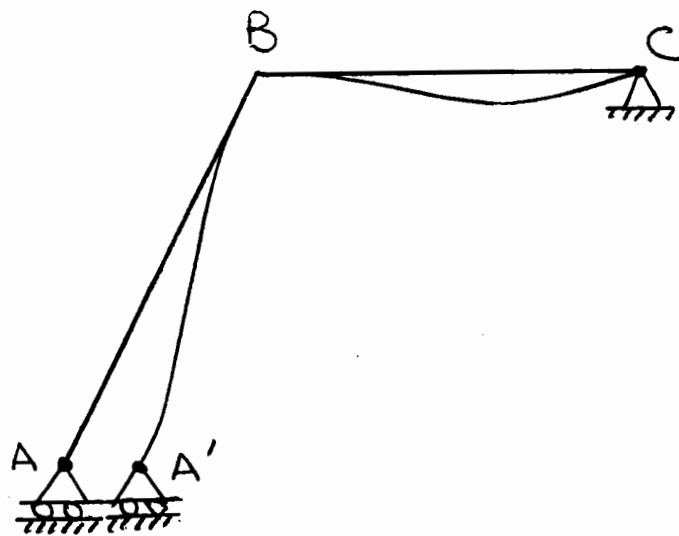
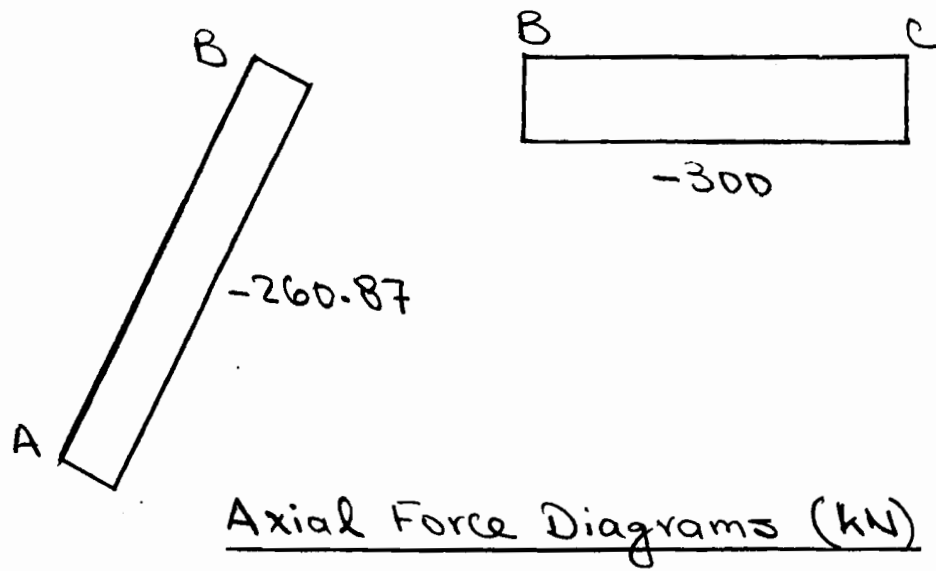


Qualitative Deflected Shape

5.61

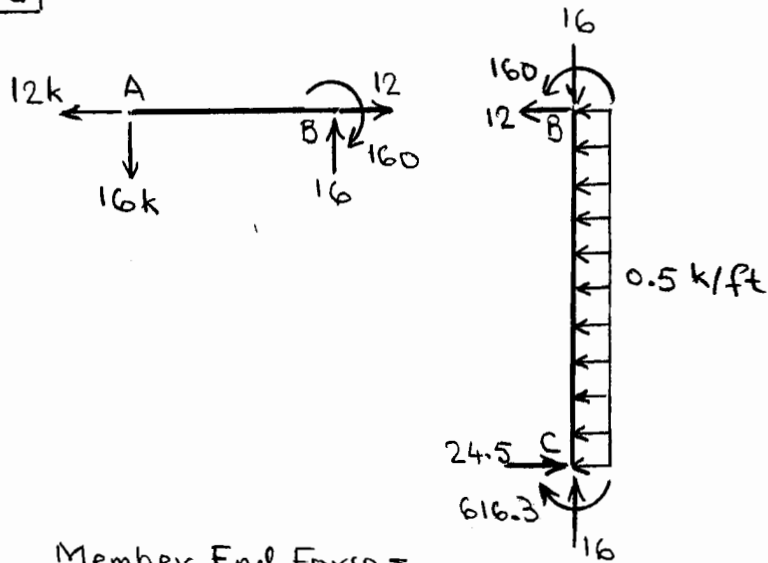


5.61 (contd.)

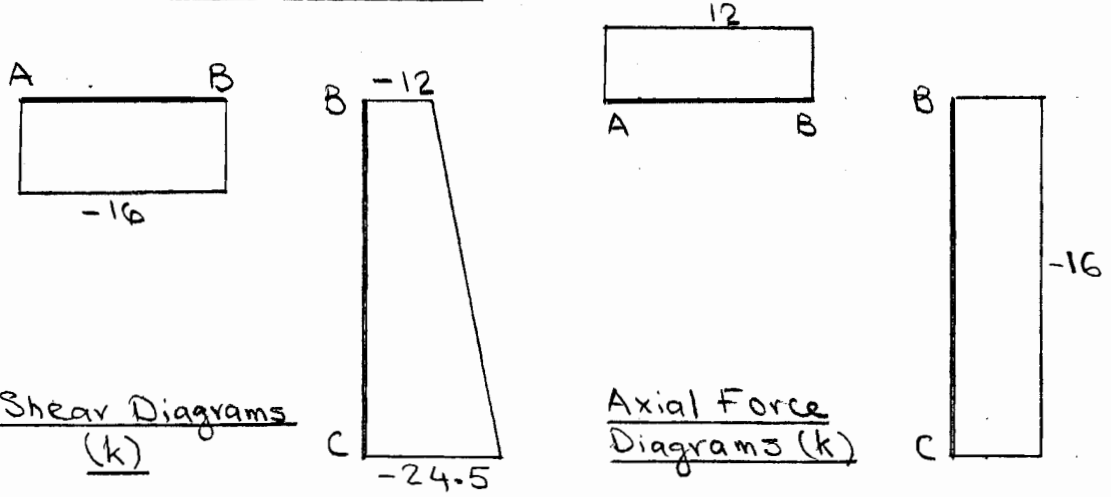


Qualitative Deflected Shape

5.62

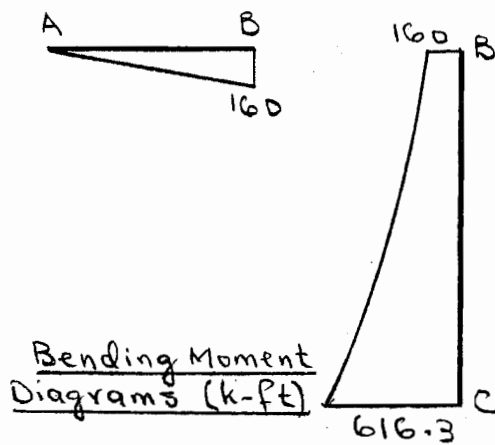


Member End Forces

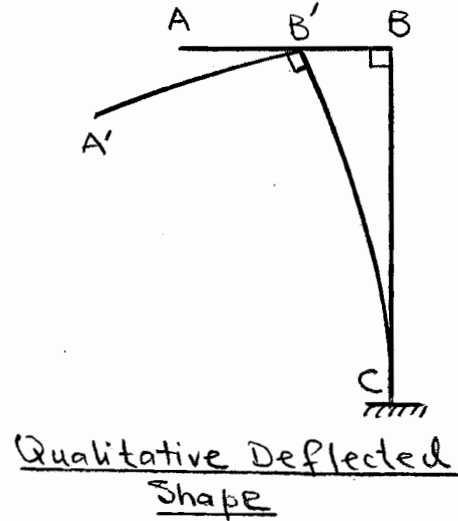


Shear Diagrams (k)

Axial Force Diagrams (k)

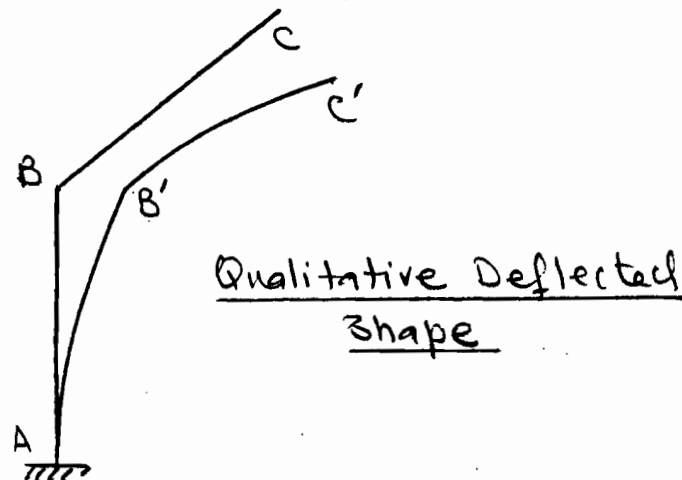
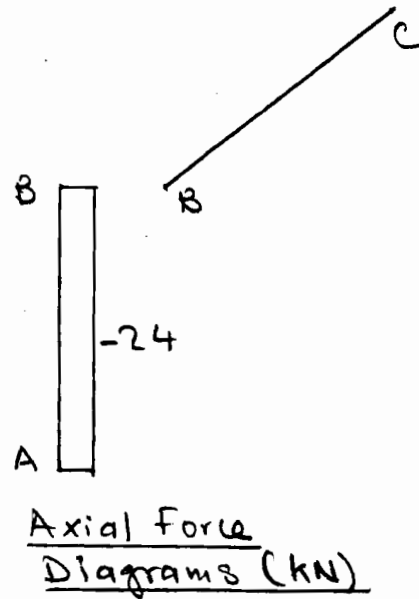
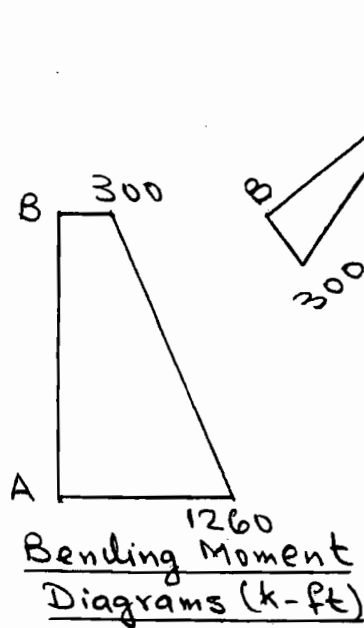
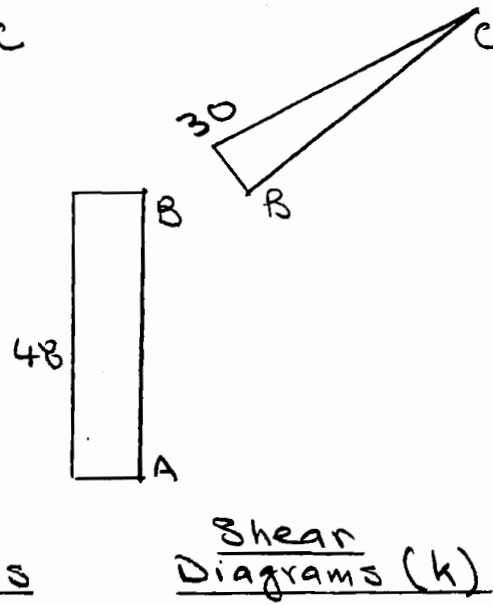
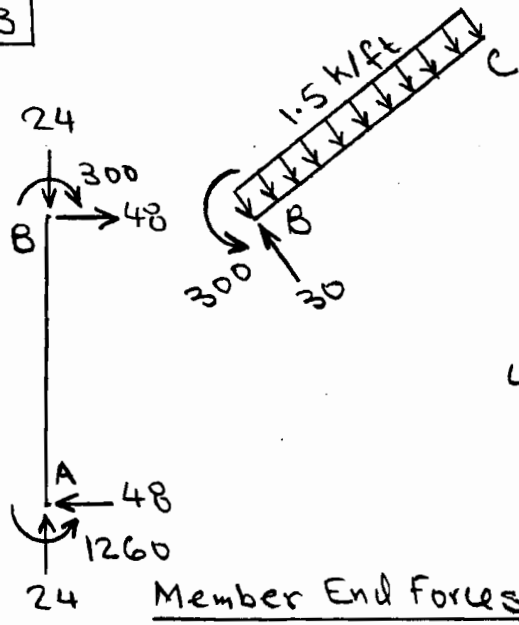


Bending Moment Diagrams (k-ft)

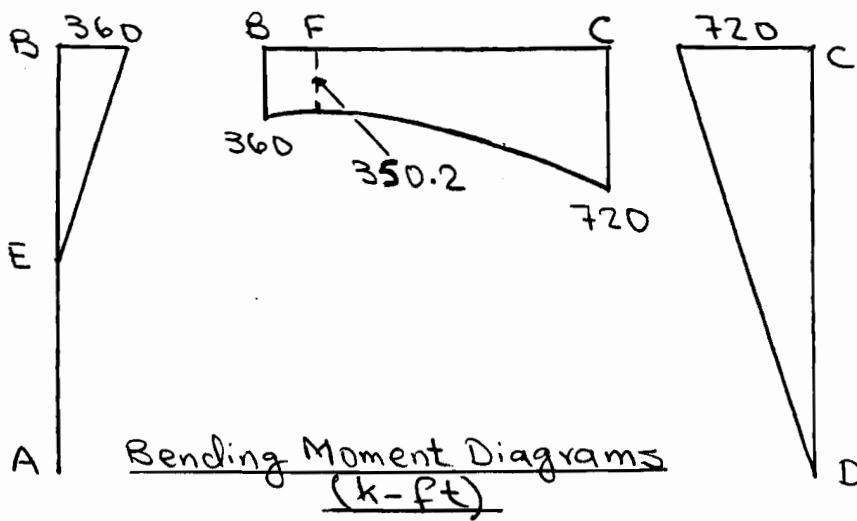
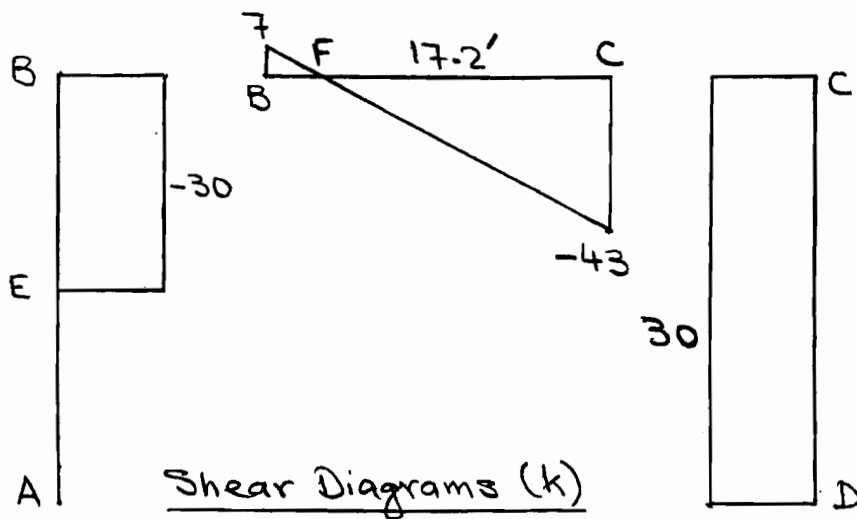
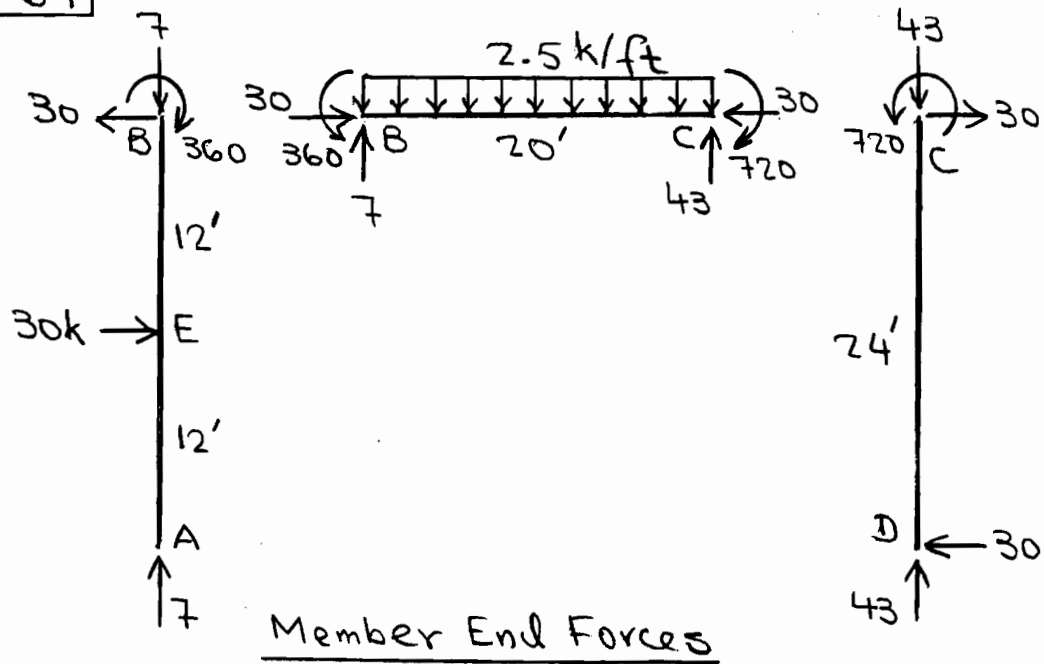


Qualitative Deflected Shape

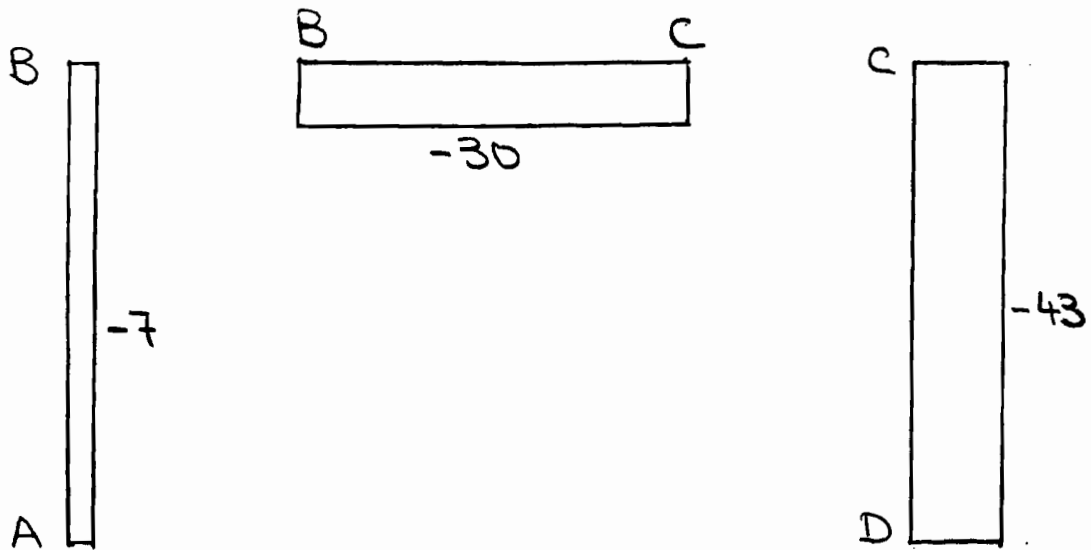
5.63



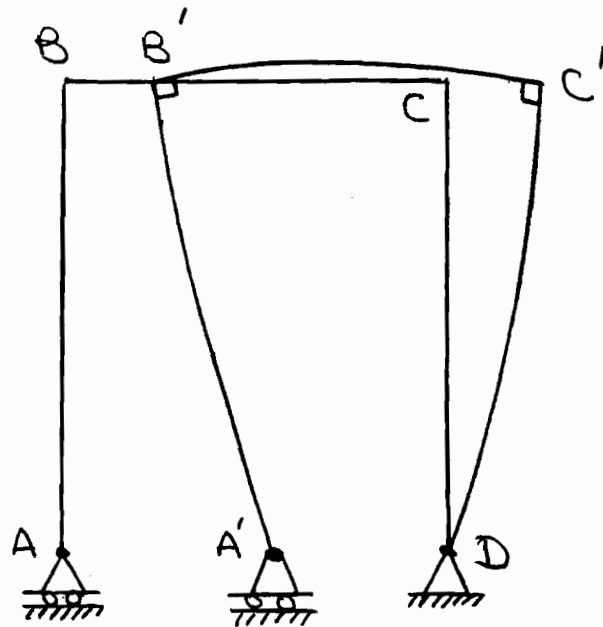
5.64



5.64 (contd.)

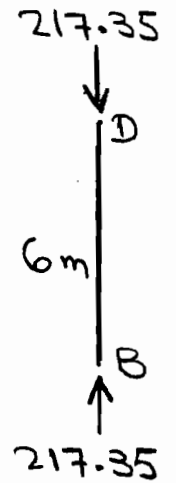
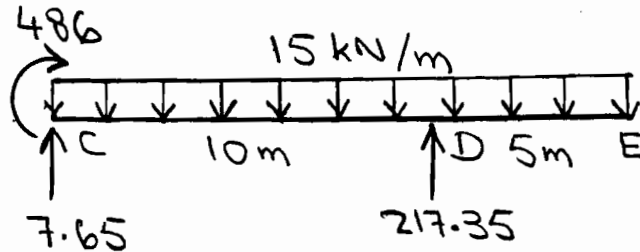
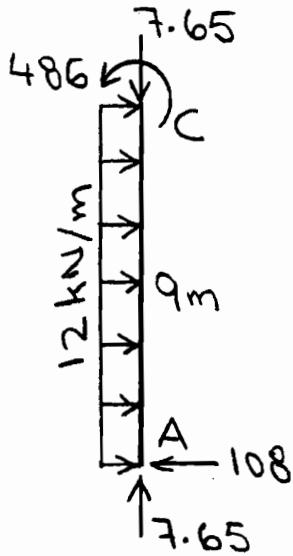


Axial Force Diagrams (k)

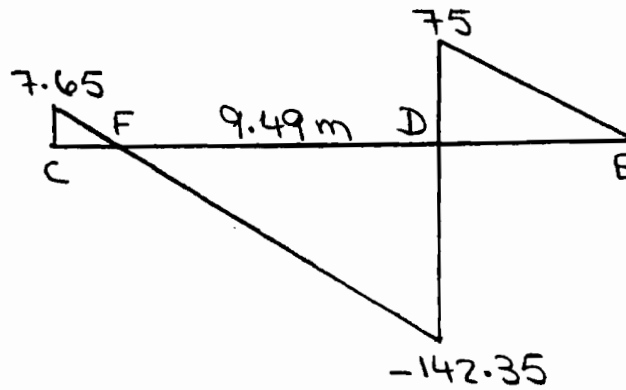
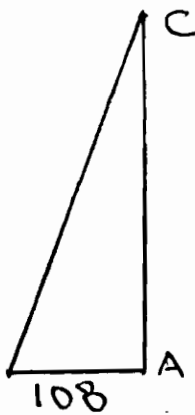


Qualitative Deflected Shape

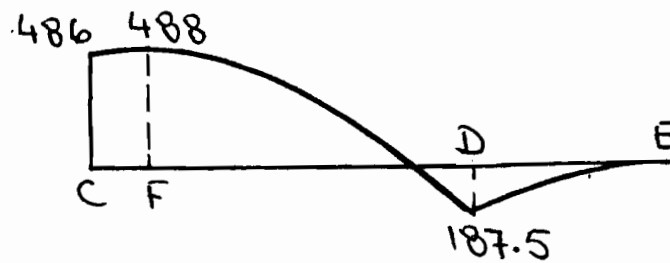
5.65



Member End Forces

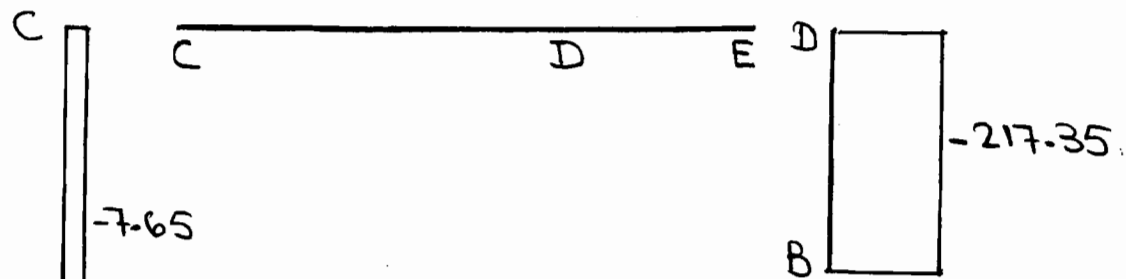


Shear Diagrams (kN)

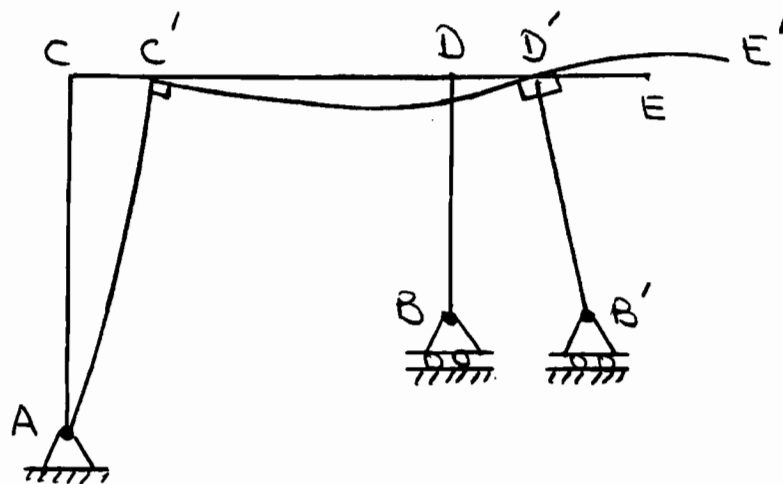


Bending Moment Diagrams (kN.m)

5.65 (contd.)

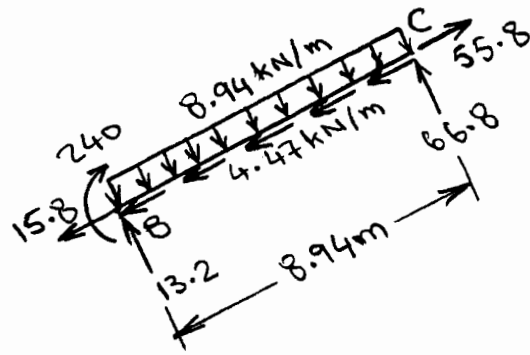
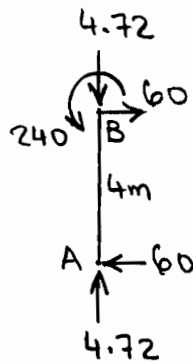


Axial Force Diagrams (kN)

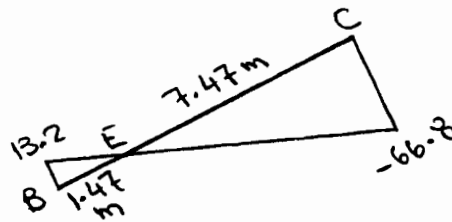
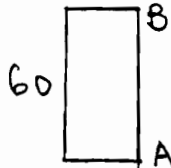
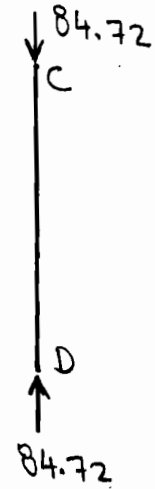


Qualitative Deflected Shape

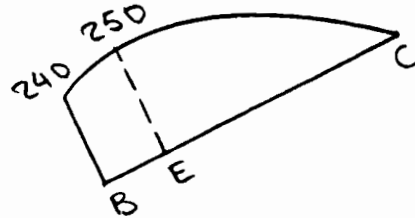
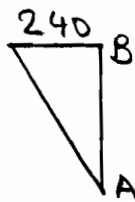
5.66



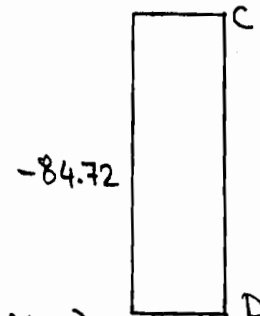
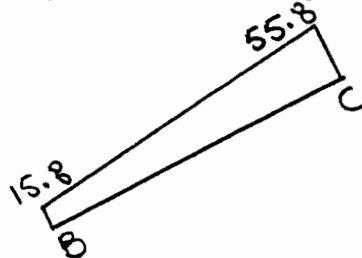
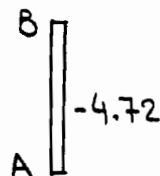
Member End Forces



Shear Diagrams (kN)

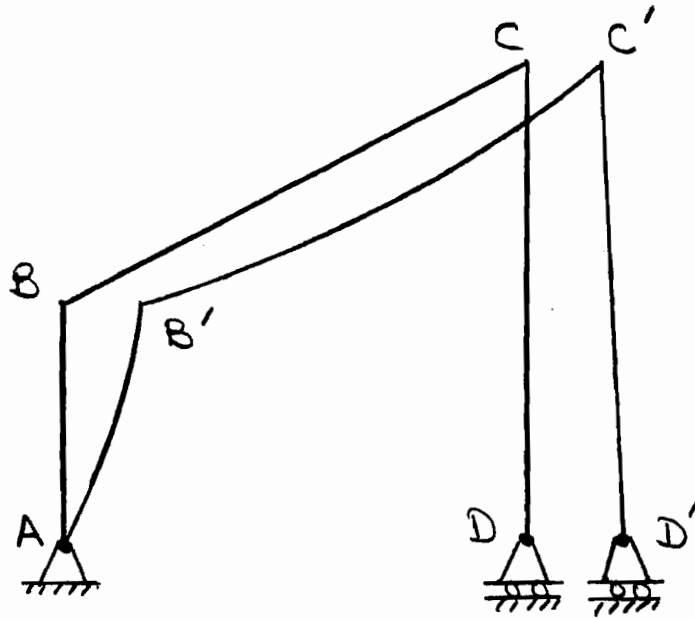


Bending Moment Diagrams (kN-m)



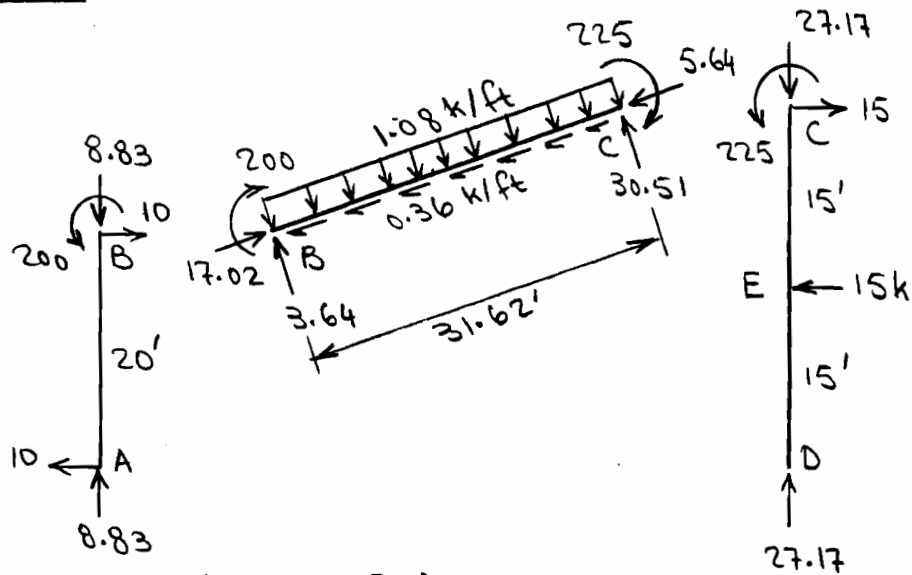
Axial Force Diagrams (kN)

5-66 (contd.)

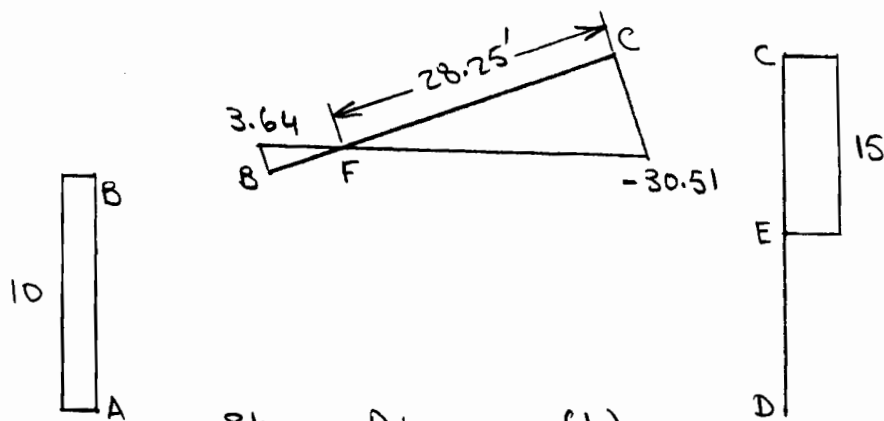


Qualitative Deflected Shape

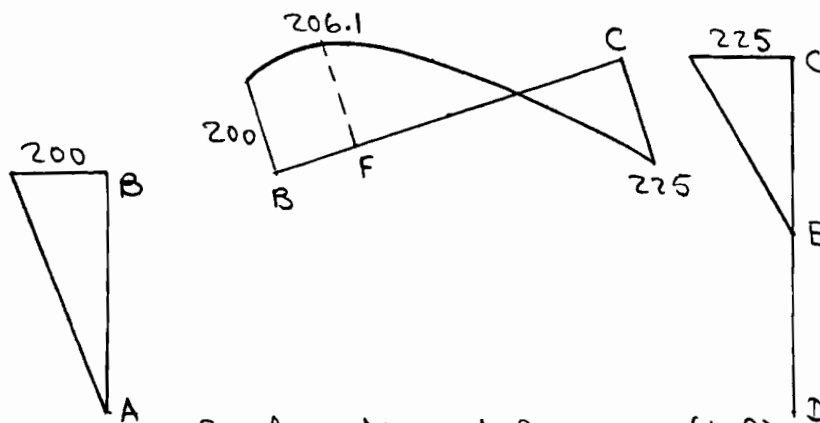
5.67



Member End Forces

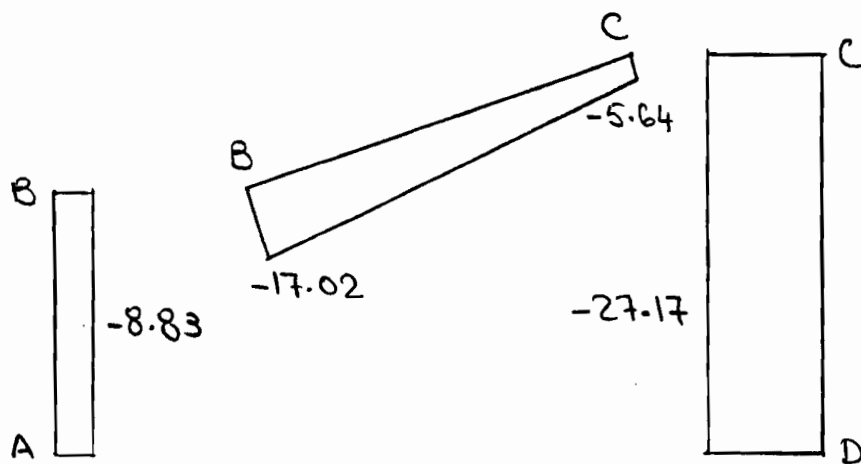


Shear Diagrams (k)

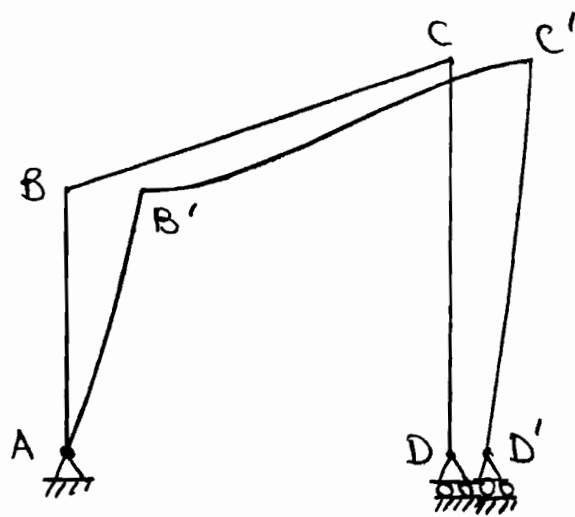


Bending Moment Diagrams (k-ft)

5.67 (contd.)

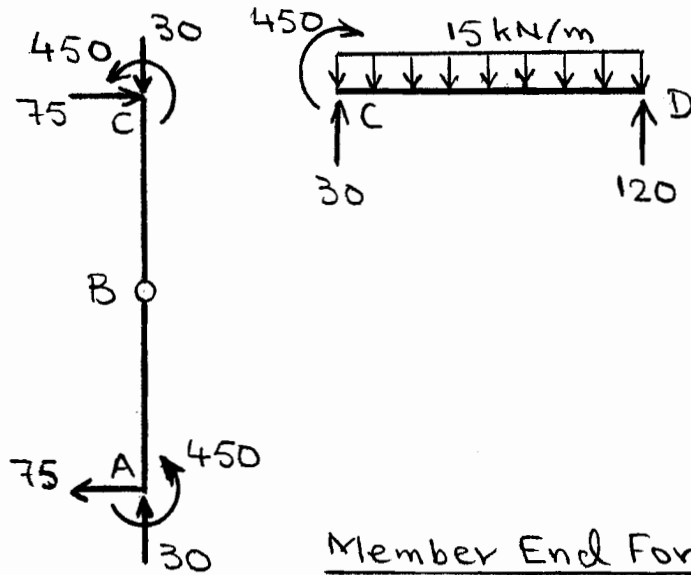


Axial Force Diagrams (k)

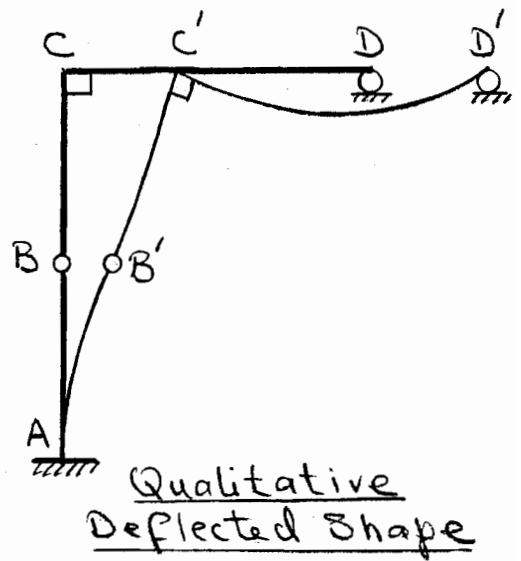
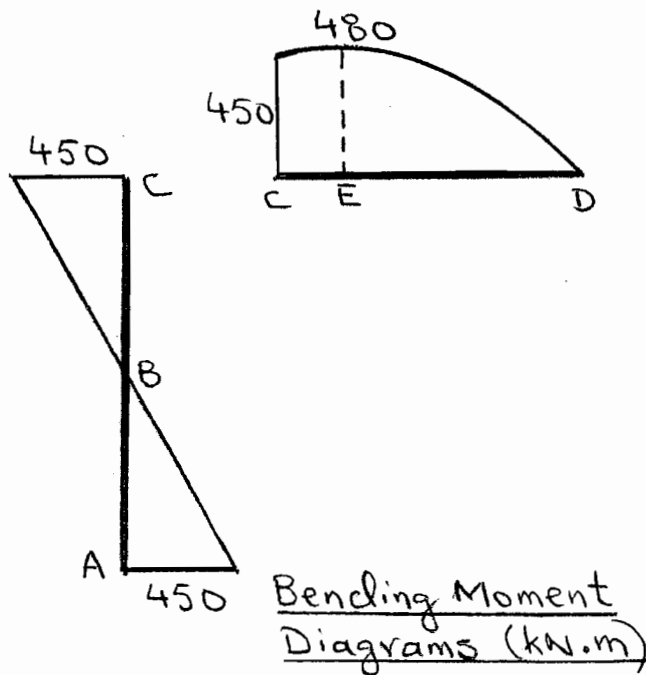
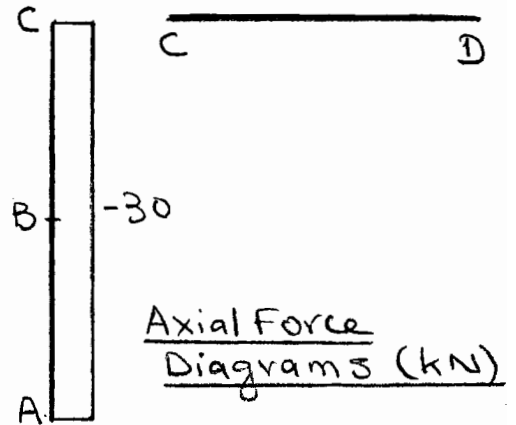
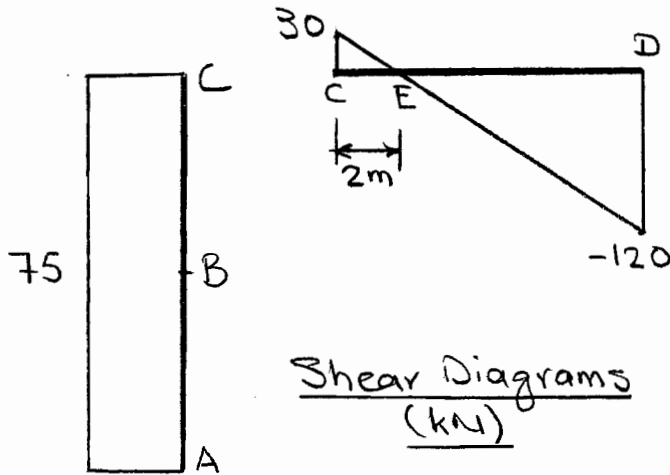


Qualitative Deflected Shape

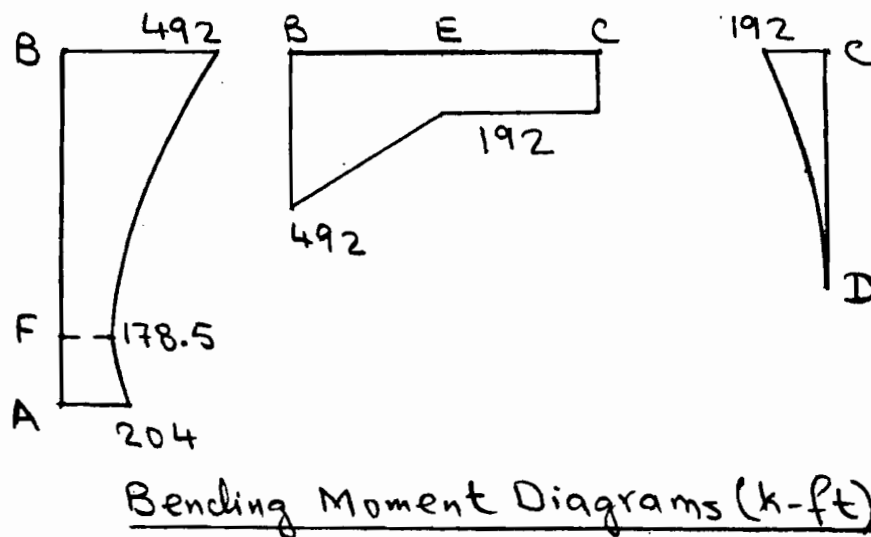
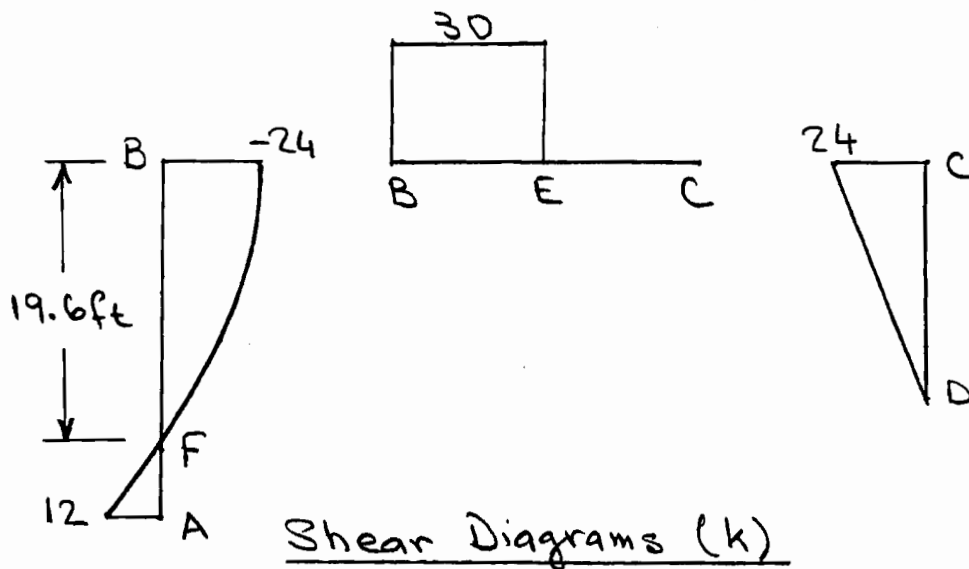
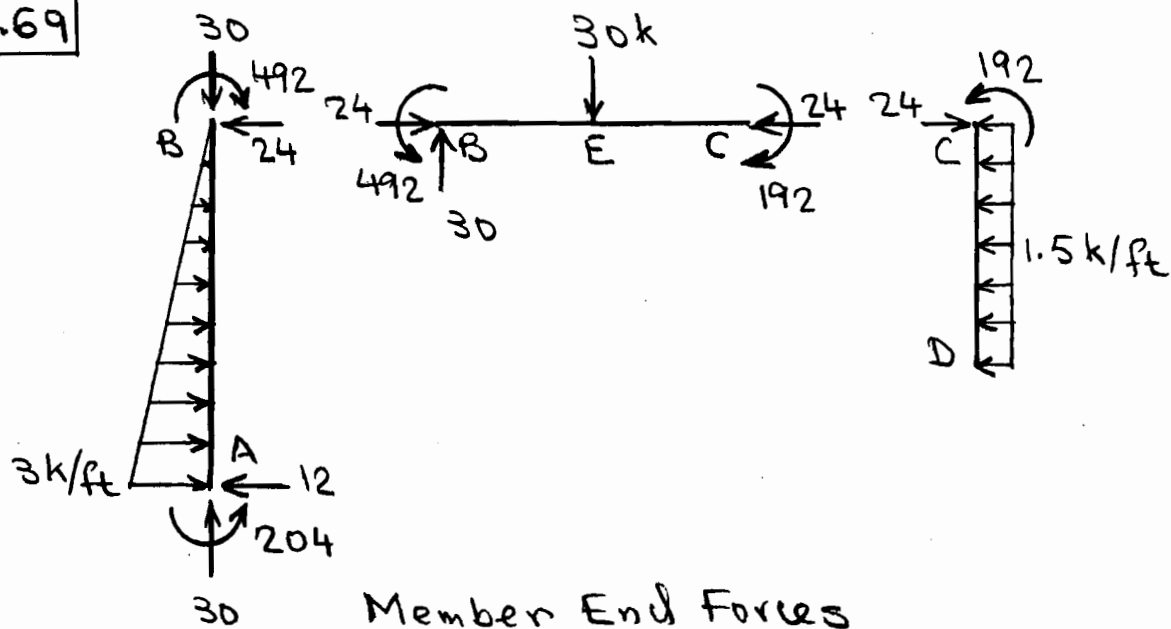
5.68



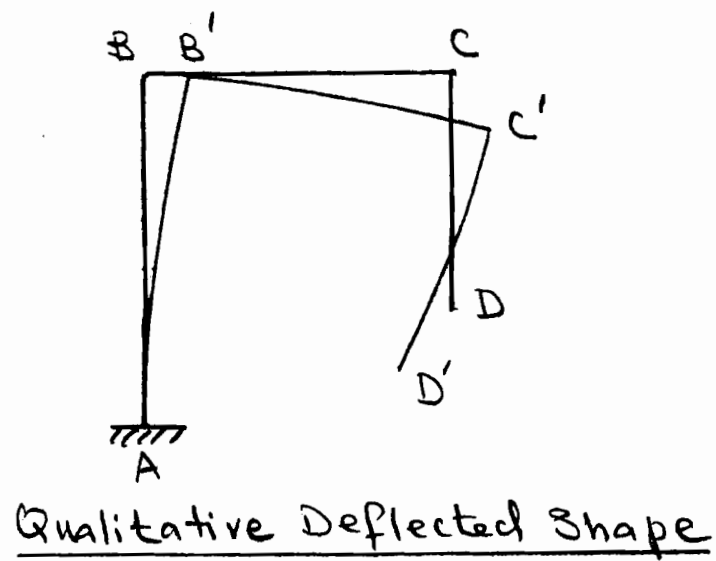
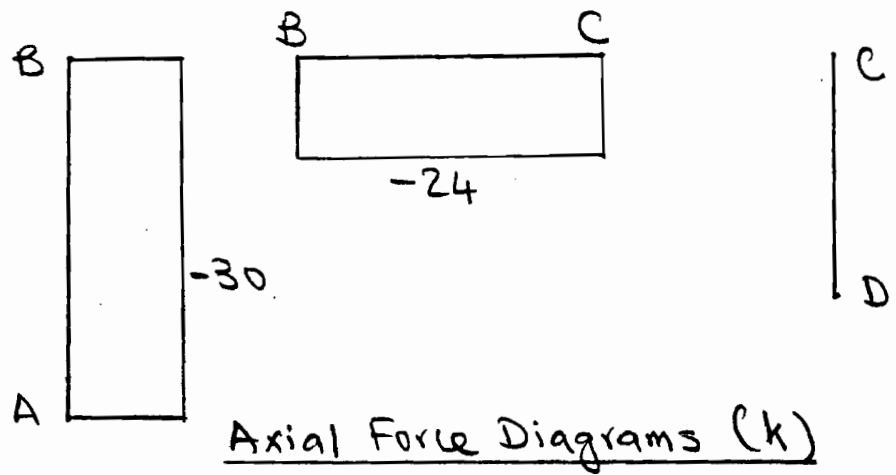
Member End Forces



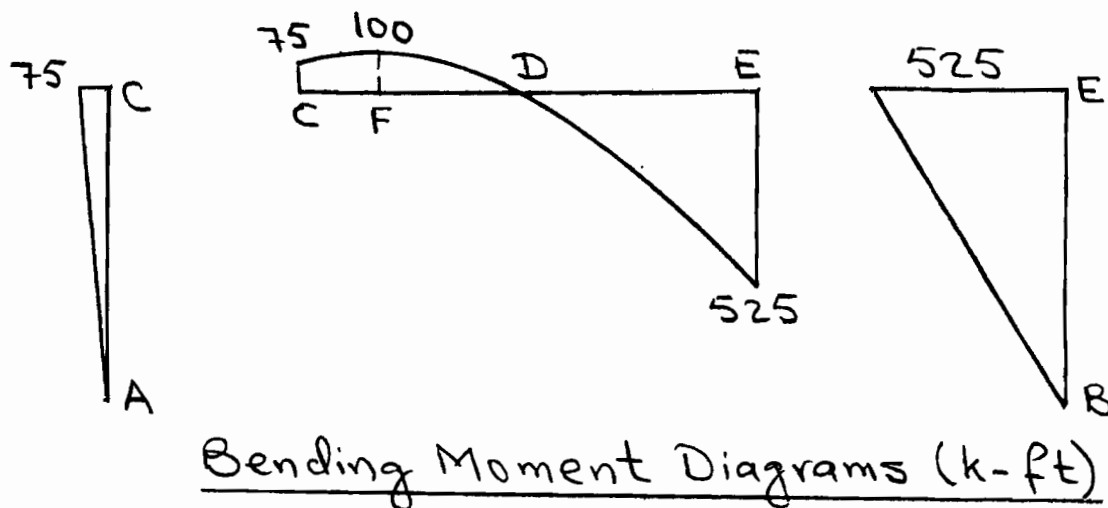
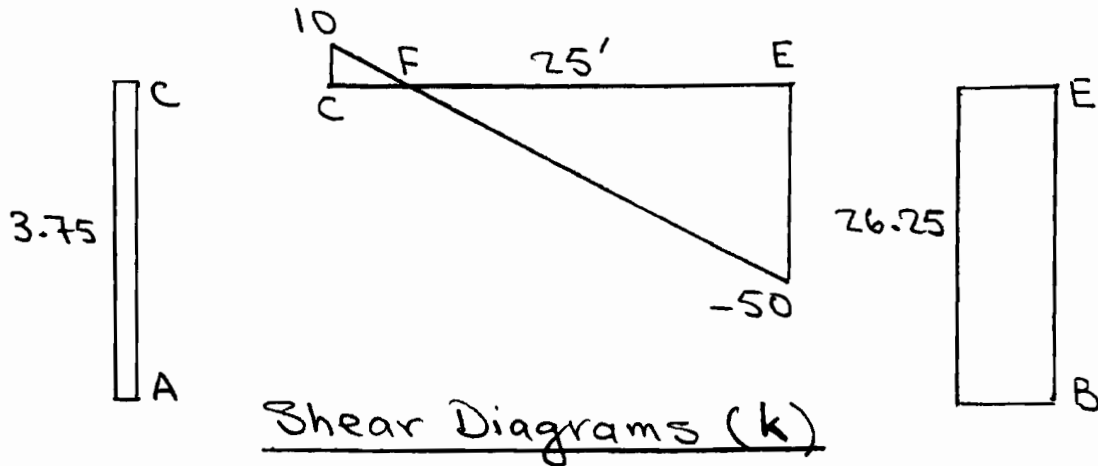
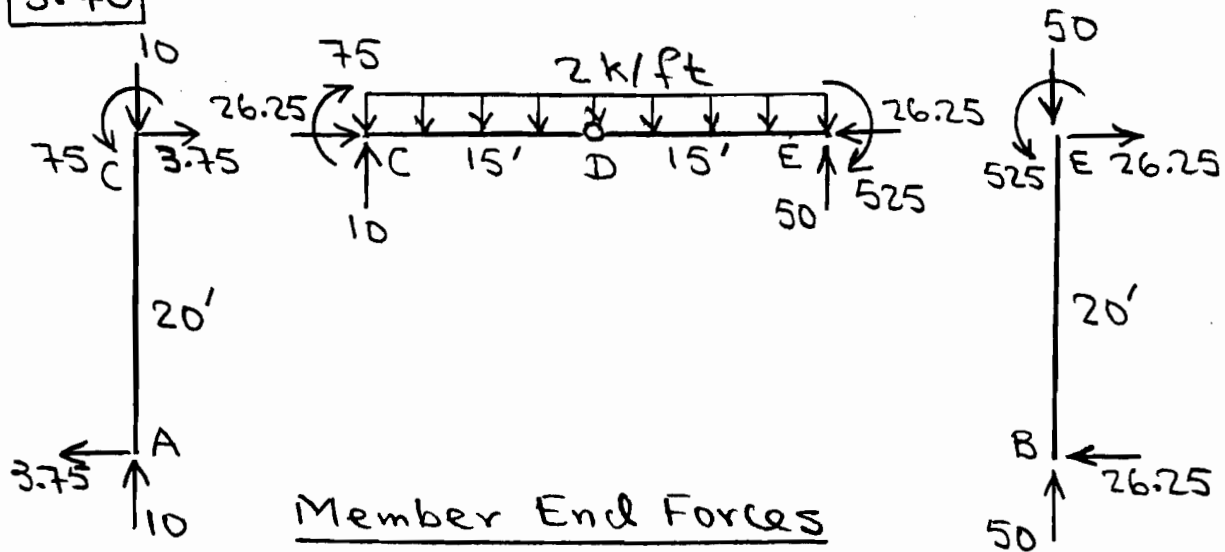
5.69



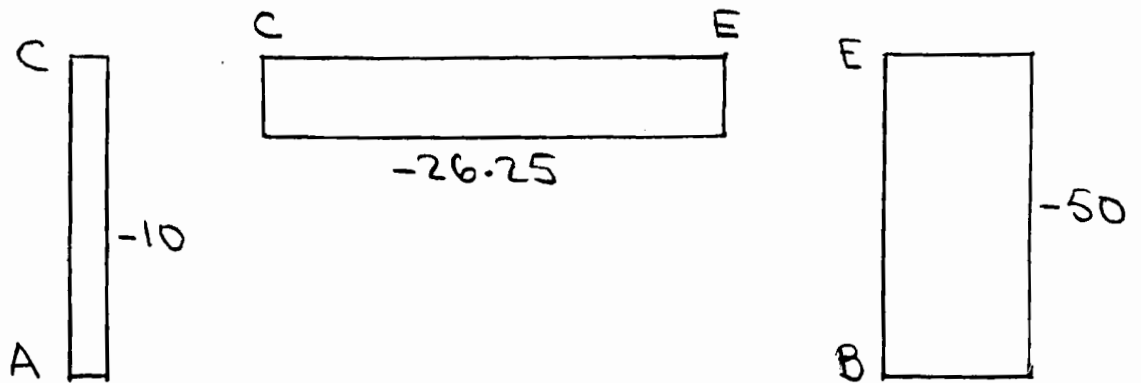
5.69 (contd.)



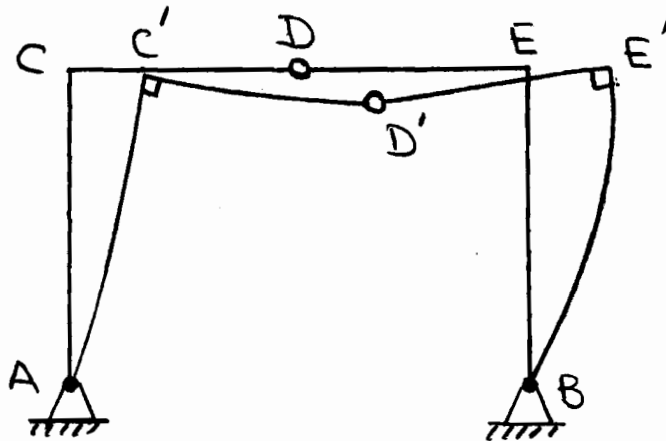
5.70



5.70 (contd.)

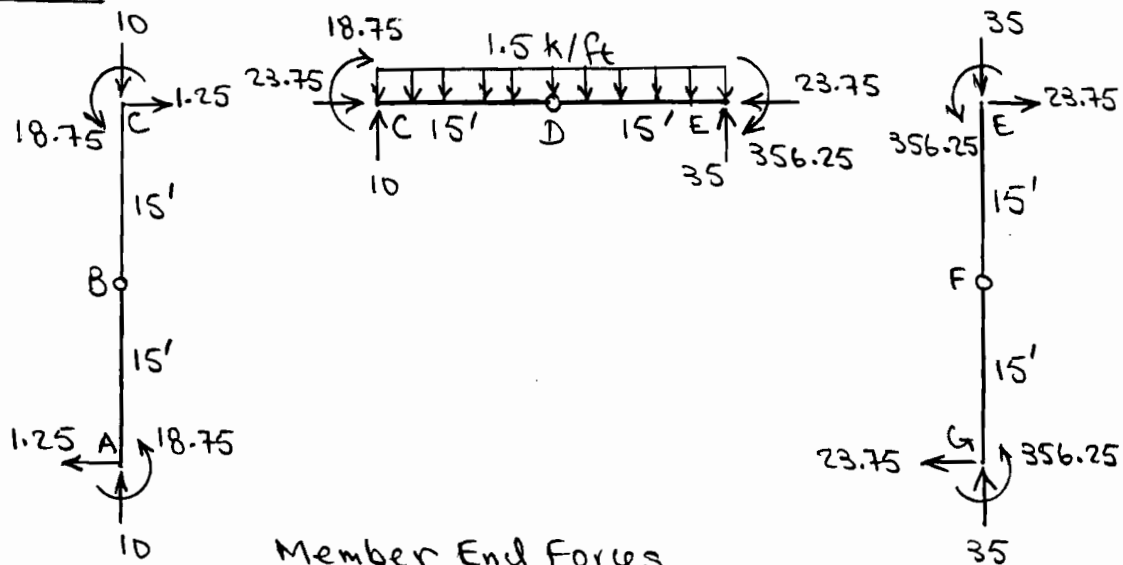


Axial Force Diagrams (k)

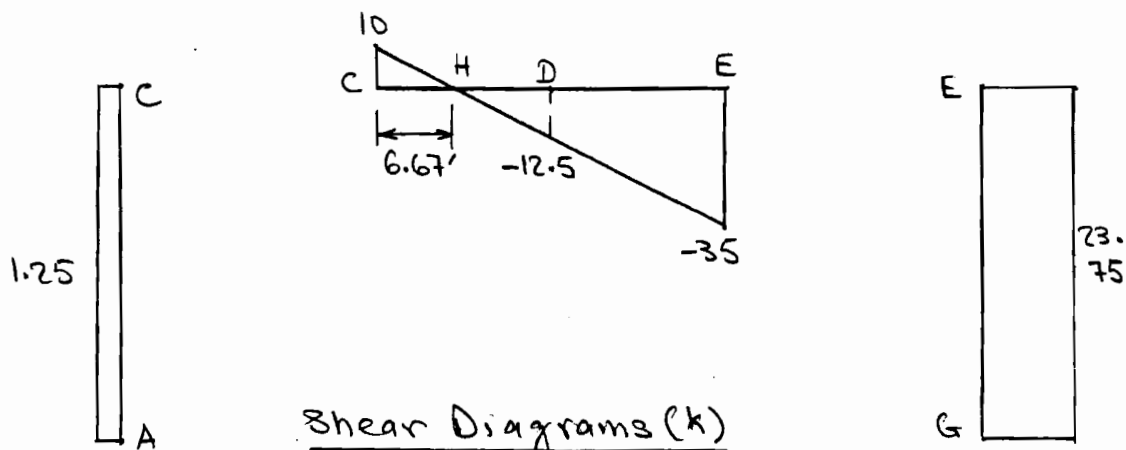


Qualitative Deflected Shape

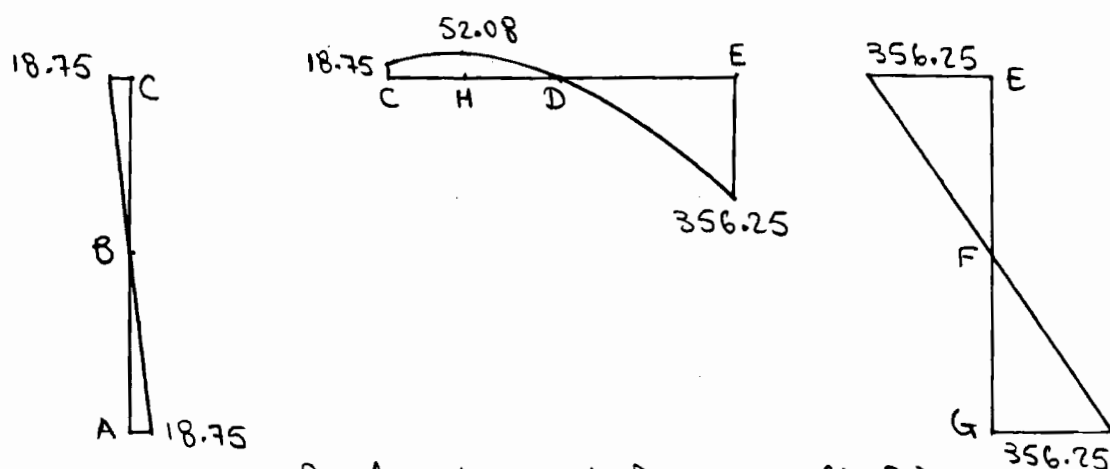
5.71



Member End Forces

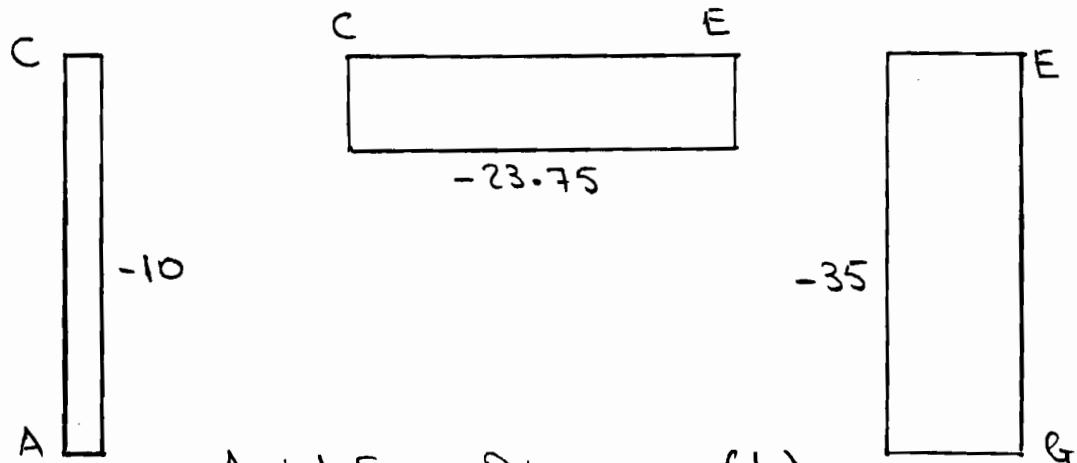


Shear Diagrams (k)

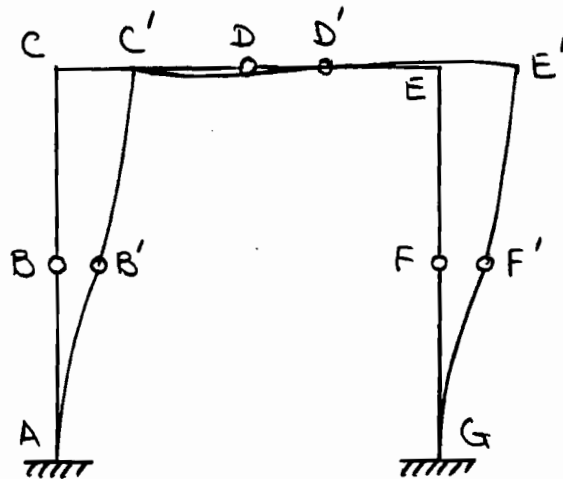


Bending Moment Diagrams (k-ft)

5.71 (Contd.)



Axial Force Diagrams (k)



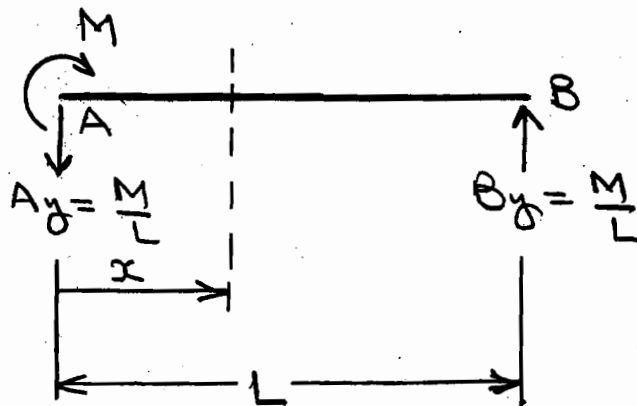
Qualitative Deflected Shape

Chapter Six

Deflections of Beams: Geometric Methods

CHAPTER 6

6.1



$$M_x = M - \frac{M}{L}(x) = \frac{M}{L}(L-x)$$

$$\frac{d^2 y}{dx^2} = \frac{M}{EIL}(L-x)$$

$$\Theta = \frac{dy}{dx} = \int \frac{M}{EIL}(L-x) dx = \frac{M}{EIL} \left(Lx - \frac{x^2}{2} \right) + C_1$$

$$\begin{aligned} y &= \int \left[\frac{M}{EIL} \left(Lx - \frac{x^2}{2} \right) + C_1 \right] dx \\ &= \frac{M}{EIL} \left(\frac{Lx^2}{2} - \frac{x^3}{6} \right) + C_1 x + C_2 \end{aligned}$$

Boundary conditions:

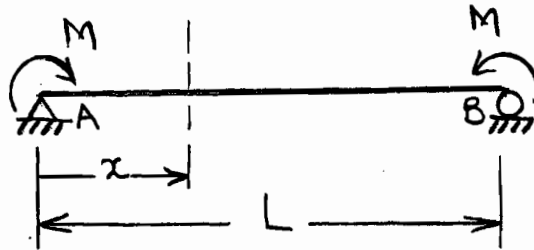
$$\text{at } x=0, \quad y=0 \quad \therefore C_2=0$$

$$\text{at } x=L, \quad y=0 \quad \therefore C_1 = -\frac{ML}{3EI}$$

$$\Theta = -\frac{M}{6EIL} (3x^2 - 6Lx + 2L^2)$$

$$y = -\frac{M}{6EIL} (x^3 - 3Lx^2 + 2L^2x)$$

6.2



$$M_x = M$$

$$\frac{d^2 y}{dx^2} = \frac{M}{EI}$$

$$\theta = \frac{dy}{dx} = \int \frac{M}{EI} dx = \frac{Mx}{EI} + C_1$$

$$y = \int \left(\frac{Mx}{EI} + C_1 \right) dx = \frac{Mx^2}{2EI} + C_1 x + C_2$$

Boundary conditions:

$$\text{at } x=0, y=0$$

$$\therefore C_2 = 0$$

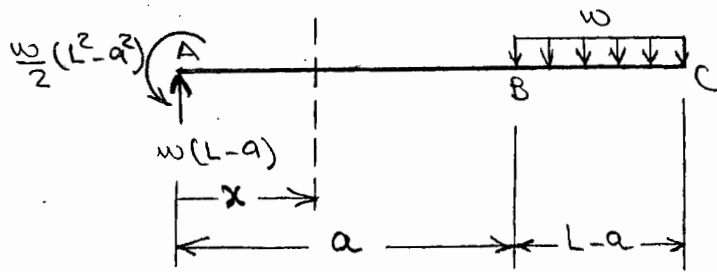
$$\text{at } x=L, y=0$$

$$\therefore C_1 = -\frac{ML}{2EI}$$

$$\theta = \frac{M}{EI} \left(x - \frac{L}{2} \right) ;$$

$$y = \frac{Mx}{2EI} (x - L)$$

6.3



Segment AB: $0 \leq x \leq a$

$$M = -\frac{w}{2}(L^2 - a^2) + w(L-a)x$$

$$\frac{d^2 y}{dx^2} = \frac{1}{EI} \left[-\frac{w}{2}(L^2 - a^2) + w(L-a)x \right]$$

$$EI \theta = -\frac{w}{2}(L^2 - a^2)x + w(L-a)\frac{x^2}{2} + C_1$$

$$EI y = -\frac{w}{2}(L^2 - a^2)\frac{x^2}{2} + w(L-a)\frac{x^3}{6} + C_1 x + C_2$$

Using the boundary conditions, $\theta = 0$ and $y = 0$ at $x = 0$, we obtain $C_1 = 0$ and $C_2 = 0$

Thus,

$$\theta = \frac{wx}{2EI} [a^2 - L^2 + (L-a)x]$$

$$y = \frac{wx^2}{2EI} \left[\frac{a^2 - L^2}{2} + \frac{(L-a)x}{3} \right]$$

Segment BC: $a \leq x \leq L$

$$M = -\frac{w}{2}(L^2 - a^2) + w(L-a)x - \frac{w}{2}(x-a)^2$$

$$EI \theta = -\frac{w}{2}(L^2 - a^2)x + w(L-a)\frac{x^2}{2} - \frac{w}{2}\left(\frac{x^3}{3} + a^2x - ax^2\right) + C_3$$

$$EI y = -\frac{w}{2}(L^2 - a^2)\frac{x^2}{2} + w(L-a)\frac{x^3}{6} - \frac{w}{2}\left(\frac{x^4}{12} + \frac{a^2x^2}{2} - \frac{ax^3}{3}\right) + C_3 x + C_4$$

By using the conditions that at $x = a$, $\theta_{B, \text{Left}} = \theta_{B, \text{Right}}$

and $y_{B, \text{Left}} = y_{B, \text{Right}}$, we obtain:

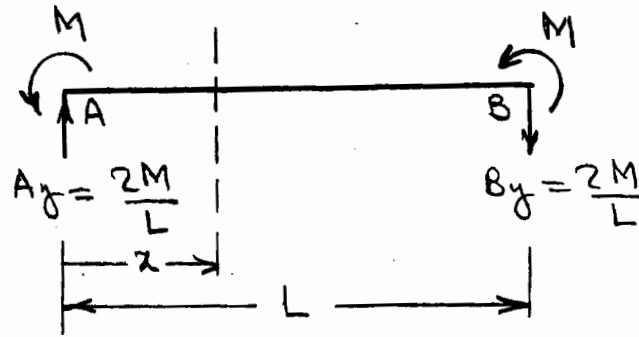
$$C_3 = \frac{wa^3}{6} \text{ and } C_4 = -\frac{wa^4}{24}$$

Thus

$$\theta = \frac{w}{2EI} \left[xL(x-L) - \frac{x^3}{3} + \frac{a^3}{3} \right]$$

$$y = \frac{w}{2EI} \left[x^2L\left(\frac{x}{3} - \frac{L}{2}\right) - \frac{x^4}{12} - \frac{a^4}{12} + \frac{a^3x}{3} \right]$$

6.4



$$M_x = \frac{2M}{L}x - M$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \left(\frac{2x}{L} - 1 \right)$$

$$\theta = \frac{dy}{dx} = \int \frac{M}{EI} \left(\frac{2x}{L} - 1 \right) dx = \frac{Mx}{EI} \left(\frac{x}{L} - 1 \right) + C_1$$

$$y = \int \left[\frac{Mx}{EI} \left(\frac{x}{L} - 1 \right) + C_1 \right] dx = \frac{Mx^2}{EI} \left(\frac{x}{3L} - \frac{1}{2} \right) + C_1x + C_2$$

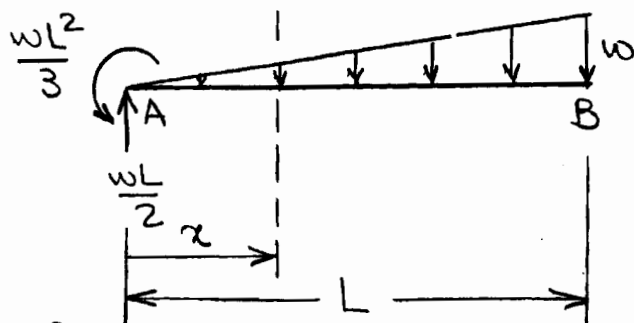
Boundary conditions:

$$\text{at } x=0, y=0 \quad \therefore C_2 = 0$$

$$\text{at } x=L, y=0 \quad \therefore C_1 = \frac{ML}{6EI}$$

$$\theta = \frac{M}{6EIL} (6x^2 - 6Lx + L^2); \quad y = \frac{Mx}{6EIL} (2x^2 - 3Lx + L^2)$$

6.5



$$M = -\frac{wL^2}{3} + \frac{wL}{2}x - \frac{1}{2}x\left(\frac{wx}{L}\right)\frac{x}{3}$$

$$EI \frac{d^2y}{dx^2} = -\frac{wL^2}{3} + \frac{wL}{2}x - \frac{wx^3}{6L}$$

$$EI \theta = -\frac{wL^2}{3}x + \frac{wL}{2}\left(\frac{x^2}{2}\right) - \frac{w}{6L}\left(\frac{x^4}{4}\right) + C_1$$

$$EI y = -\frac{wL^2}{3}\left(\frac{x^2}{2}\right) + \frac{wL}{2}\left(\frac{x^3}{6}\right) + \frac{w}{6L}\left(\frac{x^5}{20}\right) + C_1x + C_2$$

Using the boundary conditions that at $x=0$,

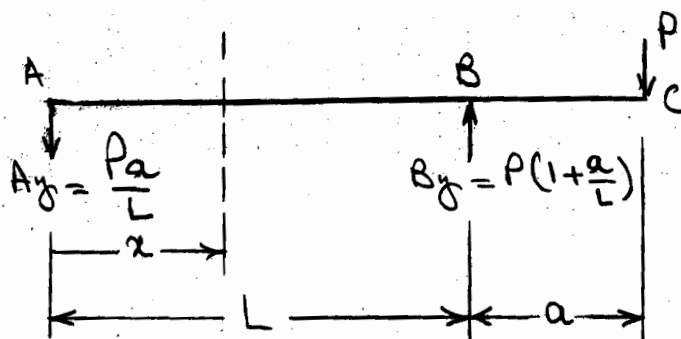
$\theta=0$ and $y=0$, we obtain $C_1=0$ and $C_2=0$.

Thus,

$$\theta = \frac{wx}{24EIL} (-8L^3 + 6L^2x - x^3)$$

$$y = \frac{wx^2}{120EIL} (-20L^3 + 10L^2x - x^3)$$

6.6



Segment AB: $0 \leq x \leq L$

$$M = -\frac{Pa}{L}x$$

$$\frac{d^2y}{dx^2} = -\frac{Pa}{EIL}$$

$$\Theta = -\frac{Pa}{EIL} \left(\frac{x^2}{2} \right) + C_1$$

$$y = -\frac{Pa}{2EIL} \left(\frac{x^3}{3} \right) + C_1x + C_2$$

Applying the boundary conditions, $y = 0$ at $x = 0$ and $y = 0$ at $x = L$, we obtain

$$C_2 = 0 \quad C_1 = \frac{PaL}{6EI}$$

Thus,

$$\Theta = \frac{PaL}{6EI} \left(1 - 3\frac{x^2}{L^2} \right); \quad y = \frac{PaLx}{6EI} \left(1 - \frac{x^2}{L^2} \right)$$

Segment BC: $L \leq x \leq (L+a)$

$$M = -P(L+a-x)$$

$$\frac{d^2y}{dx^2} = -\frac{P}{EI}$$

$$\Theta = -\frac{P}{EI} \left(Lx + ax - \frac{x^2}{2} \right) + C_3$$

$$y = -\frac{P}{EI} \left(\frac{Lx^2}{2} + \frac{ax^2}{2} - \frac{x^3}{6} \right) + C_3x + C_4$$

6.6 (contd.)

By using the condition that at $x=L$,

$\theta_{B, \text{Left}} = \theta_{B, \text{Right}}$, we obtain

$$C_3 = \frac{PL}{EI} \left(\frac{L}{2} + \frac{2a}{3} \right)$$

Next, by applying the condition that at $x=L$, $y=0$, we obtain

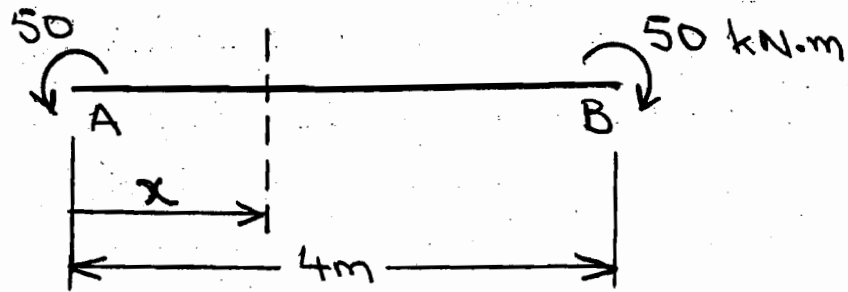
$$C_4 = -\frac{PL^2}{6EI} (L+a)$$

Thus,

$$\theta = \frac{PL}{EI} \left[\frac{x^2}{2L} - x \left(1 + \frac{a}{L} \right) + \frac{L}{2} + \frac{2a}{3} \right]$$

$$y = \frac{PL}{EI} \left[\frac{x^3}{6L} - \frac{x^2}{2} \left(1 + \frac{a}{L} \right) + x \left(\frac{L}{2} + \frac{2a}{3} \right) - \frac{L}{6} (L+a) \right]$$

6.7



$$M_x = -50$$

$$\frac{d^2y}{dx^2} = -\frac{50}{EI}$$

$$\theta = \frac{dy}{dx} = -\frac{50}{EI}x + C_1$$

$$y = -\frac{25}{EI}x^2 + C_1x + C_2$$

Boundary Conditions:

$$\text{at } x=0, \theta=0 \therefore C_1=0$$

$$\text{at } x=0, y=0 \therefore C_2=0$$

Thus,

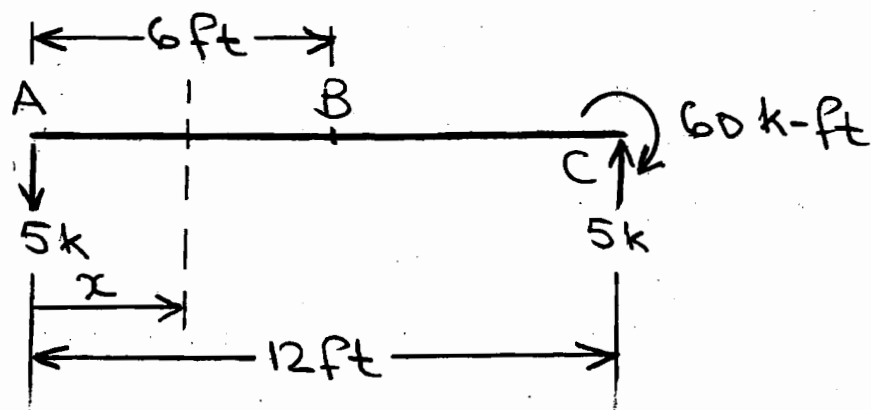
$$\theta = -\frac{50x}{EI} ; y = -\frac{25x^2}{EI}$$

At $x=4\text{m}$:

$$\theta = -\frac{50(4)}{70(164)} = -0.0174 \text{ rad} = \underline{0.0174 \text{ rad} \nabla}$$

$$y = -\frac{25(4)^2}{70(164)} = -0.0348 \text{ m} = \underline{34.8 \text{ mm} \downarrow}$$

6.8



$$M_x = -5x$$

$$\frac{d^2y}{dx^2} = -\frac{5x}{EI}$$

$$\theta = \frac{dy}{dx} = -\frac{5x^2}{2EI} + C_1$$

$$y = -\frac{5x^3}{6EI} + C_1x + C_2$$

Boundary Conditions:

$$\text{at } x=0, y=0 \quad \therefore C_2=0$$

$$\text{at } x=12', y=0 \quad \therefore C_1 = \frac{120}{EI}$$

Thus,

$$\theta = \frac{5}{2EI} (-x^2 + 48)$$

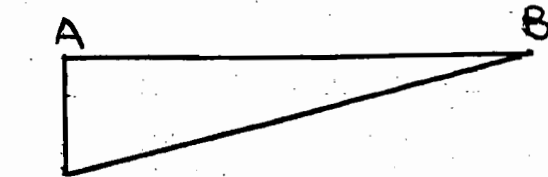
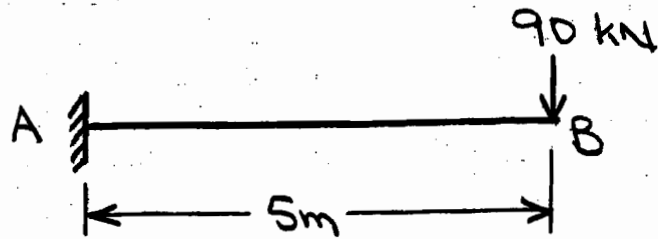
$$y = \frac{5x}{6EI} (-x^2 + 144)$$

By substituting $x=6\text{ ft}$, $E=10000(144)\text{ ksf}$
and $I = \frac{800}{(12)^4}\text{ ft}^4$, we obtain

$$\theta = 0.00054\text{ rad } \nearrow$$

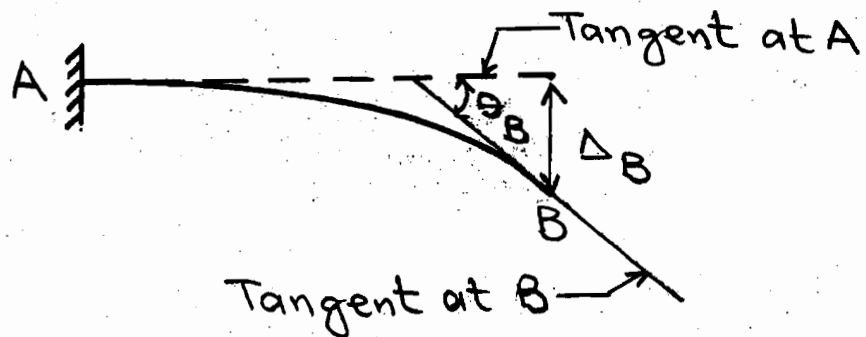
$$y = 0.00972\text{ ft} = 0.117\text{ in. } \uparrow$$

6.9



$\frac{450}{EI}$

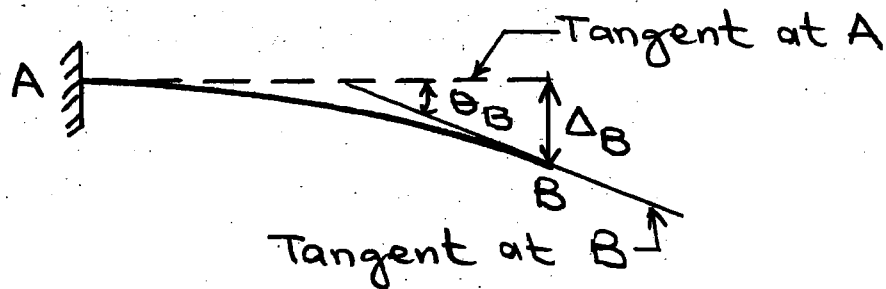
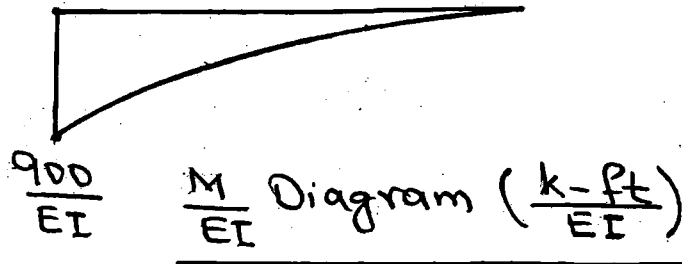
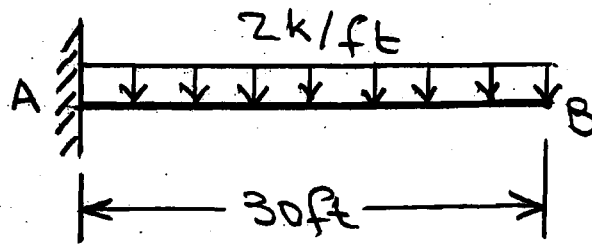
$\frac{M}{EI}$ Diagram ($\frac{kN \cdot m}{EI}$)



$$\begin{aligned}\theta_B = \theta_{BA} &= \frac{1}{2}(5)\left(\frac{450}{EI}\right) = \frac{1125 \text{ kN} \cdot \text{m}^2}{EI} \\ &= \frac{1125}{200(800)} = \underline{0.00703 \text{ rad} \downarrow}\end{aligned}$$

$$\begin{aligned}\Delta_B = \Delta_{BA} &= \frac{1125}{EI} \left(\frac{10}{3}\right) = \frac{3750 \text{ kN} \cdot \text{m}^3}{EI} \\ &= \frac{3750}{200(800)} = 0.0234 \text{ m} = \underline{23.4 \text{ mm} \downarrow}\end{aligned}$$

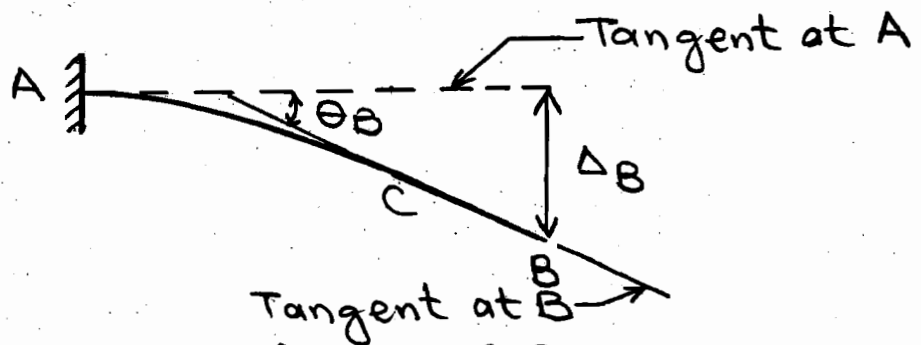
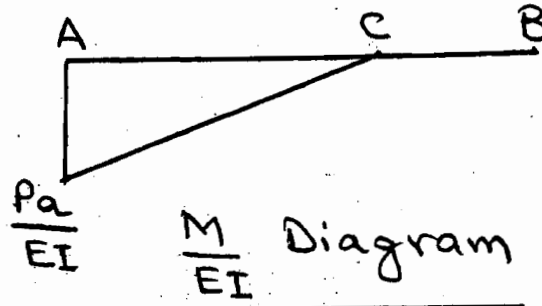
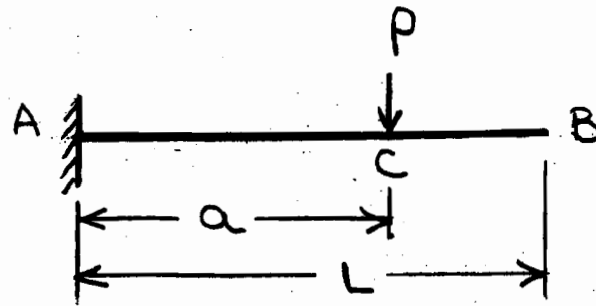
6.10



$$\begin{aligned}\theta_B = \theta_{BA} &= \frac{1}{3}(30)\left(\frac{900}{EI}\right) = \frac{9000 \text{ k-ft}^2}{EI} \\ &= \frac{9000(12)^2}{29000(3000)} = \underline{0.0149 \text{ rad} \downarrow}\end{aligned}$$

$$\begin{aligned}\Delta_B = \Delta_{BA} &= \frac{9000}{EI} \left(\frac{90}{4}\right) = \frac{202500 \text{ k-ft}^3}{EI} \\ &= \frac{202500(12)^3}{29000(3000)} = \underline{4.022 \text{ in.} \downarrow}\end{aligned}$$

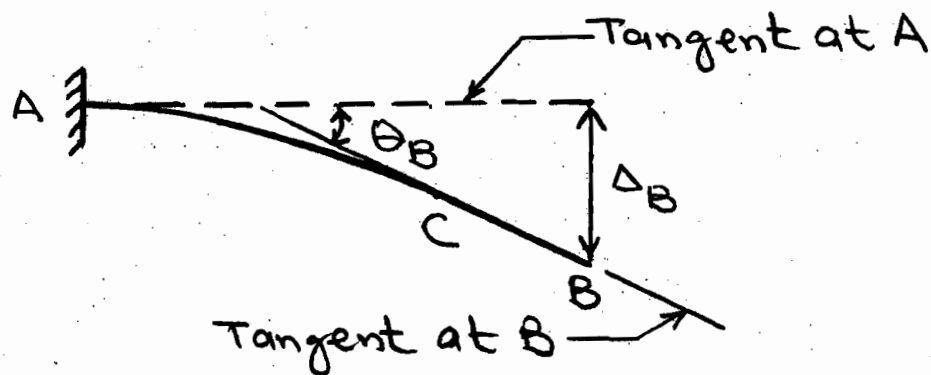
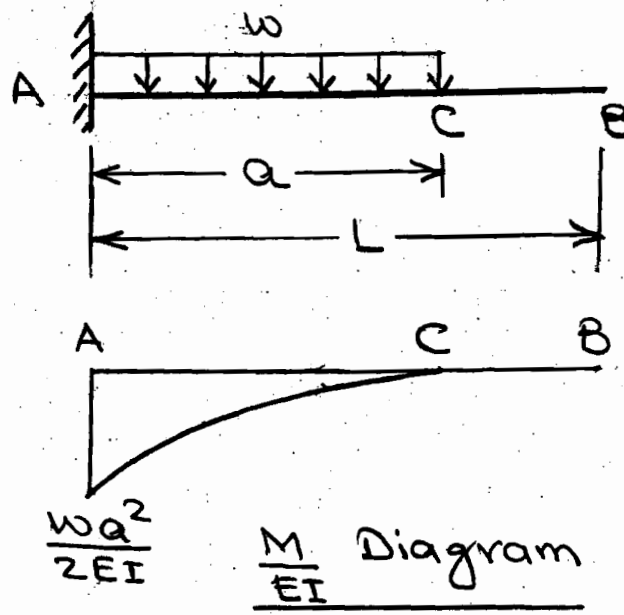
G.11



$$\theta_B = \theta_{BA} = \frac{1}{2}(a) \frac{Pa}{EI} = \frac{Pa^2}{2EI} \quad \nabla$$

$$\Delta_B = \Delta_{BA} = \frac{Pa^2}{2EI} \left(L - \frac{a}{3} \right) = \frac{Pa^2}{6EI} (3L - a) \downarrow$$

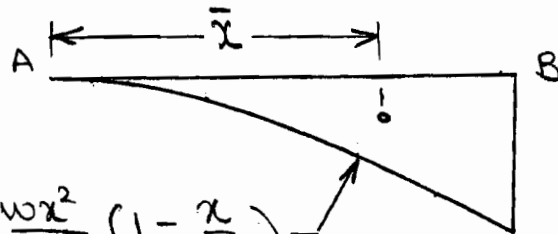
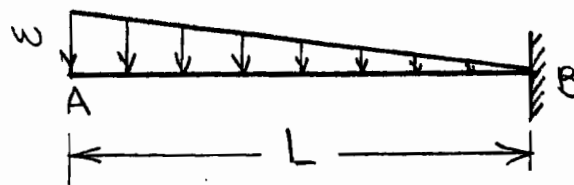
6.12



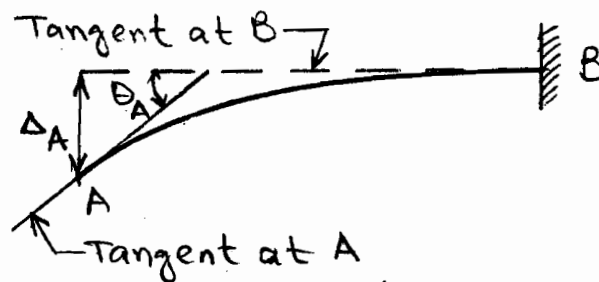
$$\theta_B = \theta_{BA} = \frac{1}{3}(a) \left(\frac{wa^2}{2EI} \right) = \frac{wa^3}{6EI} \quad \nabla$$

$$\Delta_B = \Delta_{BA} = \frac{wa^3}{6EI} \left(L - \frac{a}{4} \right) = \frac{wa^3}{24EI} (4L - a) \quad \downarrow$$

6.13



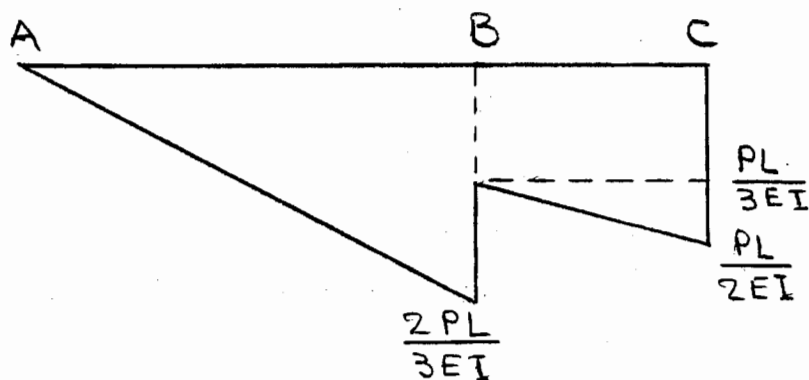
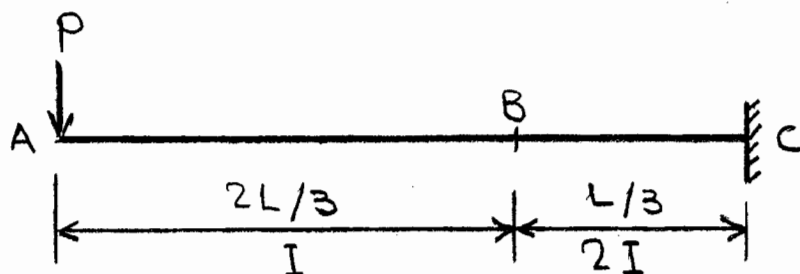
$$\frac{M}{EI} = \frac{wx^2}{2EI} \left(1 - \frac{x}{3L}\right)$$



$$\theta_A = \theta_{AB} = \int_0^L \frac{M}{EI} dx = \int_0^L \frac{wx^2}{2EI} \left(1 - \frac{x}{3L}\right) dx = \frac{WL^3}{8EI} \quad \swarrow$$

$$\begin{aligned} \Delta_A = \Delta_{AB} &= \int_0^L \frac{M}{EI} (x) dx = \int_0^L \frac{wx^2}{2EI} \left(1 - \frac{x}{3L}\right) x dx \\ &= \frac{11}{120} \frac{WL^4}{EI} \quad \downarrow \end{aligned}$$

6.14



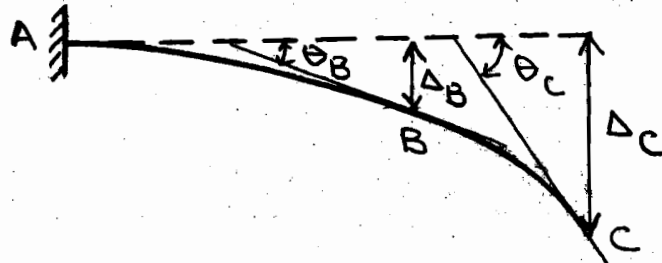
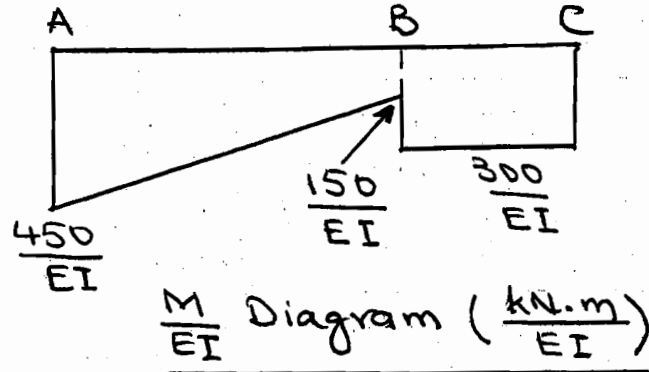
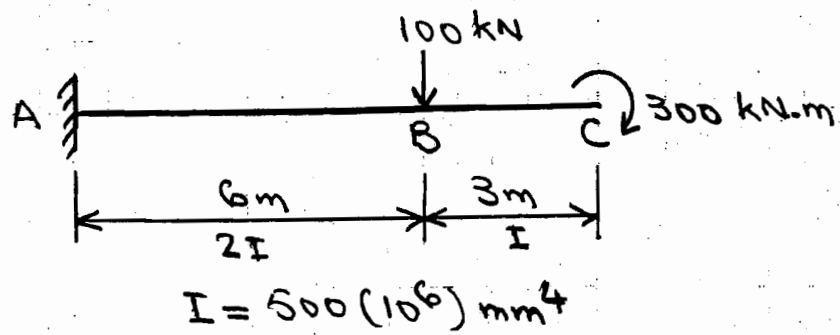
M/EI Diagram



$$\theta_A = \theta_{AC} = \frac{1}{2} \left(\frac{2L}{3} \right) \left(\frac{2PL}{3EI} \right) + \frac{L}{3} \left(\frac{PL}{3EI} \right) + \frac{1}{2} \left(\frac{L}{3} \right) \left(\frac{PL}{6EI} \right) = \frac{13PL^2}{36EI} \quad \swarrow$$

$$\Delta_A = \Delta_{AC} = \frac{2PL^2}{9EI} \left(\frac{4L}{9} \right) + \frac{PL^2}{9EI} \left(\frac{5L}{6} \right) + \frac{PL^2}{36EI} \left(\frac{8L}{9} \right) = \frac{35PL^3}{162EI} \quad \downarrow$$

6.15



$$\theta_B = \theta_{BA} = \frac{1}{EI} \left[150(6) + \frac{1}{2}(300)(6) \right] = \frac{1800 \text{ kN.m}^2}{EI}$$

$$= \frac{1800}{70(500)} = 0.0514 \text{ rad} \searrow$$

$$\Delta_B = \Delta_{BA} = \frac{1}{EI} \left[150(6)3 + \frac{1}{2}(300)(6)4 \right] = \frac{6300 \text{ kN.m}^3}{EI}$$

$$= \frac{6300}{70(500)} = 0.18 \text{ m} = 180 \text{ mm} \downarrow$$

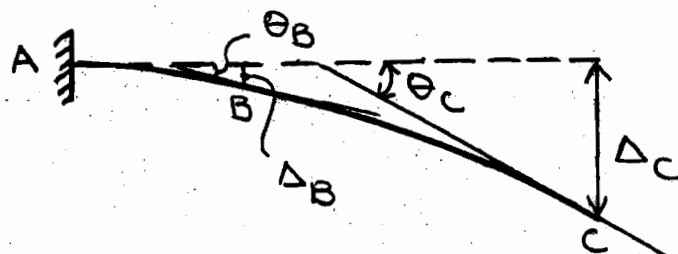
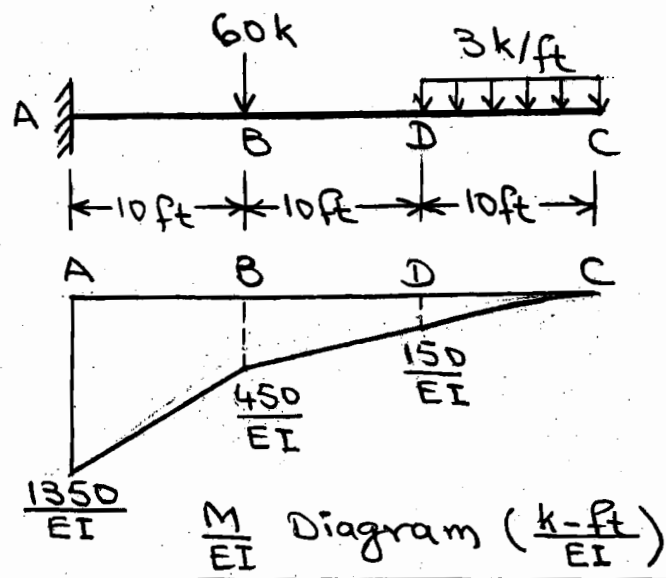
$$\theta_C = \theta_{CA} = \frac{1}{EI} [1800 + 300(3)] = \frac{2700 \text{ kN.m}^2}{EI}$$

$$= \frac{2700}{70(500)} = 0.0771 \text{ rad} \searrow$$

$$\Delta_C = \Delta_{CA} = \frac{1}{EI} \left[150(6)6 + \frac{1}{2}(300)(6)7 + 300(3)(1.5) \right]$$

$$= \frac{13050 \text{ kN.m}^3}{EI} = \frac{13050}{70(500)} = 0.373 \text{ m} = 373 \text{ mm} \downarrow$$

6.16



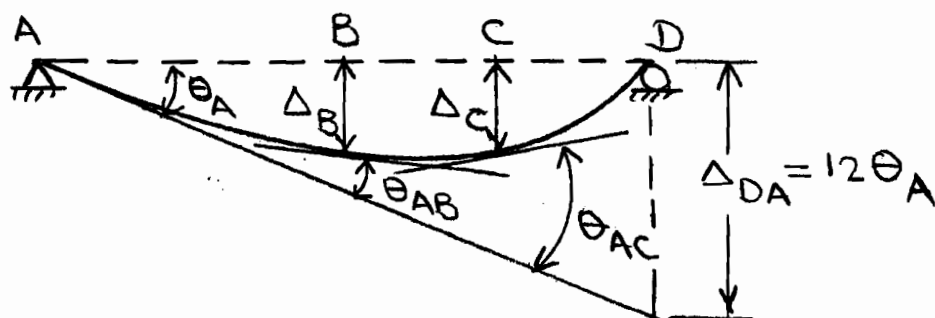
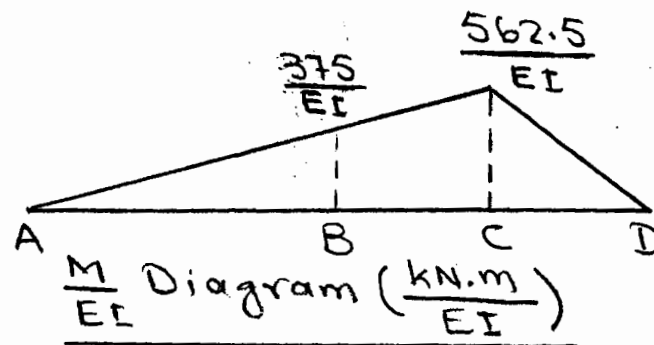
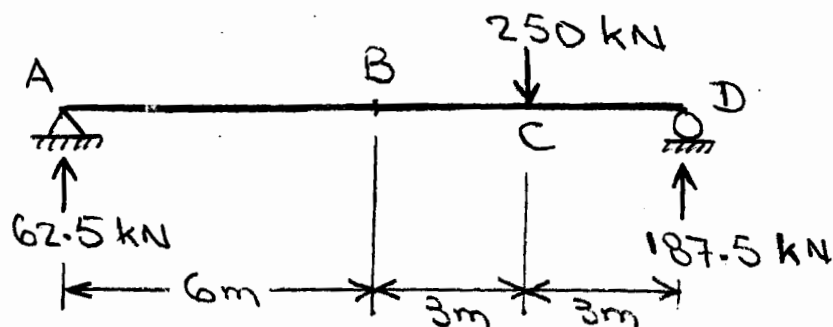
$$\begin{aligned}\theta_B = \theta_{BA} &= \frac{1}{EI} \left[450(10) + \frac{1}{2}(900)(10) \right] = \frac{9000 \text{ k-ft}^2}{EI} \\ &= \frac{9000(12)^2}{29000(4000)} = \underline{0.0112 \text{ rad} \searrow}\end{aligned}$$

$$\begin{aligned}\Delta_B = \Delta_{BA} &= \frac{1}{EI} \left[450(10)(5) + \frac{1}{2}(900)(10)\left(\frac{20}{3}\right) \right] = \frac{52500 \text{ k-ft}^3}{EI} \\ &= \frac{52500(12)^3}{29000(4000)} = \underline{0.782 \text{ in.} \downarrow}\end{aligned}$$

$$\begin{aligned}\theta_C = \theta_{CA} &= \frac{1}{EI} \left[9000 + 150(10) + \frac{1}{2}(300)(10) + \frac{1}{3}(150)(10) \right] \\ &= \frac{12500 \text{ k-ft}^2}{EI} = \frac{12500(12)^2}{29000(4000)} = \underline{0.0155 \text{ rad} \searrow}\end{aligned}$$

$$\begin{aligned}\Delta_C = \Delta_{CA} &= \frac{1}{EI} \left[450(10)(25) + \frac{1}{2}(900)(10)\left(\frac{20}{3} + 20\right) \right. \\ &\quad \left. + 150(10)(15) + \frac{1}{2}(300)(10)\left(\frac{20}{3} + 10\right) + \frac{1}{3}(150)(10)\left(\frac{30}{4}\right) \right] \\ &= \frac{283750 \text{ k-ft}^3}{EI} = \frac{283750(12)^3}{29000(4000)} = \underline{4.227 \text{ in.} \downarrow}\end{aligned}$$

6.17



$$\Delta_{DA} = \frac{1}{EI} \left[\frac{1}{2} (562.5) 9 (6) + \frac{1}{2} (562.5) 3 (2) \right] = \frac{16875}{EI}$$

$$\theta_A = \frac{\Delta_{DA}}{12} = \frac{1406.25}{EI}$$

$$\begin{aligned} \theta_B &= \theta_A - \theta_{AB} = \frac{1}{EI} \left[1406.25 - \frac{1}{2} (375) 6 \right] \\ &= \frac{281.25 \text{ kN.m}^2}{EI} = \frac{281.25}{200(462)} = 0.00304 \text{ rad} \end{aligned}$$

$$\begin{aligned} \Delta_B &= 6\theta_A - \Delta_{BA} = \frac{1}{EI} \left[6(1406.25) - \frac{1}{2} (375) 6 (2) \right] \\ &= \frac{6187.5 \text{ kN.m}^3}{EI} = \frac{6187.5}{200(462)} = 0.067 \text{ m} \end{aligned}$$

$$\Delta_B = 67 \text{ mm} \downarrow$$

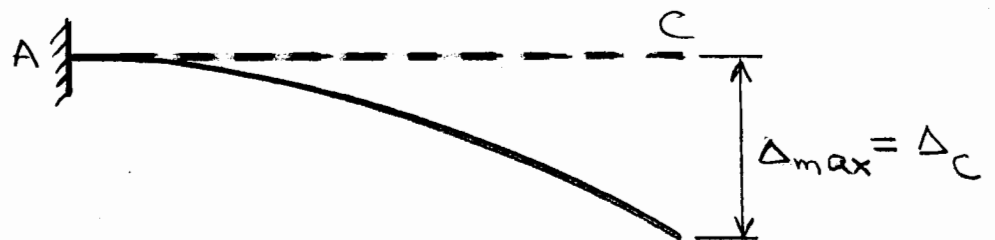
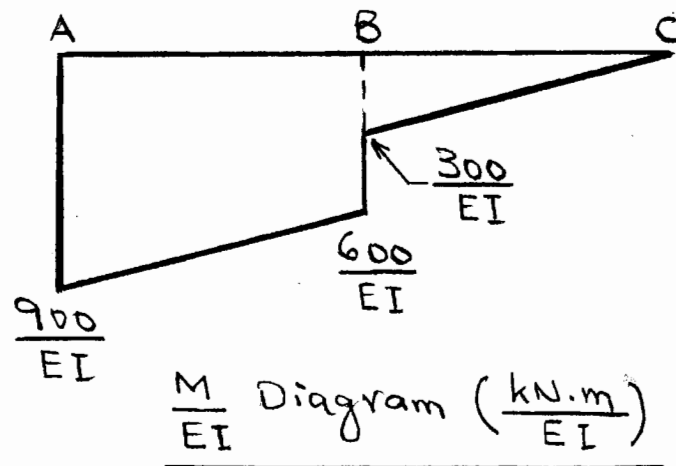
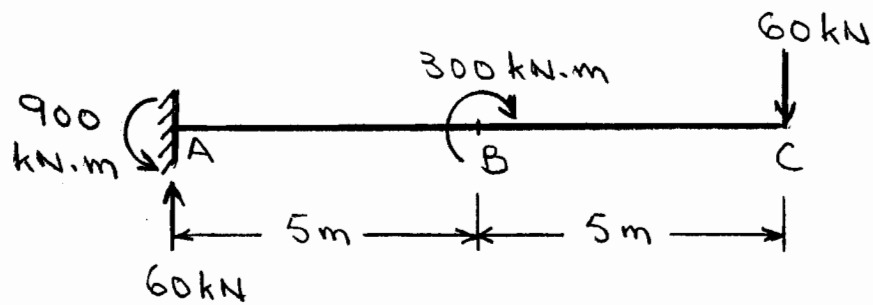
6.17 (contd.)

$$\begin{aligned}\theta_C &= \theta_{AC} - \theta_A = \frac{1}{EI} \left[\frac{1}{2} (562.5) 9 - 1406.25 \right] \\ &= \frac{1125 \text{ kN}\cdot\text{m}^2}{EI} = \frac{1125}{200(462)} = 0.0122 \text{ rad} \swarrow\end{aligned}$$

$$\begin{aligned}\Delta_C &= 9\theta_A - \Delta_{CA} = \frac{1}{EI} \left[9(1406.25) - \frac{1}{2} (562.5) 9(3) \right] \\ &= \frac{5062.5 \text{ kN}\cdot\text{m}^3}{EI} = \frac{5062.5}{200(462)} = 0.0548 \text{ m}\end{aligned}$$

$$\Delta_C = 54.8 \text{ mm} \downarrow$$

6.18



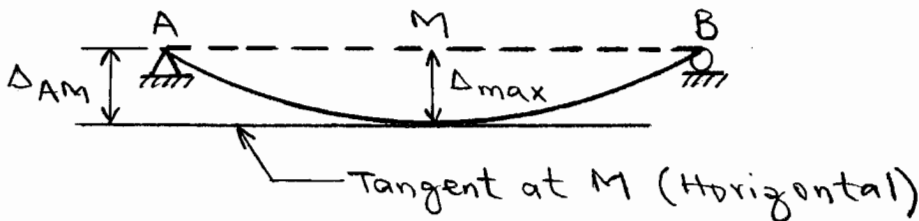
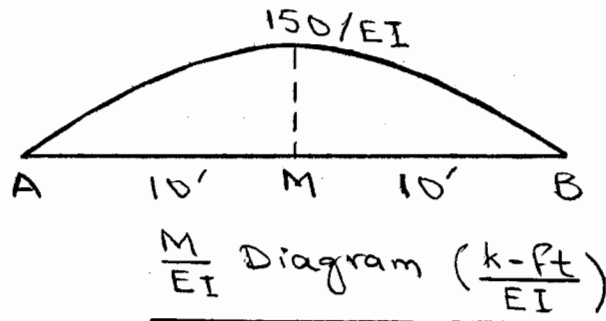
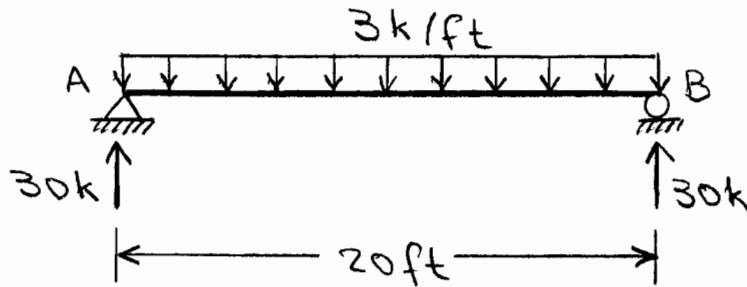
$$\Delta_{\max} = \Delta_C = \Delta_{CA} = \frac{1}{EI} \left[600(5)(7.5) + \frac{1}{2}(300)(5)\left(\frac{25}{3}\right) + \frac{1}{2}(300)(5)\left(\frac{10}{3}\right) \right] = \frac{31250 \text{ kN.m}^3}{EI}$$

$$\Delta_{\max} = \frac{L}{360}$$

$$\frac{31250}{200(10^6)I} = \frac{10}{360}$$

$$\text{From which, } I = 5625(10^6) \text{ m}^4 = \underline{5625(10^6) \text{ mm}^4}$$

6.19



The maximum deflection occurs at the beam midspan, M, where the tangent is horizontal.

$$\Delta_{\max} = \Delta_{AM} = \frac{1}{EI} \left[\frac{2}{3} (150)(10) \left(\frac{50}{8} \right) \right]$$

$$= \frac{6250 \text{ k-ft}^3}{EI}$$

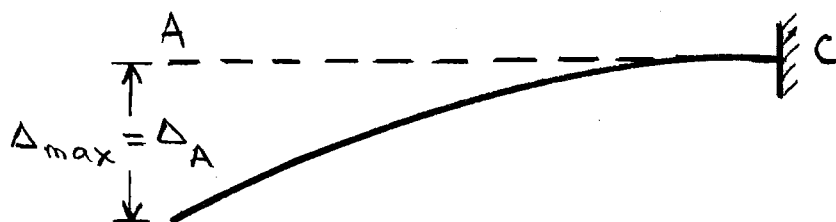
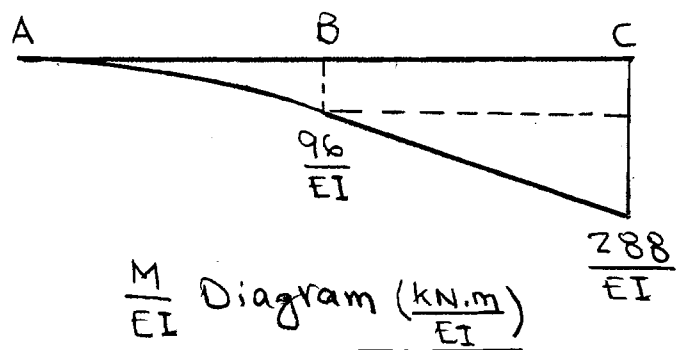
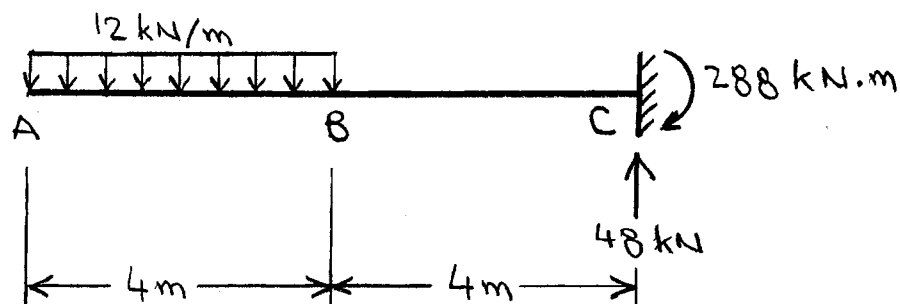
$$\Delta_{\max} = \frac{L}{360}$$

$$\frac{6250(12)^3}{29000(I)} = \frac{20(12)}{360}$$

From which,

$$\underline{I = 559 \text{ in}^4}$$

6.20



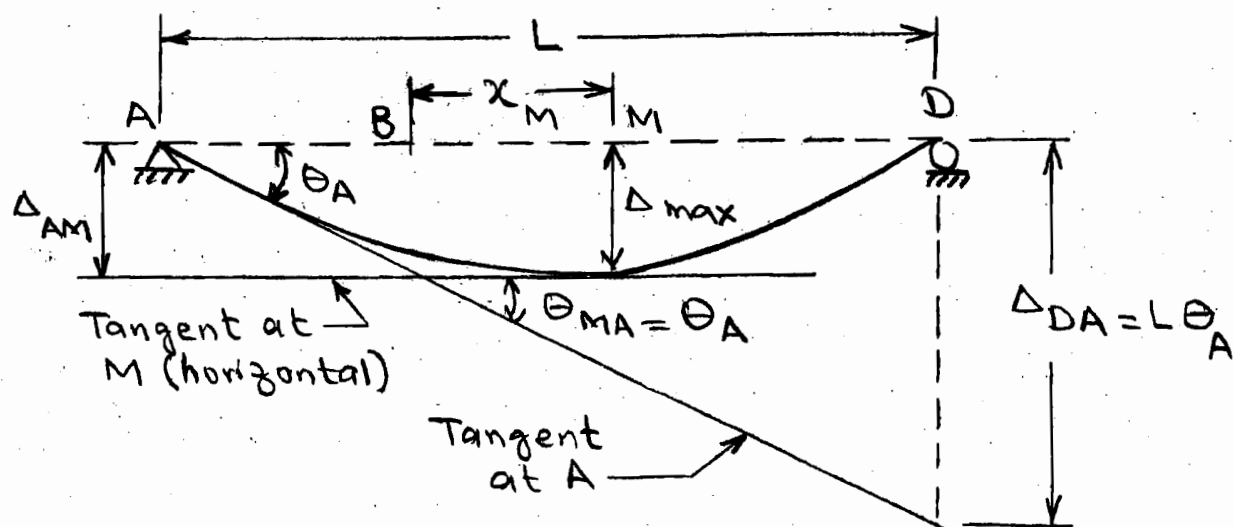
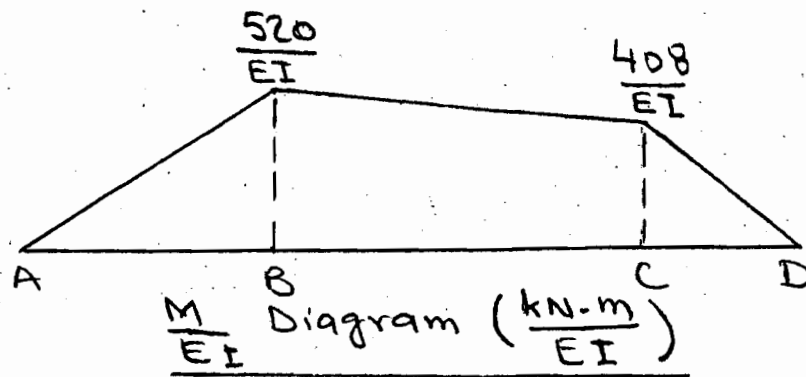
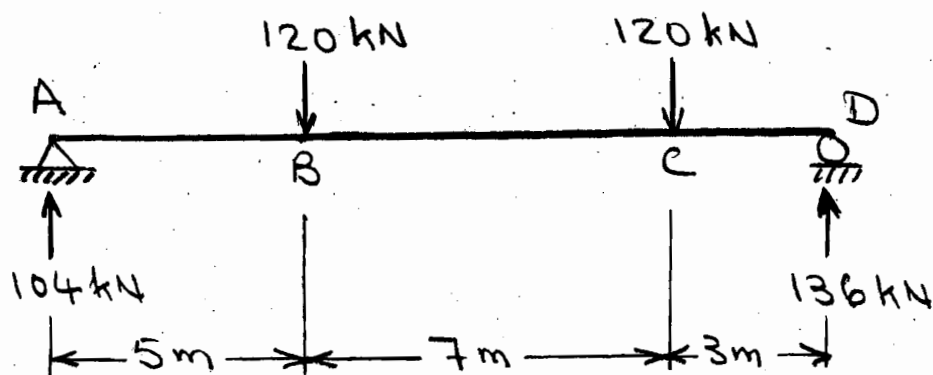
$$\Delta_{\max} = \Delta_A = \Delta_{AC} = \frac{1}{EI} \left[\frac{1}{3} (96)(4)(3) + 96(4)(6) + \frac{1}{2} (192) 4 \left(\frac{8}{3} + 4 \right) \right] = \frac{5248 \text{ kN.m}^3}{EI}$$

$$\Delta_{\max} = \frac{L}{360}$$

$$\frac{5248}{70(10^6)(I)} = \frac{8}{360}$$

$$\text{from which, } I = 3374 (10^6) \text{ m}^4 = \underline{3374 (10^6) \text{ mm}^4}$$

6.21



$$\Delta_{DA} = \frac{1}{EI} \left[\frac{1}{2} (520) (5) \left(\frac{5}{3} + 10 \right) + (408) (7) (6.5) + \frac{1}{2} (112) (7) \left(\frac{14}{3} + 3 \right) + \frac{1}{2} (408) (3) (2) \right]$$

$$= \frac{37960 \text{ kN.m}^3}{EI}$$

$$\theta_A = \frac{\Delta_{DA}}{L} = \frac{37960}{15(EI)} = \frac{2530.67 \text{ kN.m}^2}{EI}$$

6.21 (contd.)

If the maximum deflection occurs at M, then

$$\theta_{MA} = \theta_A$$

$$\frac{1}{EI} \left[\frac{1}{2}(520)(5) + 520x_M - \frac{1}{2} \left(\frac{112}{7} x_M \right) x_M \right] = \frac{2530.67}{EI}$$

$$8x_M^2 - 520x_M + 1230.67 = 0$$

$$x_M = 2.46 \text{ m}$$

$$\Delta_{\max} = \Delta_{AM} = \frac{1}{EI} \left[\frac{1}{2}(520)(5) \left(\frac{10}{3} \right) \right.$$

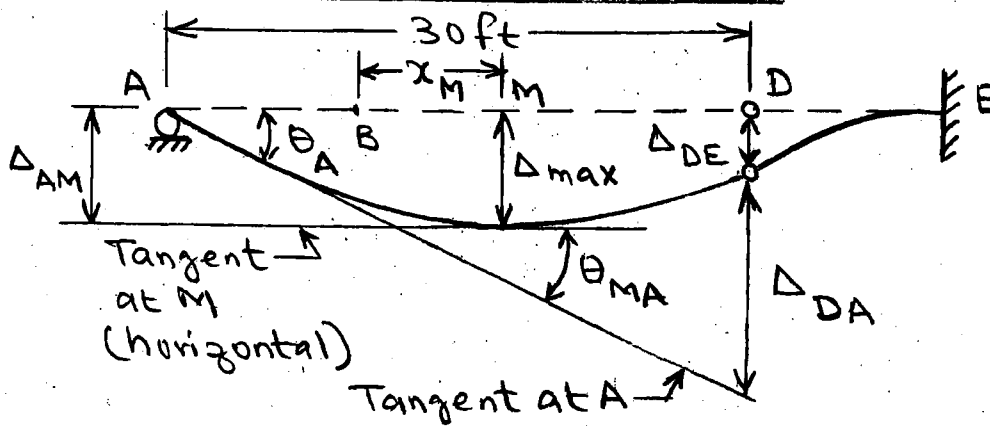
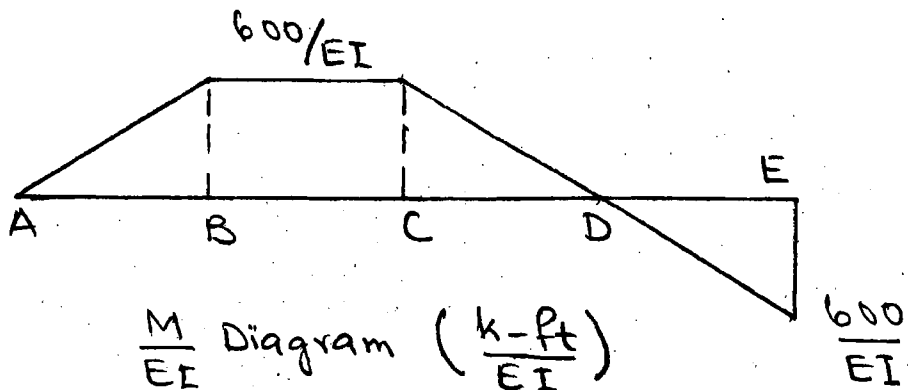
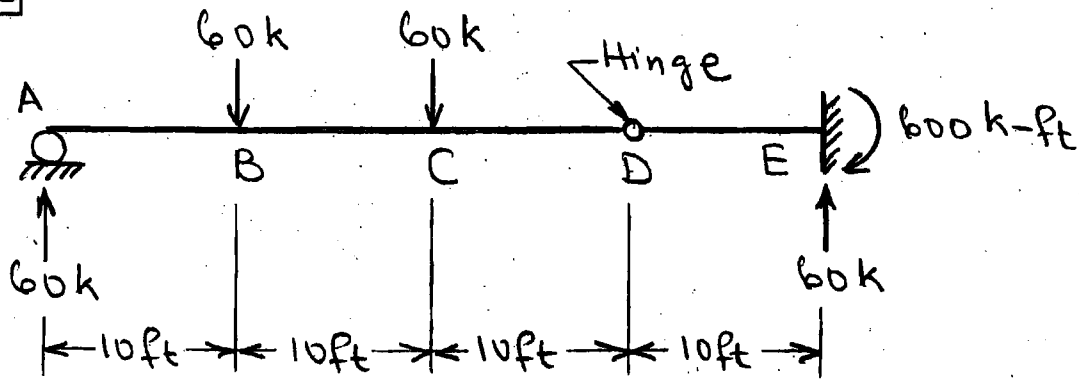
$$+ 520(2.46) \left(\frac{2.46}{2} + 5 \right) - \frac{1}{2} \left(\frac{112}{7} \right) (2.46)(2.46)(6.64) \Big]$$
$$= \frac{11981.29 \text{ kN.m}^3}{EI}$$

$$\Delta_{\max} = \frac{L}{360}$$

$$\frac{11981.29}{30(10^6)I} = \frac{15}{360}$$

$$\text{From which } I = 9585 (10^{-6}) \text{ m}^4 = \underline{9585 (10^6) \text{ mm}^4}$$

6.22



$$\Delta_{DA} = \frac{1}{EI} \left[\frac{1}{2} (600) (10) \left(\frac{10}{3} + 20 \right) + 600 (10) (15) + \frac{1}{2} (600) (10) \left(\frac{20}{3} \right) \right] = \frac{180000 \text{ k-ft}^3}{EI}$$

$$\Delta_{DE} = \frac{1}{EI} \left[\frac{1}{2} (600) (10) \left(\frac{20}{3} \right) \right] = \frac{20000 \text{ k-ft}^3}{EI}$$

$$\theta_A = \frac{\Delta_{DA} + \Delta_{DE}}{30} = \frac{180000 + 20000}{30} = \frac{6666.67}{EI}$$

6.22 (contd.)

If the maximum deflection occurs at M, then

$$\theta_{MA} = \theta_A$$

$$\frac{1}{EI} \left[\frac{1}{2} (600)(10) + 600 x_M \right] = \frac{6666.67}{EI}$$

$$x_M = 6.11 \text{ ft}$$

$$\Delta_{\max} = \Delta_{AM} = \frac{1}{EI} \left[\frac{1}{2} (600)(10) \left(\frac{20}{3} \right) + (600)(6.11)(13.06) \right] = \frac{67878 \text{ k-ft}^3}{EI}$$

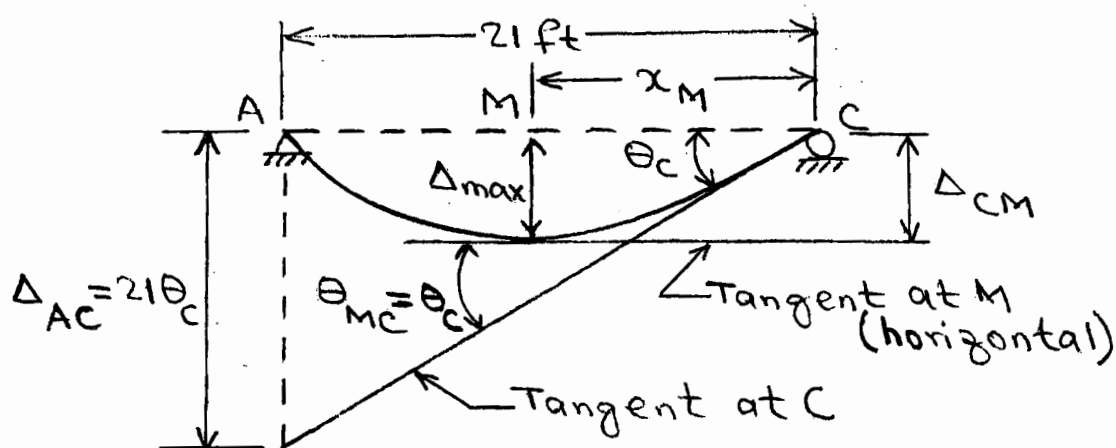
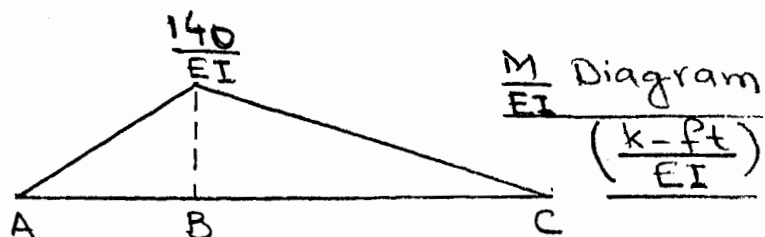
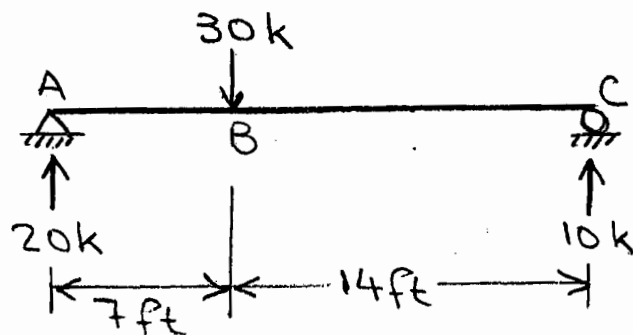
$$\Delta_{\max} = \frac{L}{360}$$

$$\frac{67878 (12)^3}{29000 (I)} = \frac{40(12)}{360}$$

from which

$$\underline{I = 3033 \text{ in}^4}$$

6.23



$$\Delta_{AC} = \frac{1}{EI} \left[\frac{1}{2} (140) 7 \left(\frac{14}{3} \right) + \frac{1}{2} (140) 14 \left(7 + \frac{14}{3} \right) \right] = \frac{13720}{EI}$$

$$\theta_c = \frac{\Delta_{AC}}{21} = \frac{653.33}{EI}$$

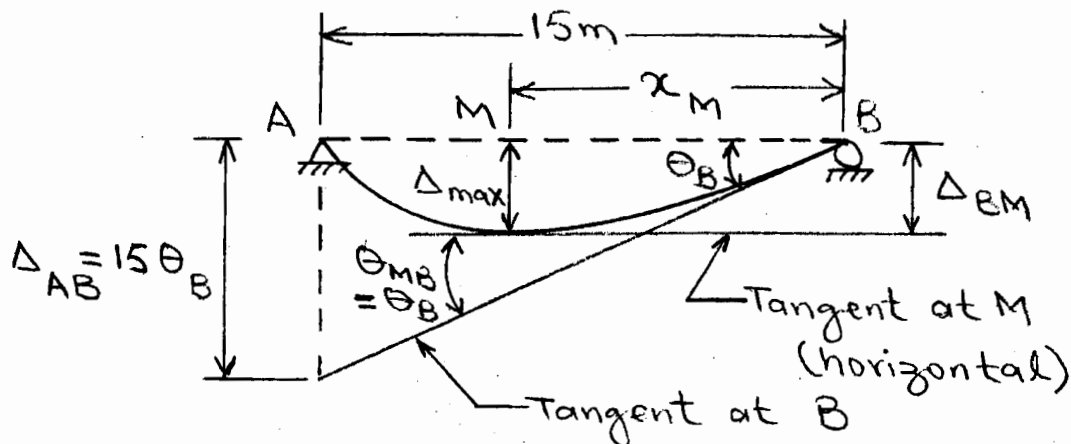
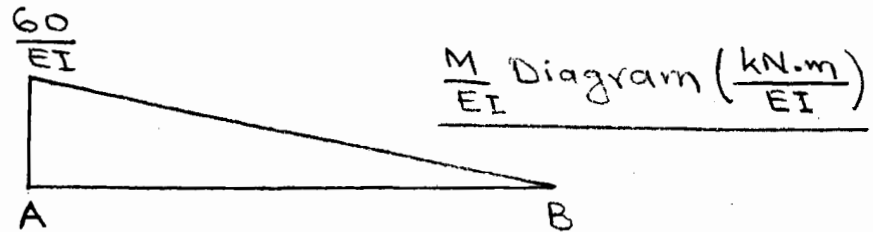
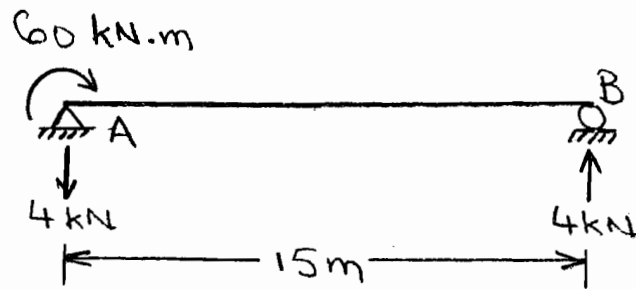
$$\theta_{Mc} = \theta_c$$

$$\frac{1}{EI} \left[\frac{1}{2} \left(\frac{140}{14} x_M \right) x_M \right] = \frac{653.33}{EI}$$

$$\text{from which, } x_M = 11.43 \text{ ft}$$

$$\begin{aligned} \Delta_{max} = \Delta_{cm} &= \frac{1}{EI} \left[\frac{1}{2} (114.3) (11.43) \frac{2}{3} (11.43) \right] \\ &= \frac{4978.81 \text{ k-ft}^3}{EI} = \frac{4978.81 (12)^3}{10000 (500)} = \underline{1.72 \text{ in.} \downarrow} \end{aligned}$$

6.24



$$\Delta_{AB} = \frac{1}{EI} \left[\frac{1}{2} (60) 15 (5) \right] = \frac{2250}{EI}$$

$$\theta_B = \frac{\Delta_{AB}}{15} = \frac{150}{EI}$$

$$\theta_{MB} = \theta_B$$

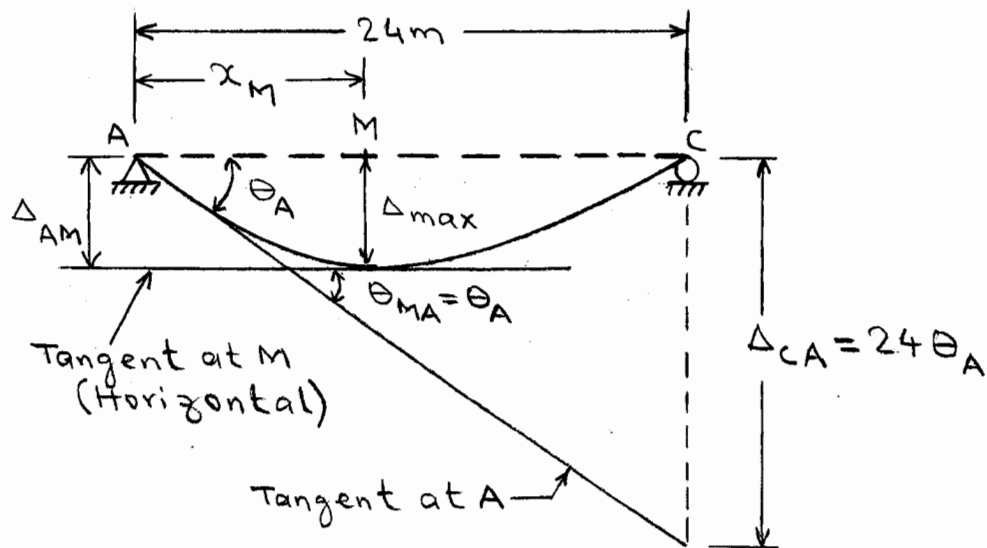
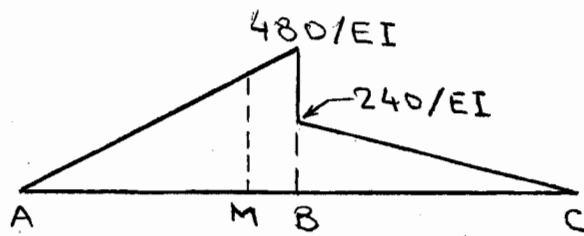
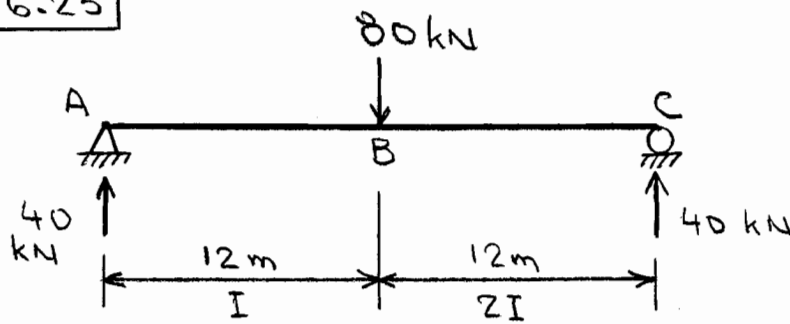
$$\frac{1}{EI} \left[\frac{1}{2} \left(\frac{60}{15} x_M \right) x_M \right] = \frac{150}{EI}$$

$$\text{from which } x_M = 8.66 \text{ m}$$

$$\begin{aligned} \Delta_{\max} = \Delta_{BM} &= \frac{1}{EI} \left[\frac{1}{2} (34.64) (8.66) \frac{2}{3} (8.66) \right] \\ &= \frac{866 \text{ kN.m}^3}{EI} = \frac{866}{70(712)} = 0.0174 \text{ m} \end{aligned}$$

$$\underline{\Delta_{\max} = 17.4 \text{ mm} \downarrow}$$

6.25



$$\Delta_{CA} = \frac{1}{EI} \left[\frac{1}{2} (480)(12)(16) + \frac{1}{2} (240)(12)(8) \right] = \frac{57600 \text{ kN.m}^3}{EI}$$

$$\theta_A = \frac{\Delta_{CA}}{24} = \frac{57600}{24(EI)} = \frac{2400 \text{ kN.m}^2}{EI}$$

If the maximum deflection occurs at M, then

$$\theta_{MA} = \theta_A$$

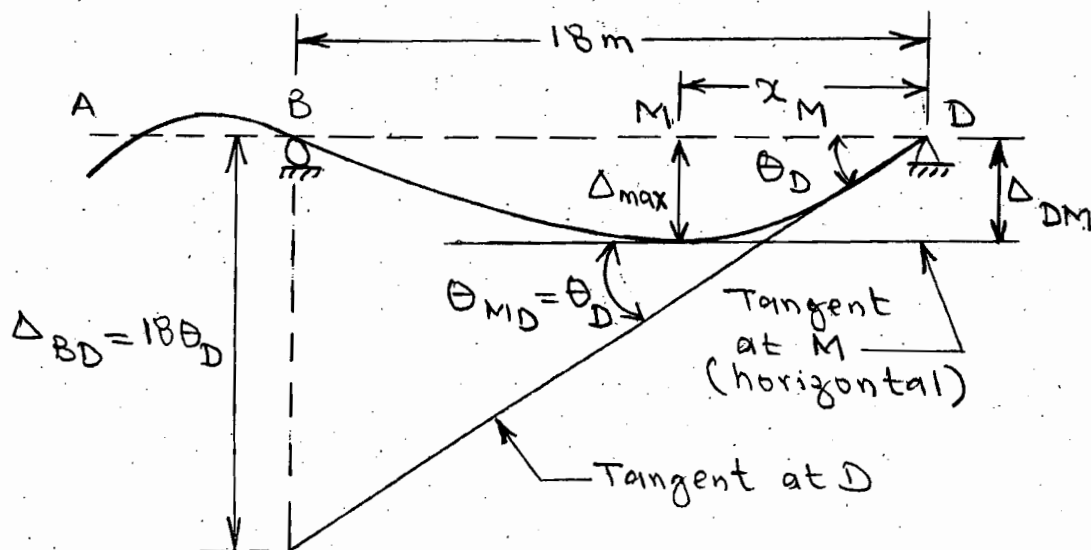
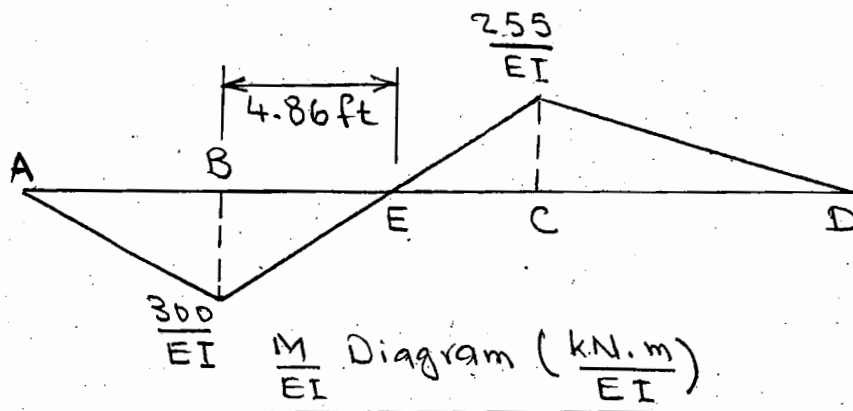
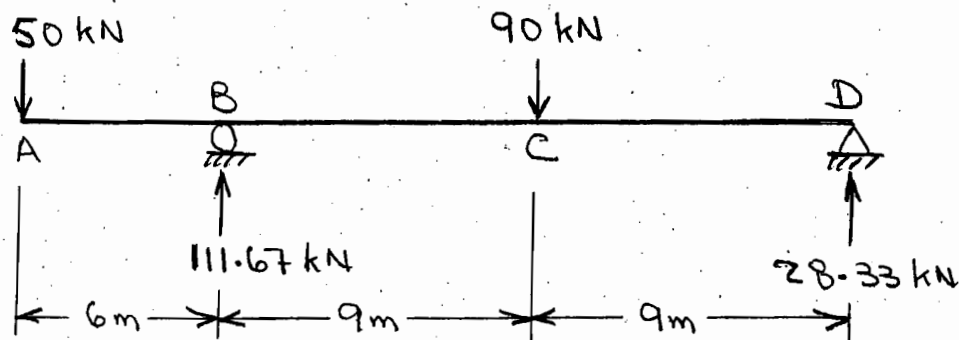
6.25 (contd.)

$$\frac{1}{EI} \left[\frac{1}{2} (40x_M) x_M \right] = \frac{2400}{EI}$$

From which, $x_M = 10.95 \text{ m}$

$$\begin{aligned} \Delta_{\max} = \Delta_{AM} &= \frac{1}{EI} \left[\frac{1}{2} (438) (10.95) \frac{2}{3} (10.95) \right] \\ &= \frac{17506 \text{ kN.m}^3}{EI} = \frac{17506}{200(600)} = 0.146 \text{ m} \\ &\quad \underline{\Delta_{\max} = 146 \text{ mm} \downarrow} \end{aligned}$$

6.26



$$\Delta_{BD} = \frac{1}{EI} \left[-\frac{1}{2} (300) (4.86) \left(\frac{4.86}{3} \right) + \frac{1}{2} (255) (4.14) (7.62) + \frac{1}{2} (255) (9) (12) \right] = \frac{16611.24 \text{ kN} \cdot \text{m}^3}{EI}$$

$$\theta_D = \frac{\Delta_{BD}}{18} = \frac{16611.24}{18(EI)} = \frac{922.85 \text{ kN} \cdot \text{m}^2}{EI}$$

$$\boxed{6.26(\text{contd.})} \quad \theta_{MD} = \theta_D$$

$$\frac{1}{EI} \left[\frac{1}{2} \left(\frac{255}{9} x_M \right) x_M \right] = \frac{922.85}{EI}$$

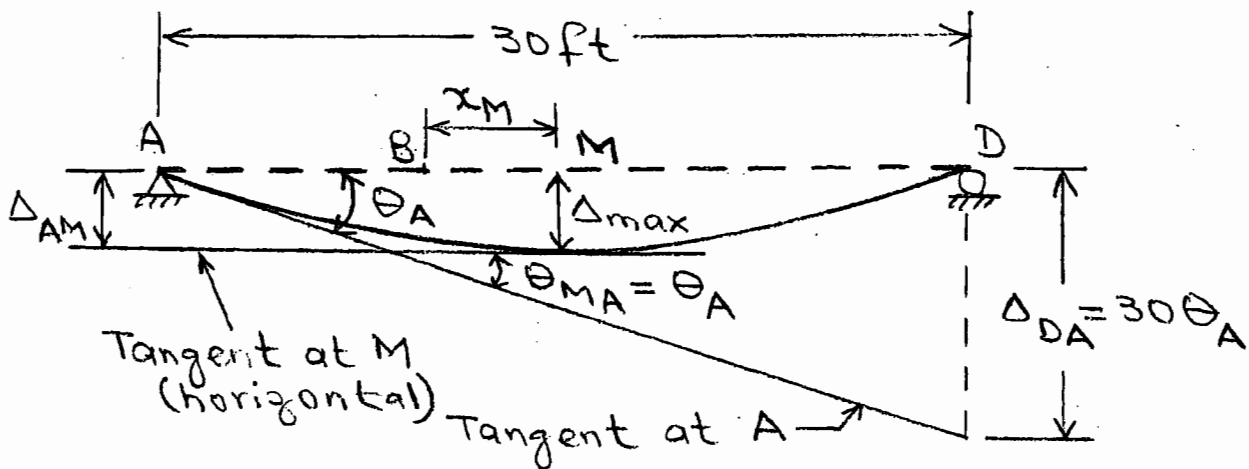
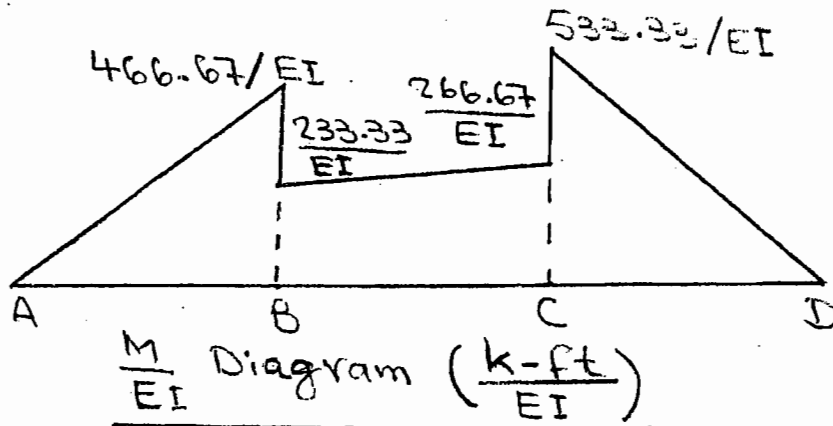
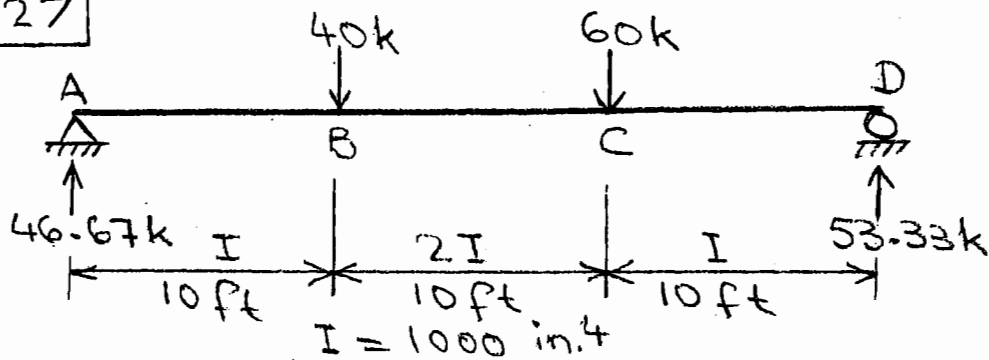
from which, $x_M = 8.07 \text{ m}$

$$\Delta_{\max} = \Delta_{DM} = \frac{1}{EI} \left[\frac{1}{2} (228.65)(8.07) \frac{2}{3} (8.07) \right]$$

$$= \frac{4963.6 \text{ kN.m}^3}{EI} = \frac{4963.6}{70(95)} = 0.746 \text{ m}$$

$$\underline{\Delta_{\max} = 746 \text{ mm} \downarrow}$$

6.27



$$\Delta_{DA} = \frac{1}{EI} \left[\frac{1}{2} (466.67) 10 \left(\frac{10}{3} + 20 \right) + \frac{1}{2} (33.33) 10 \left(\frac{10}{3} + 10 \right) + 233.33 (10) 15 + \frac{1}{2} (533.33) 10 \left(\frac{20}{3} \right) \right]$$

$$= \frac{109444.4 \text{ k-ft}^3}{EI}$$

$$\theta_A = \frac{\Delta_{DA}}{30} = \frac{3648.15 \text{ k-ft}^2}{EI}$$

6.27 (contd.)

If the maximum deflection occurs at M , then

$$\theta_{MA} = \theta_A$$

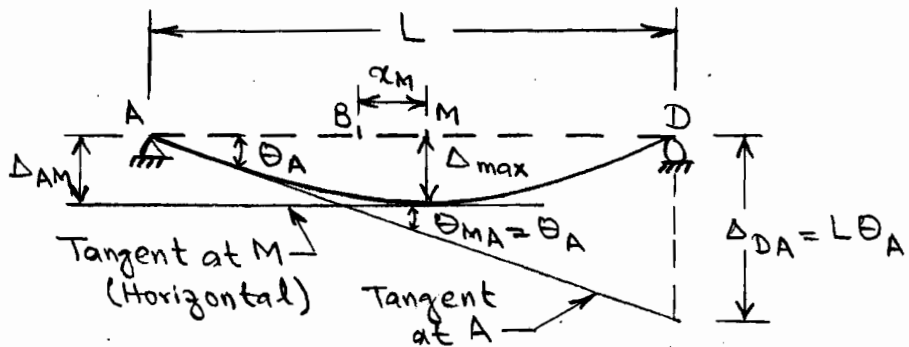
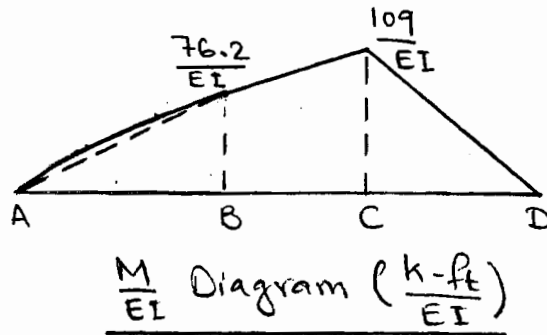
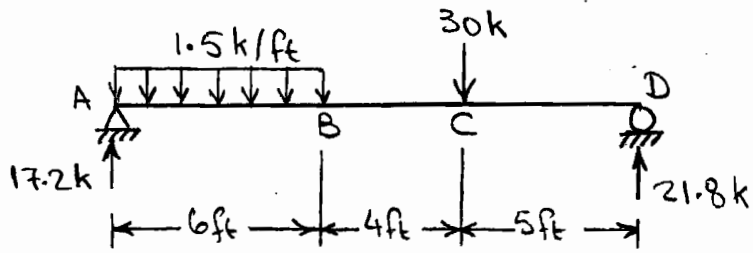
$$\frac{1}{EI} \left[\frac{1}{2} (466.67) 10 + 233.33 x_M + \frac{1}{2} \left(\frac{33.33 x_M}{10} \right) x_M \right] = \frac{3648.15}{EI}$$

$$1.67 x_M^2 + 233.33 x_M - 1314.8 = 0$$

$$x_M = 5.42 \text{ ft}$$

$$\begin{aligned} \Delta_{\max} = \Delta_{AM} &= \frac{1}{EI} \left[\frac{1}{2} (466.67) 10 \left(\frac{20}{3} \right) \right. \\ &\quad \left. + 233.33 (5.42) \left(\frac{5.42}{2} + 10 \right) \right. \\ &\quad \left. + \frac{1}{2} (18.06) (5.42) (3.61 + 10) \right] \\ &= \frac{32296 \text{ k-ft}^3}{EI} \\ &= \frac{32296 (12)^3}{29000 (1000)} = \underline{1.92 \text{ in.} \downarrow} \end{aligned}$$

6.28



$$\Delta_{DA} = \frac{1}{EI} \left[\frac{1}{2} (76.2) (6) (11) + \frac{2}{3} \frac{(1.5)(6)^2}{8} (6) (12) \right. \\ \left. + (76.2)(4)(7) + \frac{1}{2} (32.8)(4) \left(\frac{4}{3} + 5 \right) + \frac{1}{2} (109)(5) \left(\frac{10}{3} \right) \right] \\ = \frac{6296 \text{ k-ft}^3}{EI}$$

$$\textcircled{2} A = \frac{6296}{15EI} = \frac{419.73 \text{ k-ft}^2}{EI}$$

By setting $\Theta_{MA} = \Theta_A$, we write

$$\frac{1}{EI} \left[\frac{1}{2} (76.2)(6) + \frac{2}{3} \frac{(1.5)(6)^2(6)}{8} + \frac{1}{2} (152.4 + 8.22M) X_M \right] = \frac{419.73}{EI}$$

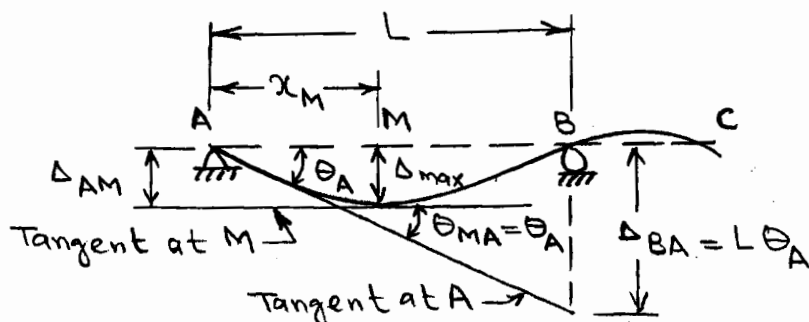
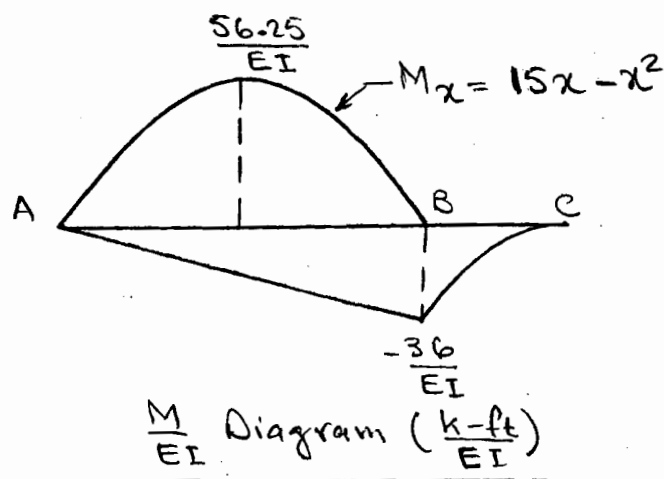
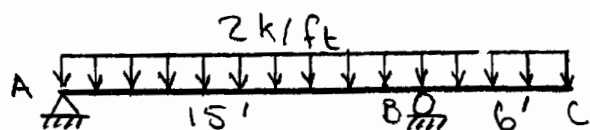
$$4.1x_M^2 + 76.2x_M - 164.13 = 0$$

$$x_M = 1.95 \text{ ft}$$

6.28(Contd.)

$$\begin{aligned}\Delta_{\max} &= \Delta_{AM} = \frac{1}{EI} \left[\frac{1}{2} (76.2) (6) (4) + \frac{2}{3} \frac{(1.5)(6)^3}{8} (3) \right. \\ &\quad \left. + 76.2 (1.95) \left(\frac{1.95}{2} + 6 \right) + \frac{1}{2} (16) (1.95) (1.3 + 6) \right] \\ &= \frac{2145.7 \text{ k-ft}^3}{EI} = \frac{2145.7 (12)^3}{(1500)(20000)} = \underline{0.124 \text{ in.} \downarrow}\end{aligned}$$

6.29



$$\Delta_{BA} = \frac{1}{EI} \left[\frac{2}{3} (56.25) 15 (7.5) - \frac{1}{2} (36) (15) 5 \right]$$

$$= \frac{2868.75 \text{ k-ft}^3}{EI}$$

$$\Theta_A = \frac{2868.75}{15 EI} = \frac{191.25 \text{ k-ft}^2}{EI}$$

$$\Theta_{MA} = \Theta_A$$

$$\frac{1}{EI} \left[\frac{1}{2} (15x_M - x_M^2) x_M + \frac{2}{3} \left(\frac{2x_M^2}{8} \right) x_M - \frac{1}{2} (2.4x_M) x_M \right] = \frac{191.25}{EI}$$

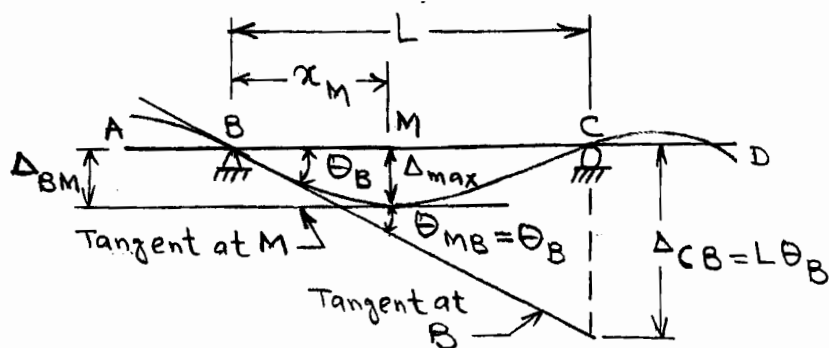
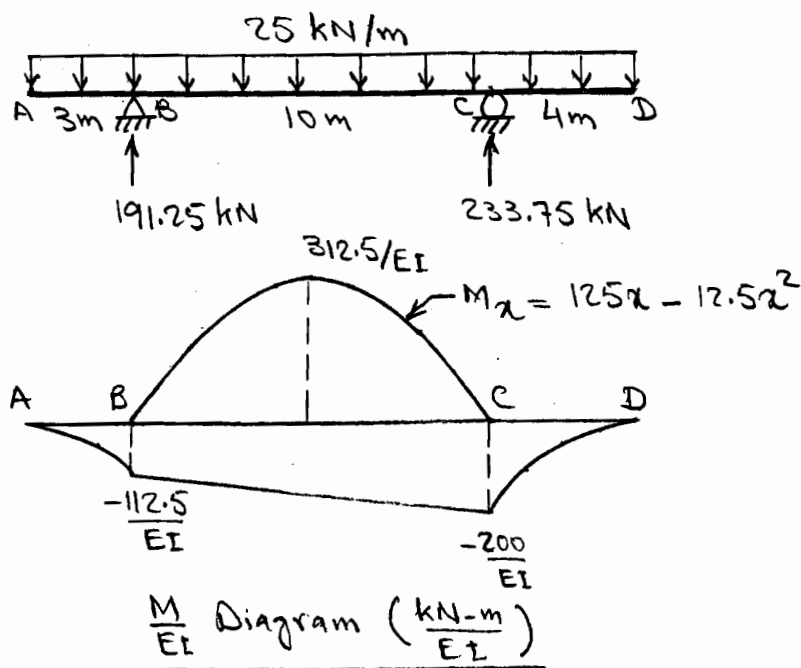
$$-\frac{x_M^3}{3} + 6.3x_M^2 = 191.25$$

$$\text{from which } x_M = 6.92 \text{ ft}$$

6.29 (contd.)

$$\begin{aligned}\Delta_{\max} &= \Delta_{AM} = \frac{1}{EI} \left[\frac{1}{2} (55.91) (6.92) (4.61) \right. \\ &\quad \left. + \frac{2}{3} (11.97) \left(\frac{6.92}{2} \right)^2 - \frac{1}{2} (16.61) (6.92) \frac{2}{3} (6.92) \right] \\ &= \frac{817.73 \text{ k-ft}^3}{EI} = \frac{817.73 (12)^3}{29000 (3500)} \\ &= \underline{0.0139 \text{ in.} \downarrow}\end{aligned}$$

6.30



$$\Delta_{CB} = \frac{1}{EI} \left[\frac{2}{3} (312.5)(10)(5) - (112.5)(10)(5) - \frac{1}{2} (87.5)(10)\left(\frac{10}{3}\right) \right]$$

$$= \frac{3333.33 \text{ kN-m}^3}{EI}$$

$$\theta_B = \frac{3333.33}{10EI} = \frac{333.33 \text{ kN-m}^2}{EI}$$

$$\theta_{MB} = \theta_A$$

$$\frac{1}{EI} \left[\frac{1}{2} (125x_M - 12.5x_M^2)x_M + \frac{2}{3} \left(\frac{25x_M^2}{8} \right)x_M - 112.5x_M - \frac{1}{2} (8.75x_M)x_M \right] = \frac{333.33}{EI}$$

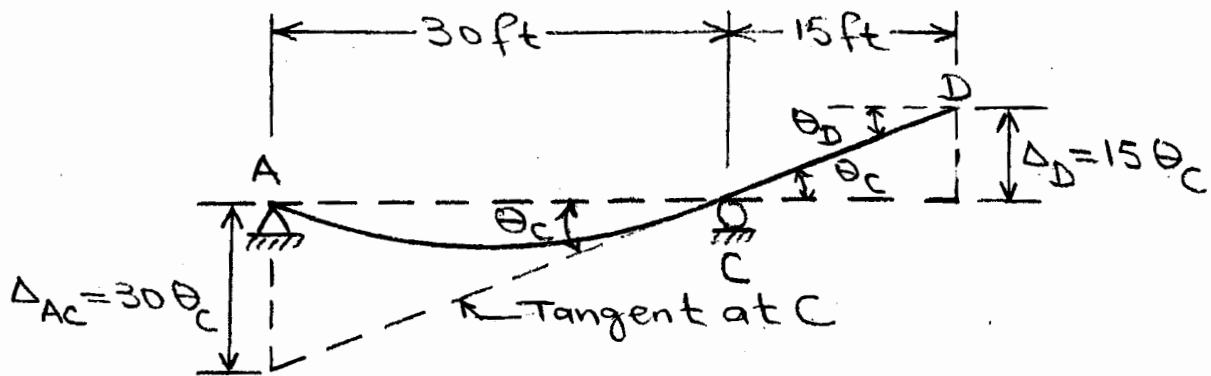
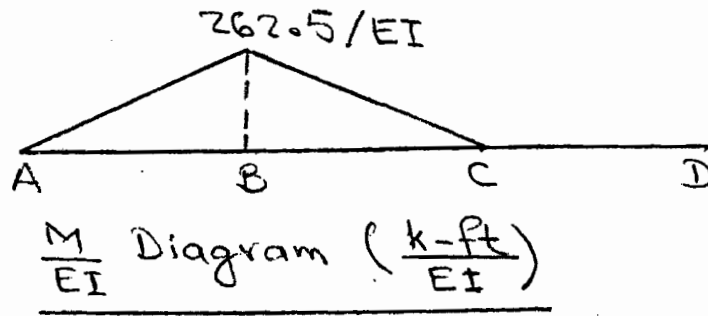
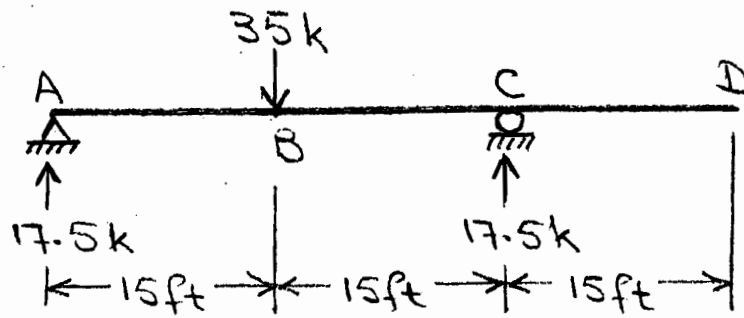
$$-4.167x_M^3 + 58.125x_M^2 - 112.5x_M = 333.33$$

$$\text{From which } x_M = 4.77 \text{ m}$$

6.30 (Contd.)

$$\begin{aligned}\Delta_{\max} &= \Delta_{BM} = \frac{1}{EI} \left[\frac{1}{2} (311.84) (4.77) (3.18) + \frac{2}{3} (71.1) \right. \\ &\quad \left. \times \frac{(4.77)^2}{2} - (112.5) \frac{(4.77)^2}{2} - \frac{1}{2} (41.74) (4.77) (3.18) \right] \\ &= \frac{1307.91 \text{ kN-m}^3}{EI} = \frac{1307.91}{(200)(500)} = 0.0131 \text{ m} \\ \Delta_{\max} &= 13.1 \text{ mm} \downarrow\end{aligned}$$

6.31



$$\Delta_{Ac} = \frac{1}{EI} \left[\frac{1}{2} (262.5) 30 (15) \right] = \frac{59062.5}{EI}$$

$$\theta_c = \frac{\Delta_{Ac}}{30} = \frac{1968.75}{EI}$$

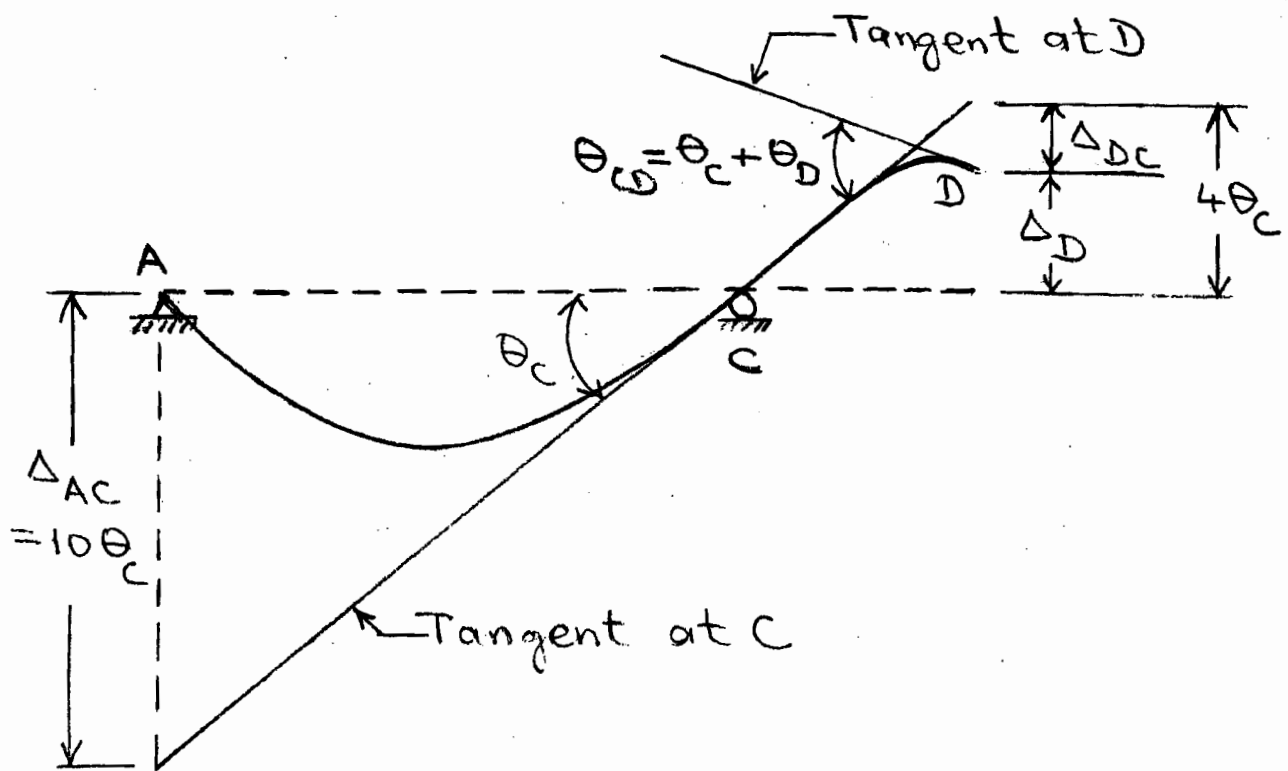
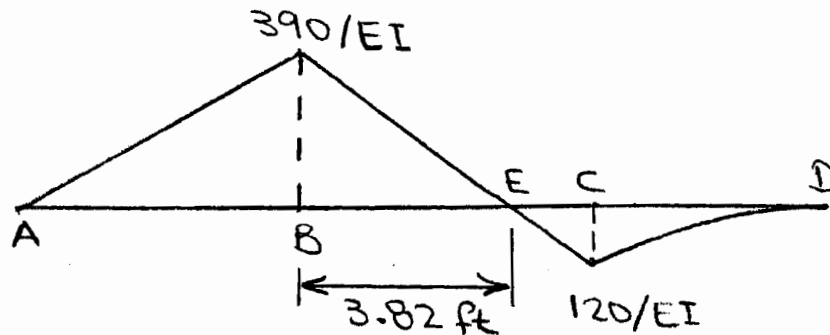
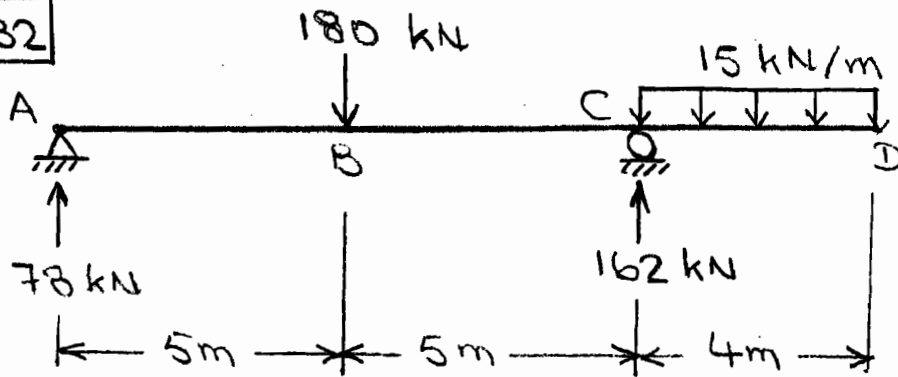
$$\theta_D = \theta_c = \frac{1968.75 \text{ k-ft}^2}{EI} = \frac{1968.75 (12)^2}{10000 (2500)}$$

$$\theta_D = 0.01134 \text{ rad} \nearrow$$

$$\Delta_D = 15\theta_c = \frac{29531.25 \text{ k-ft}^3}{EI} = \frac{29531.25 (12)^3}{10000 (2500)}$$

$$\Delta_D = 2.04 \text{ in.} \uparrow$$

6.32



6.32 (wntd.)

$$\Delta_{Ac} = \frac{1}{EI} \left[\frac{1}{2} (390) 5 \left(\frac{10}{3} \right) + \frac{1}{2} (390) 3.82 \left(\frac{3.82}{3} + 5 \right) - \frac{1}{2} (120) (1.18) (9.607) \right] = \frac{7243}{EI}$$

$$\theta_c = \frac{\Delta_{Ac}}{10} = \frac{724.3}{EI}$$

$$\theta_{CD} = \frac{1}{EI} \left[\frac{1}{3} (120) 4 \right] = \frac{160}{EI}$$

$$\begin{aligned} \theta_D &= \theta_{CD} - \theta_c = \frac{1}{EI} (160 - 724.3) = - \frac{564.3 \text{ kN.m}^2}{EI} \\ &= - \frac{564.3}{70(2340)} = -0.00345 \text{ rad} \end{aligned}$$

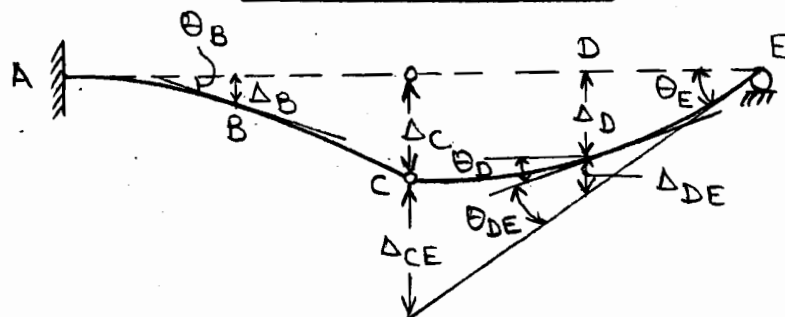
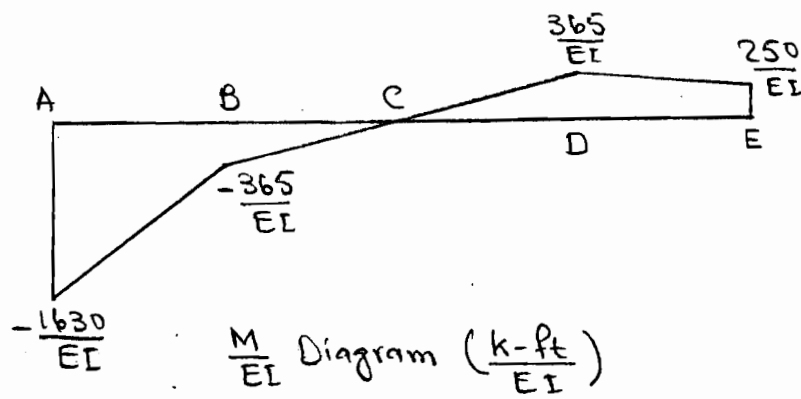
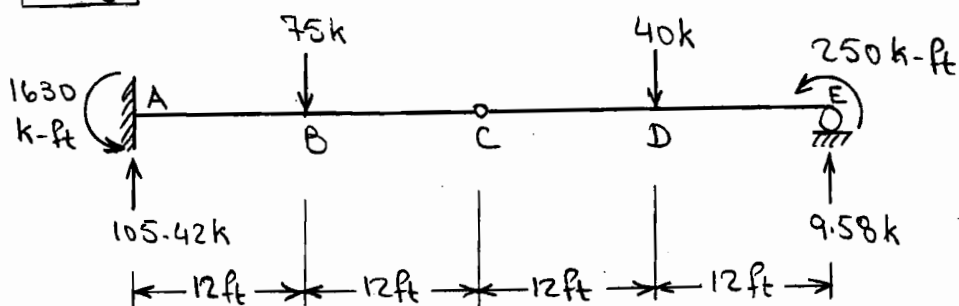
$$\underline{\theta_D = 0.00345 \text{ rad} \triangleleft}$$

$$\Delta_{DC} = \frac{1}{EI} [160(3)] = \frac{480}{EI}$$

$$\begin{aligned} \Delta_D &= 4\theta_c - \Delta_{DC} = \frac{1}{EI} [4(724.3) - 480] \\ &= \frac{2417.2 \text{ kN.m}^3}{EI} = \frac{2417.2}{70(2340)} = 0.01476 \text{ m} \end{aligned}$$

$$\underline{\Delta_D = 14.76 \text{ mm} \uparrow}$$

6.33



$$\theta_B = \theta_{BA} = \frac{1}{EI} \left[\left(\frac{1630 + 365}{2} \right) 12 \right] = \frac{11970 \text{ k-ft}^2}{EI} = \underline{0.0099 \text{ rad.} \nabla}$$

$$\Delta_B = \Delta_{BA} = \frac{1}{EI} \left[(365) 12 (6) + \frac{1}{2} (1265) (12) (8) \right] = \frac{87000}{EI} = \underline{0.86 \text{ in.} \downarrow}$$

$$\Delta_C = \Delta_{CA} = \frac{1}{EI} \left[365 (12) (18) + \frac{1}{2} (1265) (12) (20) + \frac{1}{2} (365) \times (12) (8) \right] = \frac{248160 \text{ k-ft}^3}{EI}$$

$$\Delta_{CE} = \frac{1}{EI} \left[\frac{1}{2} (365) (12) (8) + (250) (12) (18) + \frac{1}{2} (115) (12) (16) \right] = \frac{82560 \text{ k-ft}^3}{EI}$$

$$\theta_E = \frac{\Delta_C + \Delta_{CE}}{24} = \frac{248160 + 82560}{24 EI} = \frac{13780 \text{ k-ft}^2}{EI}$$

6.33 (contd.)

$$\theta_{DE} = \frac{1}{EI} \left[\left(\frac{365 + 250}{2} \right) 12 \right] = \frac{3690 \text{ k-ft}^2}{EI}$$

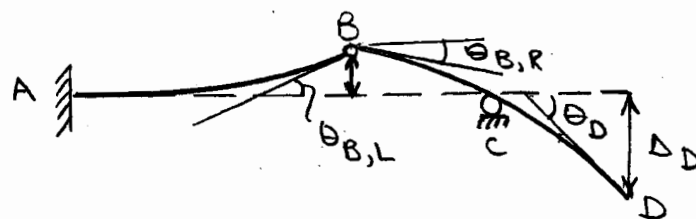
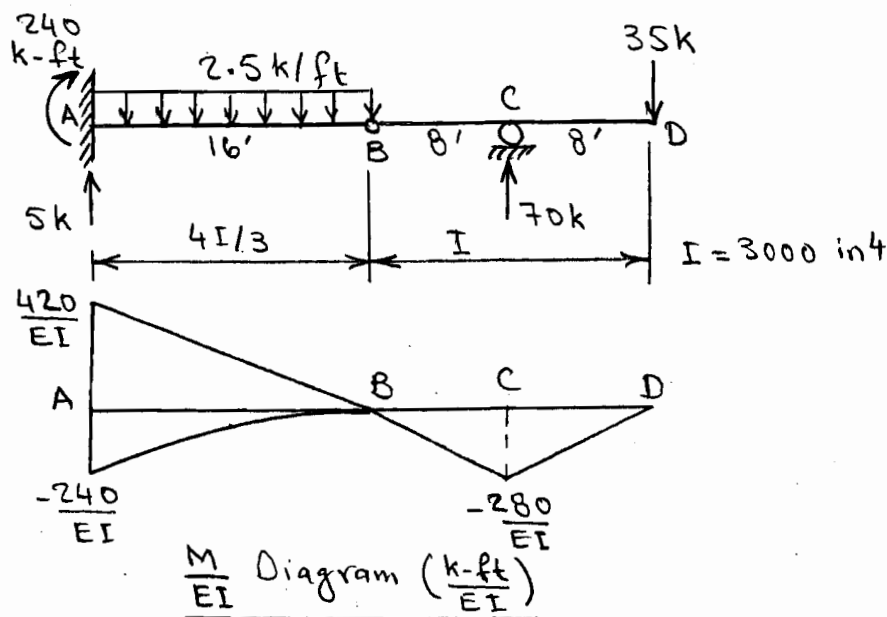
$$\theta_D = \theta_E - \theta_{DE} = \frac{13780 - 3690}{EI} = \frac{10090 \text{ k-ft}^2}{EI} = \underline{0.0084 \text{ rad.} \downarrow}$$

$$\Delta_{DE} = \frac{1}{EI} \left[(250)(12)(6) + \frac{1}{2}(115)(12)(4) \right] = \frac{20760 \text{ k-ft}^3}{EI}$$

$$\Delta_D = 12\theta_E - \Delta_{DE} = \frac{1}{EI} [12(13780) - 20760] = \frac{144600 \text{ k-ft}^3}{EI}$$

$$\underline{\Delta_D = 1.44 \text{ in.} \downarrow}$$

6.34



$$\begin{aligned}\theta_{B, \text{Left}} = \theta_{AB} &= \frac{1}{EI} \left[\frac{1}{2}(420)16 - \frac{1}{3}(240)16 \right] \\ &= \frac{2080 \text{ k-ft}^2}{EI} = \frac{2080 (12)^2}{30000 (3000)} = 0.0033 \text{ rad.} \quad \triangle\end{aligned}$$

$$\begin{aligned}\Delta_B = \Delta_{BA} &= \frac{1}{EI} \left[\frac{1}{2}(420)16(10.67) - \frac{1}{3}(240)16(12) \right] \\ &= \frac{20480 \text{ k-ft}^3}{EI} = \frac{20480 (12)^3}{30000 (3000)} = 0.39 \text{ in } \uparrow\end{aligned}$$

$$\Delta_{BC} = \frac{1}{EI} \left[\frac{1}{2}(280)8 \left(\frac{16}{3} \right) \right] = \frac{5973.33}{EI}$$

$$\Delta_{BC} + \Delta_B = 8 \theta_C$$

$$\theta_C = \frac{1}{8EI} (5973.33 + 20480) = \frac{3306.67}{EI}$$

$$\theta_{BC} = \frac{1}{EI} \left[\frac{1}{2}(280)8 \right] = \frac{1120}{EI}$$

$$\begin{aligned}\theta_{B, \text{Right}} = \theta_C - \theta_{BC} &= \frac{1}{EI} (3306.67 - 1120) \\ &= \frac{2186.67 \text{ k-ft}^2}{EI} = 0.0035 \text{ rad.} \quad \nabla\end{aligned}$$

6.34(contd.)

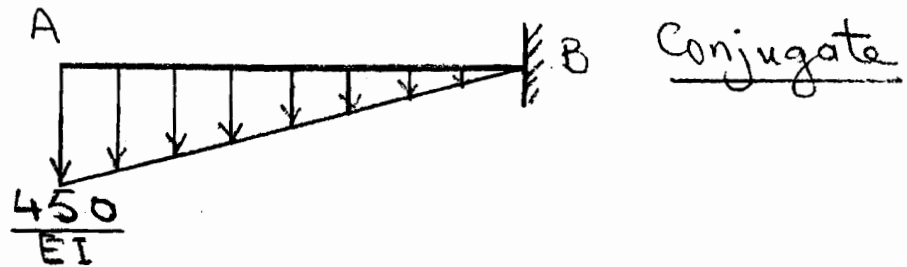
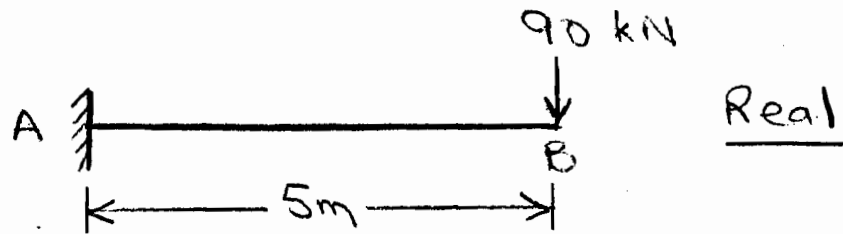
$$\theta_{CD} = \frac{1}{EI} \left[\frac{1}{2} (280) 8 \right] = \frac{1120}{EI}$$

$$\begin{aligned} \theta_D &= \theta_C + \theta_{CD} = \frac{1}{EI} (3306.67 + 1120) \\ &= \frac{4426.67 \text{ k-ft}^2}{EI} = \underline{0.0071 \text{ rad.} \searrow} \end{aligned}$$

$$\Delta_{DC} = \frac{1}{EI} \left[1120 \left(\frac{16}{3} \right) \right] = \frac{5973.33}{EI}$$

$$\begin{aligned} \Delta_D &= 8\theta_C + \Delta_{DC} = \frac{1}{EI} [8(3306.67) + 5973.33] \\ &= \frac{32426.67 \text{ k-ft}^3}{EI} = \underline{0.62 \text{ in} \downarrow} \end{aligned}$$

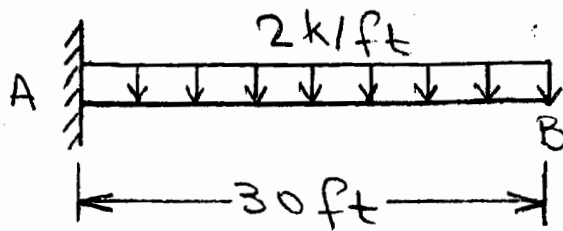
6.35



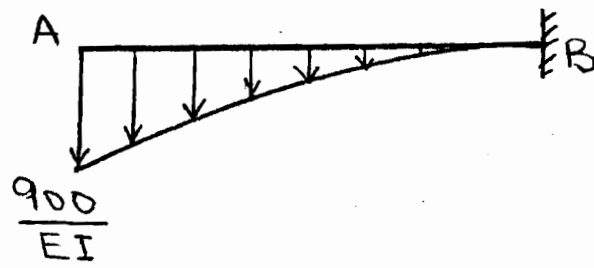
$$\begin{aligned}\theta_B &= -\frac{1}{2}(5)\left(\frac{450}{EI}\right) = -\frac{1125 \text{ kN}\cdot\text{m}^2}{EI} = -\frac{1125}{200(800)} \\ &= -0.00703 \text{ rad} = \underline{0.00703 \text{ rad} \nabla}\end{aligned}$$

$$\begin{aligned}\Delta_B &= -\frac{1}{2}(5)\left(\frac{450}{EI}\right)\left(\frac{10}{3}\right) = -\frac{3750 \text{ kN}\cdot\text{m}^3}{EI} \\ &= -\frac{3750}{200(800)} = -0.0234 \text{ m} = \underline{23.4 \text{ mm} \downarrow}\end{aligned}$$

6.36



Real



Conjugate

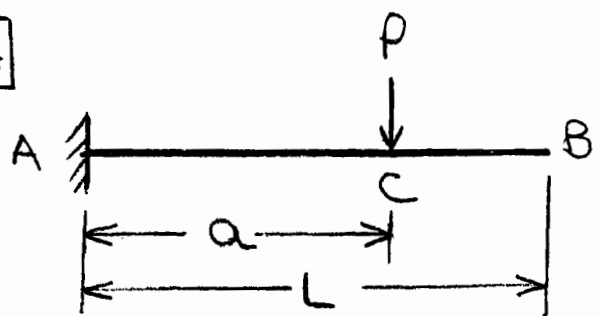
$$\begin{aligned}\Theta_B &= -\frac{1}{3} \left(\frac{900}{EI} \right) 30 = -\frac{9000 \text{ k-ft}^2}{EI} \\ &= -\frac{9000 (12)^2}{29000 (3000)} = -0.0149 \text{ rad}\end{aligned}$$

$$\underline{\Theta_B = 0.0149 \text{ rad} \nabla}$$

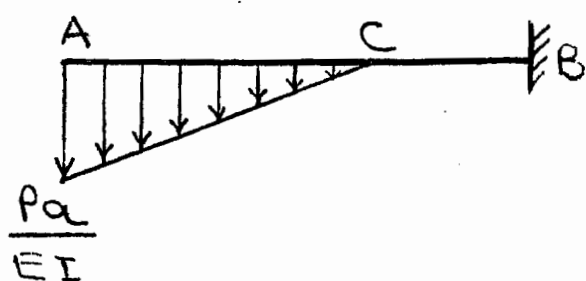
$$\begin{aligned}\Delta_B &= -\frac{9000}{EI} \left(\frac{3}{4} \right) 30 = -\frac{202500 \text{ k-ft}^3}{EI} \\ &= -\frac{202500 (12)^3}{29000 (3000)} = -4.022 \text{ in.}\end{aligned}$$

$$\underline{\Delta_B = 4.022 \text{ in.} \downarrow}$$

6.37



Real

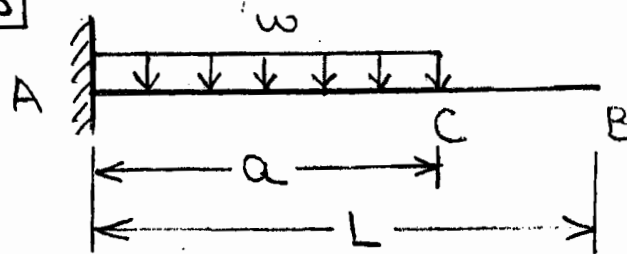


Conjugate

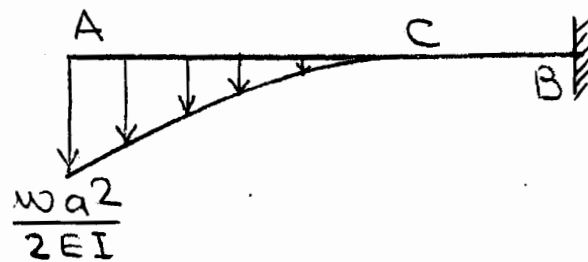
$$\theta_B = -\frac{1}{2} \left(\frac{Pa}{EI} \right) a = -\frac{Pa^2}{2EI} = \frac{Pa^2}{EI} \nabla$$

$$\begin{aligned} \Delta_B &= -\frac{Pa^2}{2EI} \left(L - \frac{a}{3} \right) = -\frac{Pa^2}{6EI} (3L - a) \\ &= \frac{Pa^2}{6EI} (3L - a) \downarrow \end{aligned}$$

6.38



Real

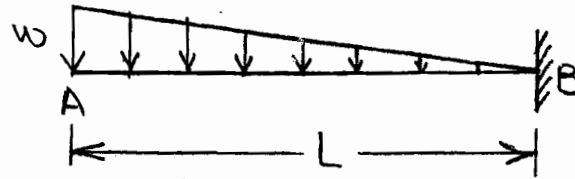


Conjugate

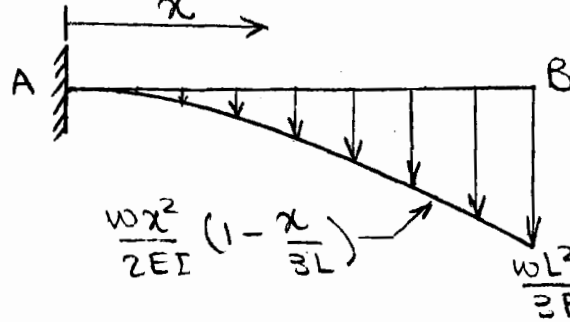
$$\theta_B = -\frac{1}{3} \left(\frac{wa^2}{2EI} \right) a = -\frac{wa^3}{6EI} = \underline{\underline{\frac{wa^3}{6EI} \downarrow}}$$

$$\begin{aligned} \Delta_B &= -\frac{wa^3}{6EI} \left(L - \frac{a}{4} \right) = -\frac{wa^3}{24EI} (4L - a) \\ &= \underline{\underline{\frac{wa^3}{24EI} (4L - a) \downarrow}} \end{aligned}$$

6.39



Real

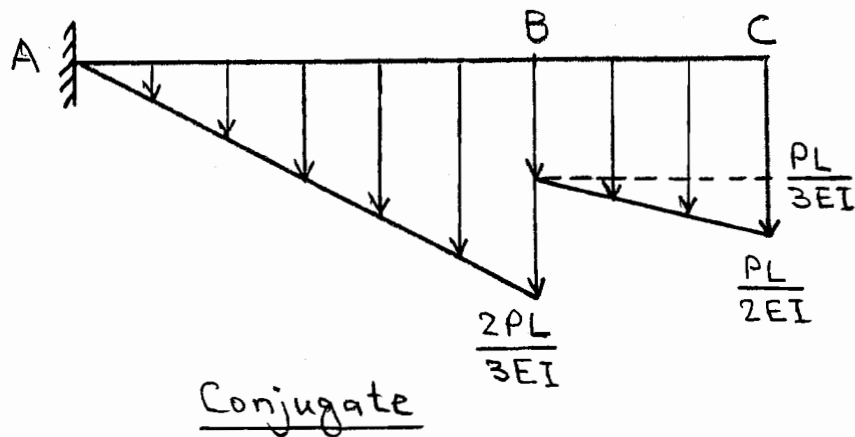
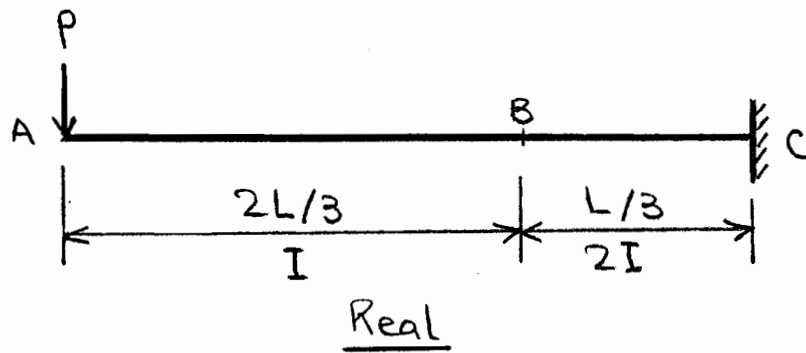


Conjugate

$$\theta_A = \int_0^L \frac{wx^2}{2EI} \left(1 - \frac{x}{3L}\right) dx = \frac{wL^3}{8EI} \nearrow$$

$$\Delta_A = \int_0^L \frac{wx^2}{2EI} \left(1 - \frac{x}{3L}\right) x dx = -\frac{11}{120} \frac{wL^4}{EI} = \underline{\underline{-\frac{11}{120} \frac{wL^4}{EI}}} \downarrow$$

6.40



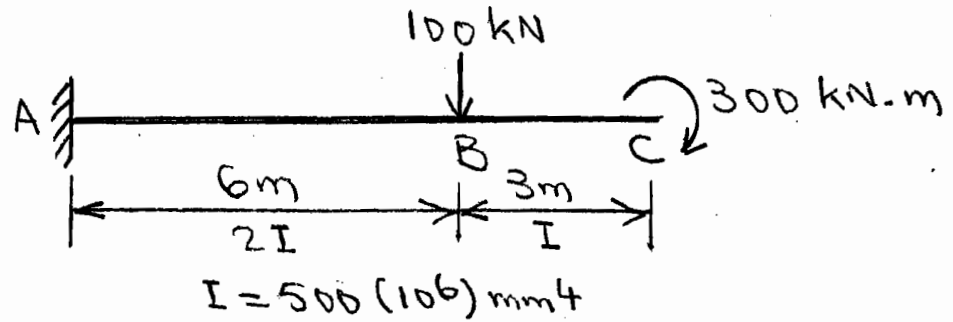
$$\theta_A = \frac{1}{2} \left(\frac{2L}{3} \right) \left(\frac{2PL}{3EI} \right) + \frac{L}{3} \left(\frac{PL}{3EI} \right) + \frac{1}{2} \left(\frac{L}{3} \right) \left(\frac{PL}{6EI} \right) = \frac{13PL^2}{36EI} \quad \triangle$$

$$\Delta_A = -\frac{2PL^2}{9EI} \left(\frac{4L}{9} \right) - \frac{PL^2}{9EI} \left(\frac{5L}{6} \right) - \frac{PL^2}{36EI} \left(\frac{8L}{9} \right) = -\frac{35PL^3}{162EI}$$

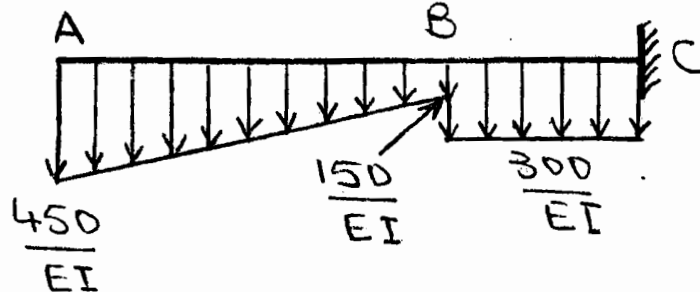
$$= \frac{35PL^3}{162EI} \quad \downarrow$$

6.41

Real



Conjugate



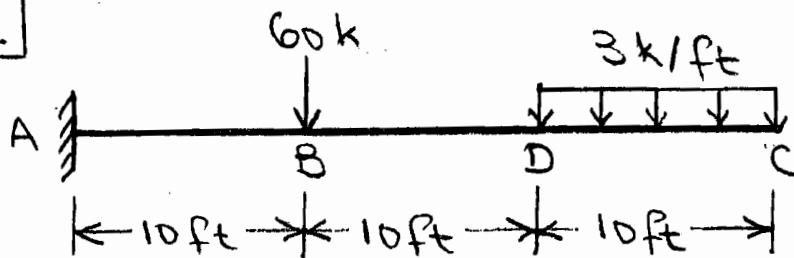
$$\begin{aligned}\theta_B &= \frac{1}{EI} \left[-150(6) - \frac{1}{2}(300)(6) \right] = -\frac{1800 \text{ kN.m}^2}{EI} \\ &= -\frac{1800}{70(500)} = -0.0514 \text{ rad} = \underline{0.0514 \text{ rad} \nabla}\end{aligned}$$

$$\begin{aligned}\Delta_B &= \frac{1}{EI} \left[-150(6)(3) - \frac{1}{2}(300)(6)(4) \right] = -\frac{6300 \text{ kN.m}^3}{EI} \\ &= -\frac{6300}{70(500)} = -0.18 \text{ m} = \underline{180 \text{ mm} \downarrow}\end{aligned}$$

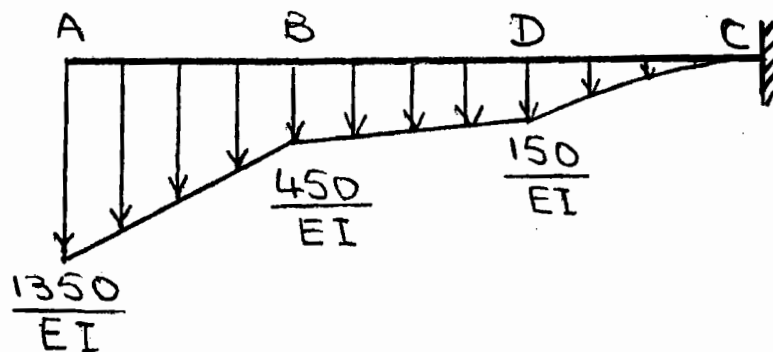
$$\begin{aligned}\theta_C &= \frac{1}{EI} \left[-1800 - 300(3) \right] = -\frac{2700 \text{ kN.m}^2}{EI} \\ &= -\frac{2700}{70(500)} = -0.0771 \text{ rad} = \underline{0.0771 \text{ rad} \nabla}\end{aligned}$$

$$\begin{aligned}\Delta_C &= \frac{1}{EI} \left[-150(6)(6) - \frac{1}{2}(300)(6)(7) - 300(3)(1.5) \right] \\ &= -\frac{13050 \text{ kN.m}^3}{EI} = -\frac{13050}{70(500)} = -0.373 \text{ m} \\ &= \underline{373 \text{ mm} \downarrow}\end{aligned}$$

6.42



Real



Conjugate

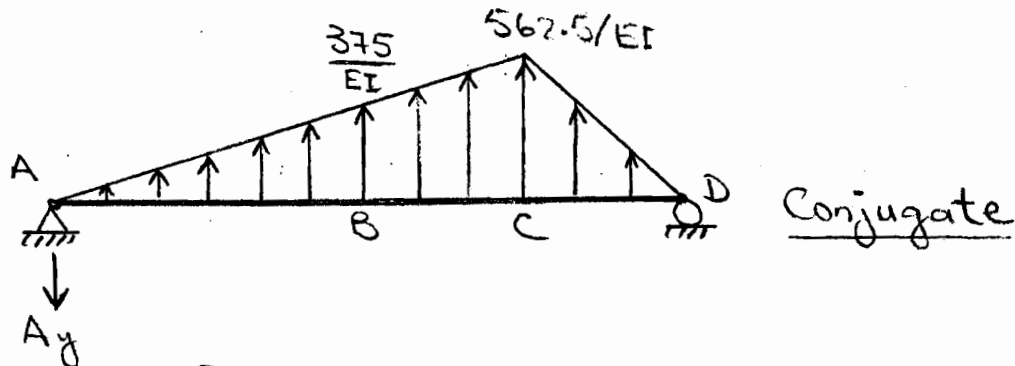
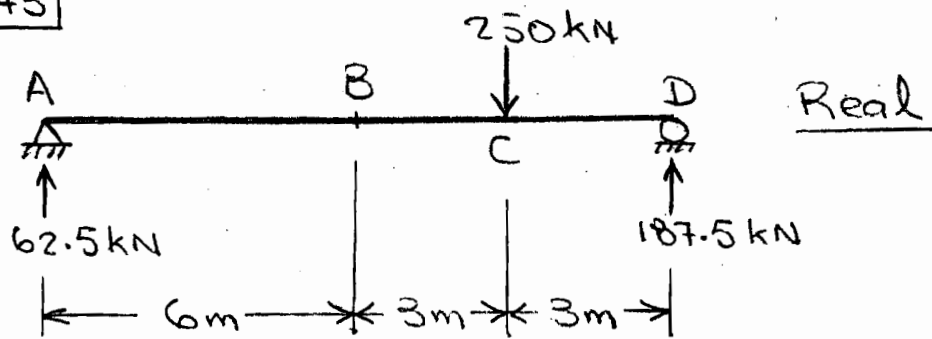
$$\begin{aligned}\theta_B &= \frac{1}{EI} \left[-450(10) - \frac{1}{2}(900)(10) \right] = -\frac{9000 \text{ k-ft}^2}{EI} \\ &= -\frac{9000(12)^2}{29000(4000)} = -0.0112 \text{ rad} = \underline{0.0112 \text{ rad} \nabla}\end{aligned}$$

$$\begin{aligned}\Delta_B &= \frac{1}{EI} \left[-450(10)5 - \frac{1}{2}(900)(10)\left(\frac{20}{3}\right) \right] = -\frac{52500 \text{ k-ft}^3}{EI} \\ &= -\frac{52500(12)^3}{29000(4000)} = -0.782 \text{ in.} = \underline{0.782 \text{ in.} \downarrow}\end{aligned}$$

$$\begin{aligned}\theta_C &= \frac{1}{EI} \left[-9000 - 150(10) - \frac{1}{2}(300)(10) - \frac{1}{3}(150)(10) \right] \\ &= -\frac{12500 \text{ k-ft}^2}{EI} = -\frac{12500(12)^2}{29000(4000)} = -0.0155 \text{ rad} \\ &= \underline{0.0155 \text{ rad} \nabla}\end{aligned}$$

$$\begin{aligned}\Delta_C &= \frac{1}{EI} \left[-450(10)25 - \frac{1}{2}(900)(10)\left(\frac{20}{3} + 20\right) \right. \\ &\quad \left. - 150(10)15 - \frac{1}{2}(300)(10)\left(\frac{20}{3} + 10\right) - \frac{1}{3}(150)(10)\left(\frac{30}{4}\right) \right] \\ &= -\frac{283750 \text{ k-ft}^3}{EI} = -\frac{283750(12)^3}{29000(4000)} \\ &= -4.227 \text{ in.} = \underline{4.227 \text{ in.} \downarrow}\end{aligned}$$

6.43



Reaction for conjugate beam:

$$+\circlearrowleft \sum M_D = 0$$

$$A_y(12) - \frac{1}{2} \left(\frac{562.5}{EI} \right) 9(6) - \frac{1}{2} \left(\frac{562.5}{EI} \right) 3(2) = 0$$

$$A_y = \frac{1406.25 \text{ kN.m}^2}{EI}$$

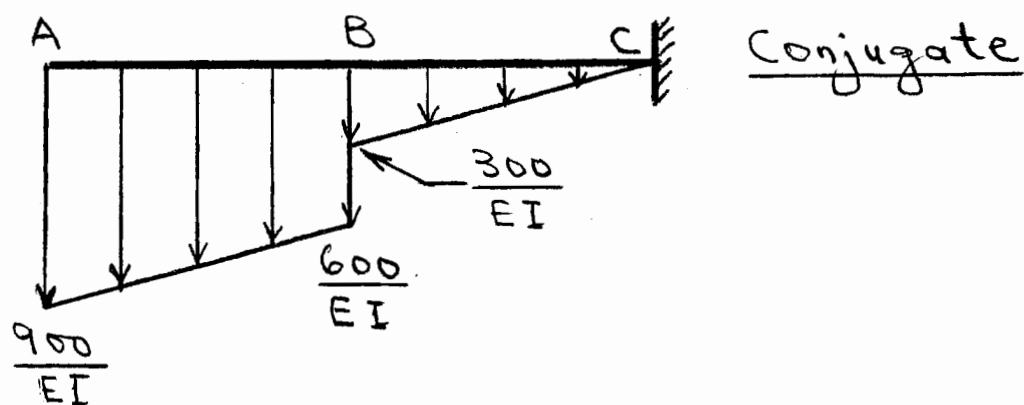
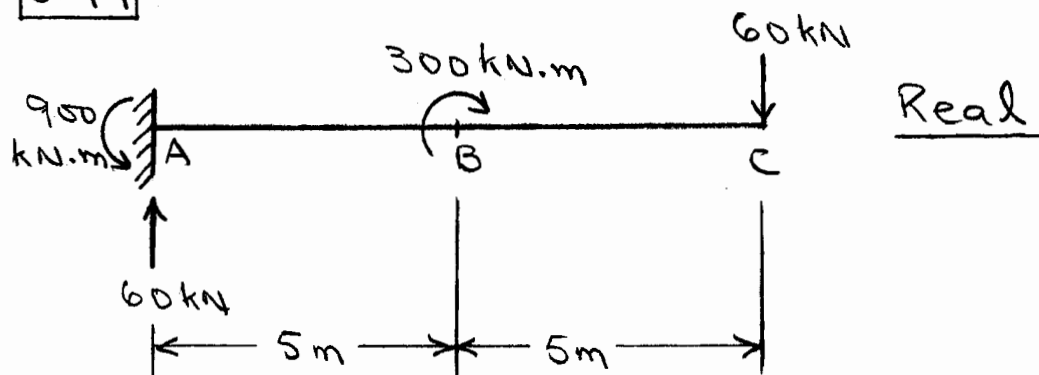
$$\begin{aligned} \theta_B &= \frac{1}{EI} \left[-1406.25 + \frac{1}{2} (375) 6 \right] = -\frac{281.25 \text{ kN.m}^2}{EI} \\ &= -\frac{281.25}{200(462)} = -0.00304 \text{ rad} = \underline{0.00304 \text{ rad} \searrow} \end{aligned}$$

$$\begin{aligned} \Delta_B &= \frac{1}{EI} \left[-1406.25(6) + \frac{1}{2} (375) 6(2) \right] = -\frac{6187.5 \text{ kN.m}^3}{EI} \\ &= -\frac{6187.5}{200(462)} = -0.007 \text{ m} = \underline{67 \text{ mm} \downarrow} \end{aligned}$$

$$\begin{aligned} \theta_C &= \frac{1}{EI} \left[-1406.25 + \frac{1}{2} (562.5) 9 \right] = \frac{1125 \text{ kN.m}^2}{EI} \\ &= \frac{1125}{200(462)} = \underline{0.0122 \text{ rad} \nearrow} \end{aligned}$$

$$\begin{aligned} \Delta_C &= \frac{1}{EI} \left[-1406.25(9) + \frac{1}{2} (562.5) 9(3) \right] = -\frac{5062.5 \text{ kN.m}^3}{EI} \\ &= -\frac{5062.5}{200(462)} = -0.00548 \text{ m} = \underline{54.8 \text{ mm} \downarrow} \end{aligned}$$

6.44



$$\Delta_{\max} = \Delta_C = -\frac{1}{EI} \left[600(5)(7.5) + \frac{1}{2}(300)(5)\left(\frac{25}{3}\right) + \frac{1}{2}(300)(5)\left(\frac{10}{3}\right) \right] = \frac{-31250 \text{ kN.m}^3}{EI}$$

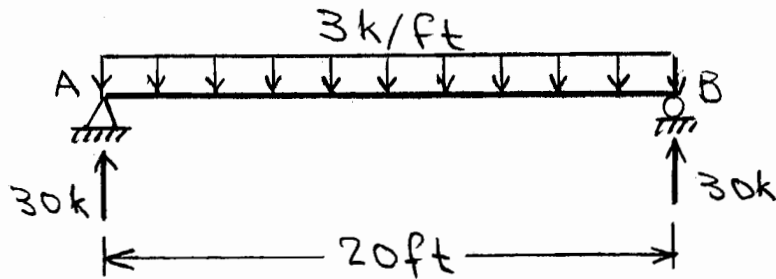
$$= \frac{31250 \text{ kN.m}^3}{EI} \downarrow$$

$$\Delta_{\max} = \frac{L}{360}$$

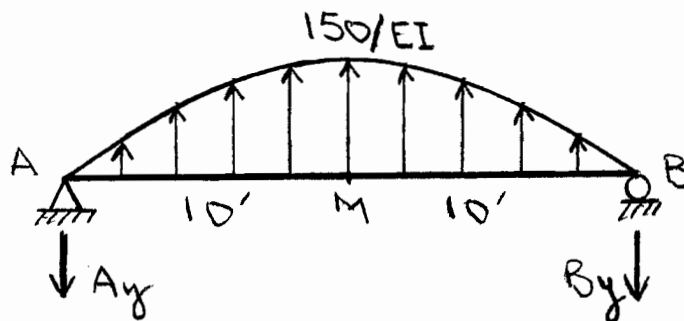
$$\frac{31250}{200(10^6)I} = \frac{10}{360}$$

$$\text{from which, } I = 5625(10^6) \text{ m}^4 = \underline{5625(10^6) \text{ mm}^4}$$

6.45



Real



Conjugate

Reactions for conjugate beam:

$$A_y = B_y = \frac{1}{2} \left(\frac{2}{3} \right) \left(\frac{150}{EI} \right) (20) = \frac{1000 \text{ k-ft}^2}{EI}$$

$$\begin{aligned} \Delta_{\max} = \Delta_M &= \frac{1}{EI} \left[-1000(10) + \frac{2}{3} (150)(10) \left(\frac{30}{8} \right) \right] \\ &= -\frac{6250 \text{ k-ft}^3}{EI} = \frac{6250 \text{ k-ft}^3}{EI} \downarrow \end{aligned}$$

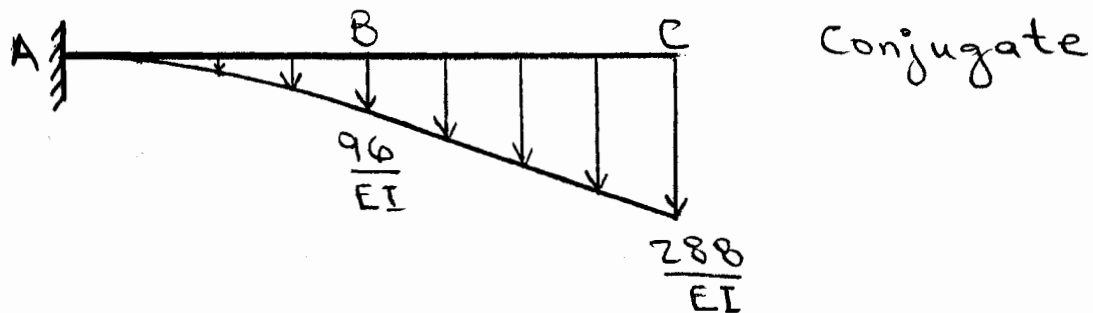
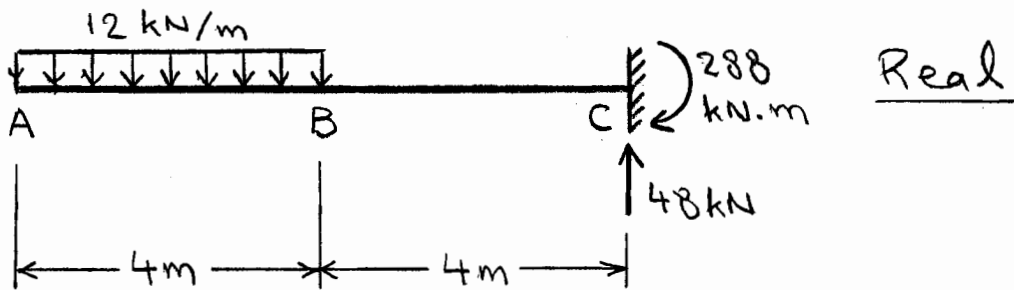
$$\Delta_{\max} = \frac{L}{360}$$

$$\frac{6250 (12)^3}{29000 (I)} = \frac{20(12)}{360}$$

From which,

$$\underline{I = 559 \text{ in}^4}$$

6.46



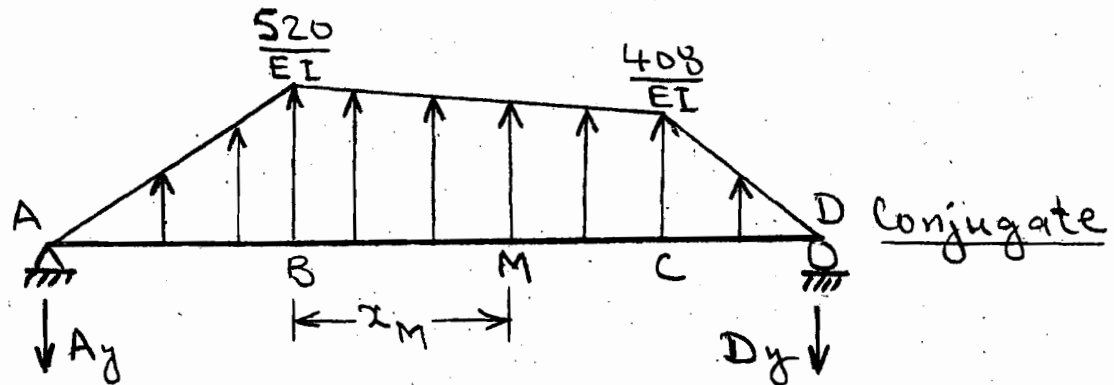
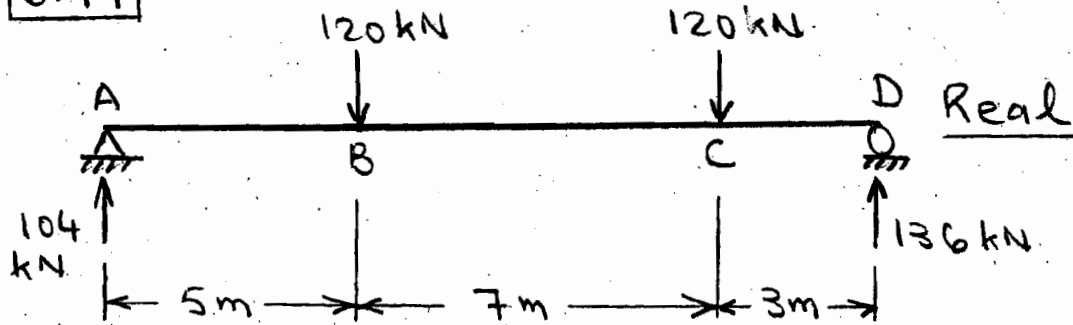
$$\begin{aligned}\Delta_{\max} = \Delta_A &= \frac{1}{EI} \left[-\frac{1}{3} (96)(4)(3) - 96(4)(6) \right. \\ &\quad \left. - \frac{1}{2} (192)(4) \left(\frac{8}{3} + 4 \right) \right] = -\frac{5248 \text{ kN.m}^3}{EI} \\ &= \frac{5248 \text{ kN.m}^3}{EI} \downarrow\end{aligned}$$

$$\Delta_{\max} = \frac{L}{360}$$

$$\frac{5248}{70(10^6)I} = \frac{8}{360}$$

$$\text{From which, } I = 3374 (10^6) \text{ m}^4 = \underline{3374 (10^6) \text{ mm}^4}$$

6.47



$$+\circlearrowleft \sum M_D = 0$$

$$A_y(15) - \frac{1}{2} \left(\frac{520}{EI} \right) 5 \left(\frac{5}{3} + 10 \right) - \left(\frac{408}{EI} \right) (7) \left(\frac{7}{2} + 3 \right) - \frac{1}{2} \left(\frac{112}{EI} \right) 7 \left(\frac{14}{3} + 3 \right) - \frac{1}{2} \left(\frac{408}{EI} \right) 3 (2) = 0$$

$$A_y = \frac{2530.67 \text{ kN.m}^2}{EI}$$

Let the maximum bending moment in the conjugate beam occur at point M, at a distance x_M from point B. Then the shear at M must be zero. Thus,

$$S_M = \frac{1}{EI} \left[-2530.67 + \frac{1}{2} (520)(5) + (520)x_M - \frac{1}{2} \left(\frac{112}{7} x_M \right) x_M \right] = 0$$

$$8x_M^2 - 520x_M + 1230.67 = 0$$

$$x_M = 2.46 \text{ m}$$

6.47 (contd.)

$$\Delta_{\max} = \Delta_M = \frac{1}{EI} \left[-2530.67(7.46) + \frac{1}{2}(520)(5) \right.$$

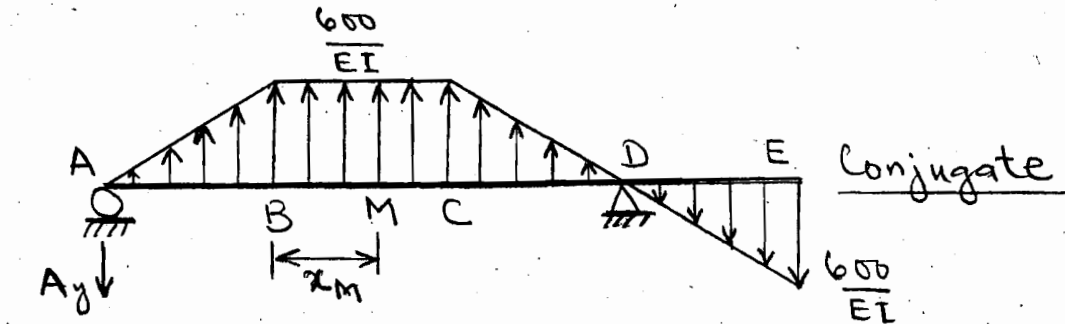
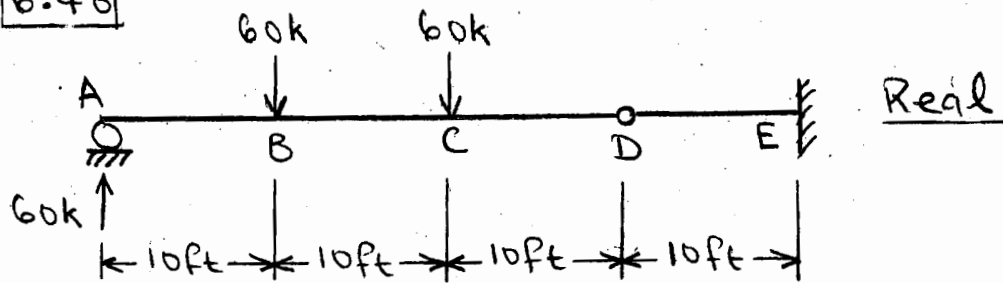
$$\left(\frac{5}{3} + 2.46 \right) + (520)(2.46) \left(\frac{2.46}{2} \right) - \frac{1}{2}(39.36)(2.46) \left(\frac{2.46}{3} \right) \Big] = \frac{-11980.41}{EI} = \frac{11980.41 \text{ kN}\cdot\text{m}^3}{EI} \downarrow$$

$$\Delta_{\max} = \frac{L}{360}$$

$$\frac{11980.41}{30(10^6)I} = \frac{15}{360}$$

$$\text{from which, } I = 9584(10^6) \text{ m}^4 = \underline{9584(10^6) \text{ mm}^4}$$

6.48



$$+\circlearrowleft \sum M_D = 0$$

$$A_y(30) - \frac{1}{2} \left(\frac{600}{EI} \right) 10 \left(\frac{10}{3} + 20 \right) - \left(\frac{600}{EI} \right) (10)(15) - \frac{1}{2} \left(\frac{600}{EI} \right) (10) \left(\frac{20}{3} \right) - \frac{1}{2} \left(\frac{600}{EI} \right) 10 \left(\frac{20}{3} \right) = 0$$

$$A_y = \frac{6666.67 \text{ k-ft}^2}{EI}$$

Let the maximum bending moment in the conjugate beam occur at point M, at a distance x_M from point B. Then the shear at M must be zero. Thus,

$$S_M = \frac{1}{EI} \left[-6666.67 + \frac{1}{2} (600) 10 + (600) x_M \right] = 0$$

$$x_M = 6.11 \text{ ft}$$

$$\Delta_{\max} = \Delta_M = \frac{1}{EI} \left[-6666.67(6.11) + \frac{1}{2} (600)(10) \left(\frac{10}{3} + 6.11 \right) + (600)(6.11) \left(\frac{6.11}{2} \right) \right]$$

$$= -\frac{67870}{EI} = \frac{67870 \text{ k-ft}^3}{EI} \downarrow$$

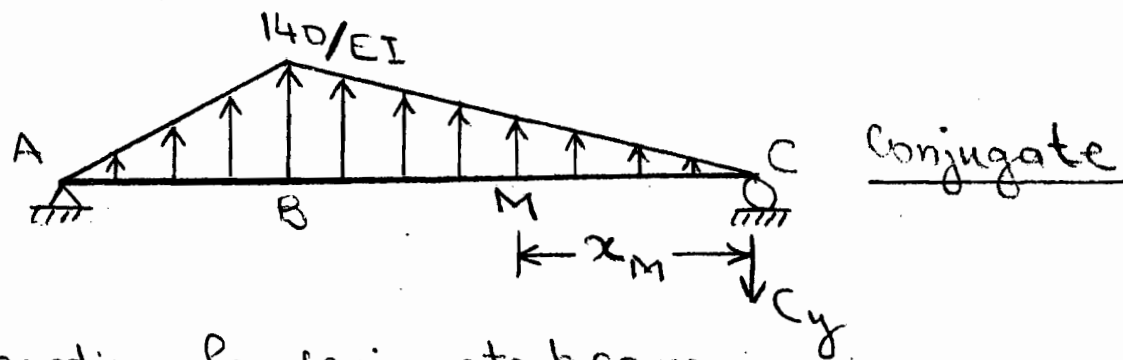
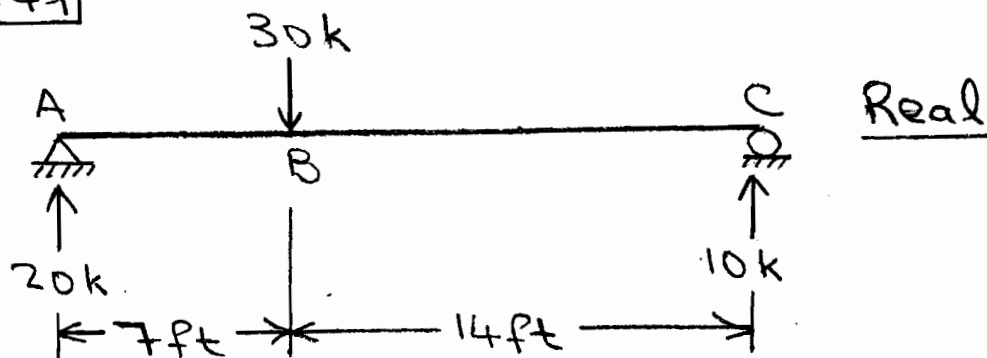
$$\Delta_{\max} = \frac{L}{360}$$

$$\frac{67870 (12)^3}{29000 (I)} = \frac{40(12)}{360}$$

from which

$$I = 3033 \text{ in}^4$$

6.49



Reaction for conjugate beam:

$$+\circlearrowleft \sum M_A = 0$$

$$-C_y(21) + \frac{1}{2} \left(\frac{140}{EI} \right) 14 \left(\frac{14}{3} + 7 \right) + \frac{1}{2} \left(\frac{140}{EI} \right) 7 \left(\frac{14}{3} \right) = 0$$

$$C_y = \frac{653.33 \text{ k-ft}^2}{EI}$$

Let the maximum bending moment in the conjugate beam occur at point M, at a distance x_M from point C. Then, the shear at M must be zero. Thus,

$$S_M = \frac{1}{EI} \left[653.33 - \frac{1}{2} (10 x_M) x_M \right] = 0$$

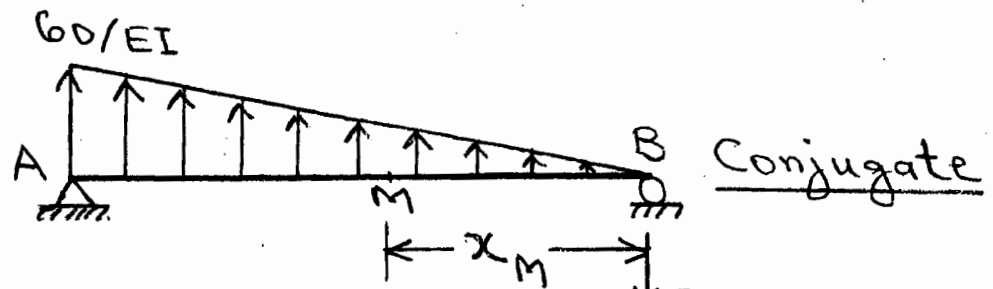
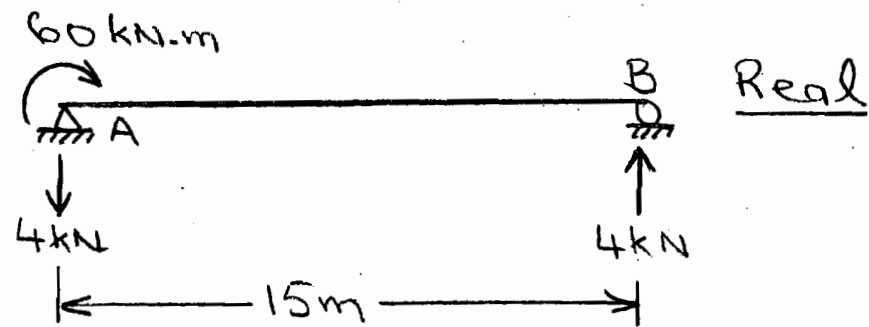
$$x_M = 11.43 \text{ ft}$$

$$\Delta_{\max} = \Delta_M = \frac{1}{EI} \left[-653.33 (11.43) + \frac{1}{2} (11.43) 11.43 \left(\frac{11.43}{3} \right) \right]$$

$$= -\frac{4978.81 \text{ k-ft}^3}{EI} = -\frac{4978.81 (12)^3}{10000 (500)}$$

$$= -1.72 \text{ in.} = \underline{1.72 \text{ in.} \downarrow}$$

6.50



Reaction for conjugate beam:

$$+\circlearrowleft \sum M_A = 0$$

$$-B_y(15) + \frac{1}{2} \left(\frac{60}{EI} \right) 15(5) = 0$$

$$B_y = \frac{150 \text{ kN} \cdot \text{m}^2}{EI}$$

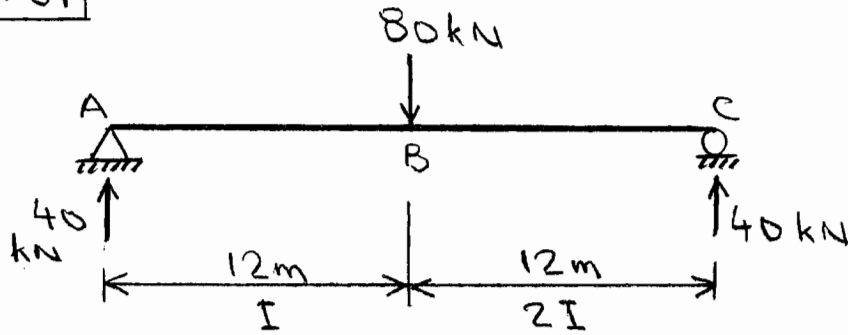
Let the maximum bending moment in the conjugate beam occur at point M, at a distance x_M from point B. Then the shear at M must be zero. Thus,

$$S_M = \frac{1}{EI} \left[150 - \frac{1}{2} (4x_M)x_M \right] = 0$$

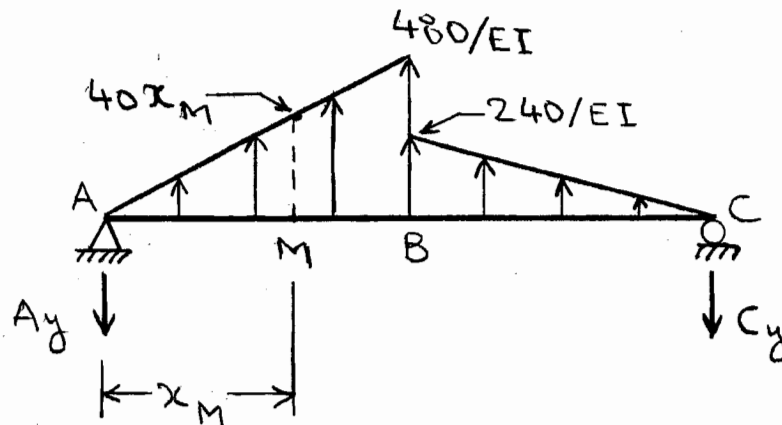
$$x_M = 8.66 \text{ m}$$

$$\begin{aligned} \Delta_{\max} = \Delta_M &= \frac{1}{EI} \left[-150(8.66) + \frac{1}{2} (34.64)(8.66) \left(\frac{8.66}{3} \right) \right] \\ &= -\frac{866 \text{ kN} \cdot \text{m}^2}{EI} = -\frac{866}{70(712)} = -0.0174 \text{ m} \\ &= \underline{17.4 \text{ mm} \downarrow} \end{aligned}$$

6.51



Real



Conjugate

$$+\circlearrowleft \sum M_C = 0$$

$$A_y(24) - \frac{1}{2} \left(\frac{480}{EI} \right) (12)(16) - \frac{1}{2} \left(\frac{240}{EI} \right) (12)(8) = 0$$

$$A_y = \frac{2400 \text{ kN}\cdot\text{m}^2}{EI}$$

$$\sum M = 0$$

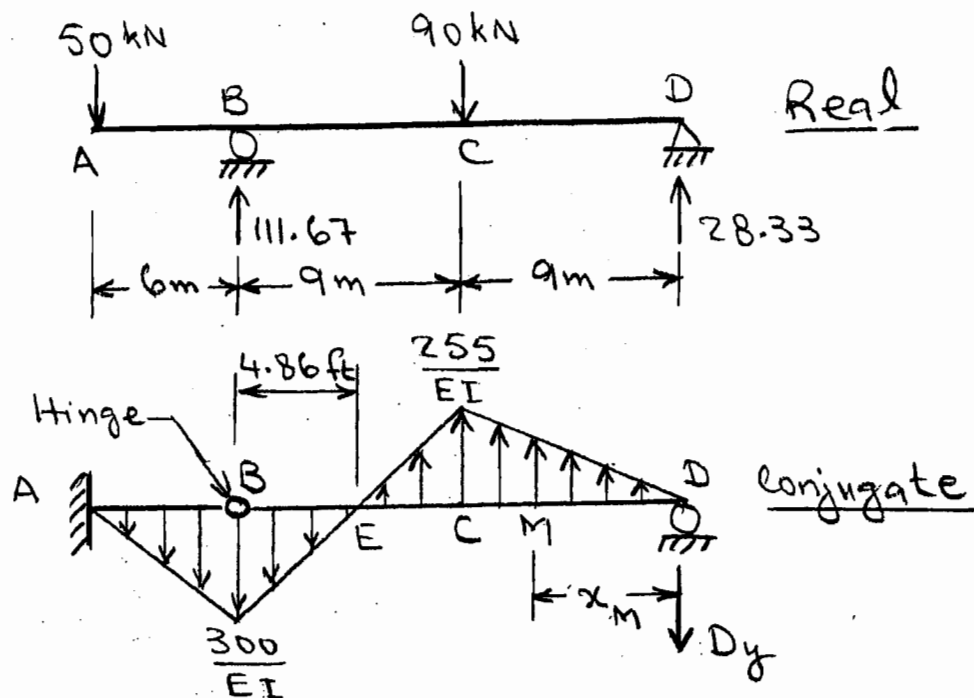
$$\frac{1}{EI} \left[-2400 + \frac{1}{2} (40x_M) x_M \right] = 0 \quad \underline{x_M = 10.95 \text{ m}}$$

$$\Delta_{\max} = \Delta_M = \frac{1}{EI} \left[-2400(10.95) + \frac{1}{2} (438)(10.95) \left(\frac{10.95}{3} \right) \right]$$

$$= -\frac{17527 \text{ kN}\cdot\text{m}^3}{EI} = -\frac{17527}{200(600)} = -0.146 \text{ m}$$

$$\underline{\Delta_{\max} = 146 \text{ mm} \downarrow}$$

6.52



$$+\circlearrowleft \sum M_B^{BD} = 0$$

$$-D_y(18) + \frac{1}{2} \left(\frac{255}{EI} \right) (9) (12) + \frac{1}{2} \left(\frac{255}{EI} \right) (4.14) (7.62) - \frac{1}{2} \left(\frac{300}{EI} \right) (4.86) \left(\frac{4.86}{3} \right) = 0$$

$$D_y = \frac{922.85 \text{ kN.m}^2}{EI}$$

Let the maximum bending moment in the conjugate beam occur at point M, at a distance x_M from point D. Then the shear at M must be zero. Thus,

$$S_M = \frac{1}{EI} \left[922.85 - \frac{1}{2} \left(\frac{255}{9} x_M \right) x_M \right] = 0$$

$$x_M = 8.07 \text{ m}$$

$$\Delta_{\max} = \Delta_M = \frac{1}{EI} \left[-922.85 (8.07) + \frac{1}{2} \left(\frac{255}{9} \right) (8.07) \cdot (8.07) \left(\frac{8.07}{3} \right) \right] = - \frac{4965 \text{ kN.m}^3}{EI}$$

$$= - \frac{4965}{70(95)} = -0.746 \text{ m} = \underline{746 \text{ mm} \downarrow}$$

6.52 (contd.)

$$1.67 x_M^2 + 233.33 x_M - 1314.8 = 0$$

$$x_M = 5.42 \text{ ft}$$

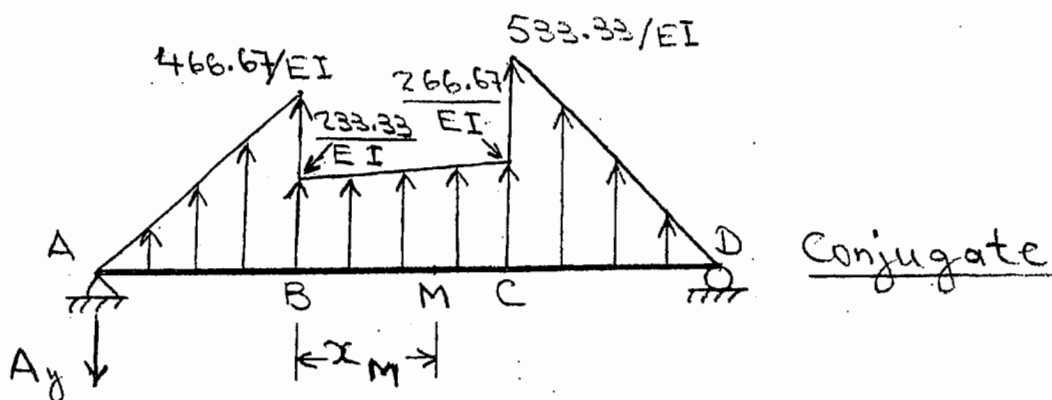
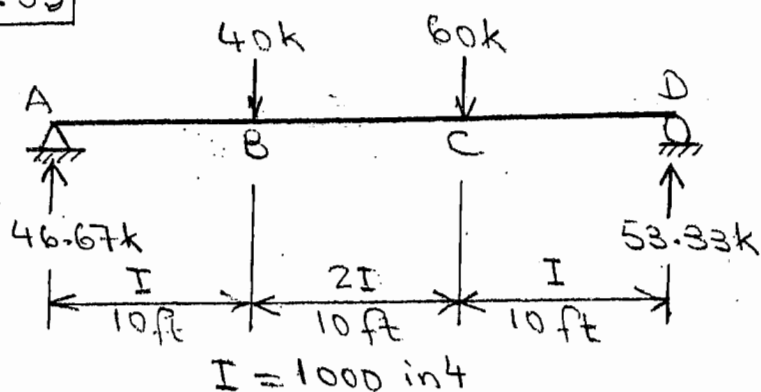
$$\Delta_{\max} = \Delta_M = \frac{1}{EI} \left[-3648.15(15.42) \right.$$

$$+ \frac{1}{2}(466.67)(10) \left(\frac{10}{3} + 5.42 \right) + 233.33(5.42) \left(\frac{5.42}{2} \right)$$

$$+ \left. \frac{1}{2}(18.06)(5.42) \left(\frac{5.42}{3} \right) \right] = -\frac{32314 \text{ k-ft}^3}{EI}$$

$$= -\frac{32314(12)^3}{29000(1000)} = -1.92 \text{ in.} = \underline{1.92 \text{ in.} \downarrow}$$

6.53



6.53 (contd.)

Reaction for conjugate beam:

$$+\circlearrowleft \sum M_D = 0$$

$$A_y (30) - \frac{1}{EI} \left[\frac{1}{2} (466.67) 10 \left(\frac{10}{3} + 20 \right) + 233.33 (10) 15 \right. \\ \left. + \frac{1}{2} (33.33) 10 \left(\frac{10}{3} + 10 \right) + \frac{1}{2} (533.33) 10 \left(\frac{20}{3} \right) \right] = 0$$

$$A_y = \frac{3648.15 \text{ k-ft}^2}{EI}$$

Let the maximum bending moment in the conjugate beam occur at point M, at a distance x_M from point B. Then the shear at M must be zero. Thus,

$$S_M = \frac{1}{EI} \left[-3648.15 + \frac{1}{2} (466.67) 10 + 233.33 x_M \right. \\ \left. + \frac{1}{2} (3.33 x_M) x_M \right] = 0$$

$$1.67 x_M^2 + 233.33 x_M - 1314.8 = 0$$

$$x_M = 5.42 \text{ ft}$$

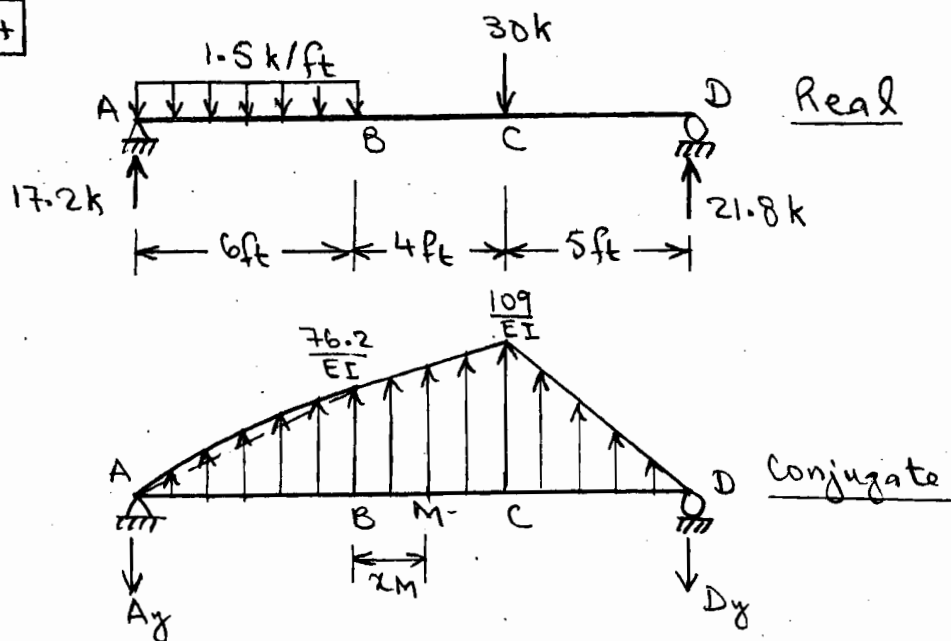
$$\Delta_{\max} = \Delta_M = \frac{1}{EI} \left[-3648.15 (5.42) \right.$$

$$+ \frac{1}{2} (466.67) 10 \left(\frac{10}{3} + 5.42 \right) + 233.33 (5.42) \left(\frac{5.42}{2} \right)$$

$$+ \frac{1}{2} (18.06) 5.42 \left(\frac{5.42}{3} \right) \left. \right] = -\frac{32314 \text{ k-ft}^3}{EI}$$

$$= -\frac{32314 (12)^3}{29000 (1000)} = -1.92 \text{ in.} = \underline{1.92 \text{ in.} \downarrow}$$

6.54



$$+\circlearrowleft \sum M_D = 0$$

$$A_y (15) - \frac{1}{EI} \left[\frac{1}{2} (76.2) 6 (11) + \frac{2}{3} \frac{(1.5)(6)^2}{8} (6) (12) \right. \\ \left. + 76.2 (4) 7 + \frac{1}{2} (32.8) (4) \left(\frac{4}{3} + 5 \right) + \frac{1}{2} (109) 5 \left(\frac{10}{3} \right) \right] \\ = 0$$

$$A_y = \frac{419.73 \text{ k-ft}^2}{EI}$$

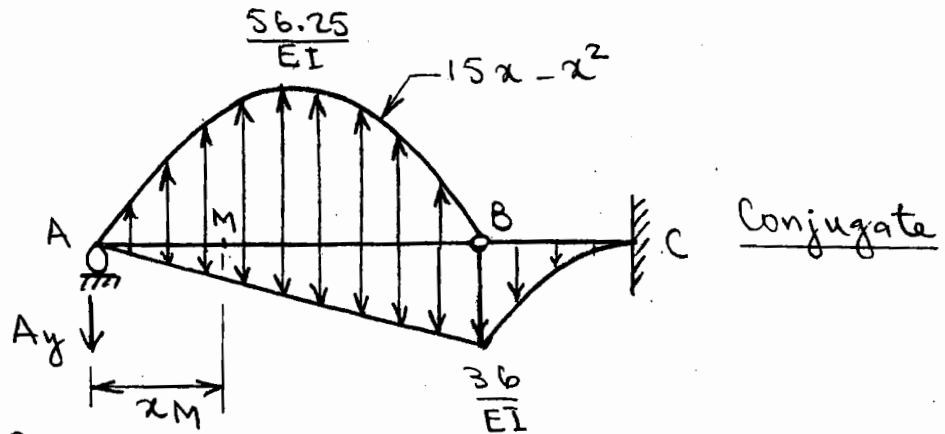
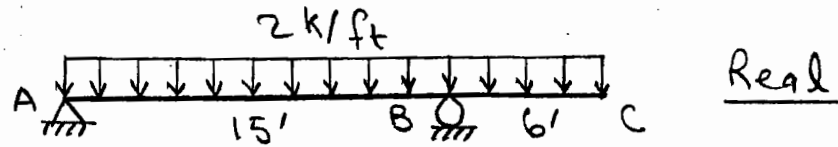
Let the maximum bending moment in the conjugate beam occur at point M. Thus,

$$8_M = \frac{1}{EI} \left[-419.73 + \frac{1}{2} (76.2) 6 + \frac{2}{3} \frac{(1.5)(6)^2}{8} (6) \right. \\ \left. + 76.2 x_M + \frac{1}{2} (8.2 x_M) x_M \right] = 0$$

$$4.1 x_M^2 + 76.2 x_M - 164.13 = 0 \\ x_M = 1.95 \text{ ft}$$

$$\Delta_{\max} = \Delta_M = \frac{1}{EI} \left[-419.73 (7.95) + \frac{1}{2} (76.2) 6 (3.95) \right. \\ \left. + \frac{2}{3} \frac{(1.5)(6)^2}{8} (6) (4.95) + 76.2 \frac{(1.95)^2}{2} \right. \\ \left. + \frac{1}{2} (15.99) (1.95) \left(\frac{1.95}{3} \right) \right] = -\frac{2145.2 \text{ k-ft}^3}{EI} \\ = -\frac{2145.2 (12)^3}{1500 (20000)} = -0.124 \text{ in} = \underline{0.124 \text{ in.} \downarrow}$$

6.55



$$+\circlearrowleft \sum M_B^{AB} = 0$$

$$A_y(15) - \frac{1}{EI} \left[\frac{2}{3} (56.25) 15 (7.5) - \frac{1}{2} (36) 15 (5) \right] = 0$$

$$A_y = \frac{191.25 \text{ k-ft}^2}{EI}$$

$$\sum M = 0$$

$$\frac{1}{EI} \left[-191.25 + \frac{1}{2} (15x_M - x_M^2) x_M + \frac{2}{3} \left(\frac{2x_M^2}{8} \right) x_M - \frac{1}{2} (2.4x_M) x_M \right] = 0$$

$$-\frac{x_M^3}{3} + 6.3x_M^2 - 191.25 = 0$$

$$\text{from which } x_M = 6.92 \text{ ft}$$

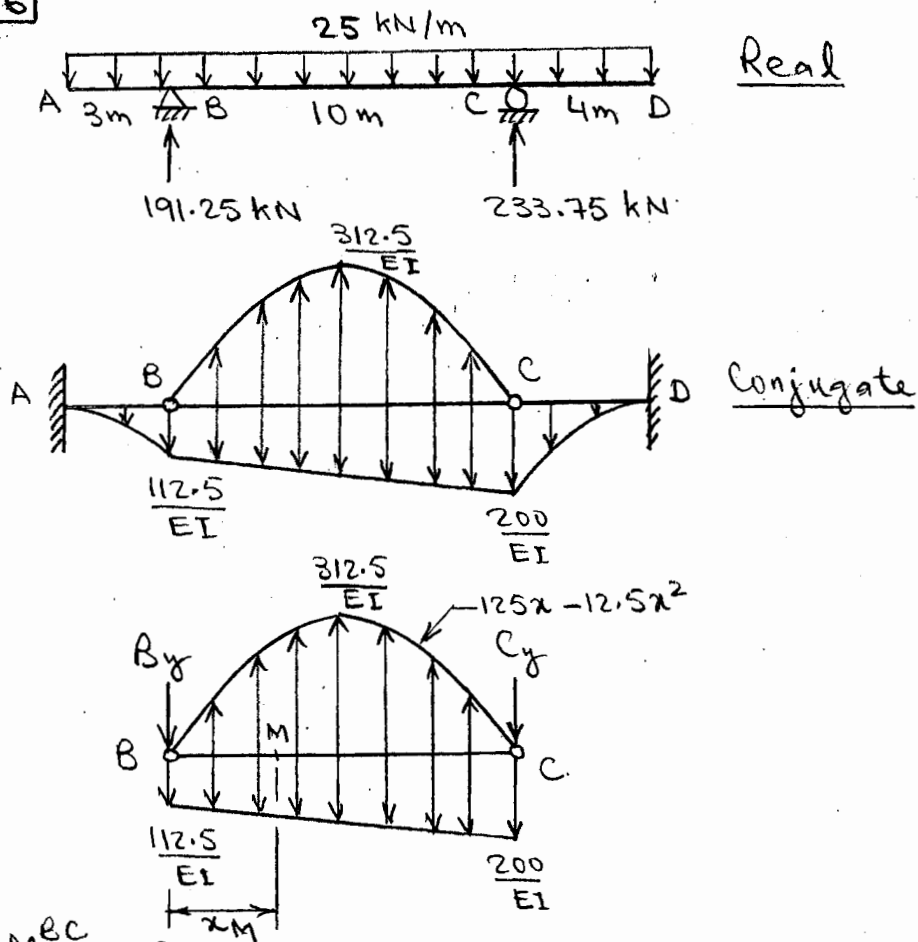
$$\Delta_{max} = \Delta_M = \frac{1}{EI} \left[-191.25 (6.92) + \frac{1}{2} (55.91) \frac{(6.92)^2}{3} \right.$$

$$\left. + \frac{2}{3} (11.97) \frac{(6.92)^2}{2} - \frac{1}{2} (16.61) 6.92 \left(\frac{6.92}{3} \right) \right]$$

$$= -\frac{818.73 \text{ k-ft}^3}{EI} = -0.0139 \text{ in}$$

$$= \underline{0.0139 \text{ in.} \downarrow}$$

6.56



$$+\circlearrowleft \sum M_C = 0$$

$$B_y(10) - \frac{1}{EI} \left[\frac{2}{3} (312.5)(10)(5) - 112.5(10)(5) - \frac{1}{2} (87.5)(10) \left(\frac{10}{3} \right) \right] = 0$$

$$B_y = \frac{333.33 \text{ kN-m}^2}{EI}$$

$$\sum M = 0$$

$$\frac{1}{EI} \left[-333.33 + \frac{1}{2} (125x_M - 12.5x_M^2)x_M + \frac{2}{3} \left(\frac{25x_M^2}{8} \right)x_M - 112.5x_M - \frac{1}{2} (8.75x_M)x_M \right] = 0$$

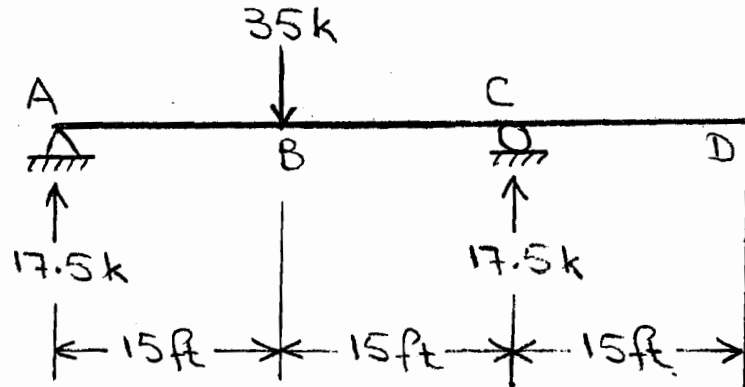
$$-4.167x_M^3 + 58.125x_M^2 - 112.5x_M - 333.33 = 0$$

$$\text{from which } x_M = 4.77 \text{ m}$$

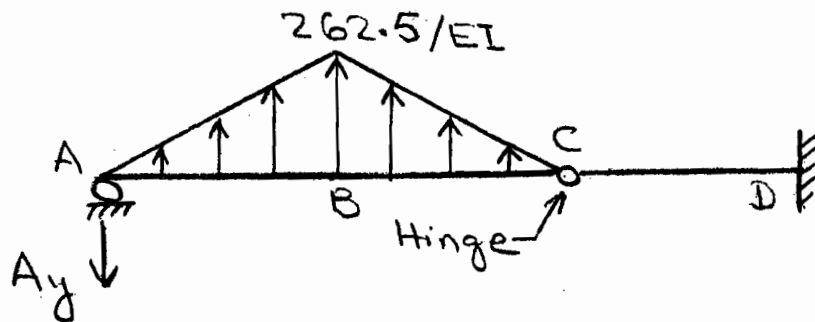
$$\Delta_{\max} = \Delta_M = \frac{1}{EI} \left[-333.33(4.77) + \frac{1}{2} (311.84) \frac{(4.77)^2}{3} + \frac{2}{3} (71.1) \frac{(4.77)^2}{2} - 112.5 \frac{(4.77)^2}{2} - \frac{1}{2} (41.74) \frac{(4.77)^2}{3} \right]$$

$$= - \frac{1306.3 \text{ kN-m}^3}{EI} = -0.0131 \text{ m} = \underline{13.1 \text{ mm} \downarrow}$$

6.57



Real



Conjugate

$$+\circlearrowleft \sum M_C^{AC} = 0$$

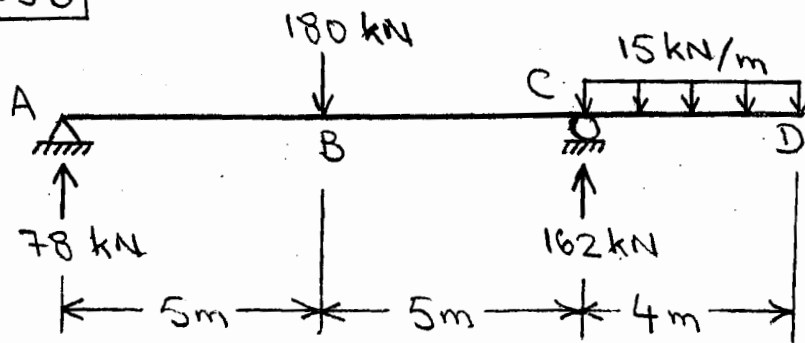
$$A_y (30) - \frac{1}{2} \left(\frac{262.5}{EI} \right) 30 (15) = 0$$

$$A_y = \frac{1968.75 \text{ k-ft}^2}{EI}$$

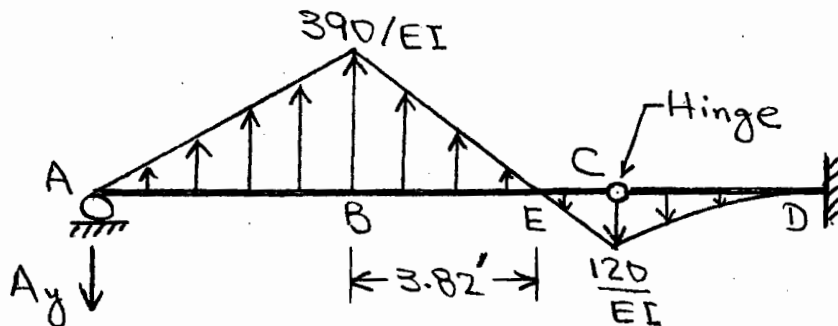
$$\begin{aligned} \theta_D &= \frac{1}{EI} \left[-1968.75 + \frac{1}{2} (262.5) 30 \right] = \frac{1968.75 \text{ k-ft}^2}{EI} \\ &= \frac{1968.75 (12)^2}{10000 (2500)} = \underline{0.01134 \text{ rad}} \end{aligned}$$

$$\begin{aligned} \Delta_D &= \frac{1}{EI} \left[-1968.75 (45) + \frac{1}{2} (262.5) 30 (30) \right] \\ &= \frac{29531.25 \text{ k-ft}^3}{EI} = \frac{29531.25 (12)^3}{10000 (2500)} \\ &= \underline{2.04 \text{ in.}} \end{aligned}$$

6.58



Real



Conjugate

$$+\circlearrowleft \sum M_C^{AC} = 0$$

$$A_y (10) - \frac{1}{EI} \left[\frac{1}{2} (390) 5 \left(\frac{5}{3} + 5 \right) + \frac{1}{2} (390) (3.82) (3.73) - \frac{1}{2} (120) 1.18 \left(\frac{1.18}{3} \right) \right] = 0$$

$$A_y = \frac{925.1 \text{ kN} \cdot \text{m}^2}{EI}$$

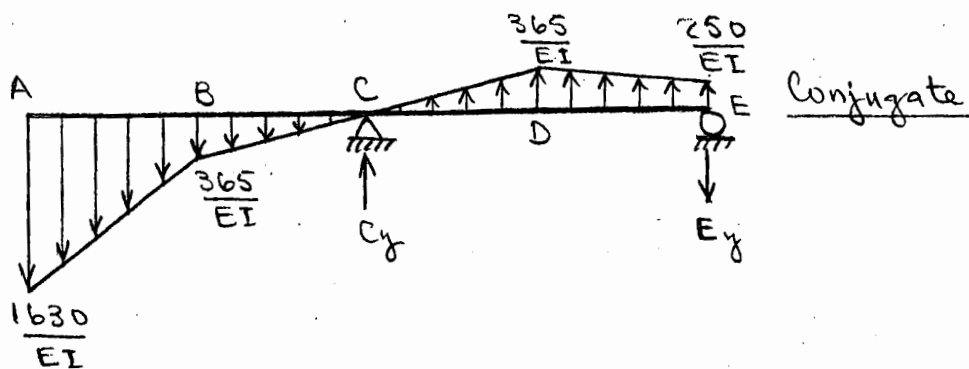
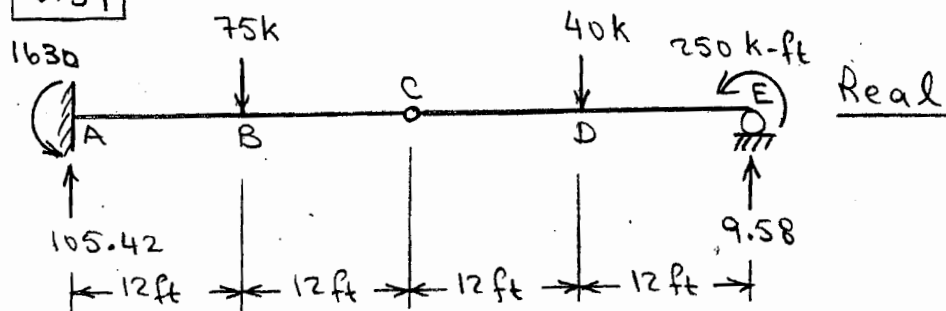
$$\theta_D = \frac{1}{EI} \left[-925.1 + \frac{1}{2} (390) 8.82 - \frac{1}{2} (120) 1.18 - \frac{1}{3} (120) 4 \right]$$

$$= \frac{564 \text{ kN} \cdot \text{m}^2}{EI} = \frac{564}{70(2340)} = 0.00344 \text{ rad} \uparrow$$

$$\Delta_D = \frac{1}{EI} \left[-925.1 (14) + \frac{1}{2} (390) 5 (10.67) + \frac{1}{2} (390) (3.82) (7.727) - \frac{1}{2} (120) 1.18 (4.393) - \frac{1}{3} (120) 4 (3) \right] = \frac{2417 \text{ kN} \cdot \text{m}^3}{EI}$$

$$= \frac{2417}{70(2340)} = 0.01476 \text{ m} = 14.76 \text{ mm} \uparrow$$

6.59



$$+\circlearrowleft \sum M_E = 0$$

$$\frac{1}{EI} \left[\frac{1}{2} (1265) 12 (44) + 365 (12) 42 + \frac{1}{2} (365) 12 (32) - \frac{1}{2} (365) 12 (16) - \frac{1}{2} (115) 12 (8) - 250 (12) 6 \right] - C_y (24) = 0$$

$$C_y = \frac{22060 \text{ k-ft}^2}{EI} \uparrow$$

$$+\uparrow \sum F_y = 0$$

$$\frac{1}{EI} \left[-\frac{1}{2} (1630 + 365) 12 + \frac{1}{2} (365 + 250) 12 + 22060 \right] - E_y = 0$$

$$E_y = \frac{13780 \text{ k-ft}^2}{EI} \downarrow$$

$$\theta_B = -\frac{1}{EI} \left[\frac{1}{2} (1630 + 365) 12 \right] = -\frac{11970 \text{ k-ft}^2}{EI} = \underline{0.0099 \text{ rad} \downarrow}$$

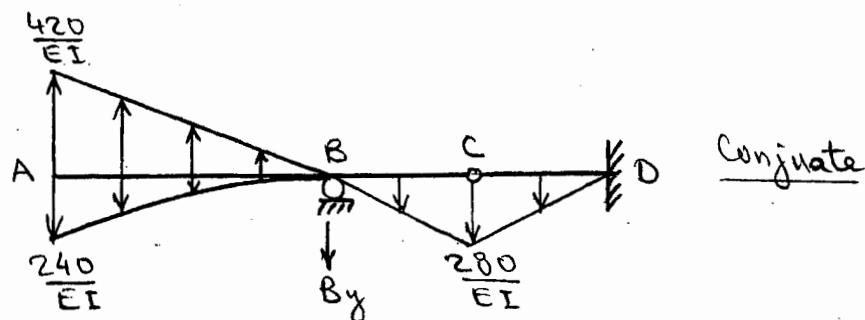
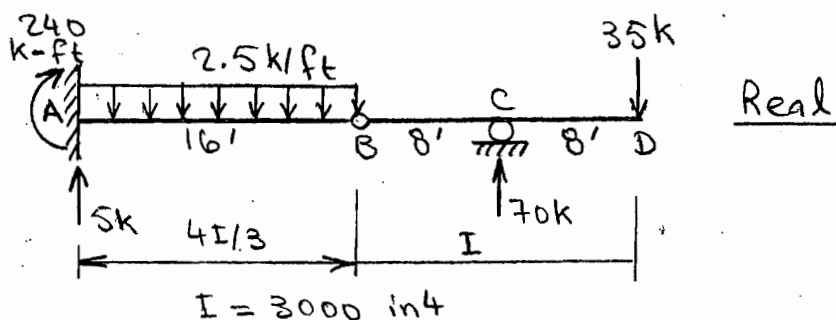
$$\Delta_B = -\frac{1}{EI} \left[365 (12) 6 + \frac{1}{2} (1265) 12 (8) \right] = -\frac{87000 \text{ k-ft}^3}{EI}$$

$$\Delta_B = 0.86 \text{ in} \downarrow$$

$$\theta_D = \frac{1}{EI} \left[13780 - \frac{1}{2} (365 + 250) 12 \right] = \frac{10090}{EI} = \underline{0.0084 \text{ rad} \uparrow}$$

$$\Delta_D = \frac{1}{EI} \left[-13780 (12) + 250 (12) 6 + \frac{1}{2} (115) 12 (4) \right] = -\frac{144600 \text{ k-ft}^3}{EI} = 1.44 \text{ in} \downarrow$$

6.60



$$+\circlearrowleft \sum M_C = 0$$

$$\frac{1}{EI} \left[\frac{1}{3} (240) 16 (20) - \frac{1}{2} (420) 16 (18.67) + \frac{1}{2} (280) 8 \left(\frac{8}{3}\right) \right]$$

$$+ B_y (8) = 0$$

$$B_y = \frac{4266.67 \text{ k-ft}^2}{EI} \downarrow$$

$$\theta_{B, \text{Left}} = \frac{1}{EI} \left[\frac{1}{2} (420) 16 - \frac{1}{3} (240) 16 \right] = \frac{2080 \text{ k-ft}^2}{EI}$$

$$= \frac{2080 (12)^2}{30000 (3000)} = 0.0033 \text{ rad. } \nearrow$$

$$\theta_{B, \text{Right}} = \frac{1}{EI} (2080 - 4266.67) = \frac{-2186.67 \text{ k-ft}^2}{EI}$$

$$= -0.0035 \text{ rad. } \searrow$$

$$\Delta_B = \frac{1}{EI} \left[\frac{1}{2} (420) 16 (10.67) - \frac{1}{3} (240) 16 (12) \right]$$

$$= \frac{20480 \text{ k-ft}^3}{EI} = \frac{20480 (12)^3}{30000 (3000)} = 0.39 \text{ in } \uparrow$$

$$\theta_D = \frac{1}{EI} \left[\frac{1}{2} (420) 16 - \frac{1}{3} (240) 16 - 4266.67 - \frac{1}{2} (280) 16 \right]$$

$$= \frac{-4426.67 \text{ k-ft}^2}{EI} = -0.0071 \text{ rad. } = 0.0071 \text{ rad. } \searrow$$

$$\Delta_D = \frac{1}{EI} \left[\frac{1}{2} (420) 16 (26.67) - \frac{1}{3} (240) 16 (28) - 4266.67 (16) - \frac{1}{2} (280) 16 (8) \right]$$

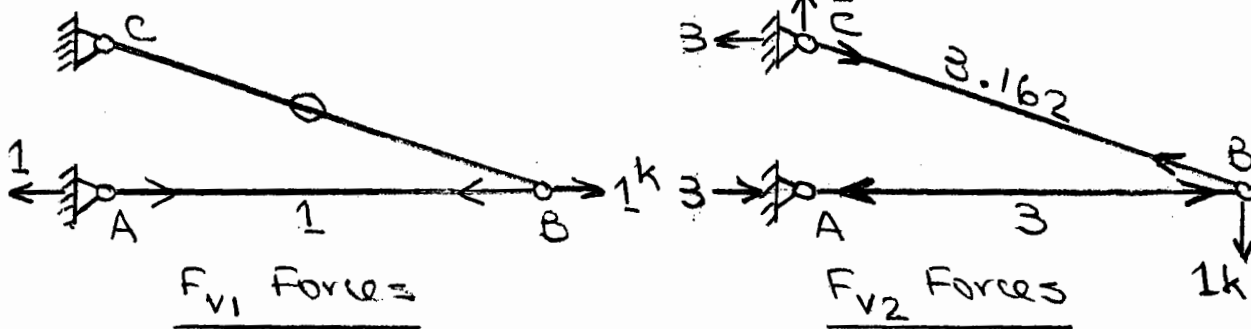
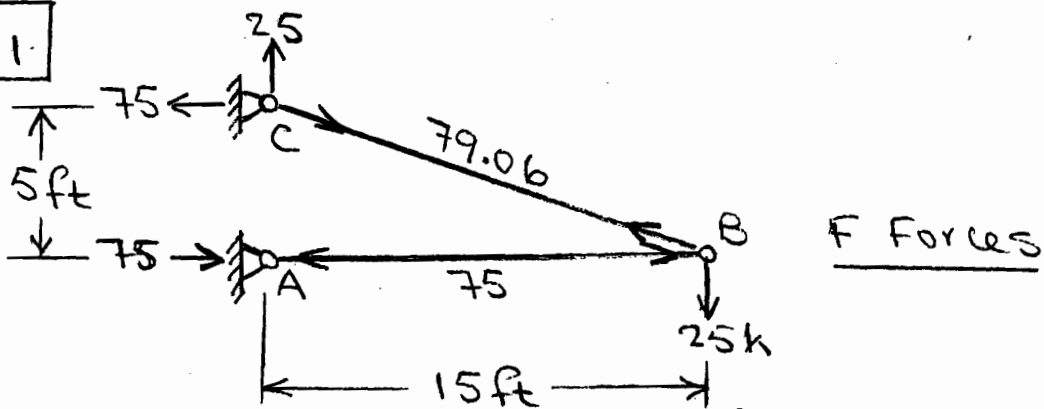
$$= \frac{-32426.67}{EI} = 0.62 \text{ in } \downarrow$$

Chapter Seven

Deflections of Trusses, Beams, and Frames: Work-Energy Methods

CHAPTER 7

7.1



Member	L (in)	F (k)	F_{v1} (k)	$F_{v1}(FL)$ (k ² -in)	F_{v2} (k)	$F_{v2}(FL)$ (k ² -in)
AB	180	-75	1	-13500	-3	40500
BC	189.74	79.06	0	0	3.162	47432.67
Σ				-13500		87932.67

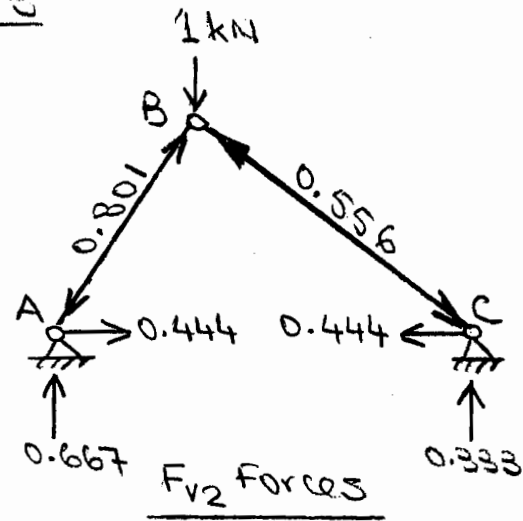
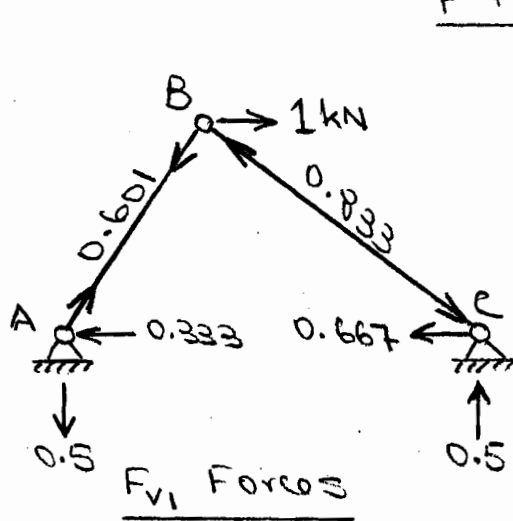
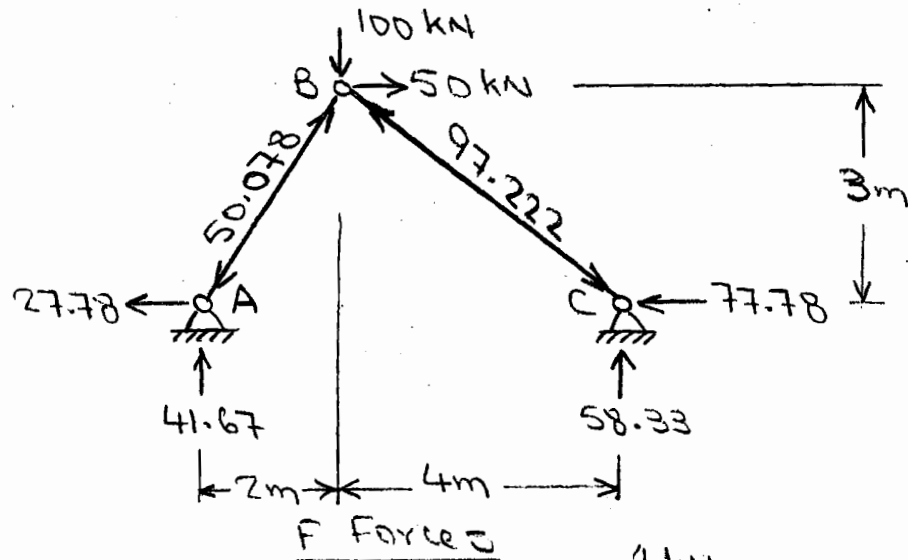
$$(1k) \Delta_{EH} = -\frac{13500}{10000(6)} = -0.225 \text{ k-in.}$$

$$\Delta_{EH} = -0.225 \text{ in.} = \underline{0.225 \text{ in.} \leftarrow}$$

$$(1k) \Delta_{BY} = \frac{87932.67}{10000(6)} = 1.466 \text{ k-in.}$$

$$\Delta_{BY} = \underline{1.466 \text{ in.} \downarrow}$$

7.2



Member	L (m)	F (kN)	F _{v1} (kN)	F _{v1} (FL) (kN ² -m)	F _{v2} (kN)	F _{v2} (FL) (kN ² -m)
AB	3.606	-50.078	0.601	-108.53	-0.801	144.65
BC	5	-97.222	-0.833	405.09	-0.556	270.06
Σ				296.56		414.71

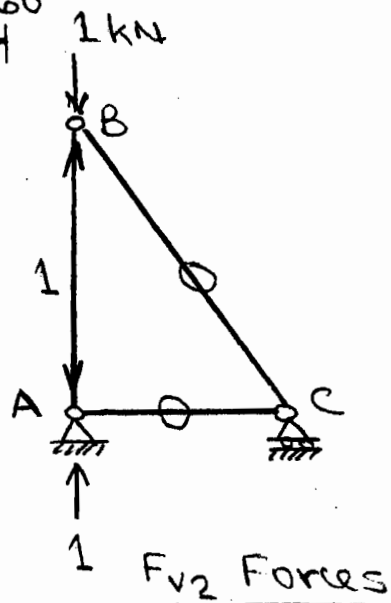
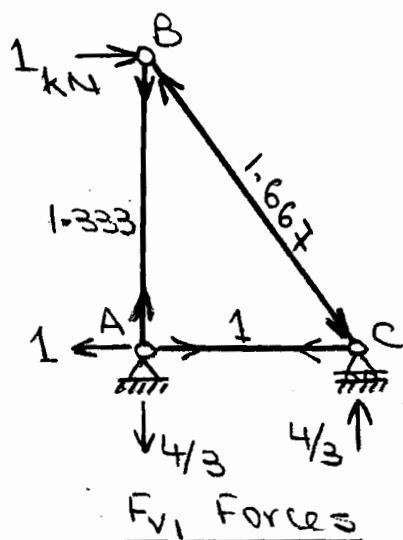
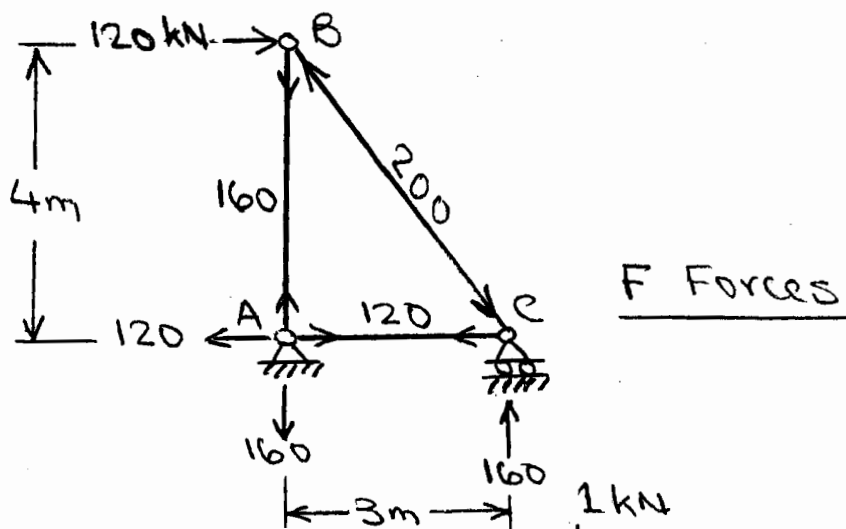
$$(1 \text{ kN}) \Delta_{BH} = \frac{296.56}{70(10^6)(0.001)} = 0.00424 \text{ kN-m}$$

$$\Delta_{BH} = 0.00424 \text{ m} = 4.24 \text{ mm} \rightarrow$$

$$(1 \text{ kN}) \Delta_{BV} = \frac{414.71}{70(10^6)(0.001)} = 0.00592 \text{ kN-m}$$

$$\Delta_{BV} = 0.00592 \text{ m} = 5.92 \text{ mm} \downarrow$$

7.3



Member	L (m)	F (kN)	F_{v1} (kN)	$F_{v1}(FL)$ (kN ² -m)	F_{v2} (kN)	$F_{v2}(FL)$ (kN ² -m)
AC	3	120	1	360	0	0
AB	4	160	1.333	853.33	-1	-640
BC	5	-200	-1.667	1666.67	0	0
Σ				2880		-640

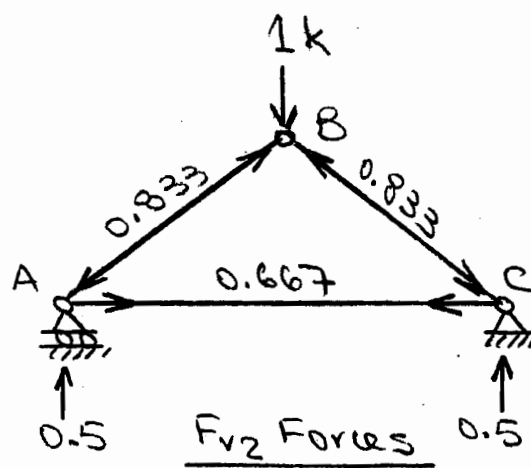
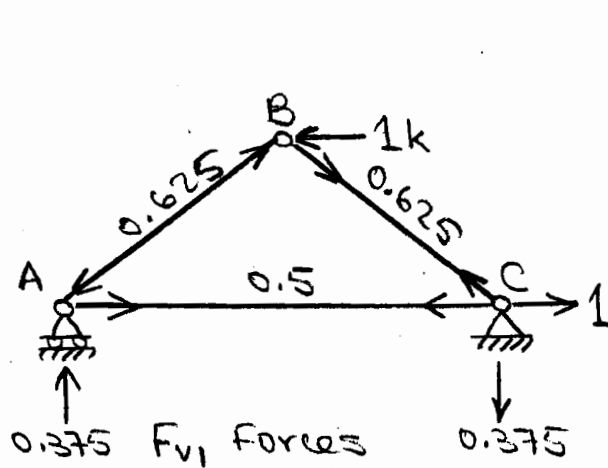
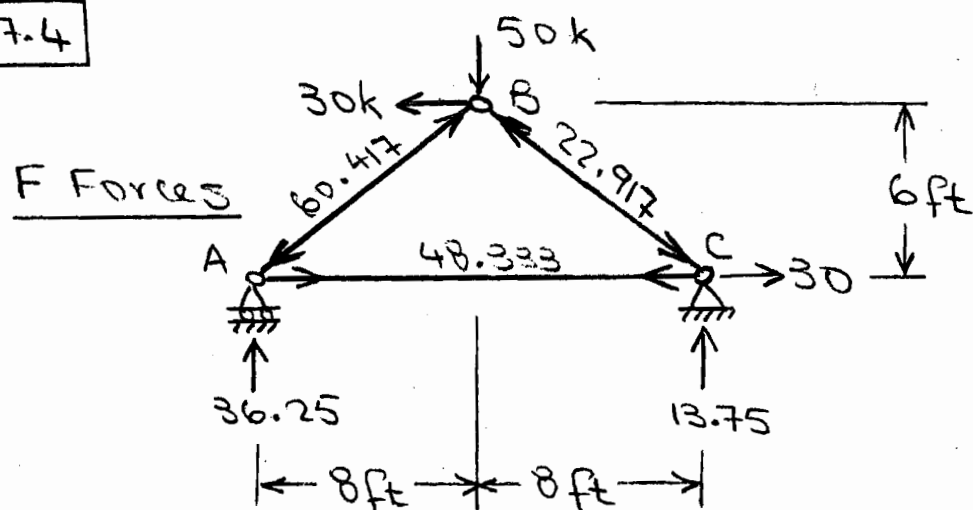
$$(1 \text{ kN}) \Delta_{BH} = \frac{2880}{200(10^6)(0.0015)} = 0.0096 \text{ kN.m}$$

$$\Delta_{BH} = 0.0096 \text{ m} = 9.6 \text{ mm} \rightarrow$$

$$(1 \text{ kN}) \Delta_{BV} = \frac{-640}{200(10^6)(0.0015)} = -0.00213 \text{ kN.m}$$

$$\Delta_{BV} = -0.00213 \text{ m} = 2.13 \text{ mm} \uparrow$$

7.4



Member	L (in.)	F (k)	F _{v1} (k)	F _{v1} (FL) (k ² -in.)	F _{v2} (k)	F _{v2} (FL) (k ² -in.)
AC	192	48.333	0.5	4640	0.667	6186.67
AB	120	-60.417	-0.625	4531.25	-0.833	6041.67
BC	120	-22.917	0.625	-1718.75	-0.833	2291.67
Σ				7452.5		14520

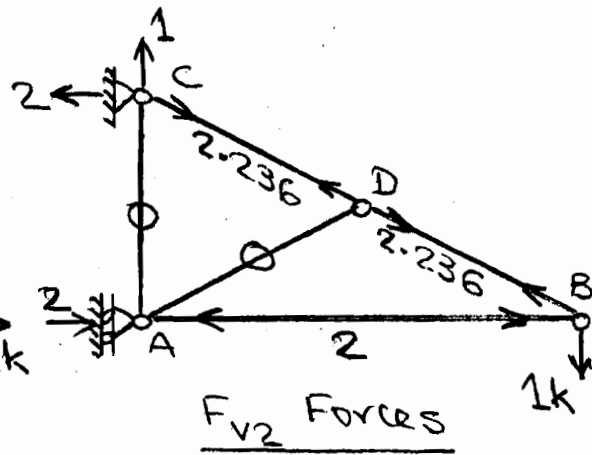
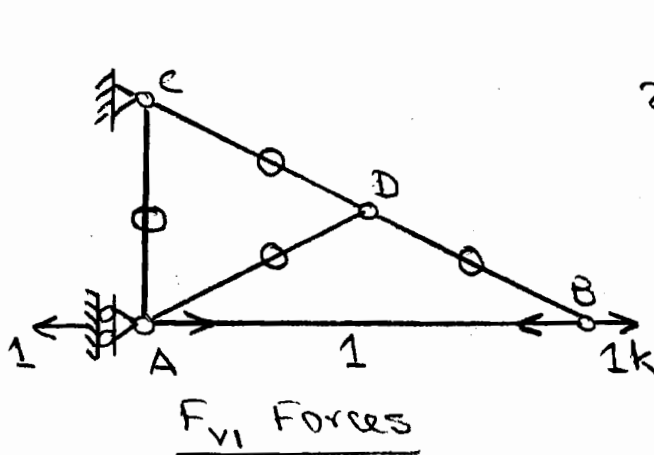
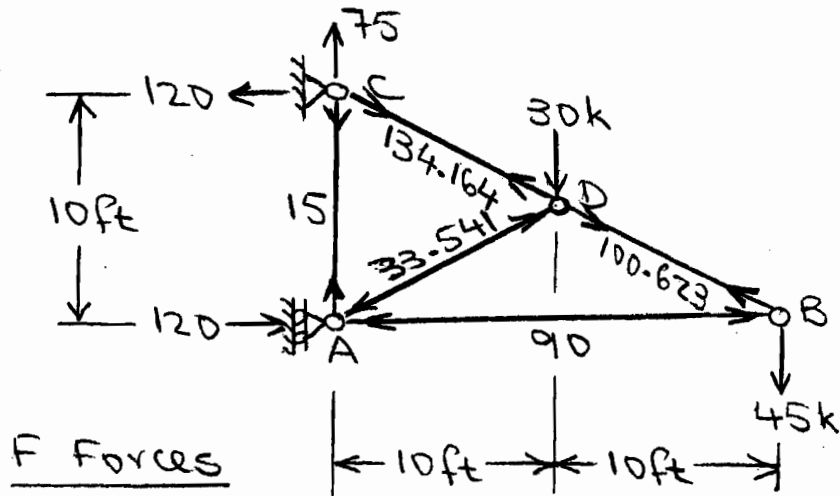
$$(1k) \Delta_{BH} = \frac{7452.5}{29000(3)} = 0.0857 \text{ k-in.}$$

$$\Delta_{BH} = 0.0857 \text{ in.} \leftarrow$$

$$(1k) \Delta_{ev} = \frac{14520}{29000(3)} = 0.167 \text{ k-in.}$$

$$\Delta_{ev} = 0.167 \text{ in.} \downarrow$$

7.5



Member	L (in.)	A (in. ²)	F (k)	F _{v1} (k)	F _{v1} (FL/A) (k ² /in.)	F _{v2} (k)	F _{v2} (FL/A) (k ² /in.)
AB	240	6	-90	1	-3600	-2	7200
AC	120	4	15	0	0	0	0
AD	134.16	4	-33.54	0	0	0	0
CD	134.16	6	134.164	0	0	2.236	6707.99
BD	134.16	6	100.623	0	0	2.236	5030.99
Σ					-3600		18938.98

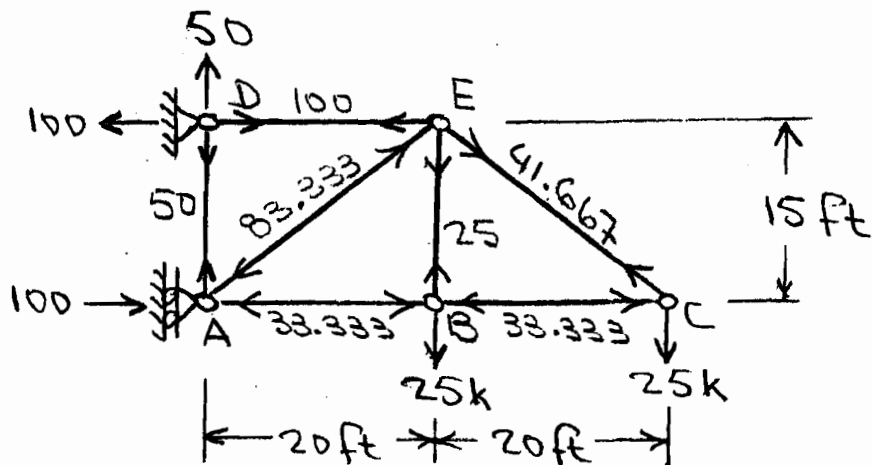
$$(1k) \Delta_{BH} = \frac{-3600}{10000} = -0.36 \text{ k-in}$$

$$\Delta_{BH} = 0.36 \text{ in.} \leftarrow$$

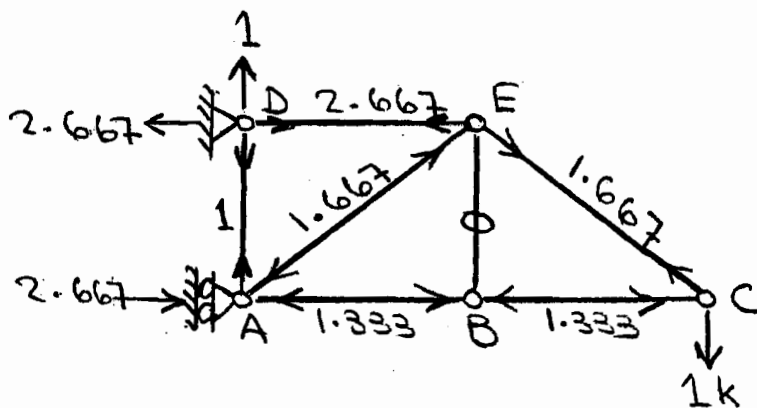
$$(1k) \Delta_{BV} = \frac{18938.99}{10000} = 1.894 \text{ k-in.}$$

$$\Delta_{BV} = 1.894 \text{ in.} \downarrow$$

7.6



F Forces

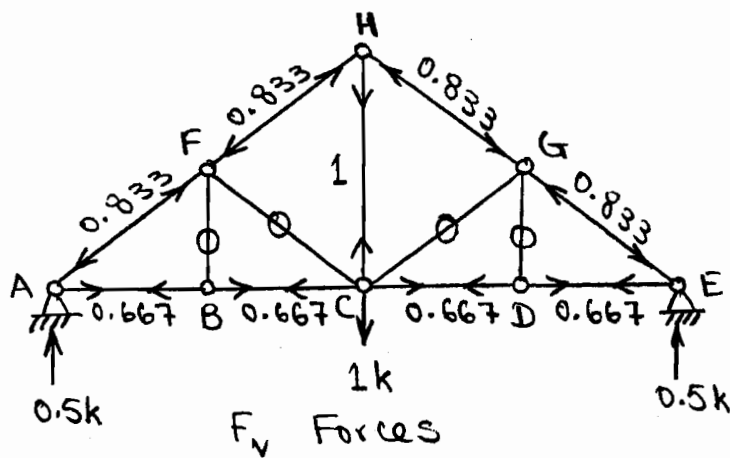
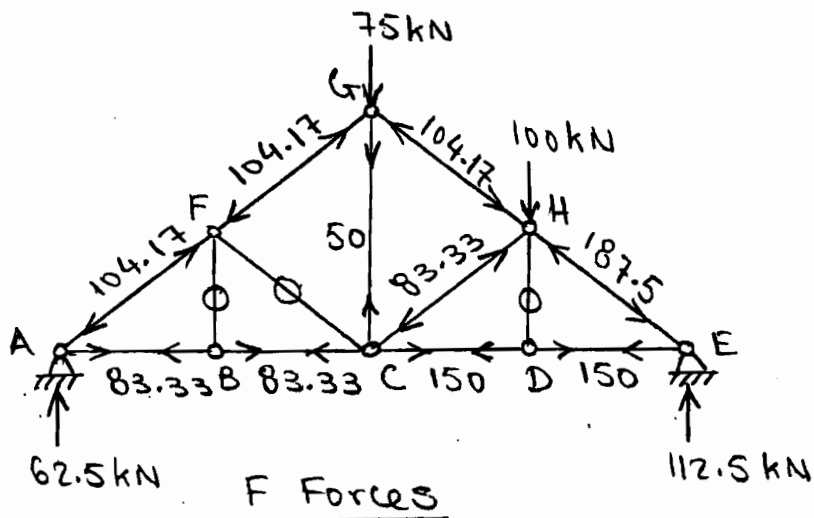


F_v Forces

Member	L (in.)	A (in ²)	F (k)	F _v (k)	F _v (FL/A) (k ² /in.)
AB	240	4	-33.333	-1.333	2666.67
BC	240	4	-33.333	-1.333	2666.67
DE	240	4	100	2.667	16000
AD	180	3	50	1	3000
BE	180	3	25	0	0
AE	300	3	-83.333	-1.667	13888.89
CE	300	4	41.667	1.667	5208.33
Σ					43430.56

$$\Delta_C = \frac{43430.56}{29000} = 1.498 \text{ in.} \downarrow$$

7.7

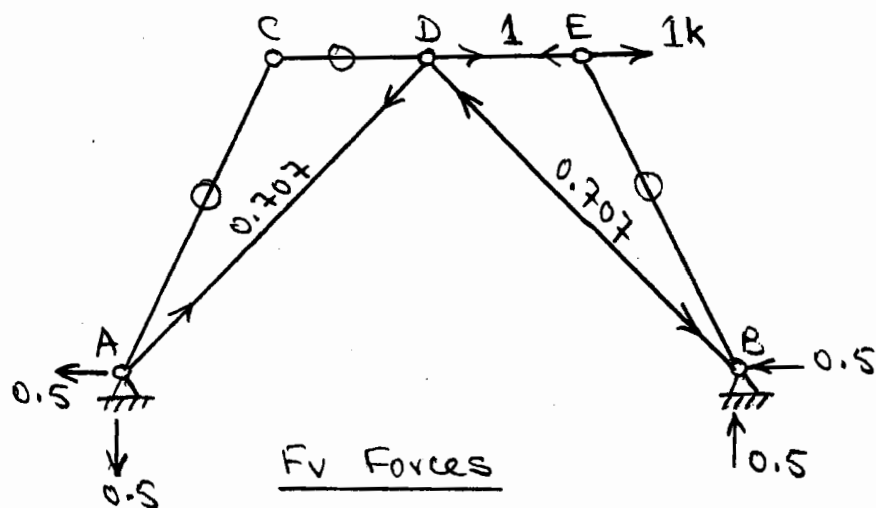
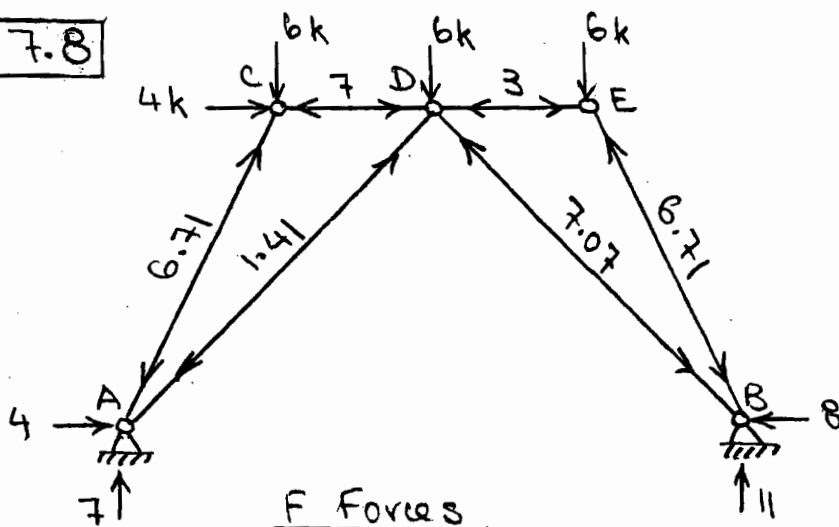


Member	L (m)	F (kN)	F _v (kN)	F _v (FL) (kN ² -m)
AB	6	83.33	0.667	333.5
BC	6	83.33	0.667	333.5
CD	6	150	0.667	600.3
DE	6	150	0.667	600.3
AF	7.5	-104.17	-0.833	650.8
FG	7.5	-104.17	-0.833	650.8
GH	7.5	-104.17	-0.833	650.8
EH	7.5	-187.5	-0.833	1171.4
CG	9	50	1	450
Σ				5441.4

$$(1 \text{ kN}) \Delta_C = \frac{5441.4}{200(10^6)(0.003)} = 0.0091 \text{ kN-m}$$

$$\Delta_C = 0.0091 \text{ m} = \underline{9.1 \text{ mm} \downarrow}$$

7.8

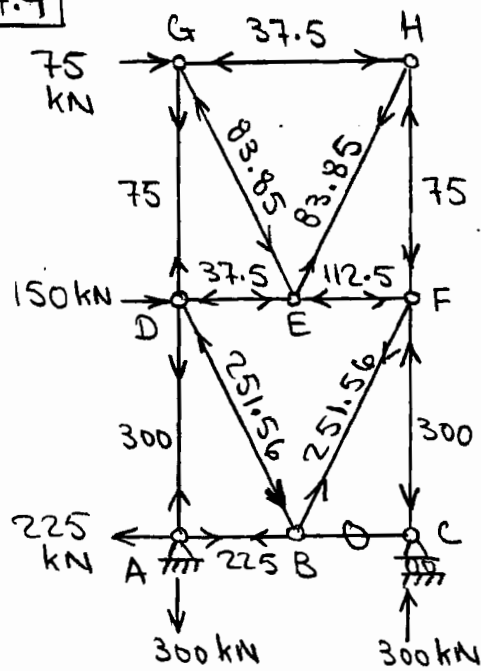


Member	L (ft)	F (k)	Fv (k)	Fv(FL) (k-ft)
AD	14.14	-1.41	0.707	-14.1
BD	14.14	-7.07	-0.707	70.68
DE	5	-3	1	-15
Σ				41.58

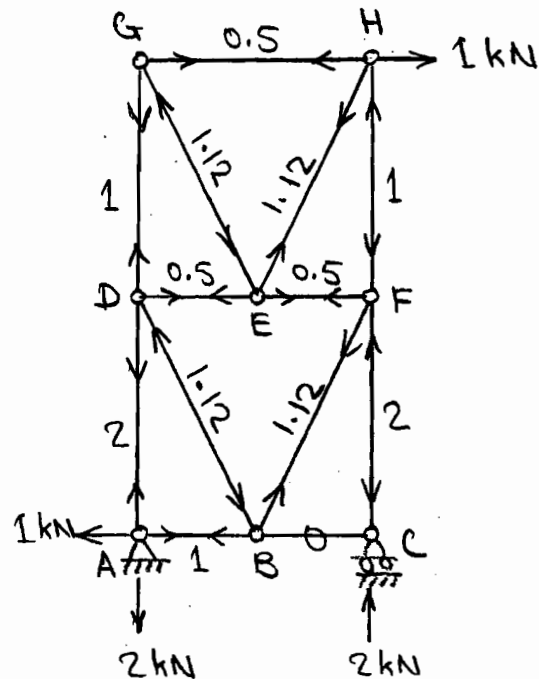
$$\Delta_{EH} = \frac{41.58}{(29000)6} = 0.000239 \text{ ft.}$$

$$= 0.0029 \text{ in.} \rightarrow$$

7.9



F Forces



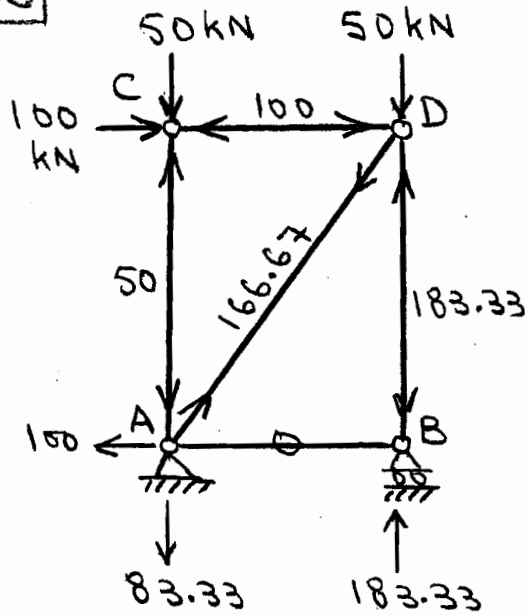
F_v Forces

Member	L (m)	A (m ²)	F (kN)	F _v (kN)	F _v (FL/A) (kN ² /m)
AD	4	0.0025	300	2	960 000
CF	4	0.0025	-300	-2	960 000
DG	4	0.0025	75	1	120 000
FH	4	0.0025	-75	-1	120 000
GH	4	0.0015	-37.5	0.5	-50 000
AB	2	0.0015	225	1	300 000
DE	2	0.0015	-37.5	0.5	-25 000
EF	2	0.0015	-112.5	0.5	-75 000
BD	4.47	0.0015	-251.56	-1.12	839 607
BF	4.47	0.0015	251.56	1.12	839 607
EG	4.47	0.0015	-83.85	-1.12	279 858
EH	4.47	0.0015	83.85	1.12	279 858
Σ					454 8930

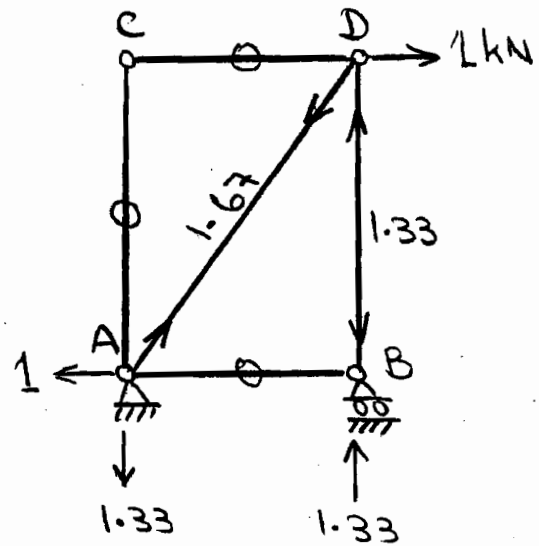
$$(1 \text{ kN}) \Delta_H = \frac{454 8930}{200 (10^6)} = 0.023 \text{ kN-m}$$

$$\Delta_H = 0.023 \text{ m} = 23 \text{ mm} \rightarrow$$

7.10



F Forces



F_v Forces

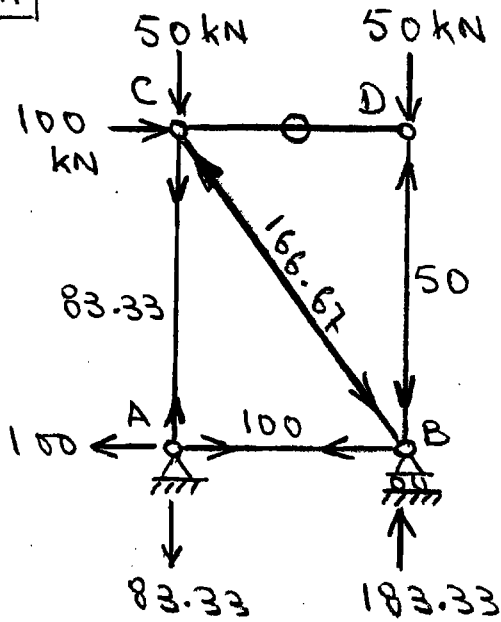
Member	L (m)	F (kN)	F _v (kN)	F _v (FL) (kN ² .m)
AB	3	0	0	0
CD	3	-100	0	0
AC	4	-50	0	0
BD	4	-183.33	-1.33	975.32
AD	5	166.67	1.67	1391.69
Σ				2367

$$(1 \text{ kN}) \Delta_D = \frac{2367 \text{ kN}^2 \cdot \text{m}}{EA}$$

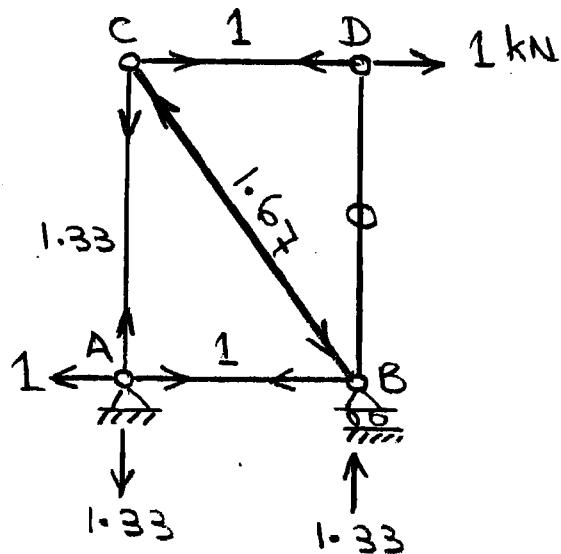
$$\Delta_D = \frac{2367 \text{ kN} \cdot \text{m}}{EA} = \frac{2367}{70(10^6)A} = 0.01 \text{ m}$$

$$\text{from which, } A = 0.003381 \text{ m}^2 = \underline{3381 \text{ mm}^2}$$

7.11



F Forces



F_v Forces

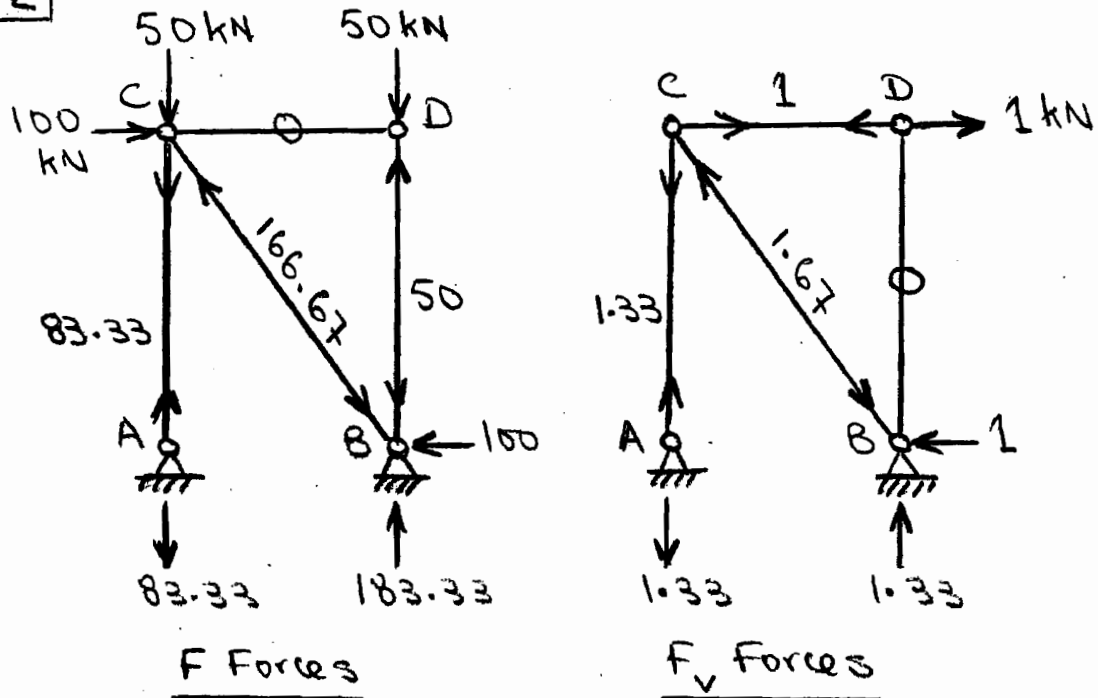
Member	L (m)	F (kN)	F _v (kN)	F _v (FL) (kN ² .m)
AB	3	100	1	300
CD	3	0	1	0
AC	4	83.33	1.33	443.32
BD	4	-50	0	0
BC	5	-166.67	-1.67	1391.69
Σ				2135

$$(1 \text{ kN}) \Delta_D = \frac{2135 \text{ kN}^2 \cdot \text{m}}{EA}$$

$$\Delta_D = \frac{2135 \text{ kN} \cdot \text{m}}{EA} = \frac{2135}{70(10^6)A} = 0.01 \text{ m}$$

$$\text{from which, } A = 0.00305 \text{ m}^2 = \underline{3050 \text{ mm}^2}$$

7.12



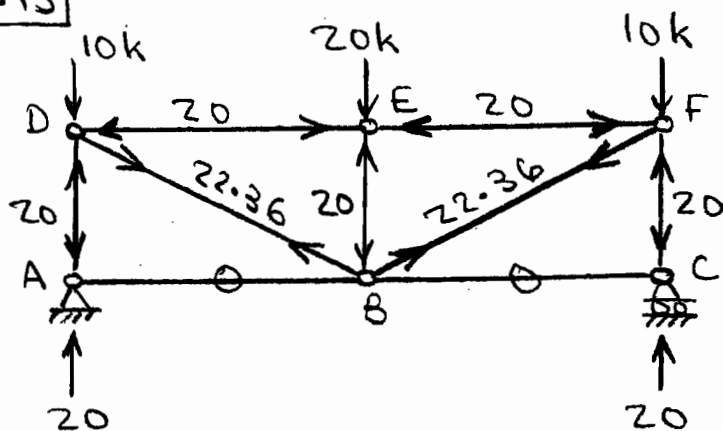
Member	L (m)	F (kN)	F _v (kN)	F _v (FL) (kN ² .m)
CD	3	0	1	0
AC	4	83.33	1.33	443.32
BD	4	-50	0	0
BC	5	-166.67	-1.67	1391.69
Σ				1835

$$(1 \text{ kN}) \Delta_D = \frac{1835 \text{ kN}^2 \cdot \text{m}}{EA}$$

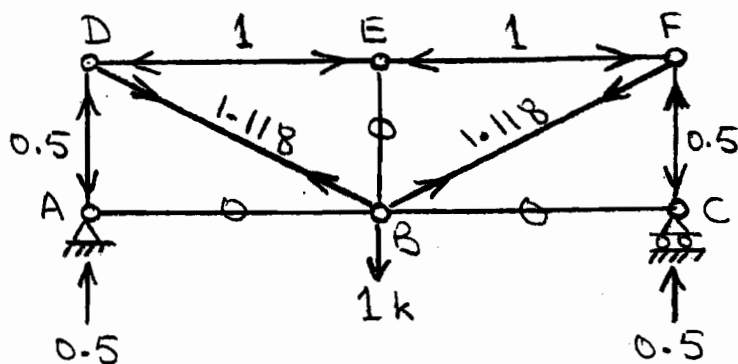
$$\Delta_D = \frac{1835 \text{ kN} \cdot \text{m}}{EA} = \frac{1835}{70(10^6)A} = 0.01 \text{ m}$$

$$\text{From which, } A = 0.002621 \text{ m}^2 = \underline{2621 \text{ mm}^2}$$

7.13



F Forces



F_v Forces

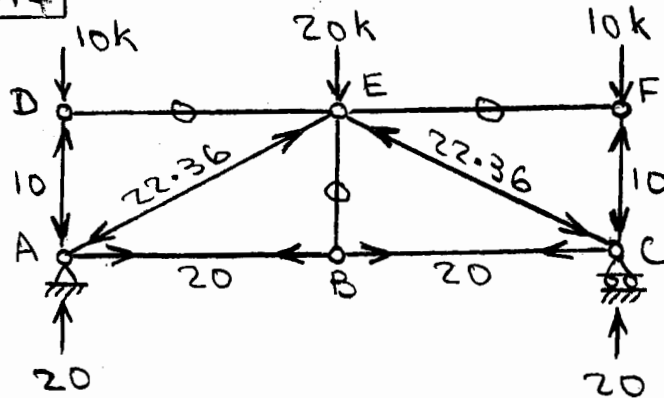
Member	L (in)	F (k)	F _v (k)	F _v (FL) (k ² -in)
AB	72	0	0	0
BC	72	0	0	0
DE	72	-20	-1	1440
EF	72	-20	-1	1440
AD	36	-20	-0.5	360
BE	36	-20	0	0
CF	36	-20	-0.5	360
BD	80.5	22.36	1.118	2012.4
BF	80.5	22.36	1.118	2012.4
Σ				7624.8

$$(1k) \Delta_B = \frac{7624.8 \text{ k}^2\text{-in}}{EA}$$

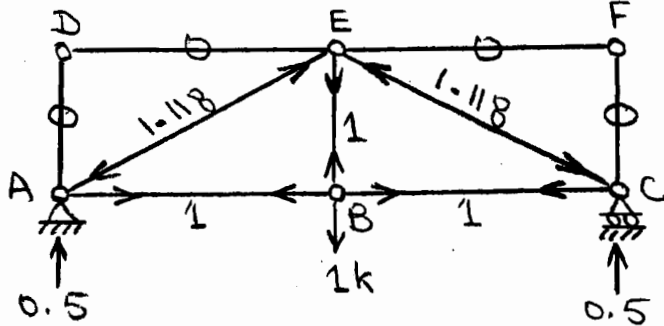
$$\Delta_B = \frac{7624.8 \text{ k-in}}{EA} = \frac{7624.8}{1600(A)} = 0.4 \text{ in}$$

from which, $A = 11.91 \text{ in}^2$

7.14



F Forces



F_v Forces

Member	L (in.)	F (k)	F _v (k)	F _v (FL) (k ² -in)
AB	72	20	1	1440
BC	72	20	1	1440
DE	72	0	0	0
EF	72	0	0	0
AD	36	-10	0	0
BE	36	0	1	0
CF	36	-10	0	0
AE	80.5	-22.36	-1.118	2012.4
CE	80.5	-22.36	-1.118	2012.4
Σ				6904.8

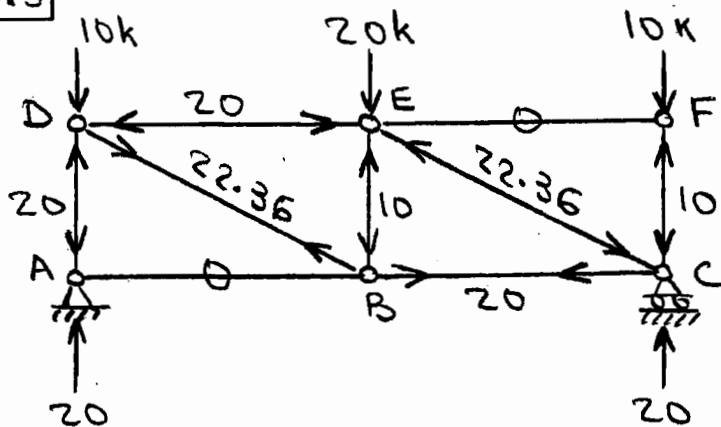
$$(1k) \Delta_B = \frac{6904.8 \text{ k}^2\text{-in}}{EA}$$

$$\Delta_B = \frac{6904.8 \text{ k-in}}{EA} = \frac{6904.8}{1600(A)} = 0.4 \text{ in}$$

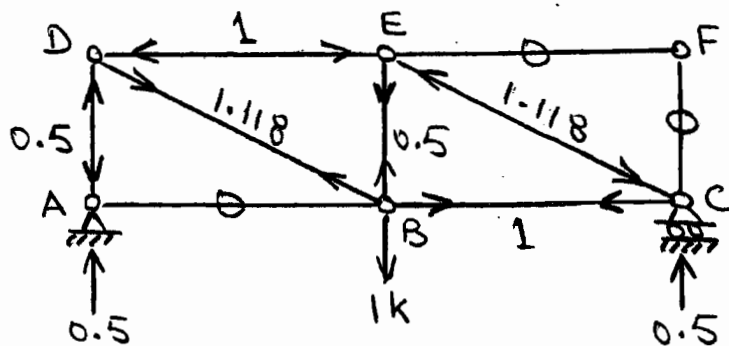
From which,

$$A = 10.79 \text{ in}^2$$

7.15



F Forces



F_v Forces

Member	L (in.)	F (k)	F _v (k)	F _v (FL) (k ² -in.)
AB	72	0	0	0
BC	72	20	1	1440
DE	72	-20	-1	1440
EF	72	0	0	0
AD	36	-20	-0.5	360
BE	36	-10	0.5	-180
CF	36	-10	0	0
BD	80.5	22.36	1.118	2012.4
CE	80.5	-22.36	-1.118	2012.4
Σ				7084.8

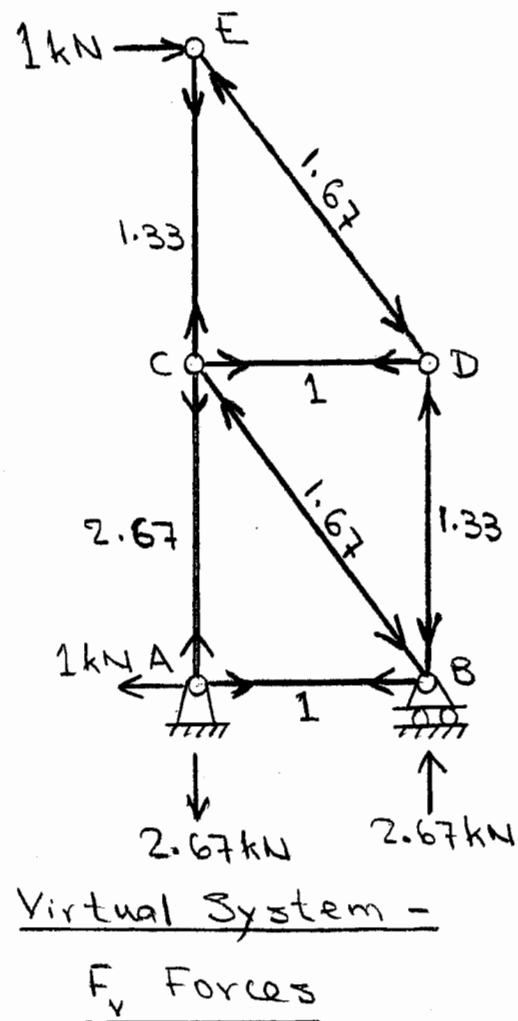
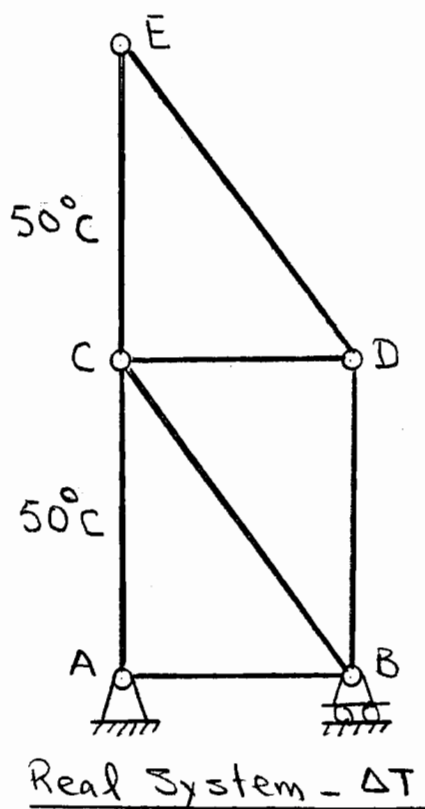
$$(1k) \Delta_B = \frac{7084.8 \text{ k}^2\text{-in.}}{EA}$$

$$\Delta_B = \frac{7084.8 \text{ k-in.}}{EA} = \frac{7084.8}{1600(A)} = 0.4 \text{ in}$$

From which,

$$A = 11.07 \text{ in}^2$$

7.16

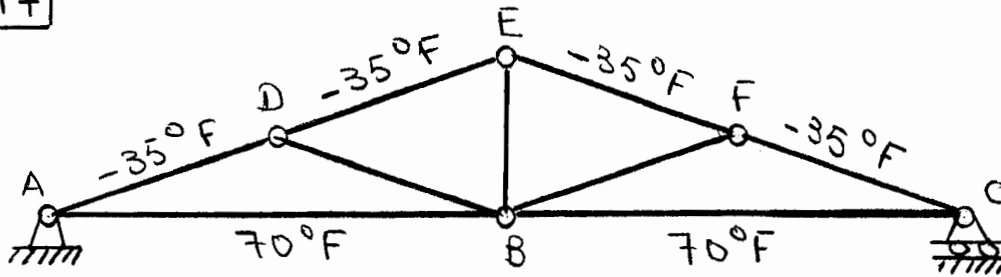


Member	F_v (kN)
AC	2.67
CE	1.33
Σ	4

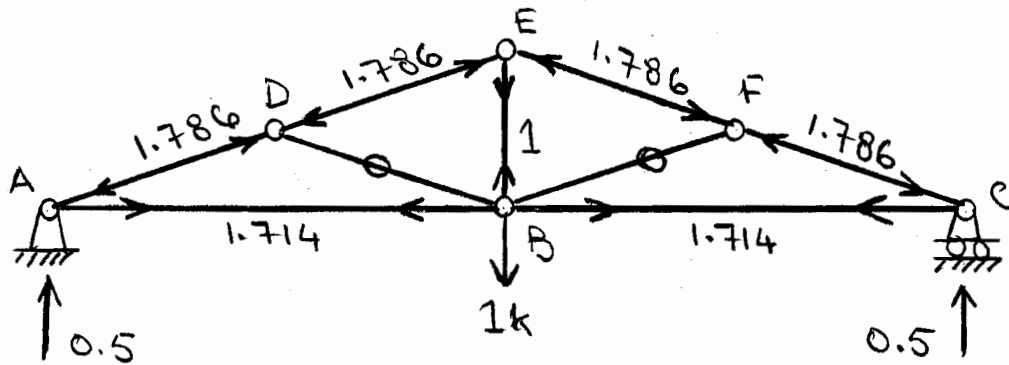
$$(1 \text{ kN}) \Delta_E = \alpha (\Delta T) L \Sigma F_v = 1.2 (10^{-5}) (50) (4) (4) \\ = 0.0096 \text{ kN.m}$$

$$\Delta_E = 0.0096 \text{ m} = \underline{9.6 \text{ mm} \rightarrow}$$

7.17



Real System - ΔT



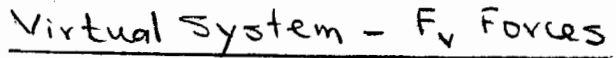
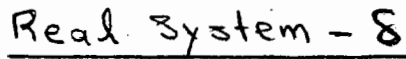
Virtual System - F_v Forces

Member	L (in.)	ΔT (°F)	F_v (k)	$F_v(\Delta T)L$ (k-in-°F)
AB	288	70	1.714	34560
BC	288	70	1.714	34560
AD	150	-35	-1.786	9375
DE	150	-35	-1.786	9375
EF	150	-35	-1.786	9375
CF	150	-35	-1.786	9375
Σ				106620

$$(1k) \Delta_B = \alpha \Sigma F_v(\Delta T)L = 6.5(10^{-6})(106620)$$

$$\Delta_B = 0.693 \text{ in.} \downarrow$$

7.18

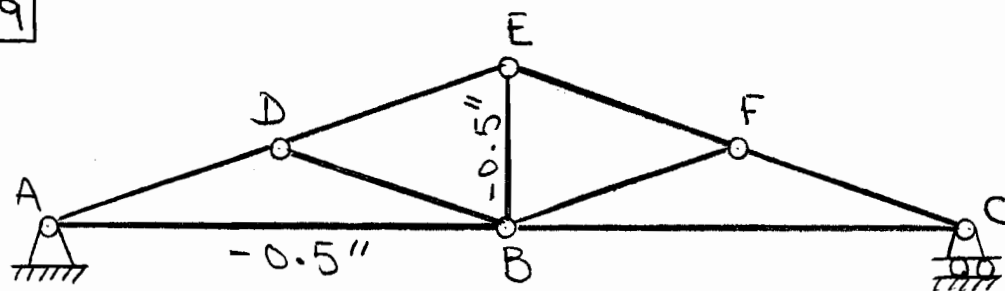


Member	δ (mm)	F_v (kN)	$F_v(\delta)$ (kN.mm)
BC	18	-1.67	-30
CE	-15	1.33	-20
Σ			-50

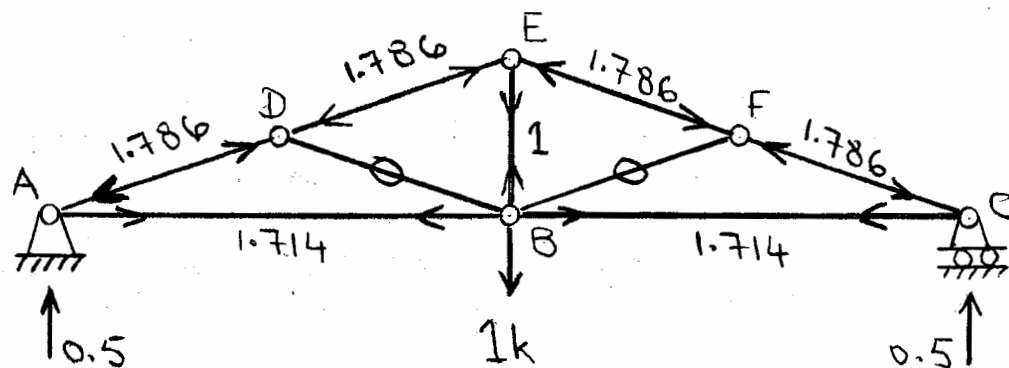
$$(1 \text{ kN}) \Delta_E = \sum F_v (\delta) = -50 \text{ kN} \cdot \text{mm}$$

$$\Delta_E = -50 \text{ mm} = \underline{50 \text{ mm} \leftarrow}$$

7.19



Real System - δ



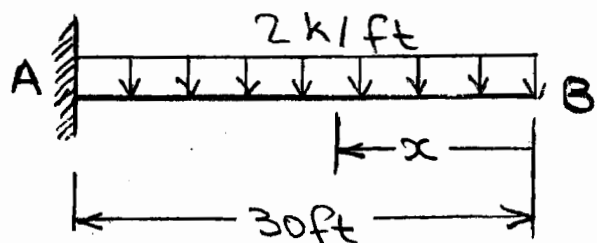
Virtual System - F_v Forces

Member	F_v (k)
AB	1.714
BE	1
Σ	2.714

$$(1k) \Delta_B = \delta \Sigma F_v = (-0.5) 2.714 = -1.357 \text{ k-in.}$$

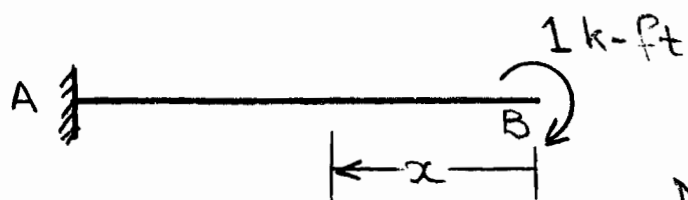
$$\Delta_B = -1.357 \text{ in.} = \underline{1.357 \text{ in.} \uparrow}$$

7.20



Real System - M

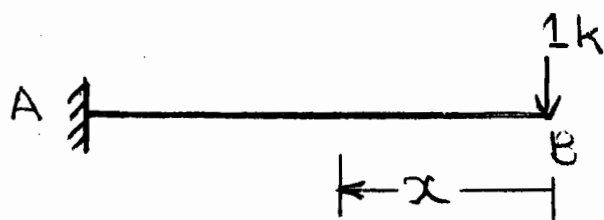
$$M = -\frac{2x^2}{2} = -x^2$$



Virtual System - M_{v1}

$$M_{v1} = -1$$

$$\begin{aligned}\theta_B &= \frac{1}{EI} \int_0^{30} -1(-x^2) dx = \frac{9000 \text{ k-ft}^2}{EI} \\ &= \frac{9000 (12)^2}{29000 (3000)} = \underline{0.0149 \text{ rad} \downarrow}\end{aligned}$$

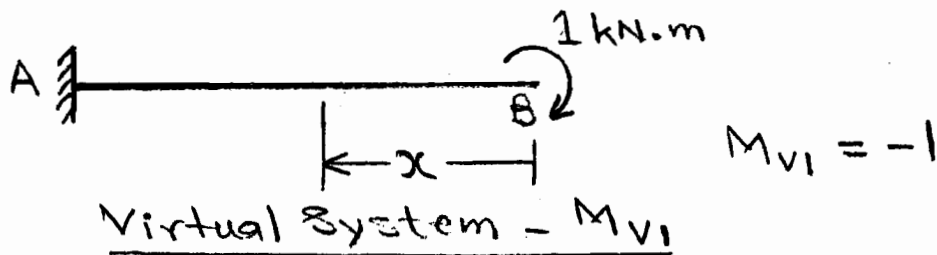
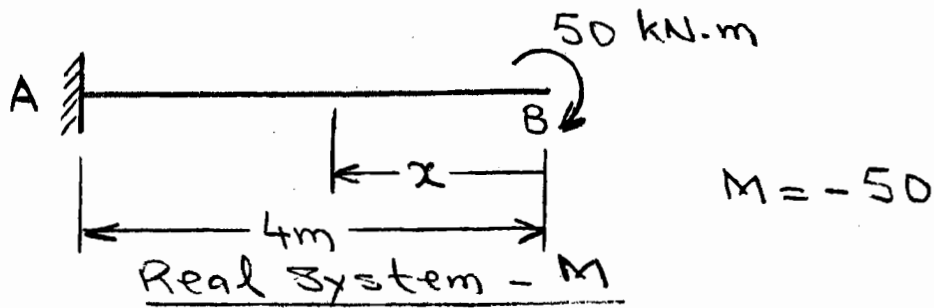


Virtual System M_{v2}

$$M_{v2} = -1(x) = -x$$

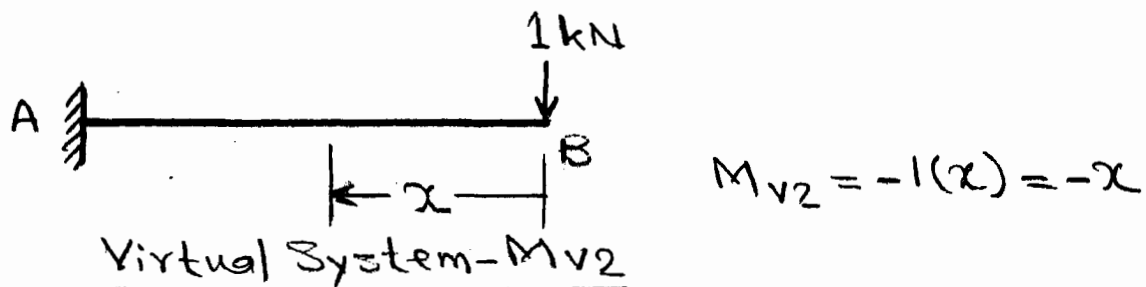
$$\begin{aligned}\Delta_B &= \frac{1}{EI} \int_0^{30} (-x)(-x^2) dx = \frac{202500 \text{ k-ft}^3}{EI} \\ &= \frac{202500 (12)^3}{29000 (3000)} = \underline{4.022 \text{ in.} \downarrow}\end{aligned}$$

7.21



$$\theta_B = \frac{1}{EI} \int_0^4 (-1)(-50) dx = \frac{200 \text{ kN}\cdot\text{m}^2}{EI}$$

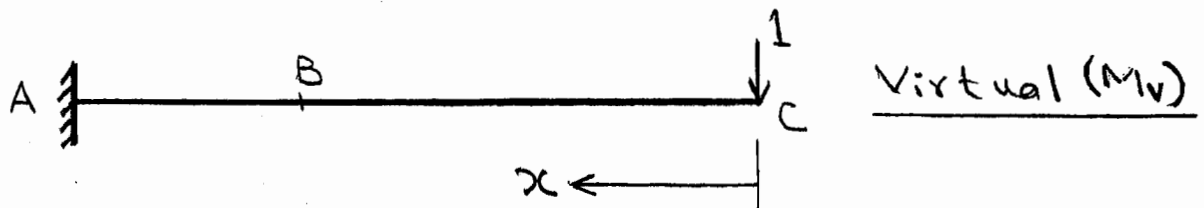
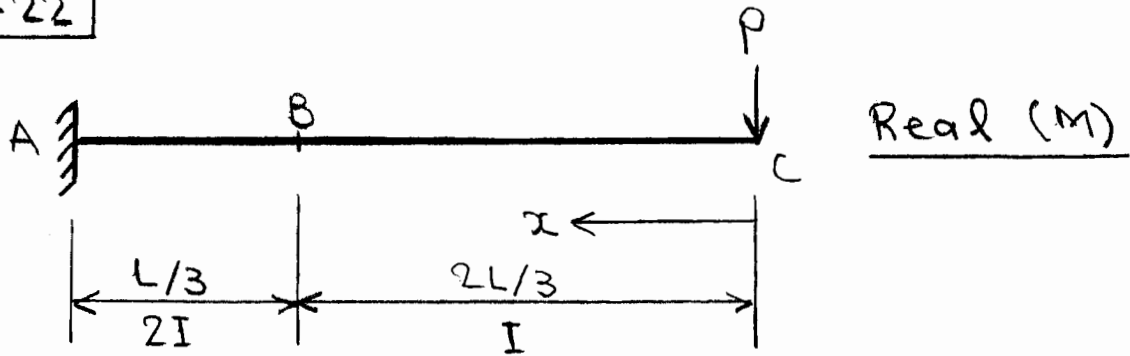
$$= \frac{200}{70(164)} = \underline{0.0174 \text{ rad} \downarrow}$$



$$\Delta_B = \frac{1}{EI} \int_0^4 (-x)(-50) dx = \frac{400 \text{ kN}\cdot\text{m}^3}{EI}$$

$$= \frac{400}{70(164)} = 0.0348 \text{ m} = \underline{34.8 \text{ mm} \downarrow}$$

7.22

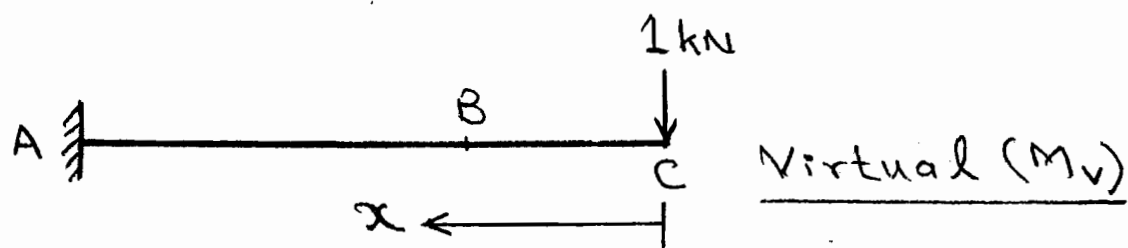
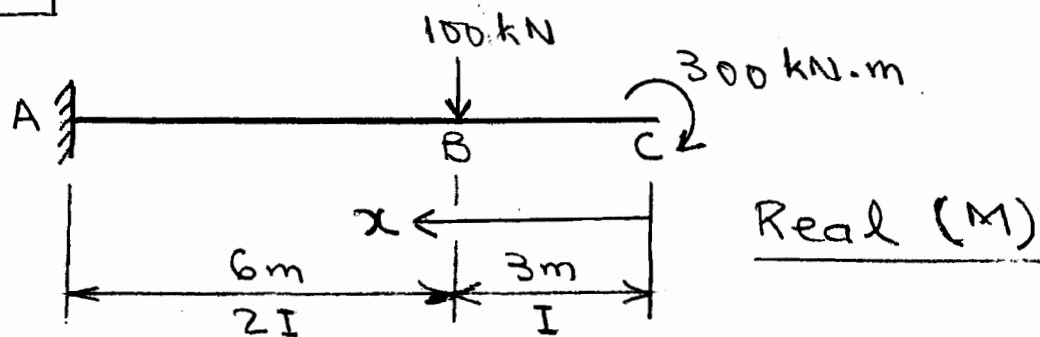


Segment	x Coordinate		M	M_v
	Origin	Limits		
CB	C	$0 - \frac{2L}{3}$	$-Px$	$-1x$
BA	C	$\frac{2L}{3} - L$	$-Px$	$-1x$

$$\Delta_C = \frac{1}{EI} \left[\int_0^{\frac{2L}{3}} (-x)(-Px) dx + \frac{1}{2} \int_{\frac{2L}{3}}^L (-x)(-Px) dx \right]$$

$$= \underline{\underline{\frac{35PL^3}{162EI} \downarrow}}$$

7.23



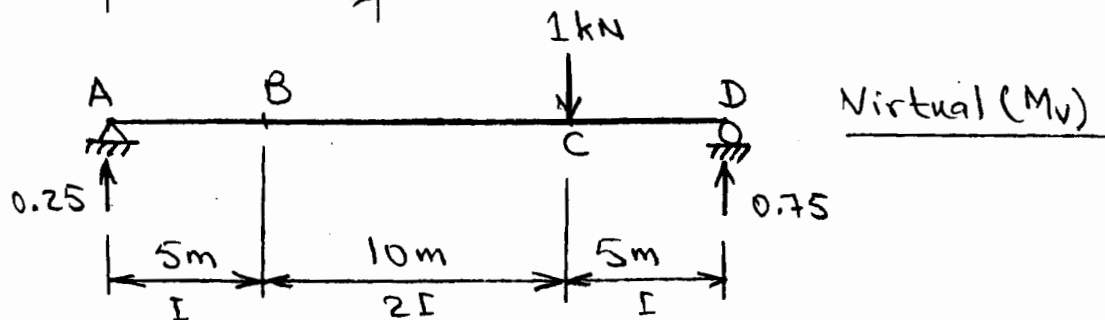
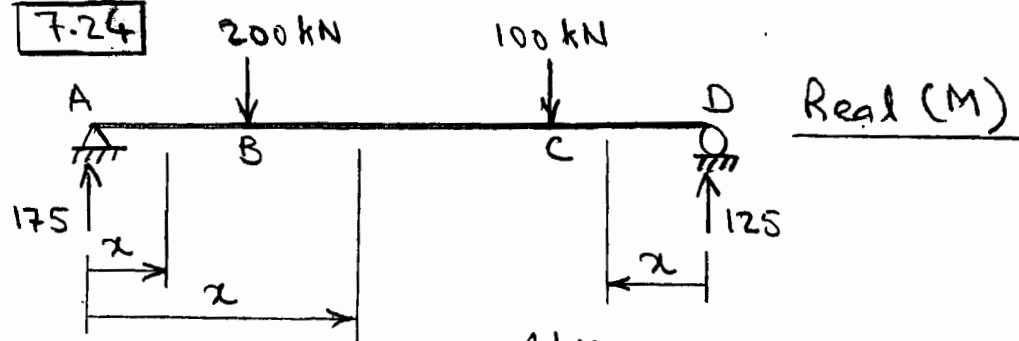
Segment	x Coordinate		M (kN.m)	M_v (kN.m)
	Origin	Limits (m)		
CB	C	0-3	-300	$-x$
BA	C	3-9	$-300 - 100(x-3)$	$-x$

$$\Delta_C = \frac{1}{EI} \left[\int_0^3 (-x)(-300) dx + \frac{1}{2} \int_3^9 (-x)(-100x) dx \right]$$

$$= \frac{13050 \text{ kN.m}^3}{EI} = \frac{13050}{70(500)} = 0.373 \text{ m}$$

$$\Delta_C = 373 \text{ mm} \downarrow$$

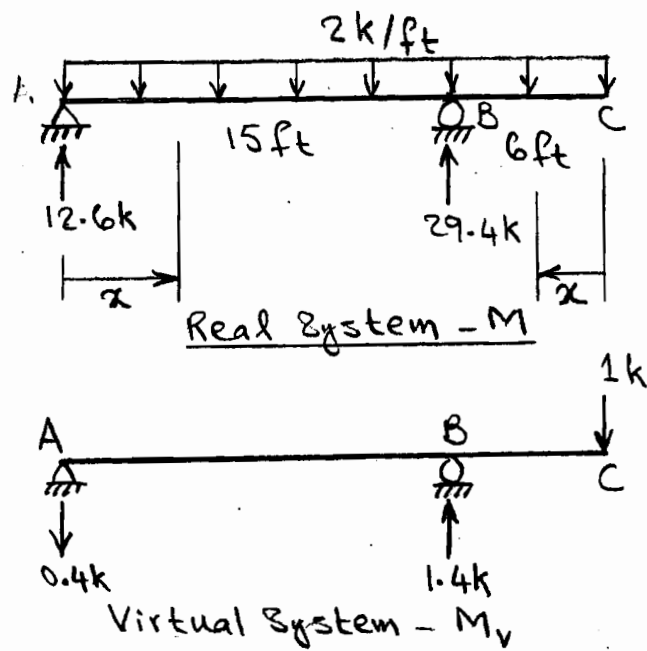
7.24



Segment	x Coordinate		M (kN-m)	M _v (kN-m)
	Origin	Limits (m)		
AB	A	0 - 5	175x	0.25x
BC	A	5 - 15	175x - 200(x - 5)	0.25x
DC	D	0 - 5	125x	0.75x

$$\begin{aligned}
 \Delta_c &= \frac{1}{EI} \left[\int_0^5 (0.25x)(175x) dx \right. \\
 &\quad \left. + \frac{1}{2} \int_5^{15} 0.25x (-25x + 1000) dx + \int_0^5 0.75x(125x) dx \right] \\
 &= \frac{14843.75 \text{ kN-m}^3}{EI} = \frac{14843.75}{250(600)} \\
 &= 0.099 \text{ m} = \underline{99 \text{ mm} \downarrow}
 \end{aligned}$$

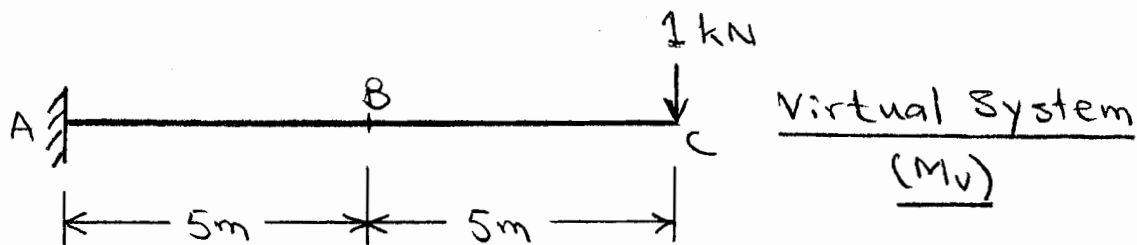
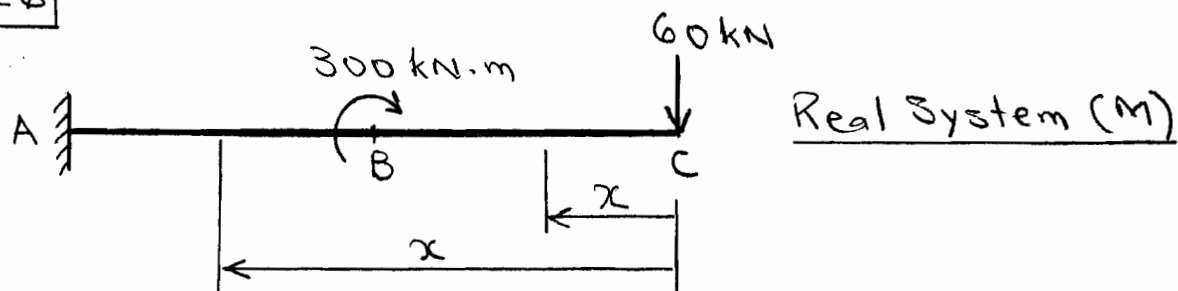
7.25



Segment	x Coordinate		M (k-ft)	M_v (k-ft)
	Origin	Limits (ft)		
AB	A	0-15	$12.6x - x^2$	$-0.4x$
CB	C	0-6	$-x^2$	$-x$

$$\begin{aligned}
 (1 \text{ k}) \Delta_c &= \frac{1}{EI} \left[\int_0^{15} -0.4x(12.6x - x^2) dx + \int_0^6 -x(-x^2) dx \right] \\
 &= - \frac{283.5 \text{ k}^2\text{-ft}^3}{EI} \\
 \Delta_c &= - \frac{283.5 (12)^3}{(29000)(3500)} = -0.0048 \text{ in.} = \underline{0.0048 \text{ in.} \uparrow}
 \end{aligned}$$

7.26



Segment	x coordinate		M (kN.m)	M _v (kN.m)
	Origin	Limits (m)		
CB	C	0-5	$-60x$	$-1x$
BA	C	5-10	$-60x-300$	$-1x$

$$\Delta_{\max} = \Delta_C = \frac{1}{EI} \left[\int_0^5 (-1x)(-60x) dx + \int_5^{10} (-1x)(-60x-300) dx \right]$$

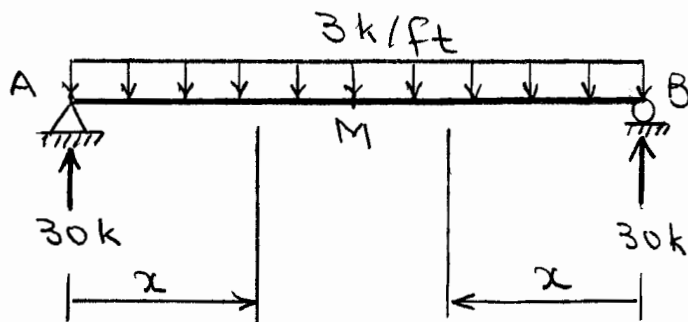
$$= \frac{31250 \text{ kN.m}^3}{EI} \downarrow$$

$$\Delta_{\max} = \frac{1}{360}$$

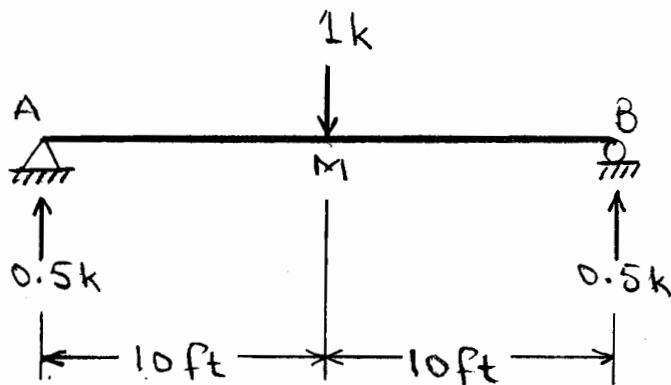
$$\frac{31250}{200(10^6)I} = \frac{1}{360}$$

From which, $I = 5625 (10^6) \text{ m}^4 = \underline{5625 (10^6) \text{ mm}^4}$

7.27



Real System (M)



Virtual System (M_v)

Segment	x Coordinate		M (k-ft)	M_v (k-ft)
	Origin	Limits (ft)		
AM	A	0-10	$30x - \frac{3x^2}{2}$	$0.5x$
BM	B	0-10	$30x - \frac{3x^2}{2}$	$0.5x$

$$\Delta_{\max} = \Delta_M = \frac{1}{EI} \left[2 \int_0^{10} (0.5x) \left(30x - \frac{3x^2}{2} \right) dx \right]$$

$$= \frac{6250 \text{ k-ft}^3}{EI}$$

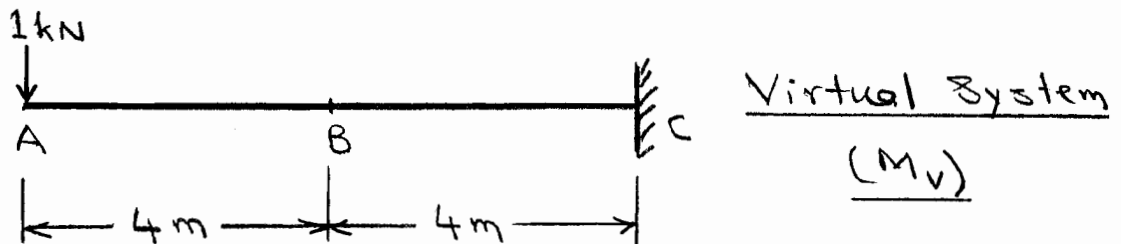
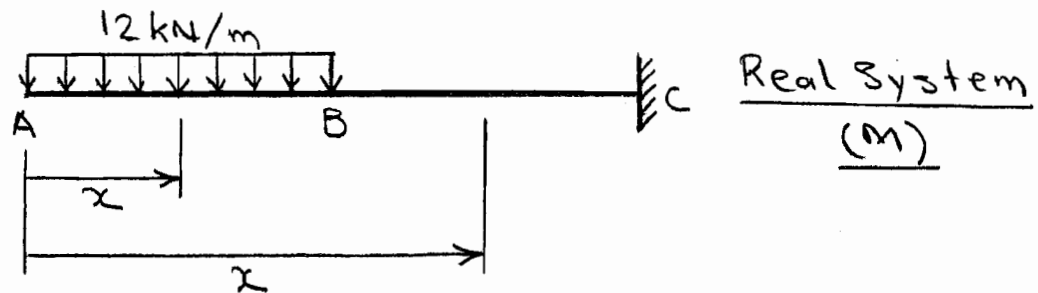
$$\Delta_{\max} = \frac{L}{360}$$

$$\frac{6250 (12)^3}{29000 (I)} = \frac{20 (12)}{360}$$

from which,

$$\underline{I = 559 \text{ in}^4}$$

7.28



Segment	x Coordinate		M (kN.m)	M _v (kN.m)
	Origin	Limits (m)		
AB	A	0-4	$-12x^2/2$	$-1x$
BC	A	4-8	$-12(4)(x-2)$	$-1x$

$$\Delta_{\max} = \Delta_A = \frac{1}{EI} \left[\int_0^4 (-1x)(-6x^2) dx + \int_4^8 (-1x)(-48)(x-2) dx \right]$$

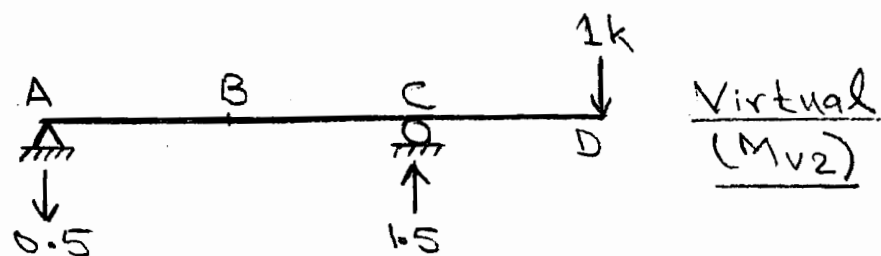
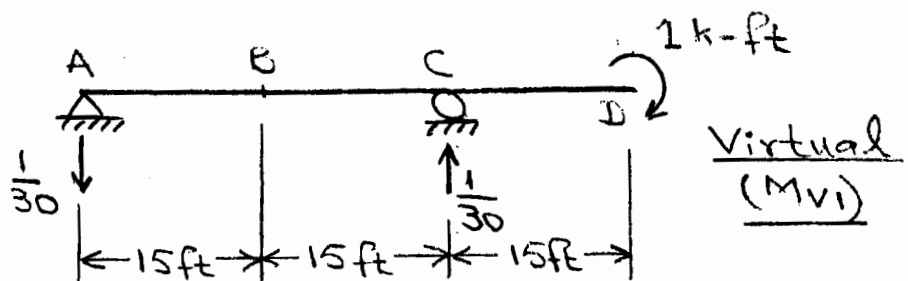
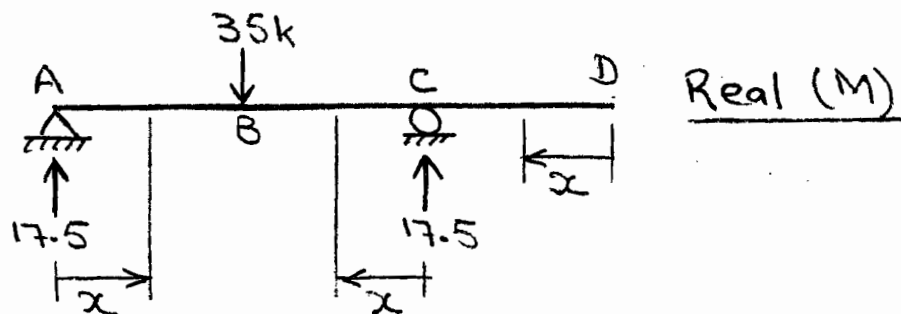
$$= \frac{5248 \text{ kN.m}^3}{EI}$$

$$\Delta_{\max} = \frac{1}{360}$$

$$\frac{5248}{70(10^6)I} = \frac{8}{360}$$

From which, $I = 3374 (10^6) \text{ m}^4 = \underline{3374 (10^6) \text{ mm}^4}$

7.29



Segment	x coordinate		M (k-ft)	Mv1 (k-ft)	Mv2 (k-ft)
	Origin	Limits (ft)			
AB	A	0-15	17.5x	-x/30	-0.5x
CB	C	0-15	17.5x	-1+(x/30)	-1(x+15)+1.5x
DC	D	0-15	0	-1	-1x

$$\theta_D = \frac{1}{EI} \left[\int_0^{15} -\frac{x}{30} (17.5x) dx + \int_0^{15} (-1 + \frac{x}{30}) (17.5x) dx \right]$$

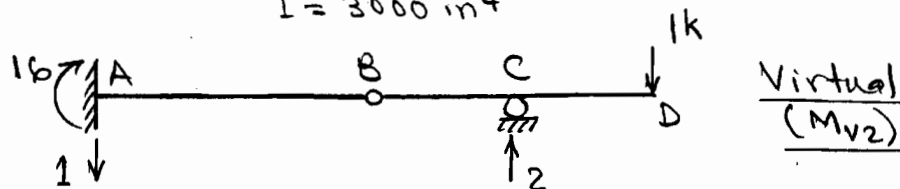
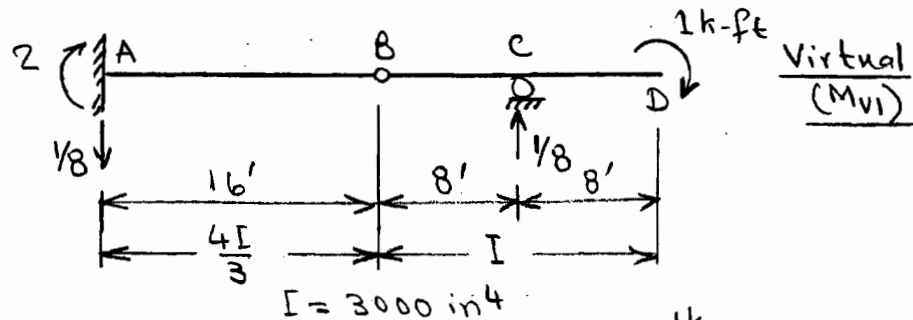
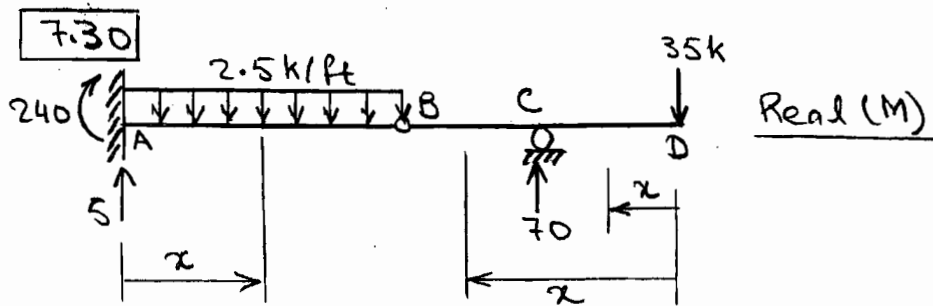
$$= - \frac{1968.75 \text{ k-ft}^2}{EI} = - \frac{1968.75 (12)^2}{10000 (2500)} = -0.01134 \text{ rad}$$

$$\theta_D = 0.01134 \text{ rad} \nearrow$$

$$\Delta_D = \frac{1}{EI} \left[\int_0^{15} (-0.5x) (17.5x) dx + \int_0^{15} (0.5x - 15) (17.5x) dx \right]$$

$$= - \frac{29531.25 \text{ k-ft}^3}{EI} = - \frac{29531.25 (12)^3}{10000 (2500)}$$

$$= -2.04 \text{ in.} = 2.04 \text{ in.} \uparrow$$



segment	x Coordinate		M (k-ft)	Mv1 (k-ft)	Mv2 (k-ft)
	Origin	Limits (ft)			
DC	D	0 - 8	$-35x$	-1	$-1x$
CB	D	8 - 16	$-35x + 70(x-8)$	$-1 + \frac{1}{8}(x-8)$	$x - 16$
AB	A	0 - 16	$240 + 5x - 1.25x^2$	$2 - \frac{x}{8}$	$16 - x$

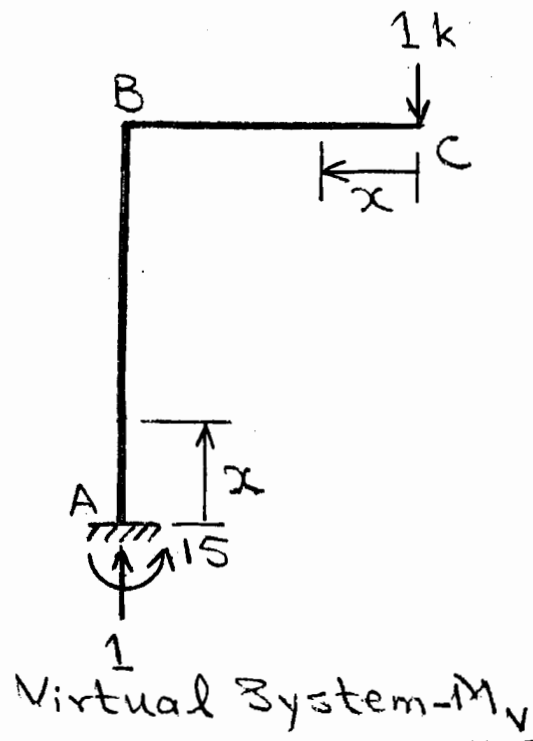
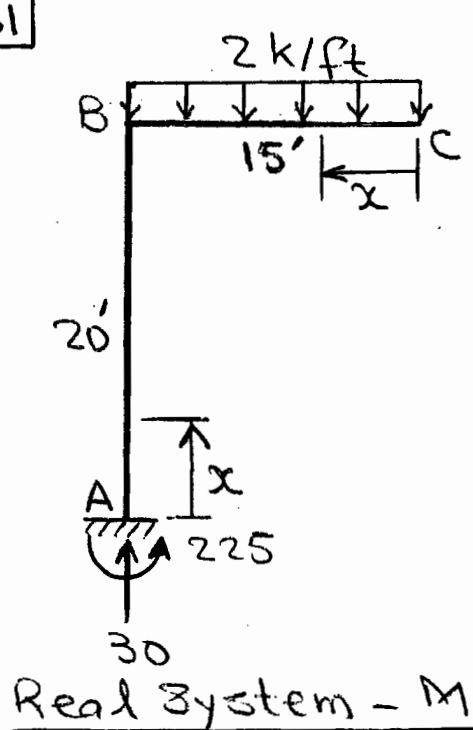
$$\theta_D = \frac{1}{EI} \left[\int_0^8 -1(-35x) dx + \int_8^{16} \left(\frac{x}{8} - 2 \right) (35x - 560) dx + \frac{3}{4} \int_0^{16} \left(2 - \frac{x}{8} \right) (240 + 5x - 1.25x^2) dx \right]$$

$$= \frac{4426.67 \text{ k-ft}^2}{EI} = \frac{4426.67 (12)^2}{30000 (3000)} = 0.0071 \text{ rad.} \swarrow$$

$$\Delta_D = \frac{1}{EI} \left[\int_0^8 -x(-35x) dx + \int_8^{16} (x-16)(35x-560) dx + \frac{3}{4} \int_0^{16} (16-x)(240+5x-1.25x^2) dx \right]$$

$$= \frac{32426.67 \text{ k-ft}^3}{EI} = \frac{32426.67 (12)^3}{30000 (3000)} = 0.62 \text{ in.} \downarrow$$

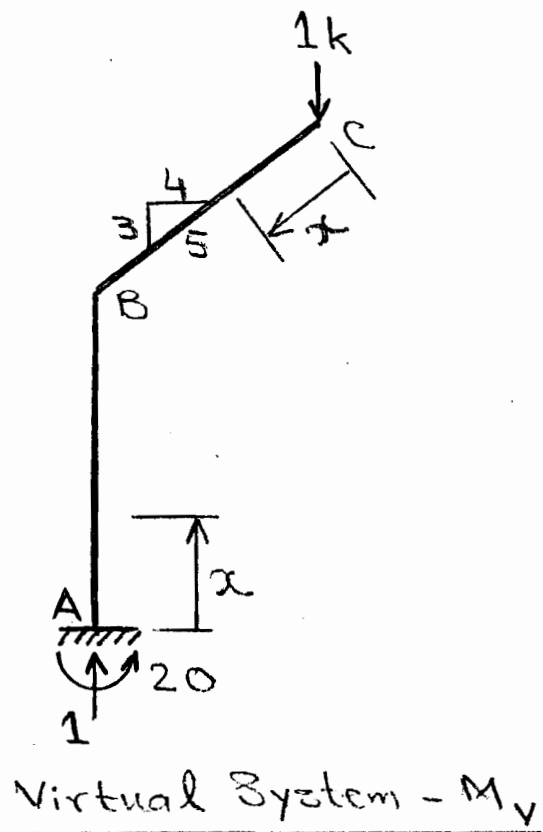
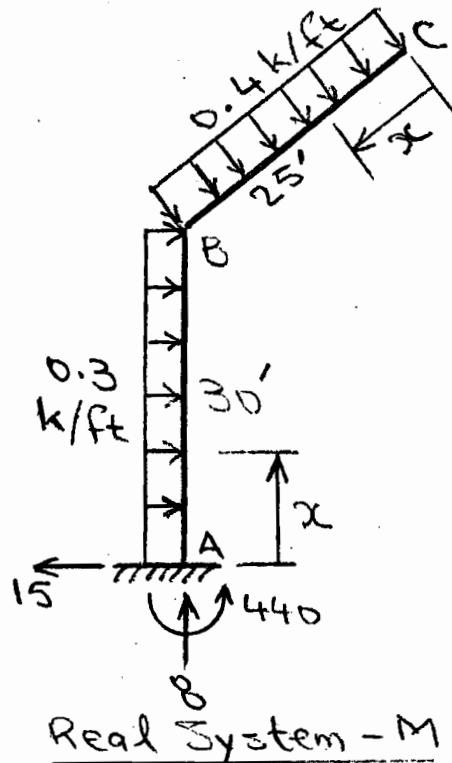
7.31



Segment	x Coordinate		M (k-ft)	M_v (k-ft)
	Origin	Limits (ft)		
AB	A	0-20	-225	-15
CB	C	0-15	$-x^2$	$-x$

$$\begin{aligned}
 (1k) \Delta_c &= \frac{1}{EI} \left[\int_0^{20} (-15)(-225) dx + \int_0^{15} (-x)(-x^2) dx \right] \\
 &= \frac{80156.25 \text{ k}^2\text{-ft}^3}{EI} \\
 \Delta_c &= \frac{80156.25 \text{ k-ft}^3}{EI} = \frac{80156.25 (12)^3}{29000 (2000)} \\
 &= \underline{2.388 \text{ in.} \downarrow}
 \end{aligned}$$

7.32

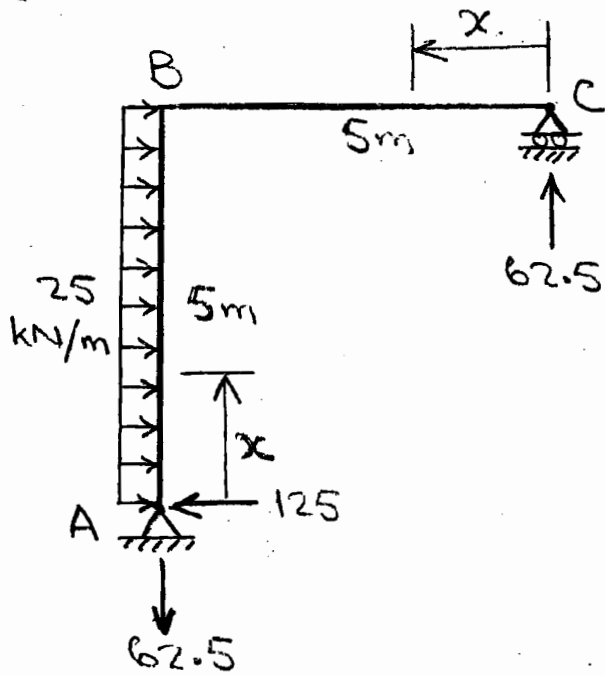


Segment	x Coordinate		M (k-ft)	M_v (k-ft)
	Origin	Limits (ft)		
AB	A	0-30	$-440 + 15x - 0.15x^2$	-20
CB	C	0-25	$-0.2x^2$	$-\frac{4}{5}x$

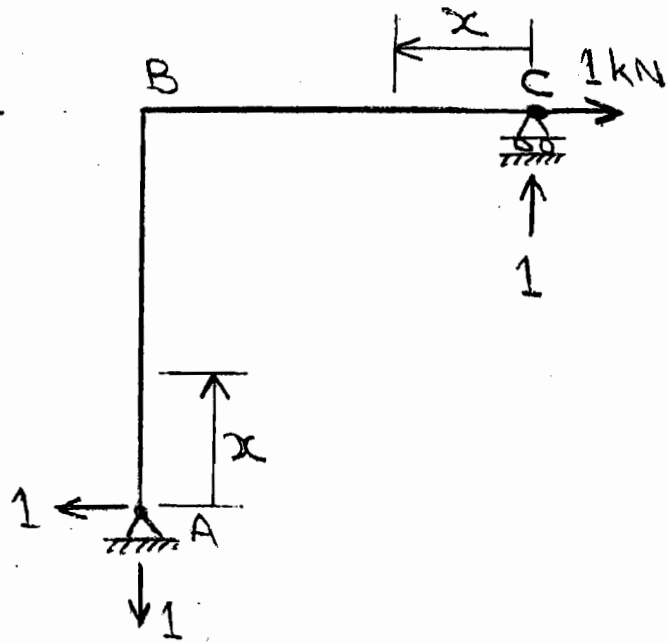
$$\begin{aligned}
 (1k) \Delta_c &= \frac{1}{EI} \left[\int_0^{30} (-20)(-440 + 15x - 0.15x^2) dx \right. \\
 &\quad \left. + \int_0^{25} \left(-\frac{4}{5}x\right)(-0.2x^2) dx \right] \\
 &= \frac{171625 \text{ k}^2\text{-ft}^3}{EI}
 \end{aligned}$$

$$\Delta_c = \frac{171625 \text{ k-ft}^3}{EI} = \frac{171625 (12)^3}{10000 (8160)} = \underline{3.63 \text{ in.} \downarrow}$$

7.33



Real System - M

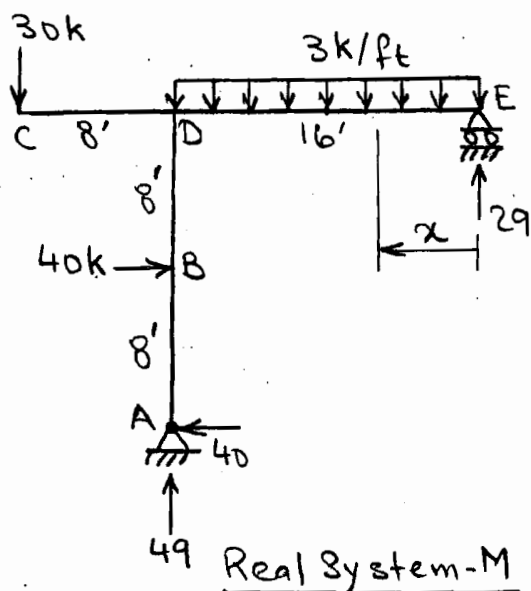


Virtual System - M_v

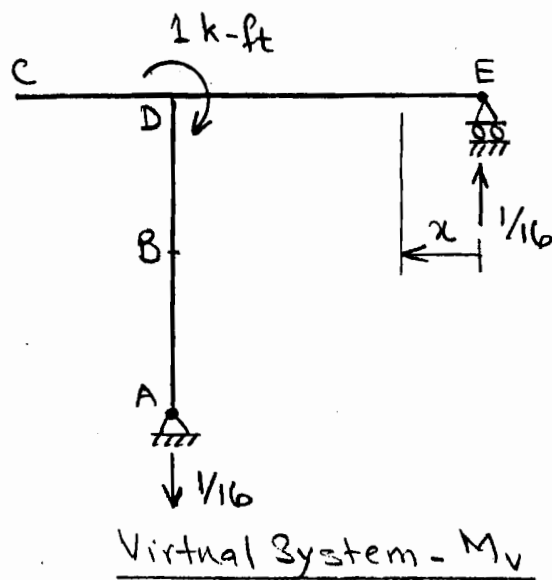
Segment	x Coordinate		M (kN.m)	M _v (kN.m)
	Origin	Limits (m)		
AB	A	0-5	$125x - 12.5x^2$	$1x$
CB	C	0-5	$62.5x$	$1x$

$$\begin{aligned}
 \Delta_C &= \frac{1}{EI} \left[\int_0^5 x(125x - 12.5x^2) dx + \int_0^5 x(62.5x) dx \right] \\
 &= \frac{5859.375 \text{ kN.m}^3}{EI} = \frac{5859.375}{70(1030)} \\
 &= 0.0813 \text{ m} = \underline{81.3 \text{ mm} \rightarrow}
 \end{aligned}$$

7.34



$$M = 29x - 1.5x^2$$



$$M_v = x/16$$

$$\begin{aligned} \theta_D &= \frac{1}{2EI} \int_0^{16} \left(\frac{x}{16}\right)(29x - 1.5x^2) dx = \frac{469.33 \text{ k-ft}^2}{EI} \\ &= \frac{469.33 (12)^2}{2000(10000)} = \underline{0.0034 \text{ rad}} \end{aligned}$$

[illegible]

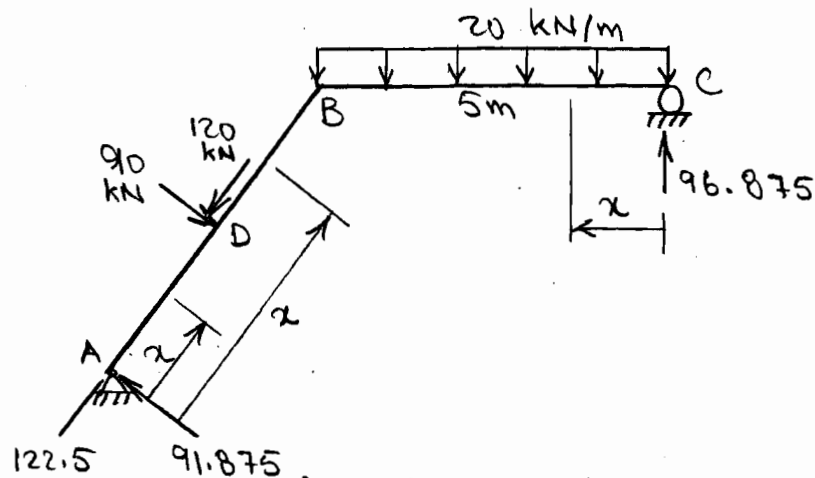
Diagram of a frame structure. The structure consists of a horizontal beam CDE and a vertical column AB. The beam has a horizontal reaction of 1k at C and a vertical reaction of 1k at E. The column has a horizontal reaction of 1k at A and a vertical reaction of 1k at B. The height of the column is 2x, and the distance from the column to the point of application of the 1k vertical reaction at E is x.

Segment	x coordinate		M (k-ft)	M _v (k-ft)
	Origin	Limits (ft)		
AB	A	0-8	402	12
BD	A	8-16	320	12
ED	E	0-16	$292 - \frac{3}{2}x^2$	12

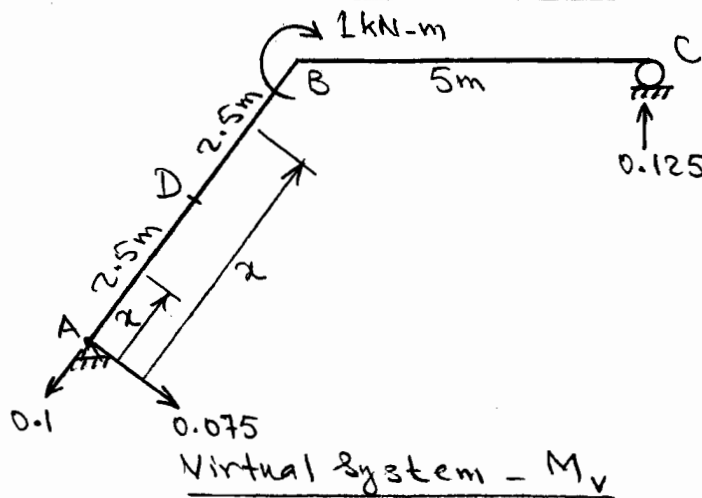
$$(1k) \Delta_E = \frac{1}{2EI} \left[\int_0^8 x(40x) dx + \int_8^{16} x(320) dx + \int_0^{16} x(29x - \frac{3}{2}x^2) dx \right] = \frac{26282.67 \text{ k}^2\text{-ft}^3}{EI}$$

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7.36



Real System - M

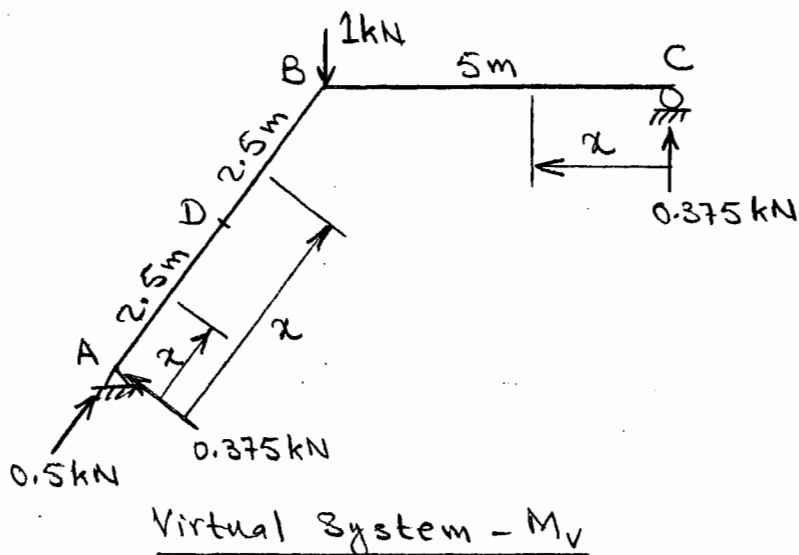
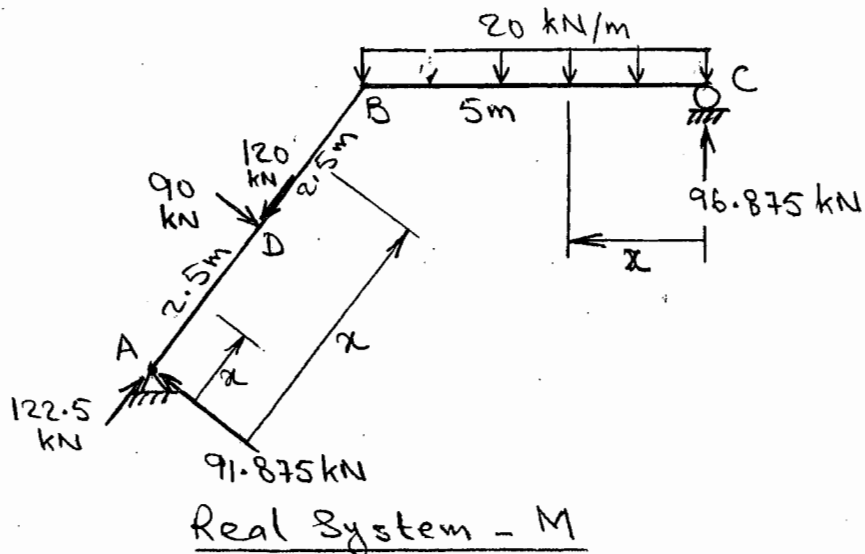


Virtual System - M_v

Segment	x Coordinate		M (kN-m)	M_v (kN-m)
	Origin	Limits (m)		
AD	A	0-2.5	$91.875x$	$-0.075x$
DB	A	2.5-5	$1.875x + 225$	$-0.075x$
CB	C	0-5	$96.875x - 10x^2$	$0.125x$

$$\begin{aligned}
 \theta_B &= \frac{1}{EI} \left[\int_0^{2.5} -0.075x(91.875x) dx \right. \\
 &\quad + \int_{2.5}^5 -0.075x(1.875x + 225) dx \\
 &\quad \left. + \int_0^5 0.125x(96.875x - 10x^2) dx \right] \\
 &= \frac{110.01 \text{ kN-m}^2}{EI} = \frac{110.01}{200(500)} = 0.0011 \text{ rad.} \checkmark
 \end{aligned}$$

7.37

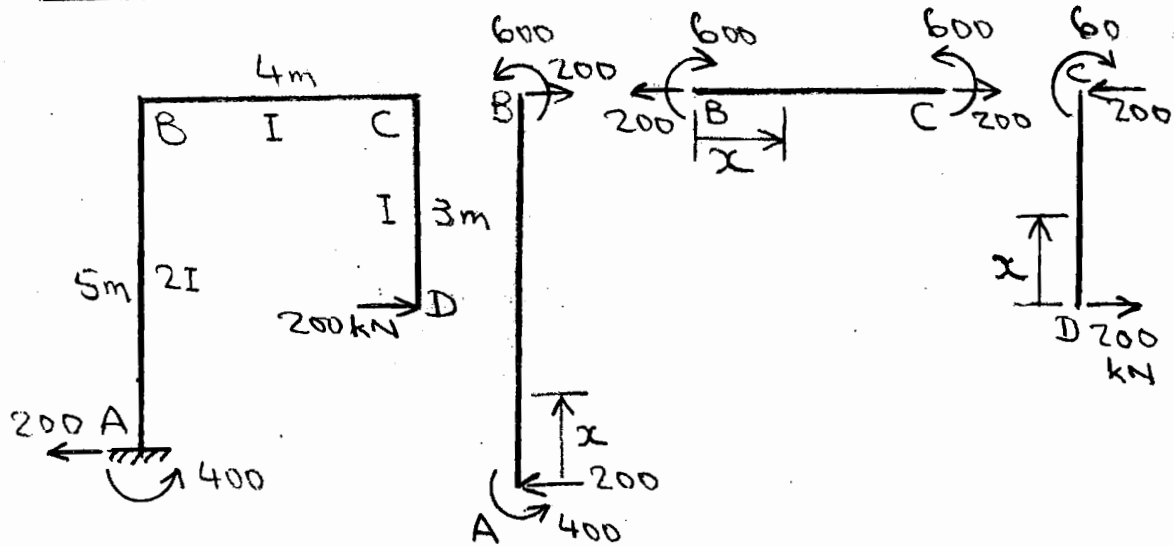


Segment	x Coordinate		M (kN.m)	M_v (kN.m)
	Origin	Limits (m)		
AD	A	0-2.5	$91.875x$	$0.375x$
DB	A	2.5-5	$1.875x + 225$	$0.375x$
CB	C	0-5	$96.875x - 10x^2$	$0.375x$

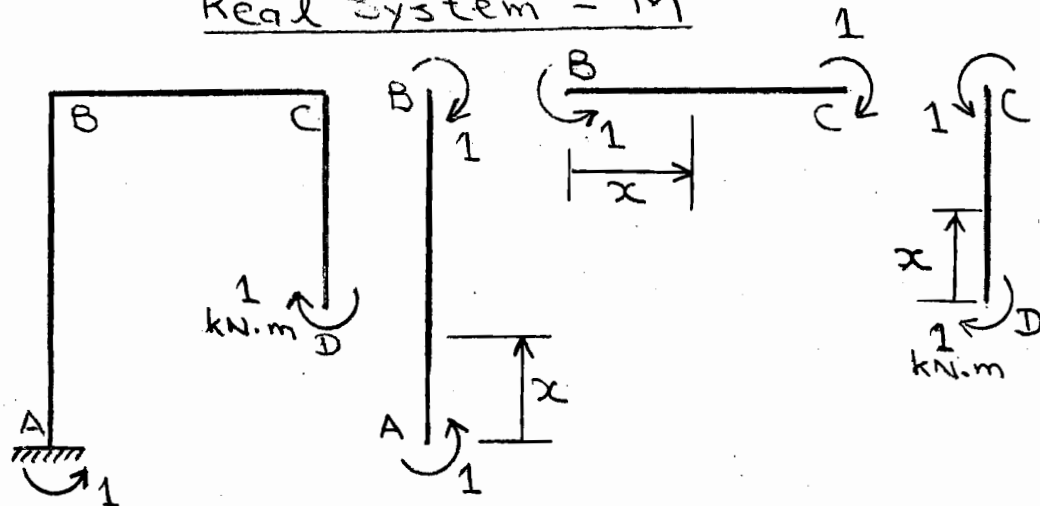
$$\begin{aligned}
 (1 \text{ kN}) \Delta_B &= \frac{1}{EI} \left[\int_0^{2.5} 0.375x(91.875x) dx + \int_{2.5}^5 0.375x(1.875x + 225) dx + \int_0^5 0.375x(96.875x - 10x^2) dx \right] \\
 &= \frac{1923.83 \text{ kN}^2 \cdot \text{m}^3}{EI}
 \end{aligned}$$

$$\Delta_B = \frac{1923.83}{200(500)} = 0.0192 \text{ m} = 19.2 \text{ mm} \downarrow$$

7.38



Real System - M

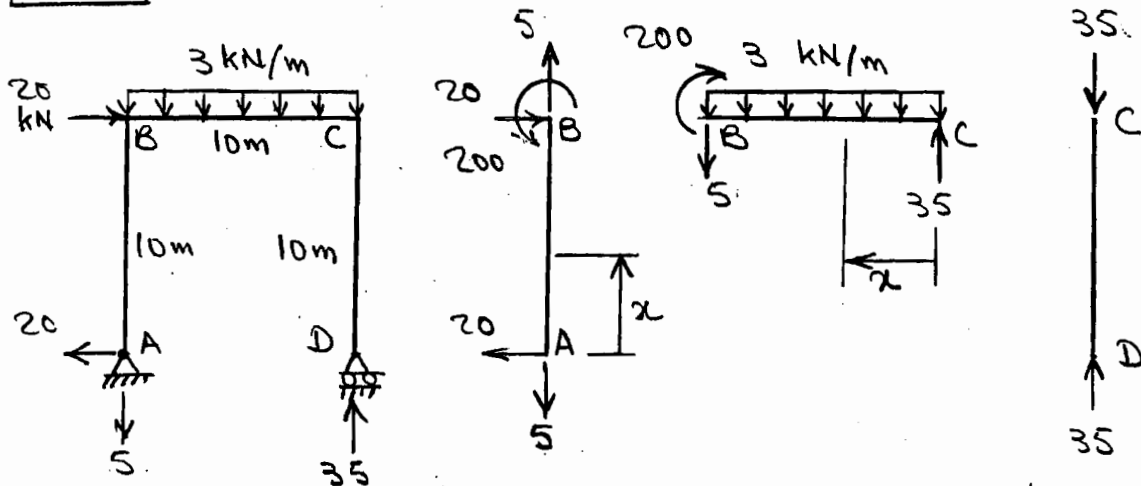


Virtual System - M_v

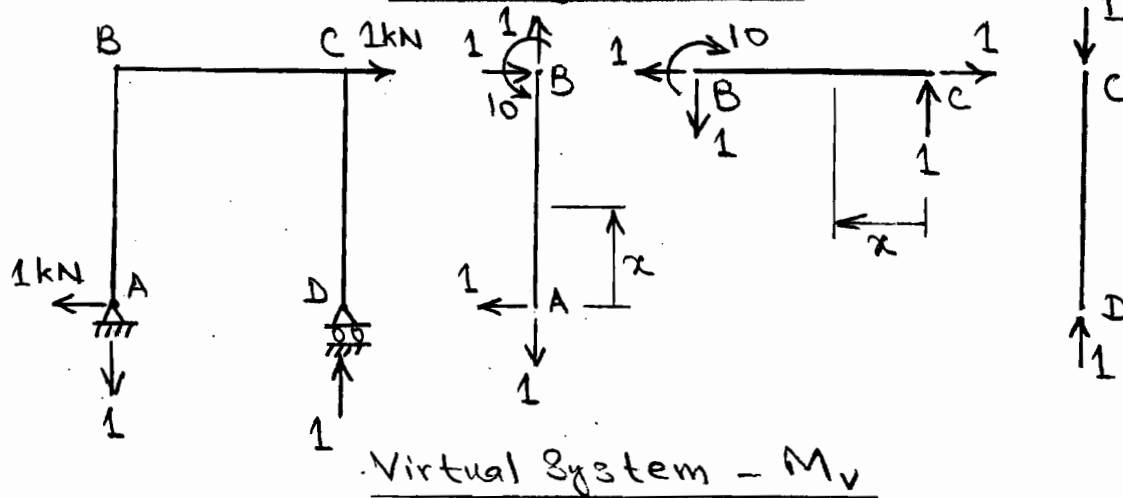
Segment	x Coordinate		M (kN.m)	M_v (kN.m)
	Origin	Limits (m)		
AB	A	0-5	$-400 + 200x$	-1
BC	B	0-4	600	-1
DC	D	0-3	$-200x$	1

$$\begin{aligned}
 \theta_D &= \frac{1}{EI} \left[\frac{1}{2} \int_0^5 -1(-400 + 200x) dx + \int_0^4 -1(600) dx \right. \\
 &\quad \left. + \int_0^3 1(-200x) dx \right] \\
 &= -\frac{3550 \text{ kN.m}^2}{EI} = -\frac{3550}{70(1296)} = -0.0393 \text{ rad} \\
 &= 0.0393 \text{ rad} \quad \swarrow
 \end{aligned}$$

7.39



Real System - M



Virtual System - M_v

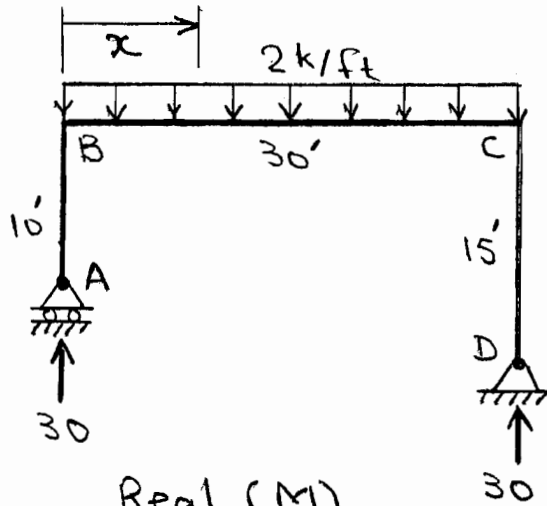
Segment	x coordinate		M (kN.m)	M_v (kN.m)
	Origin	Limits (m)		
AB	A	0-10	$20x$	$1x$
CB	C	0-10	$35x - 1.5x^2$	$1x$

$$(1\text{ kN}) \Delta_C = \frac{1}{EI} \left[\int_0^{10} x(20x) dx + \int_0^{10} x(35x - 1.5x^2) dx \right]$$

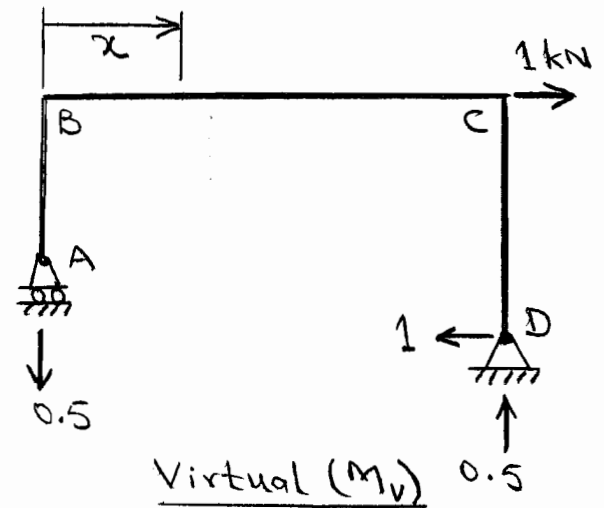
$$= \frac{14583.33 \text{ kN}^2 \cdot \text{m}^3}{EI}$$

$$\Delta_C = \frac{14583.33}{200(400)} = 0.182 \text{ m} \rightarrow$$

7.40



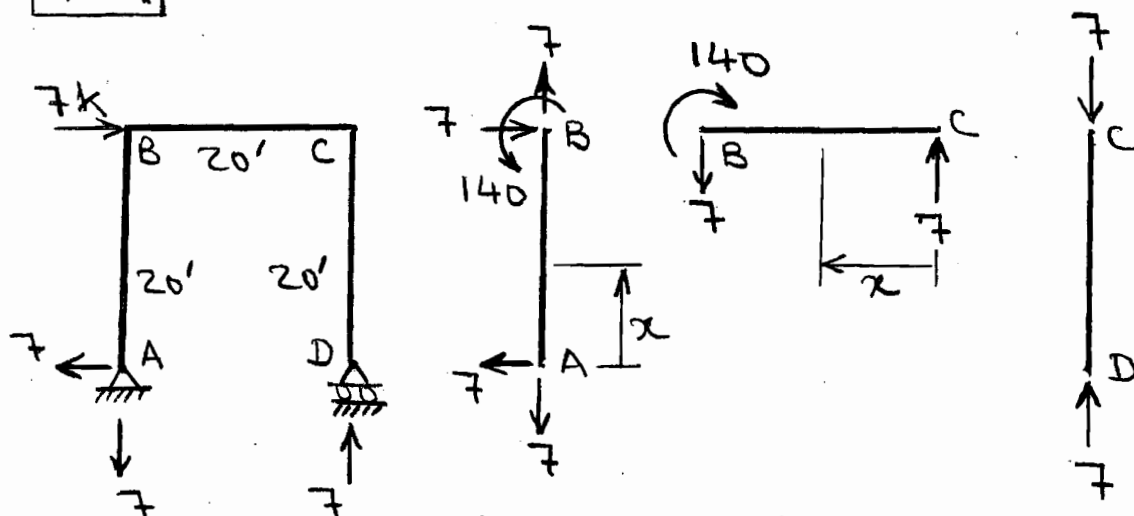
$$M = 30x - \frac{2x^2}{2}$$



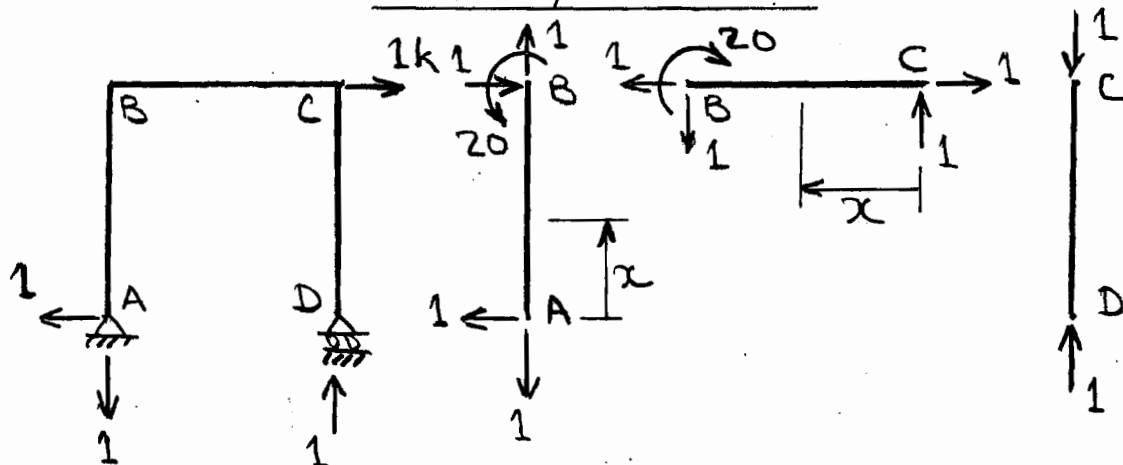
$$M_v = -0.5x$$

$$\begin{aligned} \Delta_c &= \frac{1}{EI} \int_0^{30} (-0.5x)(30x - x^2) dx = - \frac{33750 \text{ k-ft}^3}{EI} \\ &= - \frac{33750 (12)^3}{29000 (1500)} = -1.34 \text{ in.} = \underline{1.34 \text{ in.} \leftarrow} \end{aligned}$$

7.41



Real System - M



Virtual System - M_v

Segment	x coordinate		M (k-ft)	M _v (k-ft)
	Origin	Limits (ft)		
AB	A	0-20	7x	1x
CB	C	0-20	7x	1x

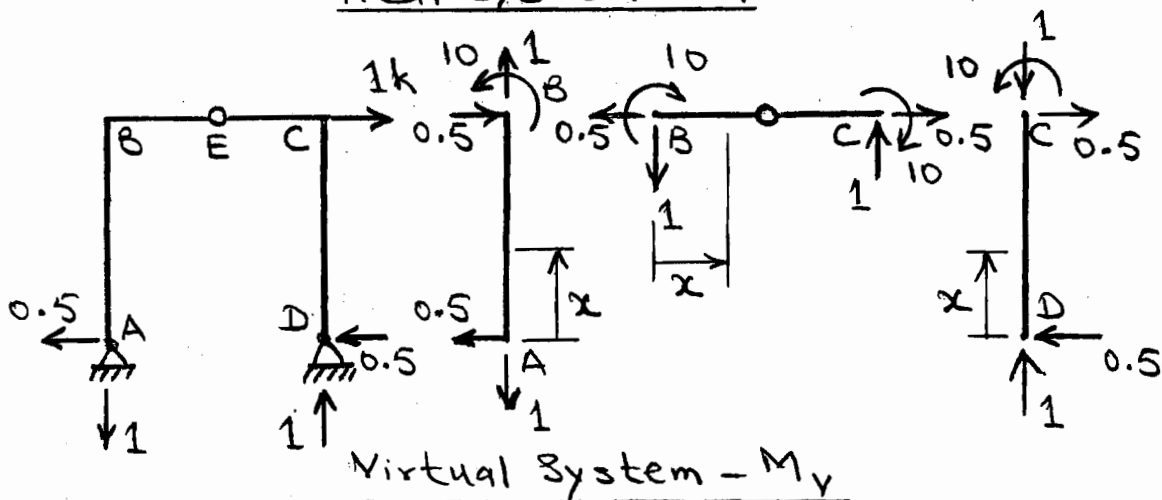
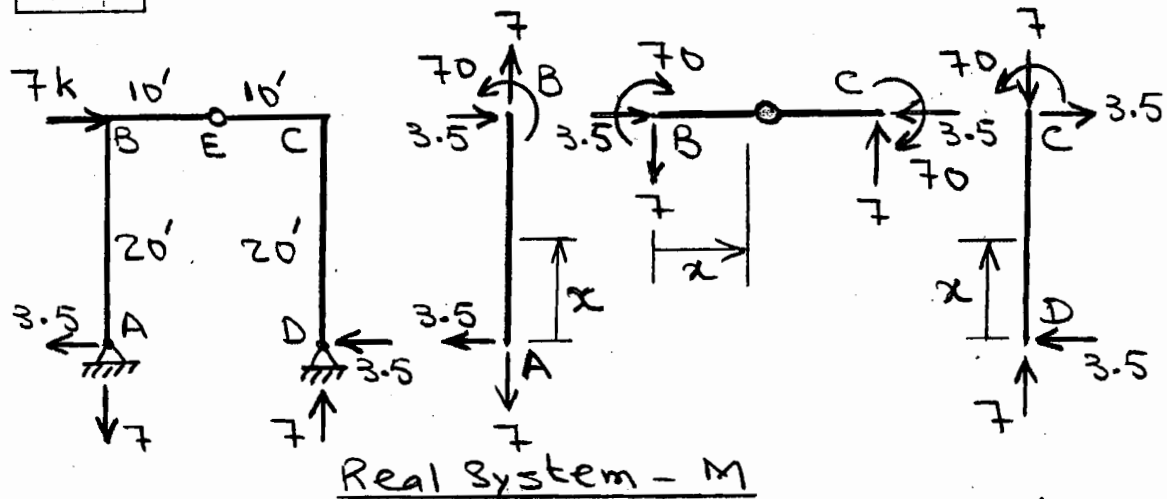
$$(1k) \Delta_C = \frac{1}{EI} \left[\int_0^{20} (1x)(7x) dx + \int_0^{20} (1x)(7x) dx \right]$$

$$= \frac{37333 \text{ k}^2 \cdot \text{ft}^3}{EI}$$

$$\Delta_C = \frac{37333 \text{ k} \cdot \text{ft}^3}{EI} = \frac{37333 (12)^3}{29000 (I)} = 1 \text{ in.}$$

from which, $I = 2225 \text{ in}^4$

7.42



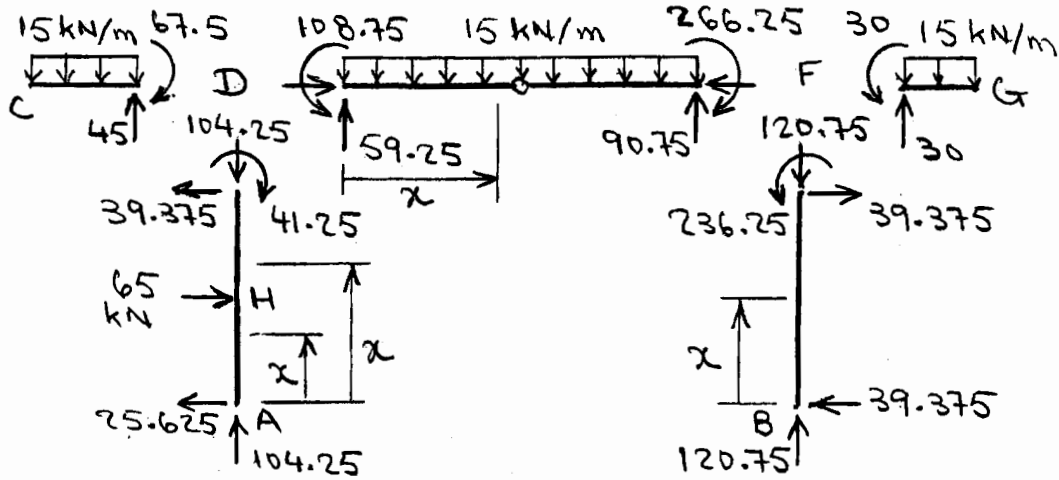
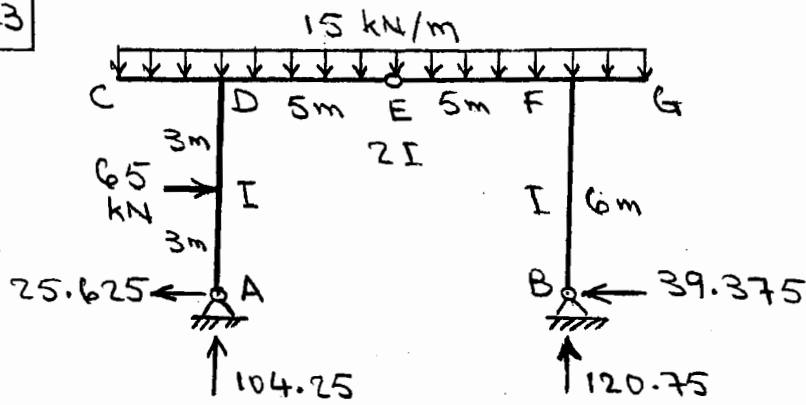
Segment	x coordinate		M (k-ft)	M_v (k-ft)
	Origin	Limits (ft)		
AB	A	0-20	$3.5x$	$0.5x$
BC	B	0-20	$70 - 7x$	$10 - 1x$
DC	D	0-20	$3.5x$	$0.5x$

$$(1k) \Delta_C = \frac{1}{EI} \left[2 \int_0^{20} (0.5x)(3.5x) dx + \int_0^{20} (10-x)(70-7x) dx \right] = \frac{14000 \text{ k}^2\text{-ft}^3}{EI}$$

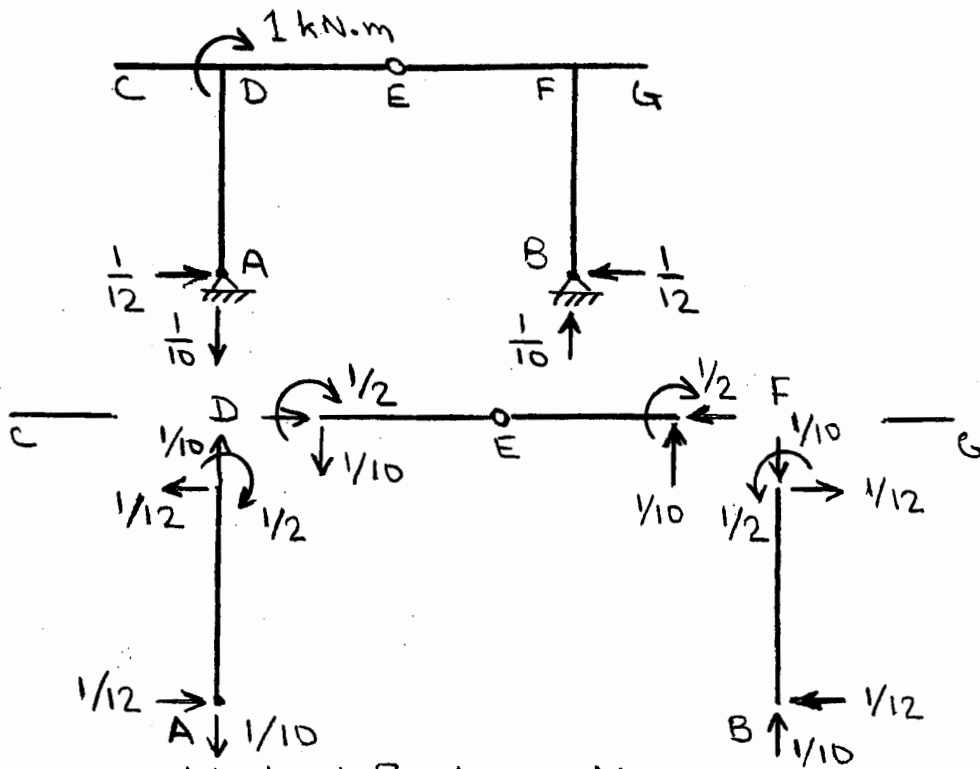
$$\Delta_C = \frac{14000 \text{ k-ft}^3}{EI} = \frac{14000(12)^3}{29000(I)} = 1 \text{ in.}$$

From which, $I = 834 \text{ in}^4$

7.43



Real System - M_1



Virtual System - M_v

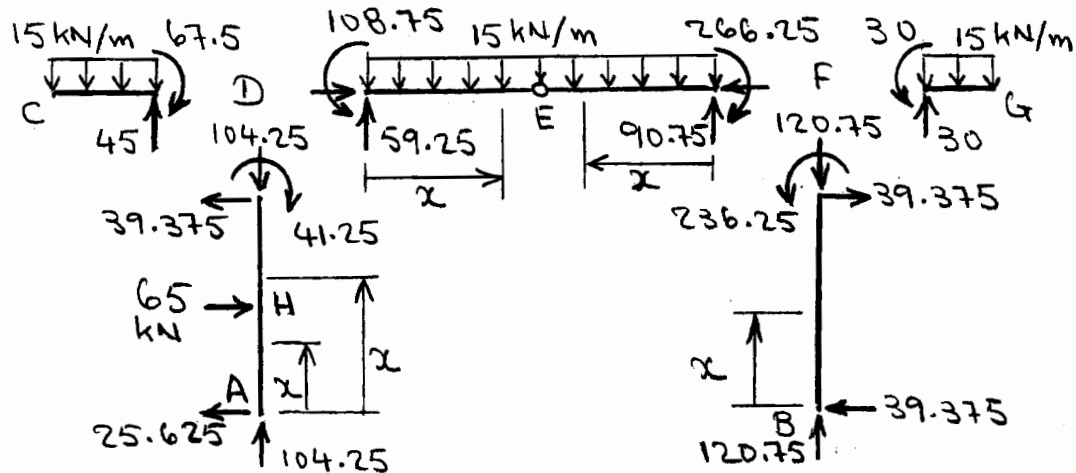
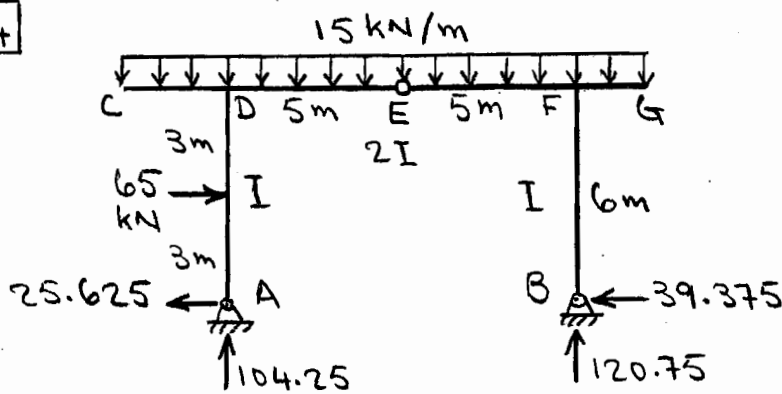
7.43 (contd.)

Segment	x Coordinate		M (kN.m)	M _v (kN.m)
	Origin	Limits (m)		
AH	A	0-3	25.625x	-x/12
HD	A	3-6	25.625x - 65(x-3)	-x/12
BF	B	0-6	39.375x	x/12
DF	D	0-10	59.25x - 108.75 - 7.5x ²	$\frac{1}{2} - \frac{x}{10}$

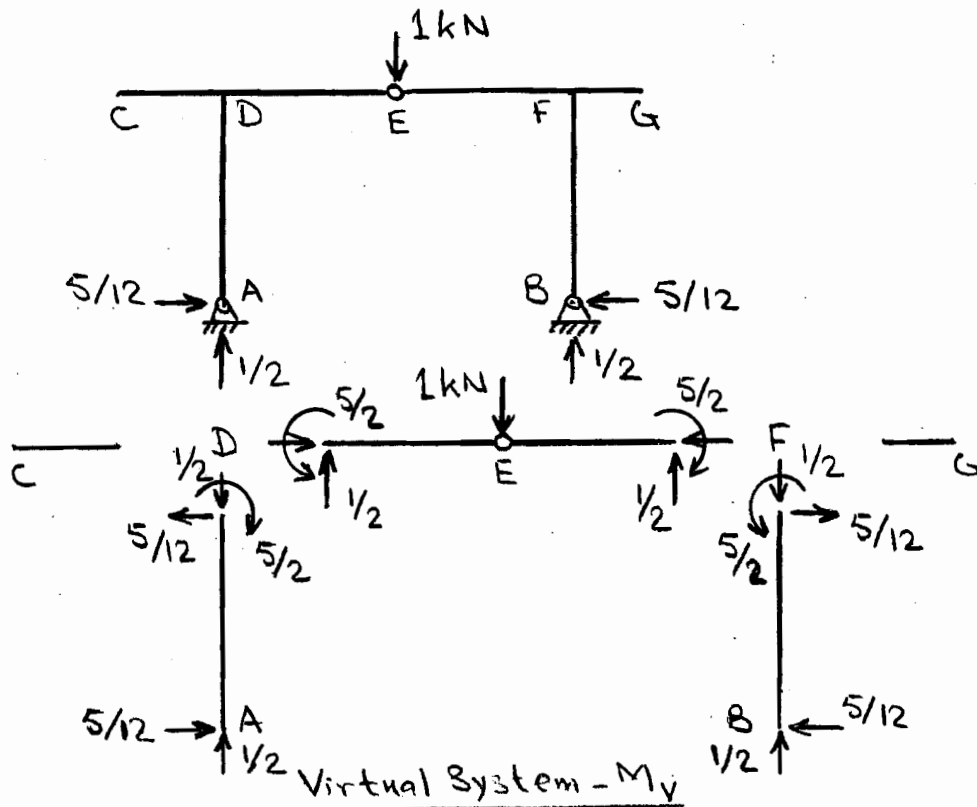
$$\begin{aligned}
 (1 \text{ kN.m}) \theta_D &= \frac{1}{EI} \left[\int_0^3 \left(-\frac{x}{12}\right) (25.625x) dx + \right. \\
 &\quad \int_3^6 \left(-\frac{x}{12}\right) (25.625x - 65x + 195) dx + \int_0^6 \frac{x}{12} (39.375x) dx \\
 &\quad \left. + \frac{1}{2} \int_0^{10} \left(\frac{1}{2} - \frac{x}{10}\right) (59.25x - 108.75 - 7.5x^2) dx \right] \\
 &= \frac{270 \text{ kN}^2 \cdot \text{m}^3}{EI}
 \end{aligned}$$

$$\theta_D = \frac{270}{200(350)} = \underline{0.00386 \text{ rad.} \nabla}$$

7.44



Real System - M



Virtual System - M_v

7.44 (contd.)

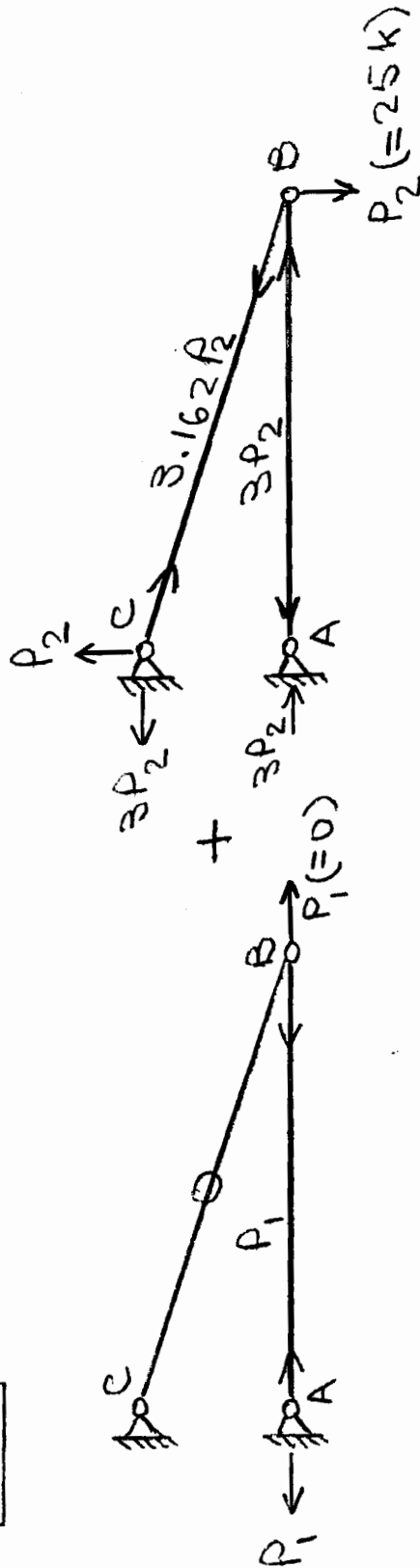
Segment	x coordinate		M (kN.m)	M _v (kN.m)
	Origin	Limits (m)		
AH	A	0-3	25.625x	-5x/12
HD	A	3-6	25.625x - 65(x-3)	-5x/12
BF	B	0-6	39.375x	5x/12
DE	D	0-5	59.25x - 108.75 - 7.5x ²	$\frac{x}{2} - \frac{5}{2}$
FE	F	0-5	90.75x - 266.25 - 7.5x ²	$\frac{x}{2} - \frac{5}{2}$

$$\begin{aligned}
 (1 \text{ kN}) \Delta_E &= \frac{1}{EI} \left[\int_0^3 \left(-\frac{5x}{12} \right) (25.625x) dx \right. \\
 &+ \int_3^6 \left(-\frac{5x}{12} \right) (25.625x - 65x + 195) dx \\
 &+ \int_0^6 \frac{5x}{12} (39.375x) dx + \frac{1}{2} \int_0^5 \left(\frac{x}{2} - \frac{5}{2} \right) (59.25x - 108.75 \\
 &\quad \left. - 7.5x^2) dx + \frac{1}{2} \int_0^5 \left(\frac{x}{2} - \frac{5}{2} \right) (90.75x - 266.25 - 7.5x^2) dx \right] \\
 &= \frac{1607.8125 \text{ kN}^2 \cdot \text{m}^3}{EI}
 \end{aligned}$$

$$\Delta_E = \frac{1607.8125}{200(350)} = 0.023 \text{ m} \downarrow$$

$$\Delta_E = 23 \text{ mm} \downarrow$$

7.45

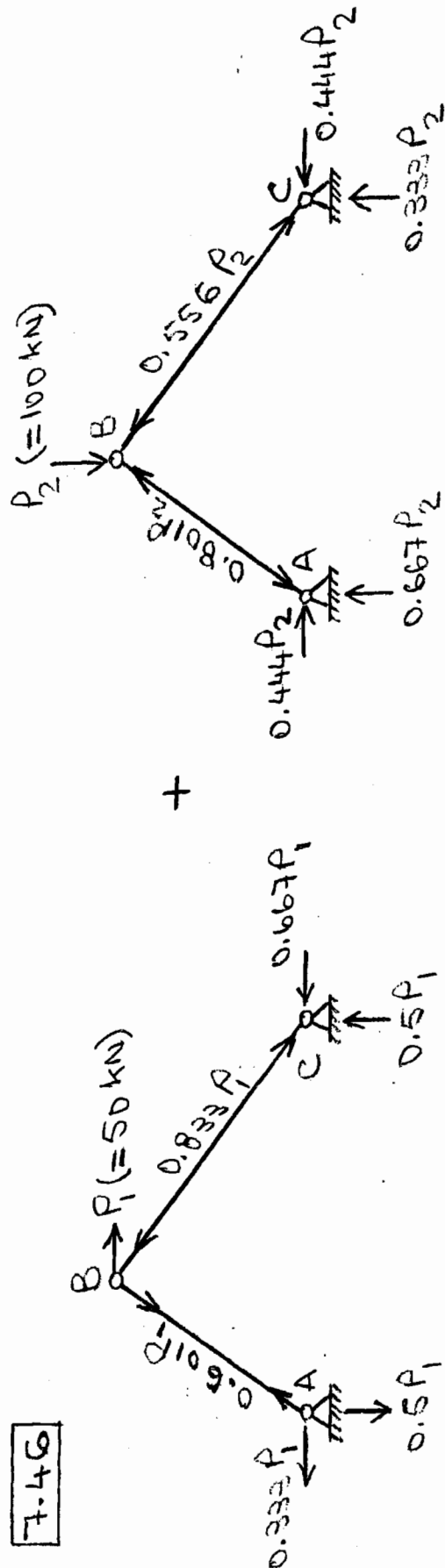


Member	L (in.)	F	$\frac{\partial F}{\partial P_1}$	$\frac{\partial F}{\partial P_2}$	$P_1=0, P_2=25k$	
					$\frac{\partial F}{\partial P_1} FL$	$\frac{\partial F}{\partial P_2} FL$
AB	180	$P_1 - 3P_2$	1	-3	-13500	40500
BC	189.74	$3.162 P_2$	0	3.162	0	47432.67
Σ					-13500	87932.67

$$\Delta_{BH} = -\frac{13500}{10000(6)} = -0.225 \text{ in.} = 0.225 \text{ in.} \rightarrow$$

$$\Delta_{BV} = \frac{87932.67}{10000(6)} = 1.466 \text{ in.} \downarrow$$

7.46

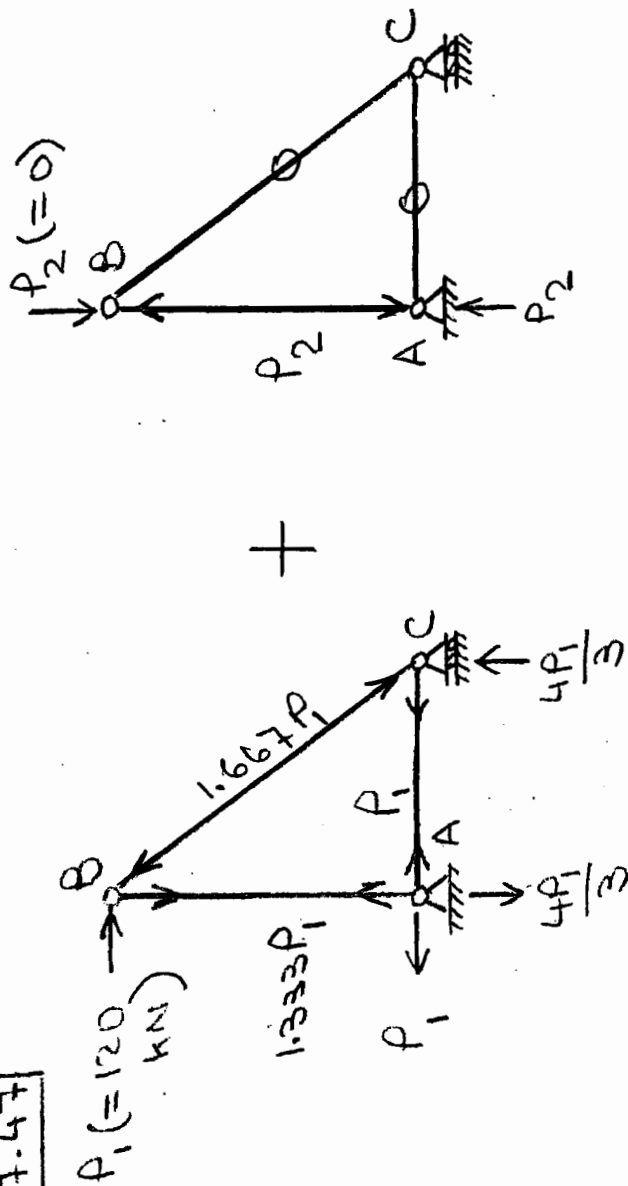


Member	L (m)	F	$\frac{\partial F}{\partial P_1}$	$\frac{\partial F}{\partial P_2}$	$P_1 = 50 \text{ kN}, P_2 = 100 \text{ kN}$ $\frac{\partial F}{\partial P_1} FL$ $\frac{\partial F}{\partial P_2} FL$
AB	3.606	$0.601P_1 - 0.801P_2$	0.601	-0.801	-108.53 144.65
BC	5	$-0.833P_1 - 0.556P_2$	-0.833	-0.556	405.09 270.06
Σ					296.56 414.71

$$\Delta_{BH} = \frac{296.56}{70(10^6)(0.001)} = 0.00424 \text{ m} = 4.24 \text{ mm} \rightarrow$$

$$\Delta_{BV} = \frac{414.71}{70(10^6)(0.001)} = 0.00592 \text{ m} = 5.92 \text{ mm} \downarrow$$

7.47

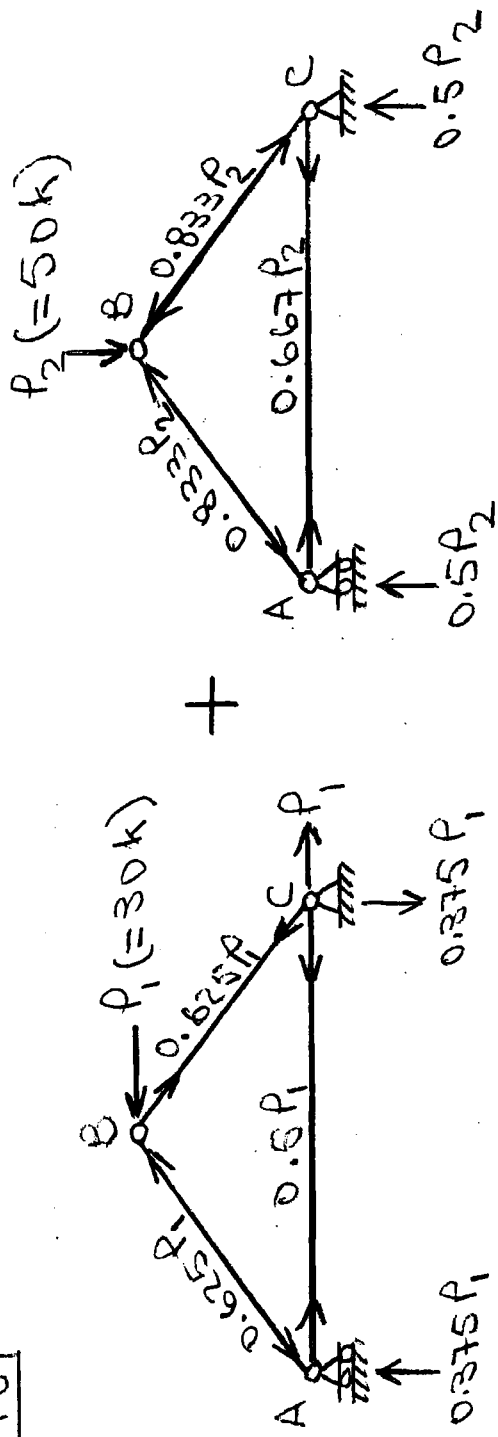


Member	L (m)	F	$\frac{\partial F}{\partial P_1}$	$\frac{\partial F}{\partial P_2}$	$P_1 = 120 \text{ kN}, P_2 = 0$ $\frac{\partial F}{\partial P_1} FL$ $\frac{\partial F}{\partial P_2} FL$
AC	3	P_1	1	0	360
AB	4	$1.333P_1 - P_2$	1.333	-1	853.33
BC	5	$-1.667P_1$	-1.667	0	1666.67
Σ					2880
					-640

$$\Delta_{BH} = \frac{2880}{200(10^6)(0.0015)} = 0.0096 \text{ m} = 9.6 \text{ mm} \rightarrow$$

$$\Delta_{BY} = -\frac{640}{200(10^6)(0.0015)} = -0.00213 \text{ m} = 2.13 \text{ mm} \uparrow$$

7.48

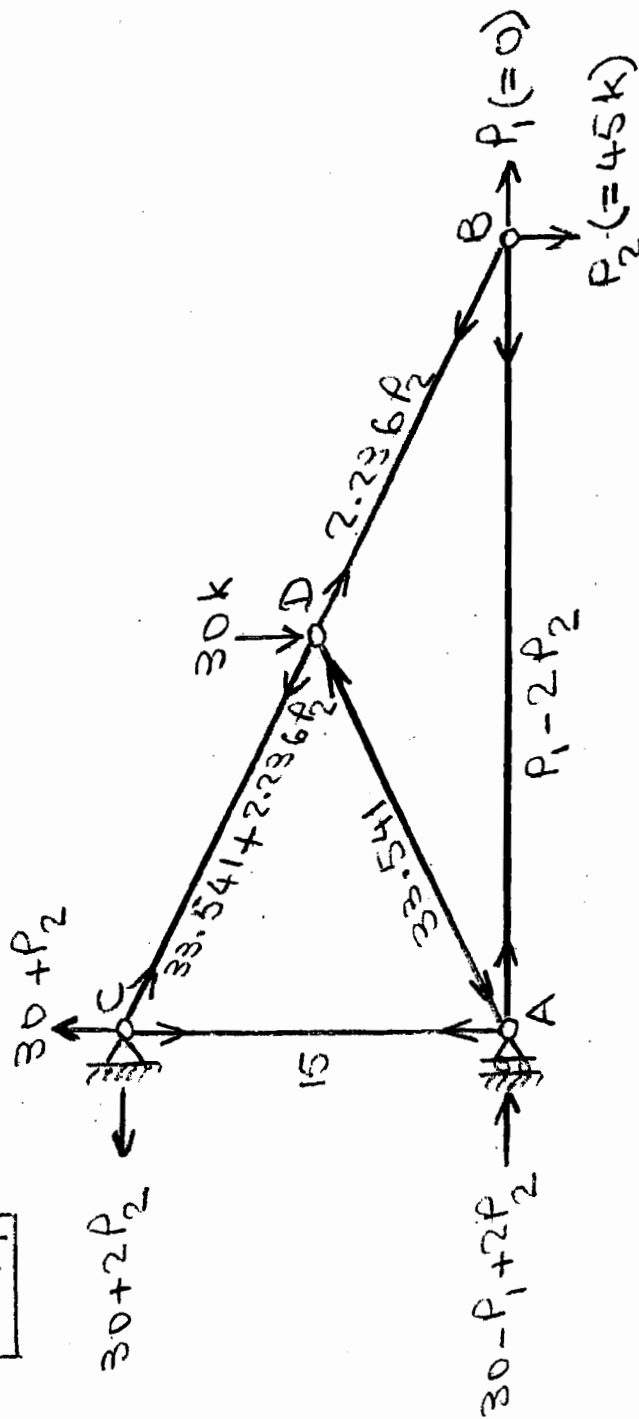


Member	L (in.)	F	$\frac{\partial F}{\partial P_1}$	$\frac{\partial F}{\partial P_2}$	$P_1 = 30k, P_2 = 50k$
AC	192	$0.5P_1 + 0.667P_2$	0.5	0.667	4840
AB	120	$-0.625P_1 - 0.833P_2$	-0.625	-0.833	4531.25
BC	120	$0.625P_1 - 0.833P_2$	0.625	-0.833	-1718.75
Σ					7452.5
					14520

$$\Delta_{BH} = \frac{7452.5}{29000(3)} = 0.0857 \text{ in.} \leftarrow$$

$$\Delta_{BV} = \frac{14520}{29000(3)} = 0.167 \text{ in.} \downarrow$$

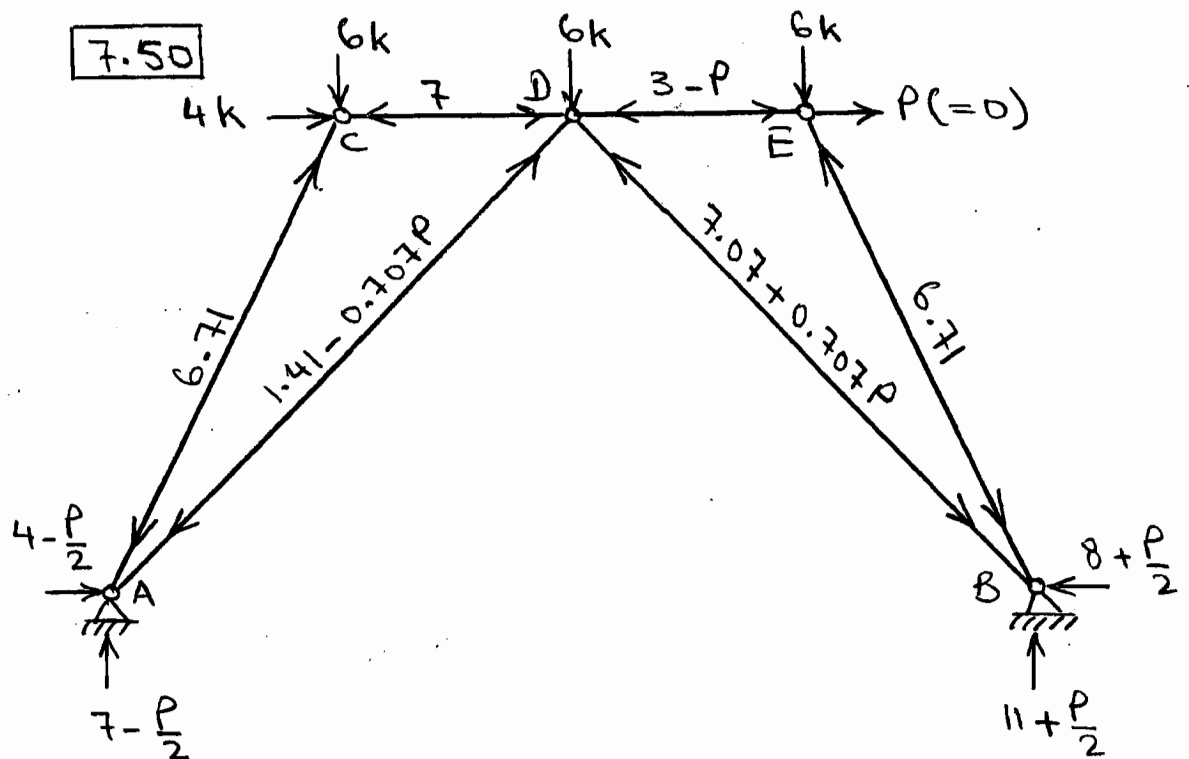
7.49



Member	L (in.)	A (in²)	F	$\frac{\partial F}{\partial P_1}$	$\frac{\partial F}{\partial P_2}$	$\frac{\partial F}{\partial P_1} \left(\frac{FL}{A} \right)$	$\frac{\partial F}{\partial P_2} \left(\frac{FL}{A} \right)$
AB	240	4	$P_1 - 2P_2$	1	-2	-3600	7200
AC	120	4	15	0	0	0	0
AD	134.16	4	-33.541	0	0	0	0
CD	134.16	4	$33.541 + 2.236P_2$	0	2.236	0	6707.99
BD	134.16	4	$2.236P_2$	0	2.236	0	5030.99
Σ						-3600	18938.98

$$\Delta_{BH} = \frac{-3600}{10000} = -0.36 \text{ in.} = 0.36 \text{ in.} \leftarrow$$

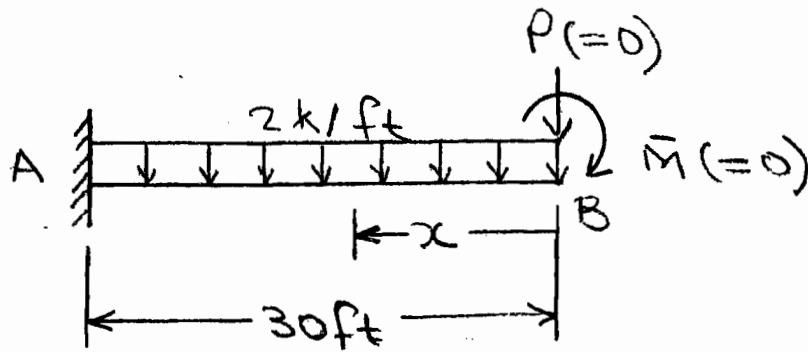
$$\Delta_{BV} = \frac{18938.98}{10000} = 1.894 \text{ in.} \downarrow$$



Member	L (in.)	F	$\frac{\partial F}{\partial P}$	P = 0 $\frac{\partial F}{\partial P}$ (FL)
AC	134.16	-6.71	0	0
BE	134.16	-6.71	0	0
AD	169.71	-1.41 + 0.707P	0.707	-169.18
BD	169.71	-7.07 - 0.707P	-0.707	848.29
CD	60	-7	0	0
DE	60	-3 + P	1	-180
Σ				499.11

$$\Delta_{EH} = \frac{499.11}{29000(6)} = 0.0029 \text{ in.} \rightarrow$$

7.51



$$M = -x^2 - \bar{M} - Px$$

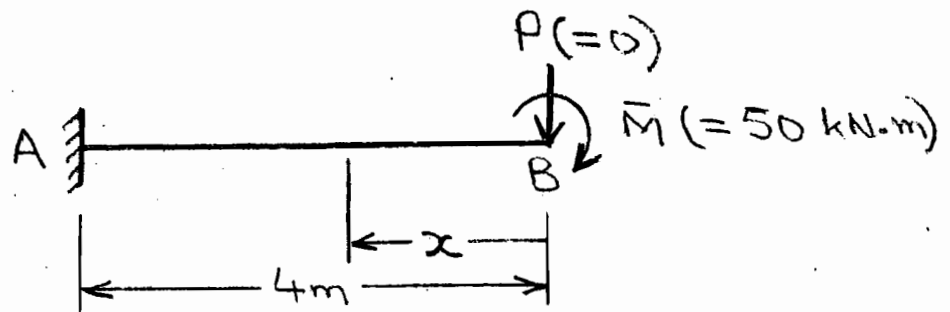
$$\frac{\partial M}{\partial \bar{M}} = -1$$

$$\begin{aligned}\theta_B &= \frac{1}{EI} \left[\int_0^{30} (-1)(-x^2) dx \right] = \frac{9000 \text{ k-ft}^2}{EI} \\ &= \frac{9000 (12)^2}{29000 (3000)} = \underline{0.0149 \text{ rad} \searrow}\end{aligned}$$

$$\frac{\partial M}{\partial P} = -x$$

$$\begin{aligned}\Delta_B &= \frac{1}{EI} \left[\int_0^{30} (-x)(-x^2) dx \right] = \frac{202500 \text{ k-ft}^3}{EI} \\ &= \frac{202500 (12)^3}{29000 (3000)} = \underline{4.022 \text{ in.} \downarrow}\end{aligned}$$

7.52



$$M = -\bar{M} - Px$$

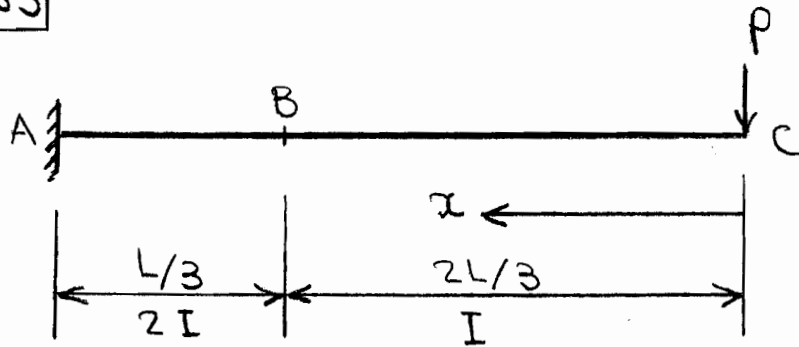
$$\frac{\partial M}{\partial \bar{M}} = -1$$

$$\begin{aligned} \theta_B &= \frac{1}{EI} \left[\int_0^4 (-1)(-50) dx \right] = \frac{200 \text{ kN.m}^2}{EI} \\ &= \frac{200}{70(164)} = \underline{0.0174 \text{ rad} \nabla} \end{aligned}$$

$$\frac{\partial M}{\partial P} = -x$$

$$\begin{aligned} \Delta_B &= \frac{1}{EI} \left[\int_0^4 (-x)(-50) dx \right] = \frac{400 \text{ kN.m}^3}{EI} \\ &= \frac{400}{70(164)} = 0.0348 \text{ m} = \underline{34.8 \text{ mm} \downarrow} \end{aligned}$$

7.53

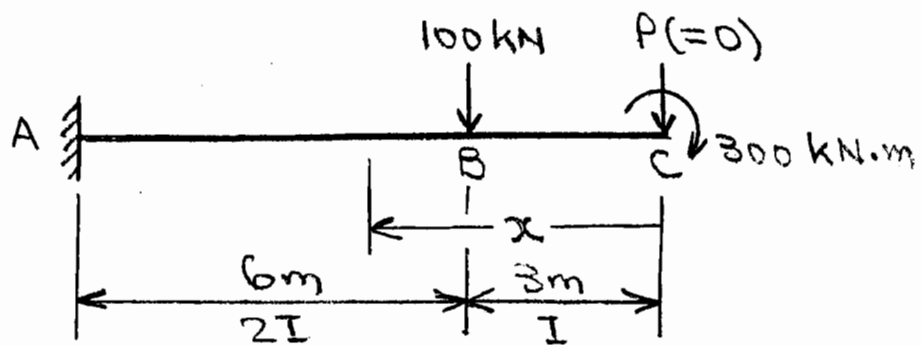


Segment	x Coordinate		M	$\frac{\partial M}{\partial P}$
	Origin	Limits		
CB	C	$0 - \frac{2L}{3}$	$-Px$	$-x$
BA	C	$\frac{2L}{3} - L$	$-Px$	$-x$

$$\Delta_C = \frac{1}{EI} \left[\int_0^{\frac{2L}{3}} (-x)(-Px) dx + \frac{1}{2} \int_{\frac{2L}{3}}^L (-x)(-Px) dx \right]$$

$$= \frac{35 PL^3}{162 EI} \downarrow$$

7.54



Segment	x Coordinate		M	$\frac{\partial M}{\partial P}$
	Origin	Limits (m)		
CB	C	0-3	$-300 - Px$	$-x$
BA	C	3-9	$-300 - Px - 100(x-3)$	$-x$

$$\begin{aligned}
 \Delta_C &= \frac{1}{EI} \left[\int_0^3 (-x)(-300) dx + \frac{1}{2} \int_3^9 (-x)(-100x) dx \right] \\
 &= \frac{13050 \text{ kN}\cdot\text{m}^3}{EI} = \frac{13050}{70(500)} \\
 &= 0.373 \text{ m} = \underline{373 \text{ mm}} \downarrow
 \end{aligned}$$

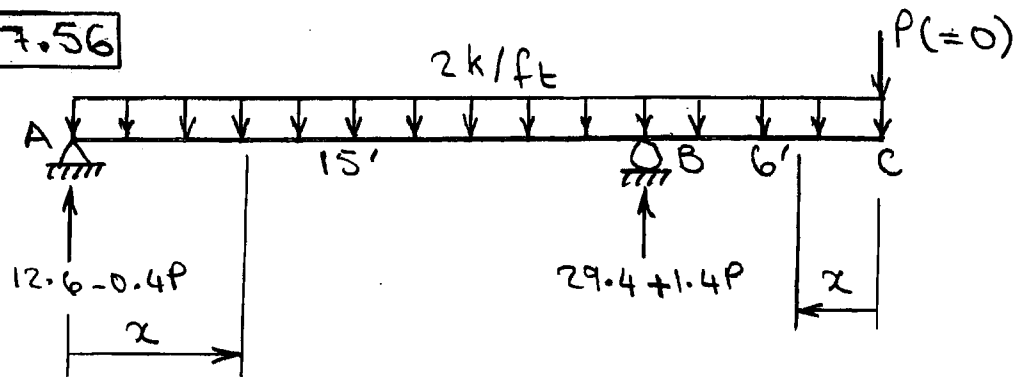
7.55

Diagram of a continuous beam with three spans. The beam is supported by a pin support at A and roller supports at B, C, and D. The beam has a constant moment of inertia I in the first and third spans, and a moment of inertia of $2I$ in the middle span. The beam is subjected to a downward point load of 200 kN at the midpoint of the first span (B) and a downward point load of $P (= 100 \text{ kN})$ at the midpoint of the third span (C). The reaction at A is labeled as $150 + \frac{P}{4}$ and the reaction at D is labeled as $50 + \frac{3P}{4}$. The distance from A to B is labeled as 5 m , from B to C as 10 m , and from C to D as 5 m . The distance from A to the midpoint of the first span is labeled as x , and the distance from the midpoint of the third span to D is labeled as x .

Substituting $P = 100 \text{ kN}$ and integrating, we obtain

323

7.56

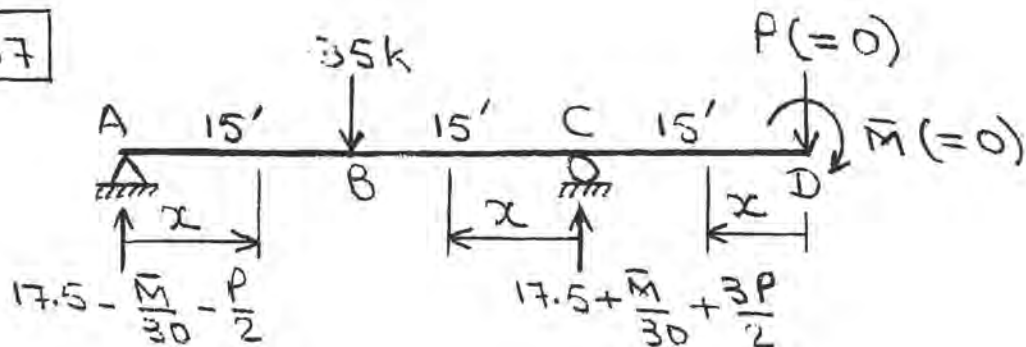


Segment	x Coordinate		M	$\frac{\partial M}{\partial P}$
	Origin	Limits		
AB	A	0-15	$(12.6 - 0.4P)x - x^2$	$-0.4x$
CD	C	0-6	$-Px - x^2$	$-x$

Substituting $P=0$ and integrating, we obtain

$$\begin{aligned}
 \Delta_C &= \frac{1}{EI} \left[\int_0^{15} (-0.4x)(12.6x - x^2) dx \right. \\
 &\quad \left. + \int_0^6 -x(-x^2) dx \right] \\
 &= - \frac{283.5 \text{ k-ft}^3}{EI} = - \frac{283.5(12)^3}{29000(3500)} \\
 &= -0.0048 \text{ in.} = \underline{0.0048 \text{ in.} \uparrow}
 \end{aligned}$$

7.57



Segment	x Coordinate		M	$\frac{\partial M}{\partial \bar{M}}$	$\frac{\partial M}{\partial P}$
	Origin	Limits (ft)			
AB	A	0-15	$(17.5 - \frac{\bar{M}}{30} - \frac{P}{2})x$	$-x/30$	$-x/2$
CB	C	0-15	$(17.5 + \frac{\bar{M}}{30} + \frac{3P}{2})x - \bar{M}$ $- P(x+15)$	$\frac{x}{30} - 1$	$\frac{x}{2} - 15$
DC	D	0-15	$-\bar{M} - Px$	-1	$-x$

Substituting $\bar{M} = P = 0$, and integrating, we obtain

$$\theta_D = \frac{1}{EI} \left[\int_0^{15} -\frac{x}{30} (17.5x) dx + \int_0^{15} \left(\frac{x}{30} - 1 \right) (17.5x) dx \right]$$

$$= -\frac{1968.75 \text{ k-ft}^2}{EI} = -\frac{1968.75 (12)^2}{10000 (2500)}$$

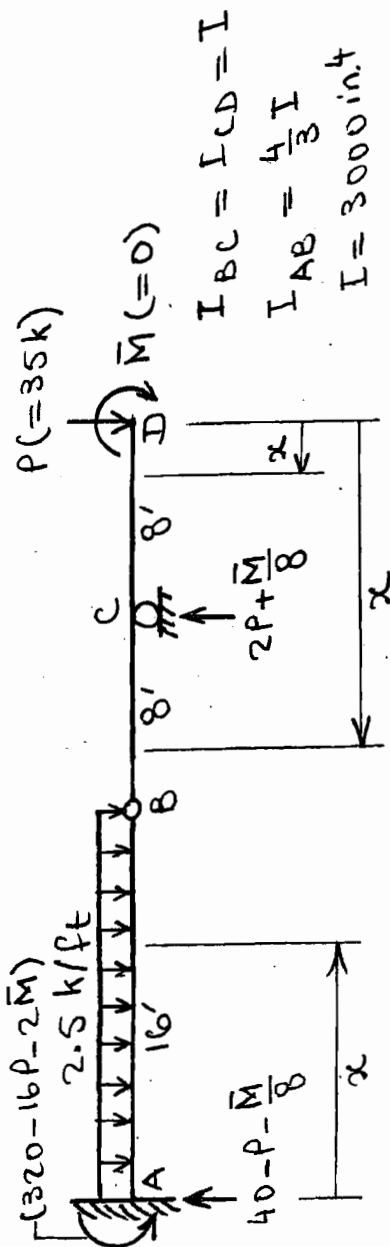
$$= -0.01134 \text{ rad} = \underline{0.01134 \text{ rad} \downarrow}$$

$$\Delta_D = \frac{1}{EI} \left[\int_0^{15} -\frac{x}{2} (17.5x) dx + \int_0^{15} \left(\frac{x}{2} - 15 \right) (17.5x) dx \right]$$

$$= -\frac{29531.25 \text{ k-ft}^3}{EI} = -\frac{29531.25 (12)^3}{10000 (2500)}$$

$$= -2.04 \text{ in.} = \underline{2.04 \text{ in.} \uparrow}$$

7.58



Segment	x Coordinate		M	$\frac{\partial M}{\partial \bar{M}}$	$\frac{\partial M}{\partial P}$
	Origin	Limits			
DC	D	0 - 8	$-Px - \bar{M}$	-1	-x
CB	D	8 - 16	$-Px - \bar{M} + (2P + \frac{\bar{M}}{8})(x - 8)$	$\frac{x}{8} - 2$	$x - 16$
AB	A	0 - 16	$-(320 - 16P - 2\bar{M}) + (40 - P - \frac{\bar{M}}{8})x - 1.25x^2$	$2 - \frac{x}{8}$	$16 - x$

Substituting $\bar{M} = 0$ and $P = 35 \text{ k}$, and integrating, we obtain

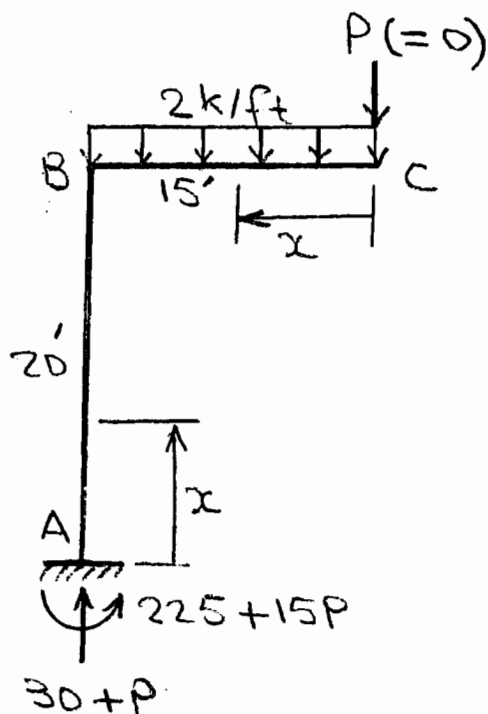
$$\theta_D = \frac{1}{EI} \left[\int_0^8 -1(-35x) dx + \int_8^{16} (\frac{x}{8} - 2)(35x - 560) dx \right.$$

$$\left. + \frac{3}{4} \int_0^{16} (2 - \frac{x}{8})(240 + 5x - 1.25x^2) dx \right] = \frac{4426.67 \text{ k-ft}^2}{EI} = 0.0071 \text{ rad.} \downarrow$$

$$\Delta_D = \frac{1}{EI} \left[\int_0^8 -x(-35x) dx + \int_8^{16} (x - 16)(35x - 560) dx \right.$$

$$\left. + \frac{3}{4} \int_0^{16} (16 - x)(240 + 5x - 1.25x^2) dx \right] = \frac{32426.67 \text{ k-ft}^3}{EI} = 0.62 \text{ in.} \downarrow$$

7.59

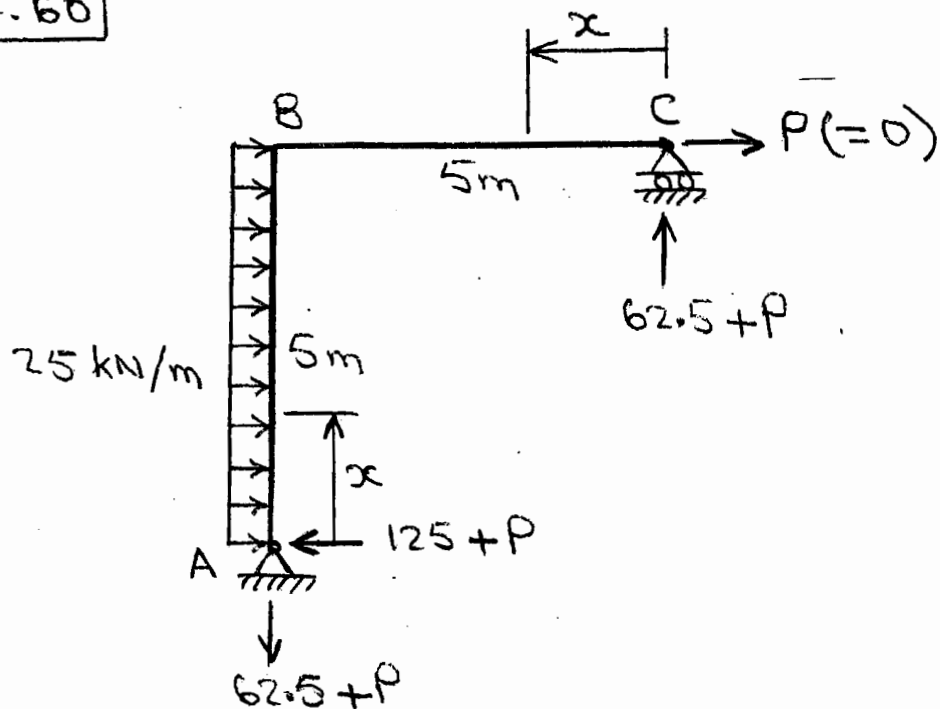


Segment	x Coordinate		M	$\frac{\partial M}{\partial P}$
	Origin	Limits (ft)		
AB	A	0-20	$-(225 + 15P)$	-15
CB	C	0-15	$-x^2 - Px$	-x

Substituting $P=0$ and integrating, we obtain

$$\begin{aligned}
 \Delta_C &= \frac{1}{EI} \left[\int_0^{20} (-15)(-225) dx + \int_0^{15} (-x)(-x^2) dx \right] \\
 &= \frac{80156.25 \text{ k-ft}^3}{EI} = \frac{80156.25 (12)^3}{29000 (2000)} \\
 &= \underline{2.388 \text{ in.} \downarrow}
 \end{aligned}$$

7.60

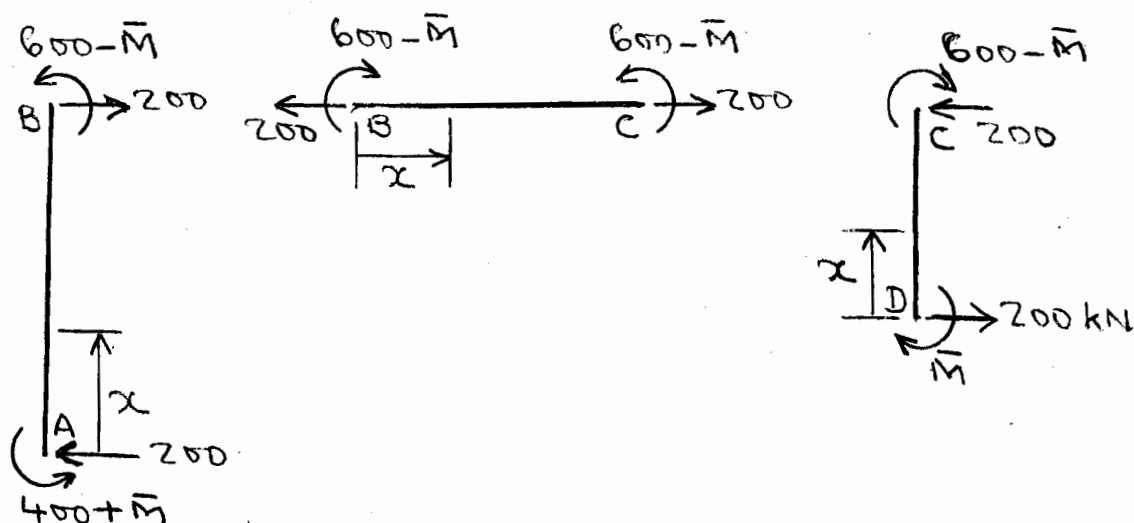
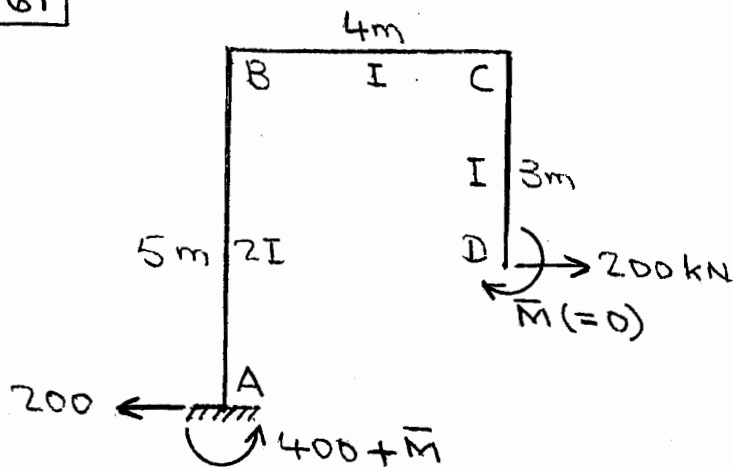


Segment	x Coordinate		M	$\frac{\partial M}{\partial P}$
	Origin	Limit (m)		
AB	A	0-5	$(125 + P)x - 12.5x^2$	x
CB	C	0-5	$(62.5 + P)x$	x

Substituting $P = 0$ and integrating, we obtain

$$\begin{aligned}
 \Delta_C &= \frac{1}{EI} \left[\int_0^5 x(125x - 12.5x^2) dx + \int_0^5 x(62.5x) dx \right] \\
 &= \frac{5859.375 \text{ kN}\cdot\text{m}^3}{EI} = \frac{5859.375}{70(1030)} \\
 &= 0.0813 \text{ m} = \underline{81.3 \text{ mm} \rightarrow}
 \end{aligned}$$

7.61



Segment	x Coordinate		M	$\frac{\partial M}{\partial \bar{M}}$
	Origin	Limits (m)		
AB	A	0-5	$-(400 + \bar{M}) + 200x$	-1
BC	B	0-4	$600 - \bar{M}$	-1
DC	D	0-3	$-200x + \bar{M}$	1

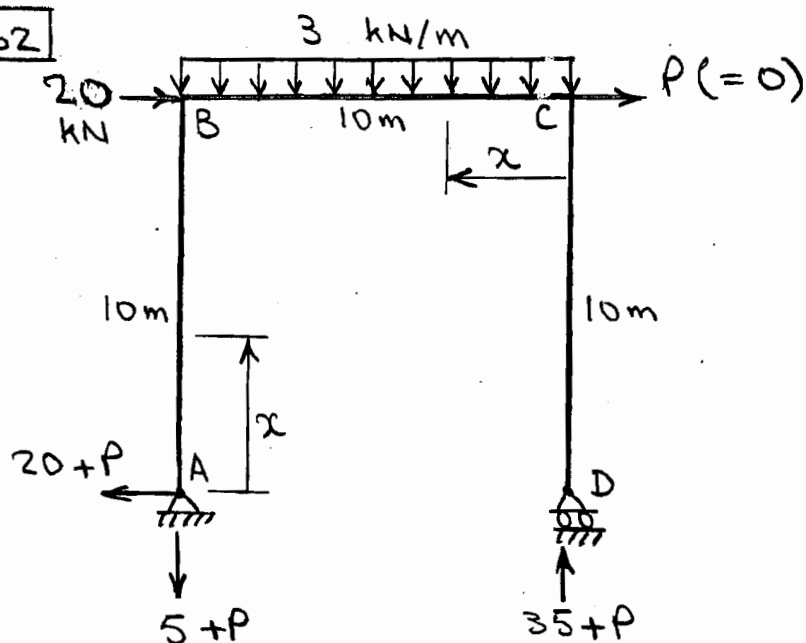
Substituting $\bar{M}=0$ and integrating, we obtain

$$\theta_D = \frac{1}{EI} \left[\frac{1}{2} \int_0^5 -1(-400 + 200x) dx + \int_0^4 -1(600) dx + \int_0^3 1(-200x) dx \right]$$

$$= -\frac{3550 \text{ kN}\cdot\text{m}^2}{EI} = -\frac{3550}{70(1290)} = -0.0393 \text{ rad}$$

0.0393 rad \triangle

7.62



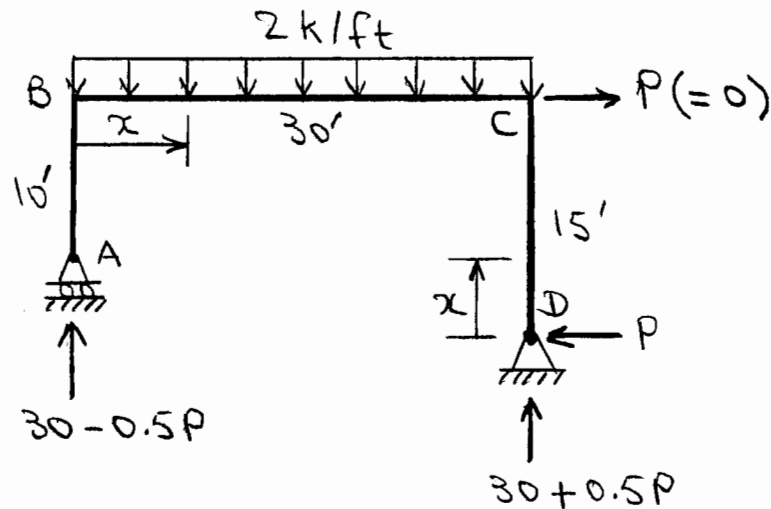
Segment	x Coordinate		M	$\frac{\partial M}{\partial P}$
	Origin	Limits		
AB	A	0-10	$(20 + P)x$	x
CB	C	0-10	$(35 + P)x - 1.5x^2$	x

Substituting $P=0$ and integrating, we obtain

$$\Delta_C = \frac{1}{EI} \left[\int_0^{10} x(20x) dx + \int_0^{10} x(35x - 1.5x^2) dx \right]$$

$$= \frac{14583.33 \text{ kN-m}^3}{EI} = 0.182 \text{ m} \rightarrow$$

7.63



Segment	x Coordinate		M	$\frac{\partial M}{\partial P}$
	Origin	Limits		
BC	B	0-30	$(30 - 0.5P)x - x^2$	$-0.5x$
DC	D	0-15	Px	x

Substituting $P=0$ and integrating, we obtain

$$\Delta_C = \frac{1}{EI} \int_0^{30} (-0.5x)(30x - x^2) dx = - \frac{33750 \text{ k-ft}^3}{EI}$$

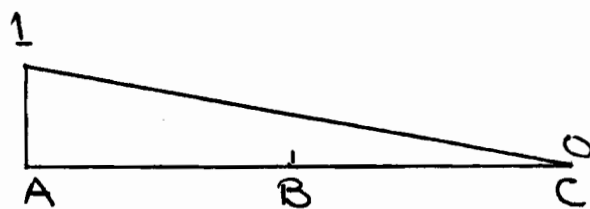
$$= - \frac{33750 (12)^3}{29000 (1500)} = -1.34 \text{ in.} = \underline{1.34 \text{ in.} \leftarrow}$$

Chapter Eight

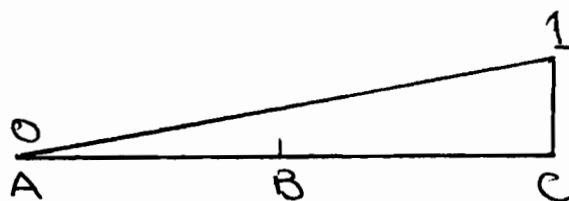
Influence Lines

CHAPTER 8

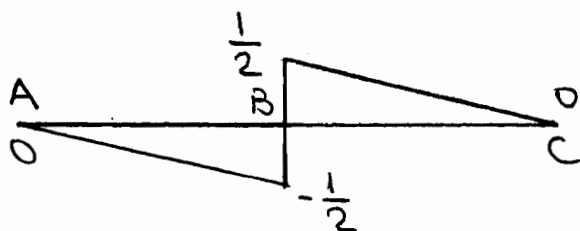
8.1



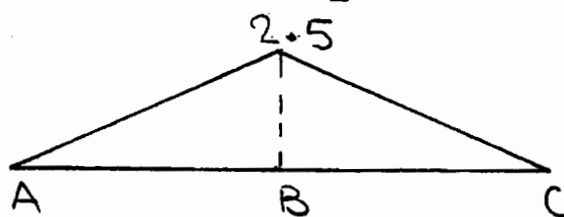
A_y



C_y

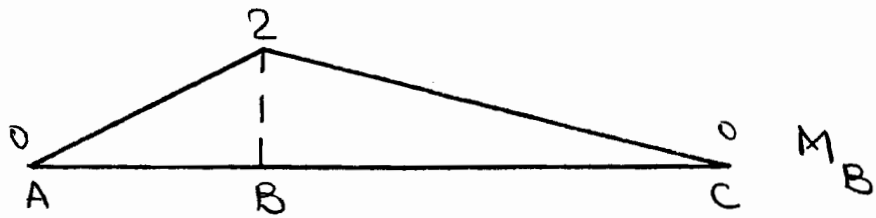
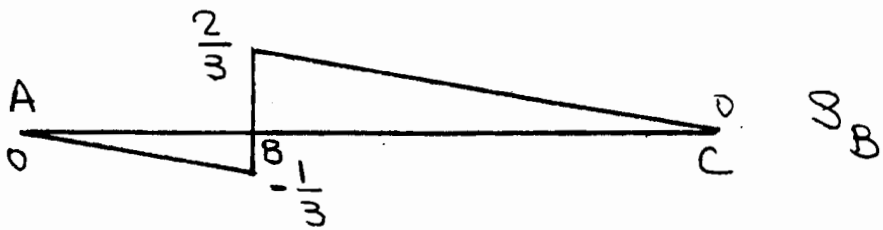
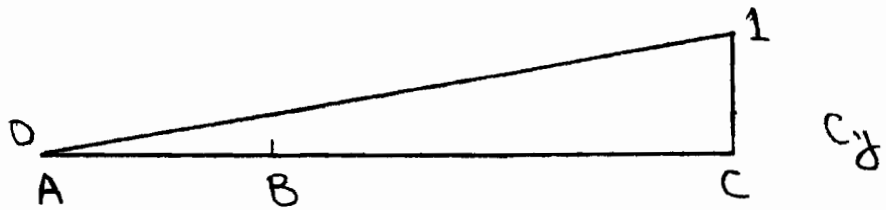
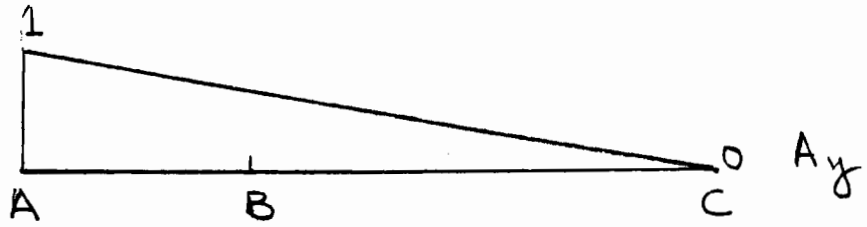


S_B

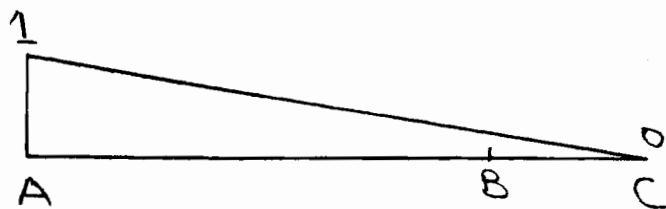


M_B

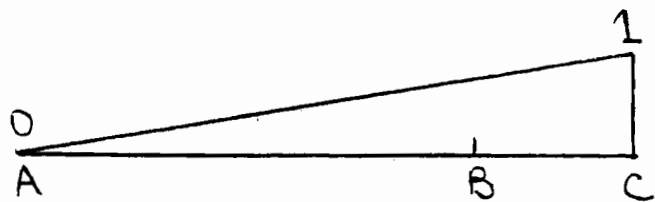
8.2



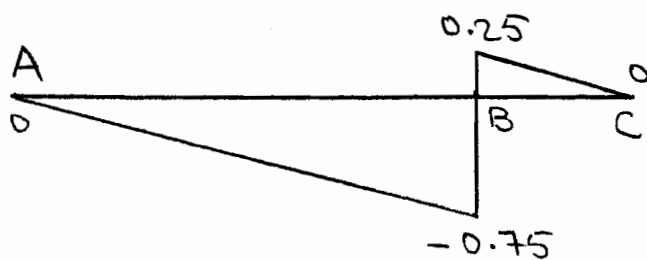
8.3



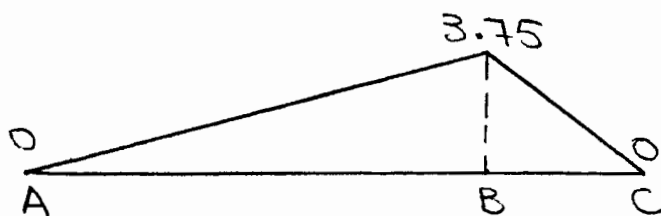
A_y



C_y

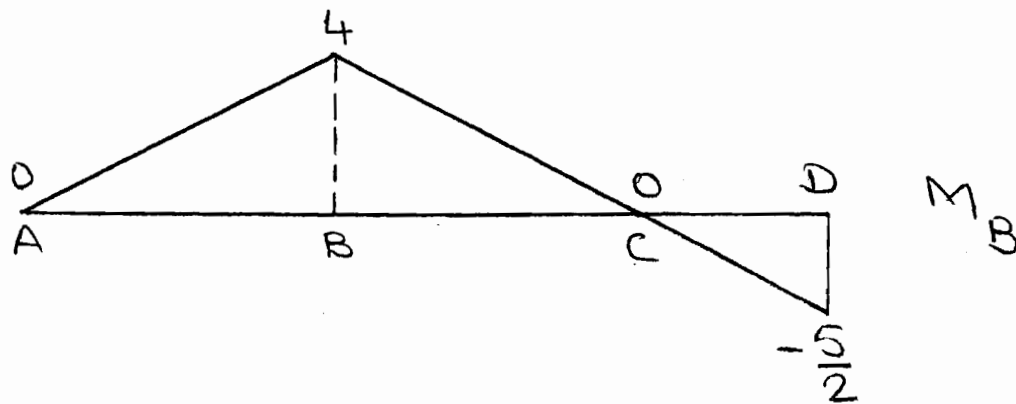
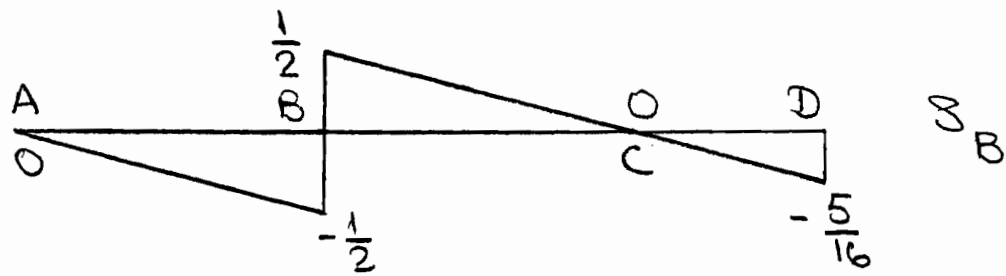
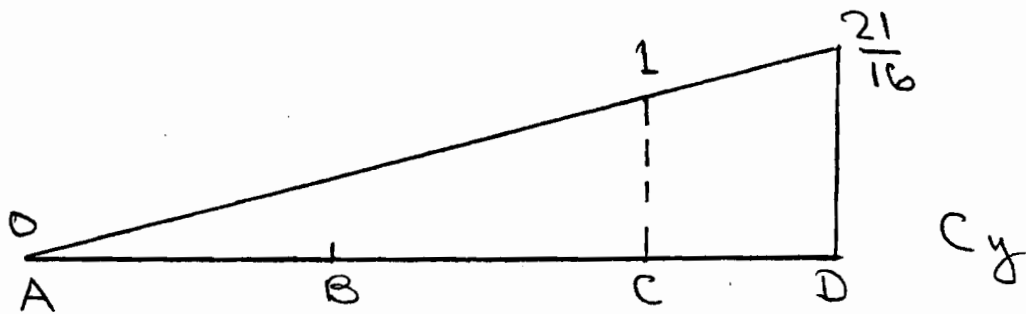
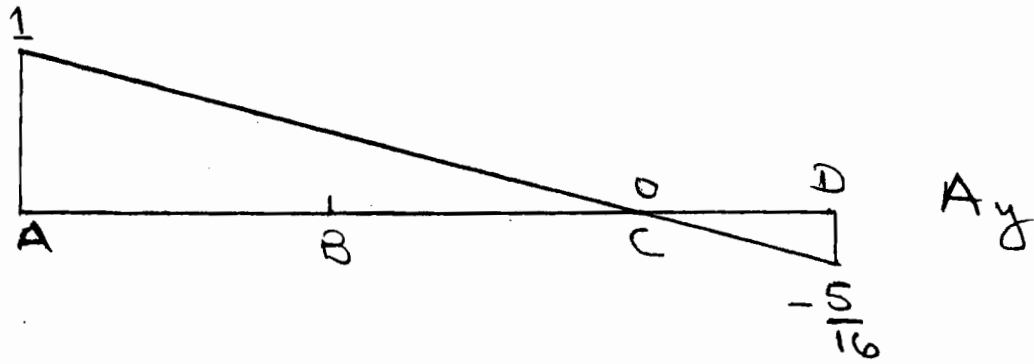


S_B

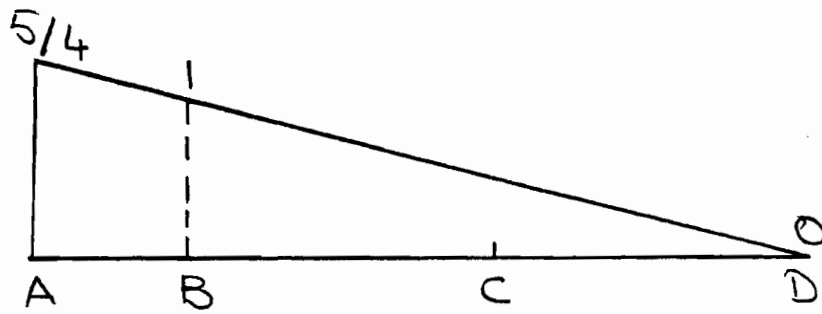


M_B

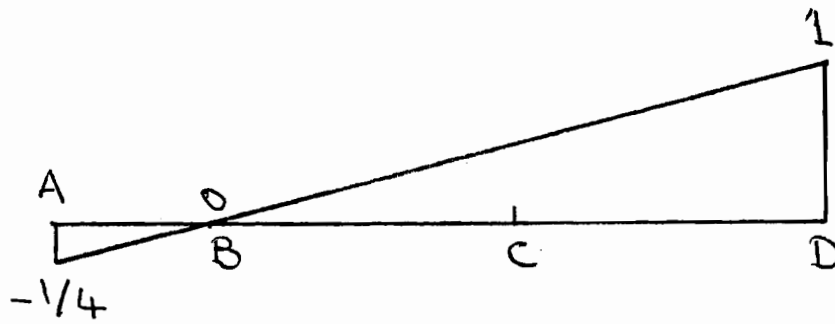
8.4



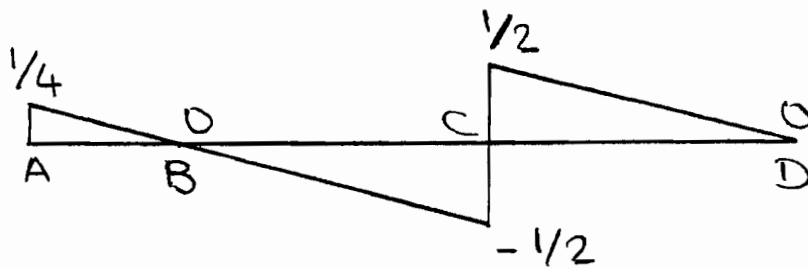
8.5



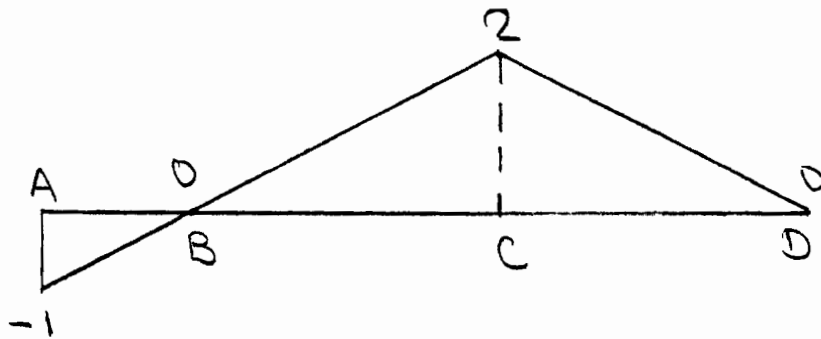
B_y



D_y

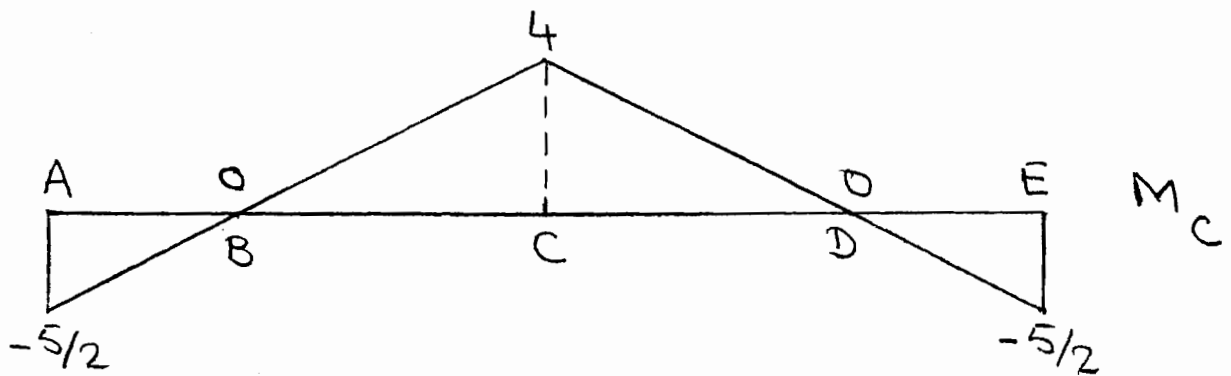
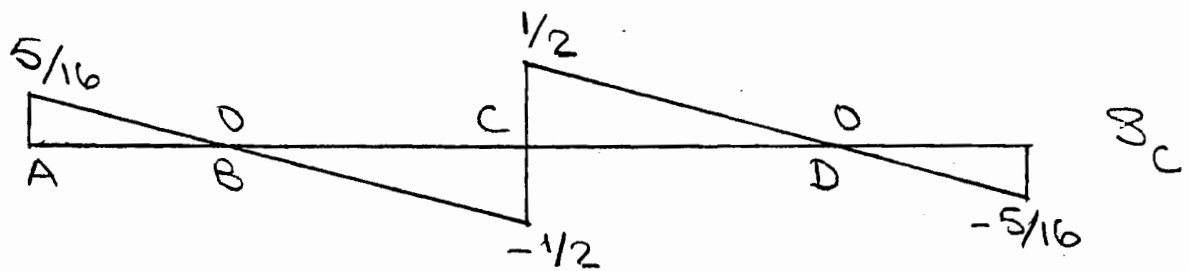
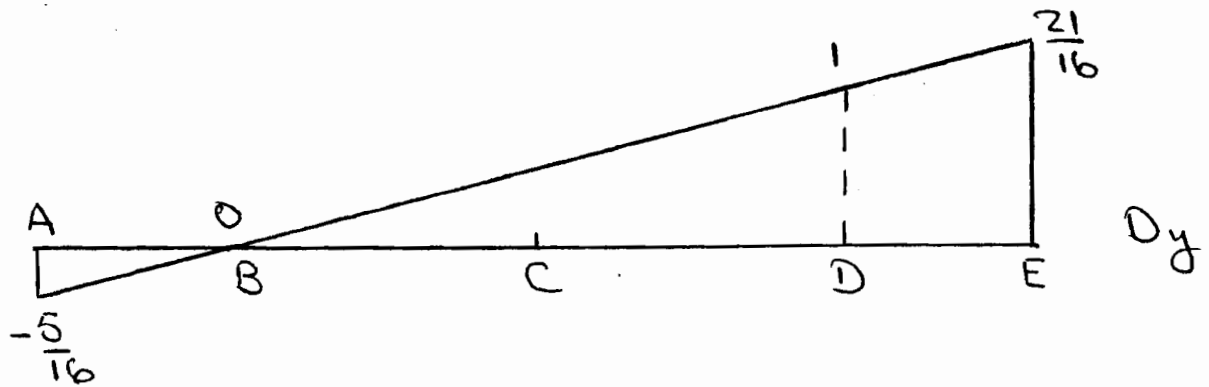
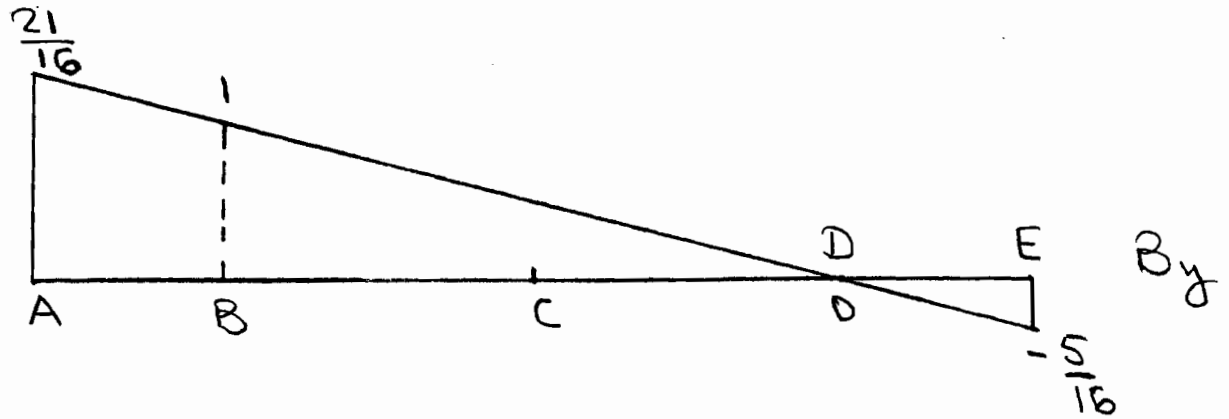


B_c

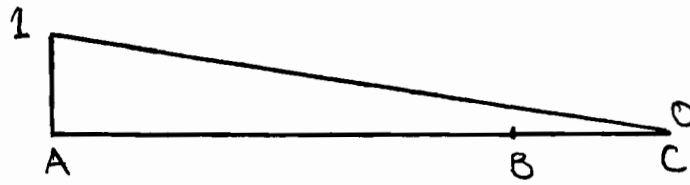


M_c

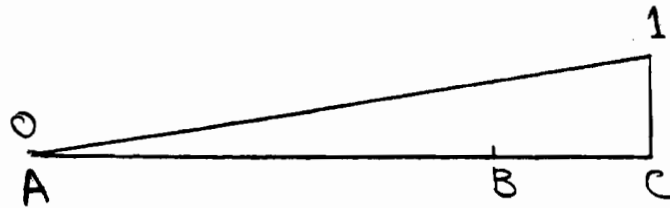
8.6



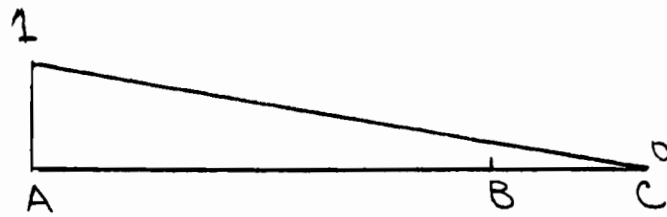
8.7



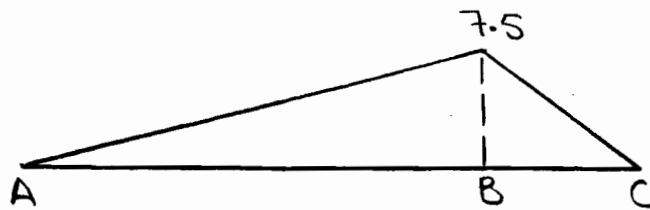
A_y



C_y

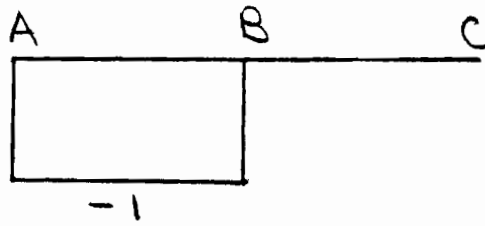


$S_{A,R}$

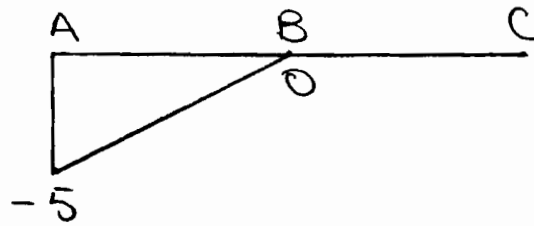


M_B

8.8

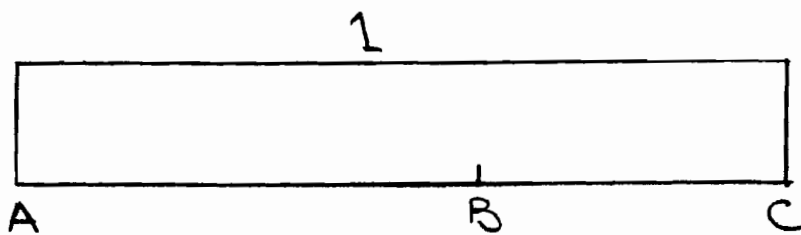


S_B

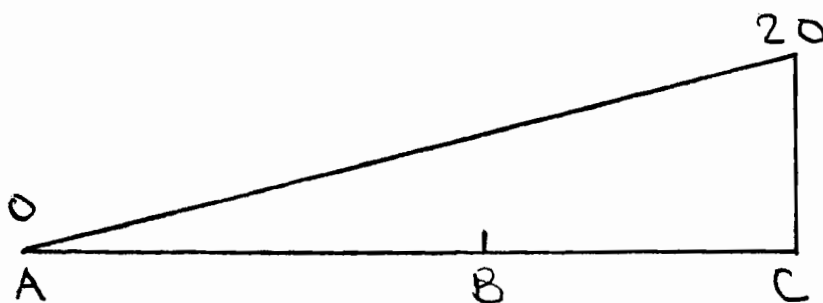


M_B

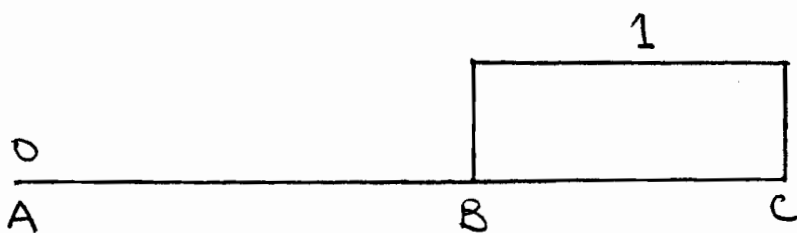
8.9



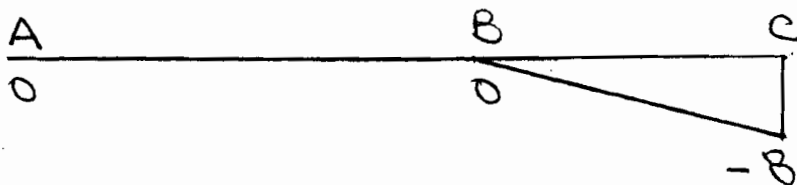
A_y



$M_A (+ \curvearrowright)$

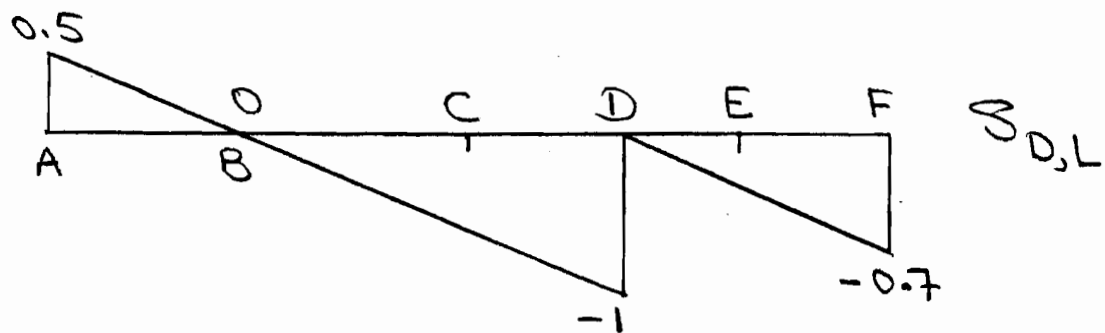
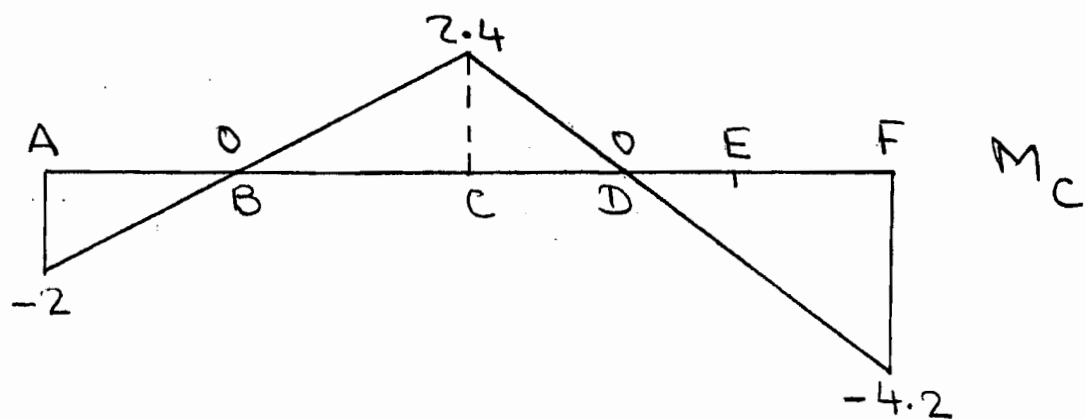
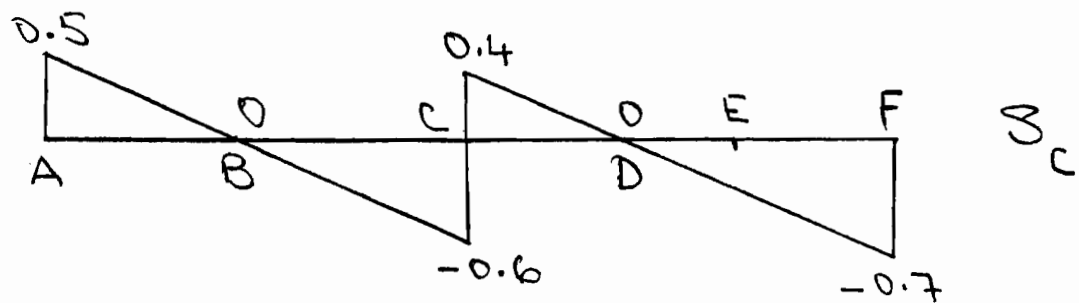


S_B

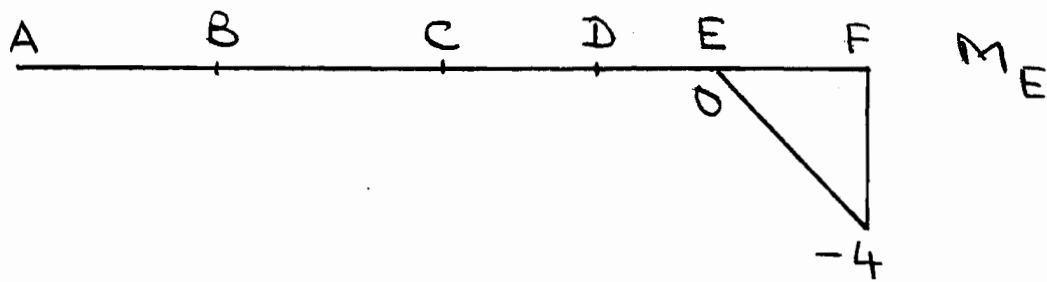
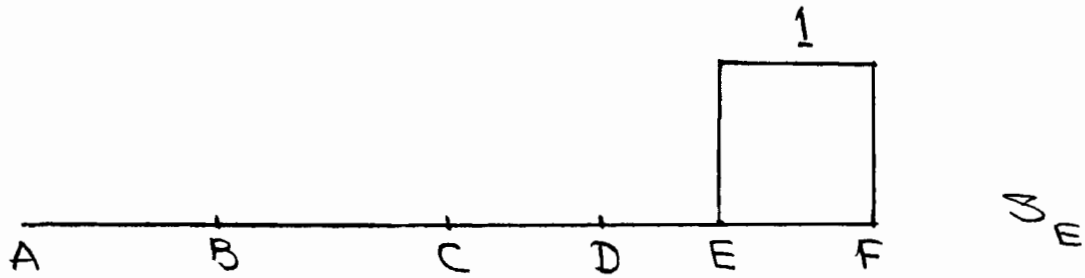


M_B

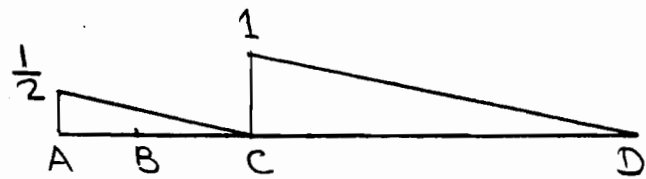
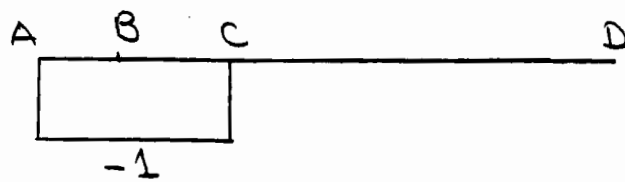
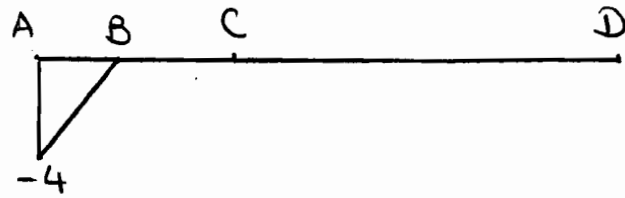
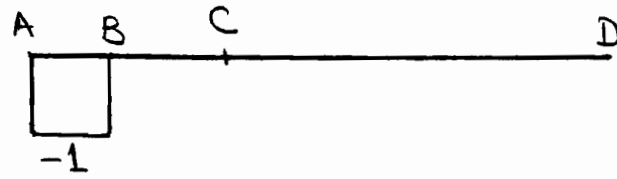
8.10



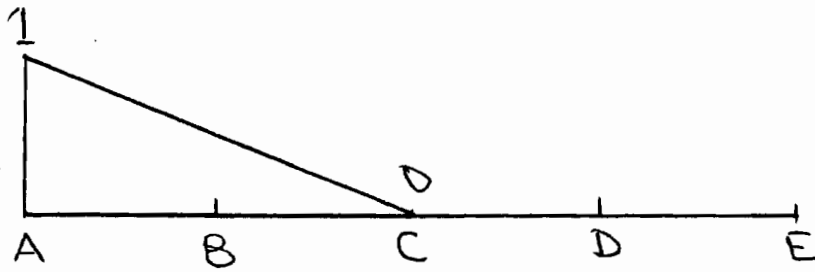
8.11



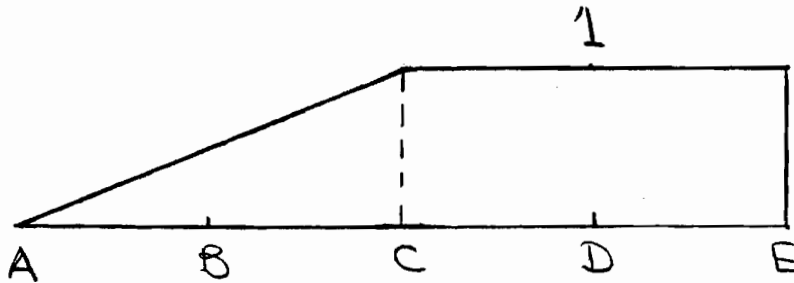
8.12



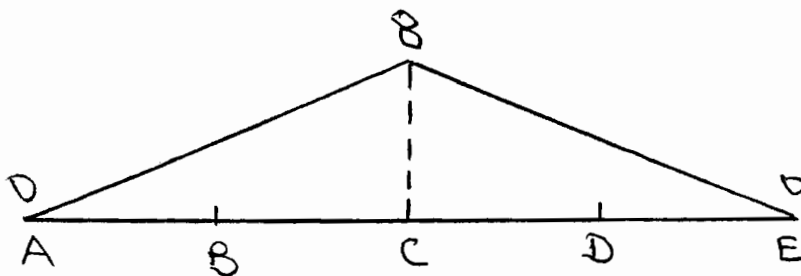
8.13



A_y

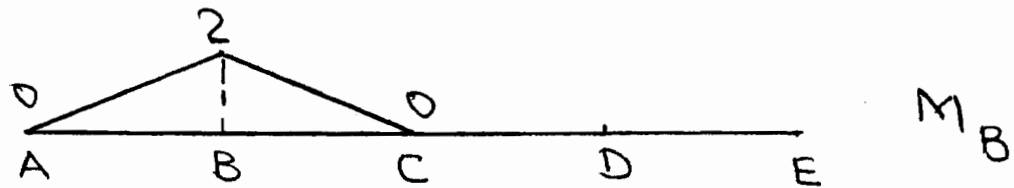
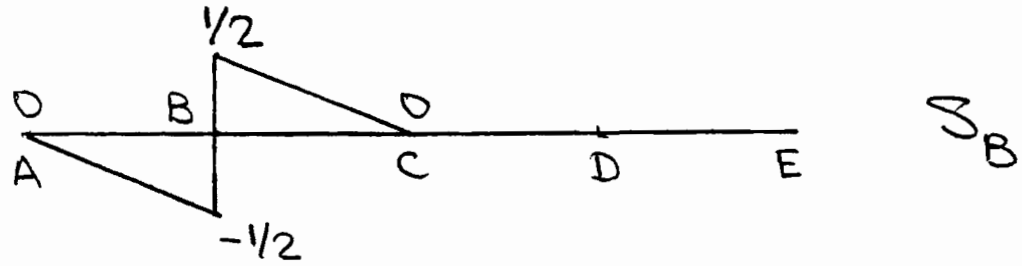


E_y

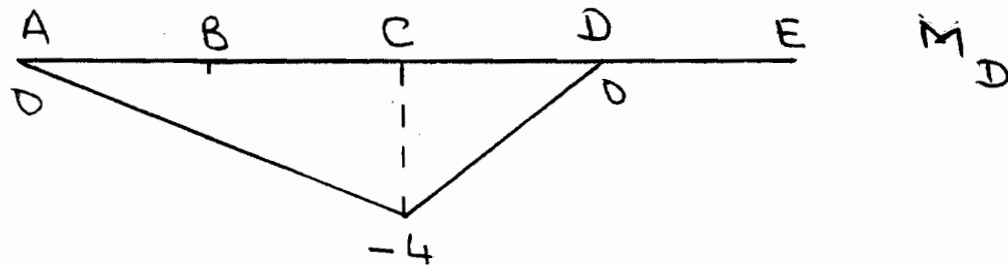
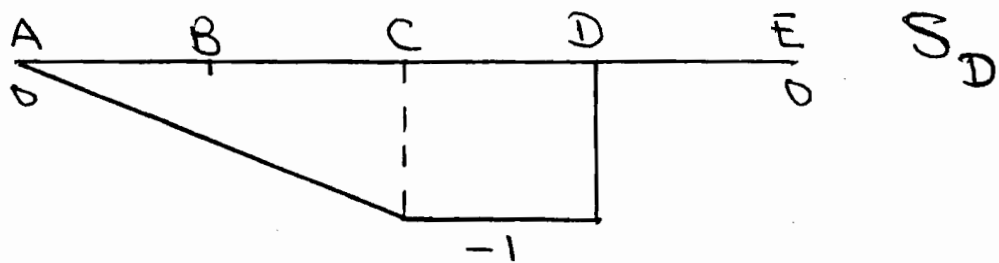


$M_E (+ \curvearrowright)$

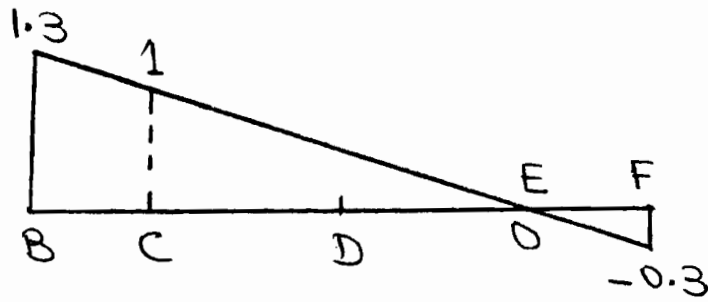
8.14



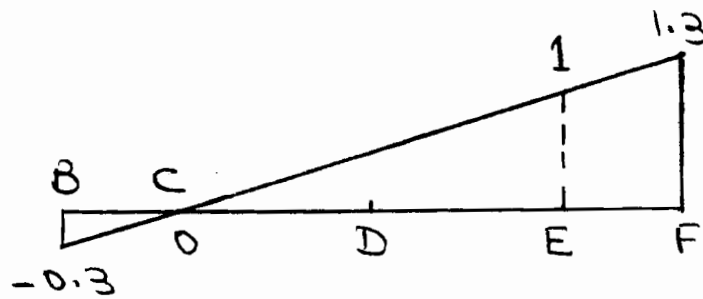
8.15



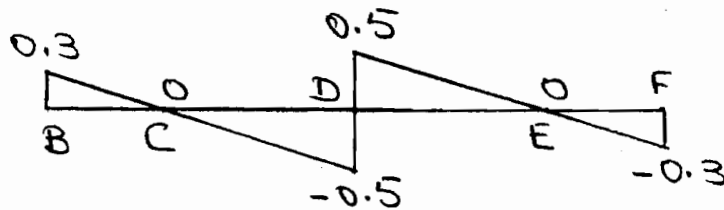
8.16



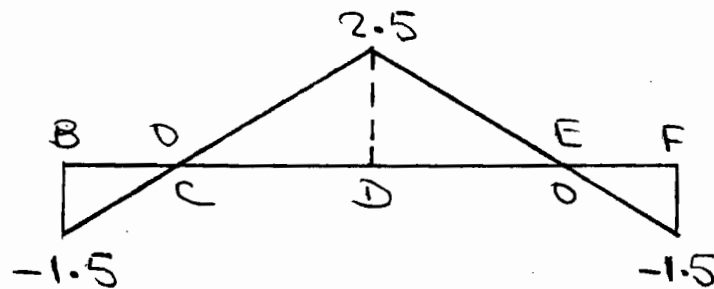
A_y



E_y

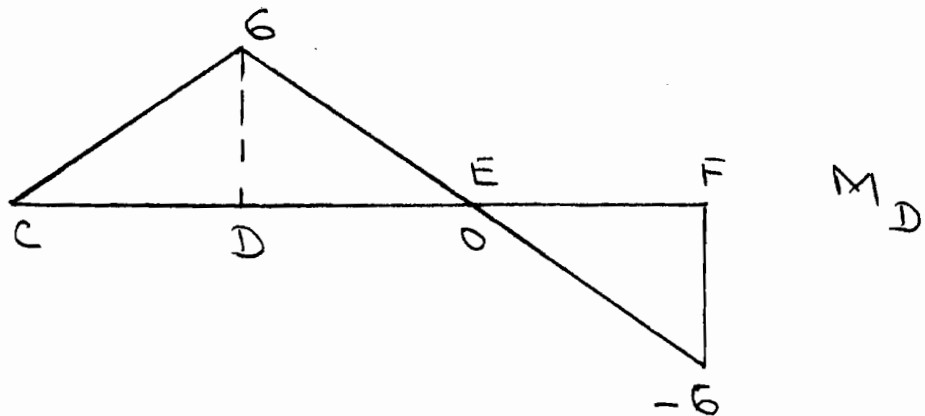
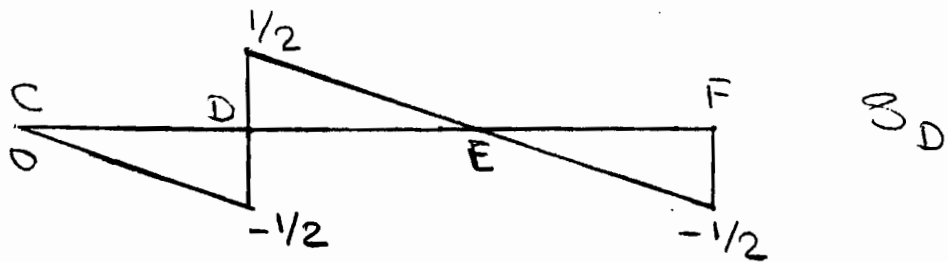
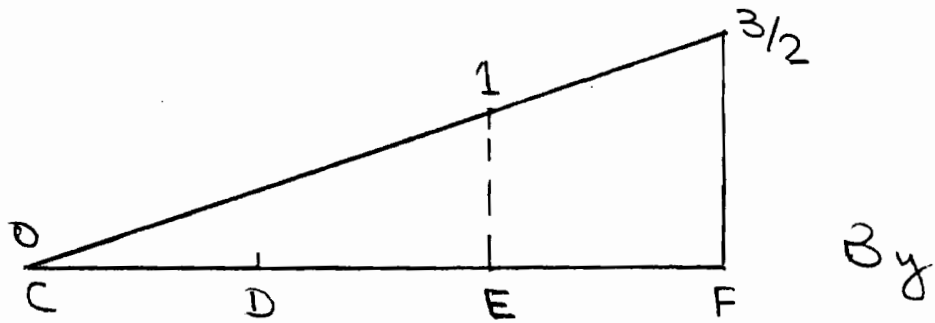
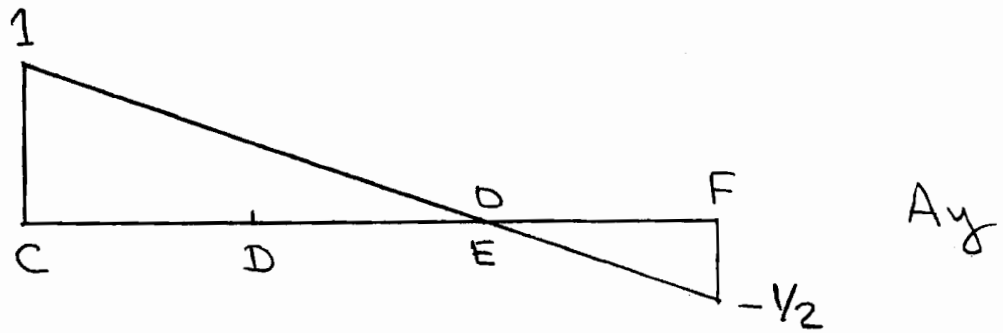


S_D

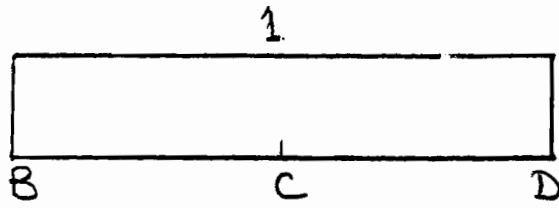


M_D

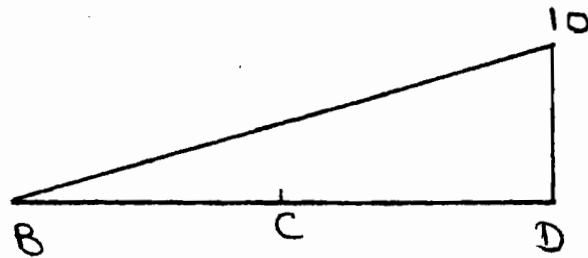
8.17



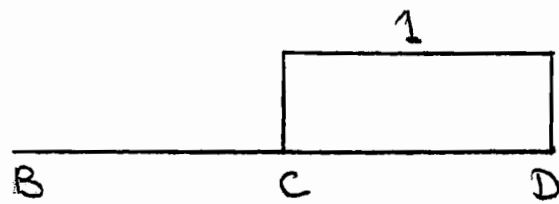
8.18



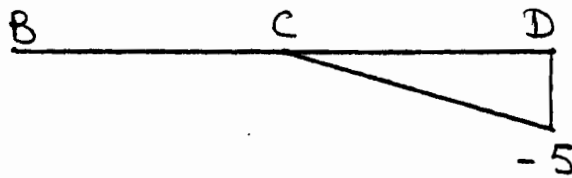
A_y



$M_A (+\curvearrowright)$

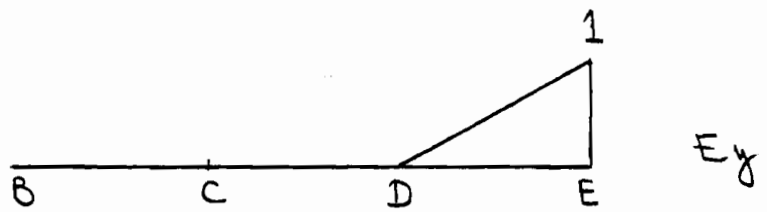


S_C

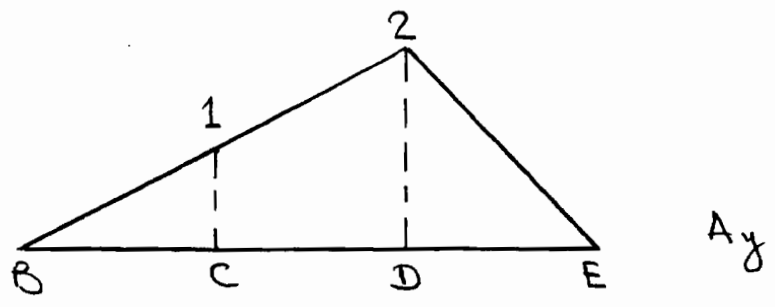


M_C

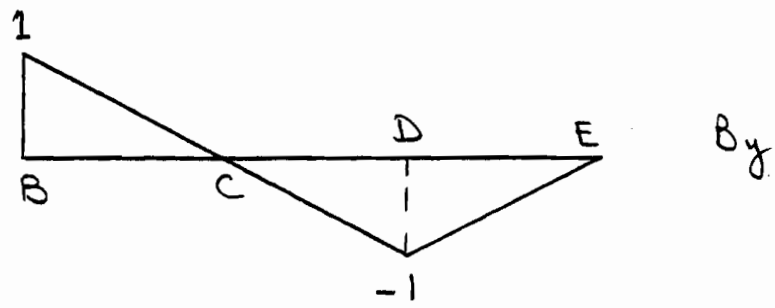
8.19



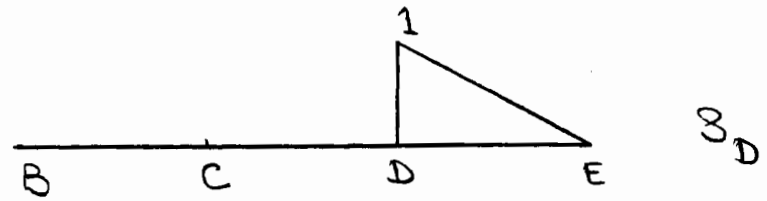
E_y



A_y

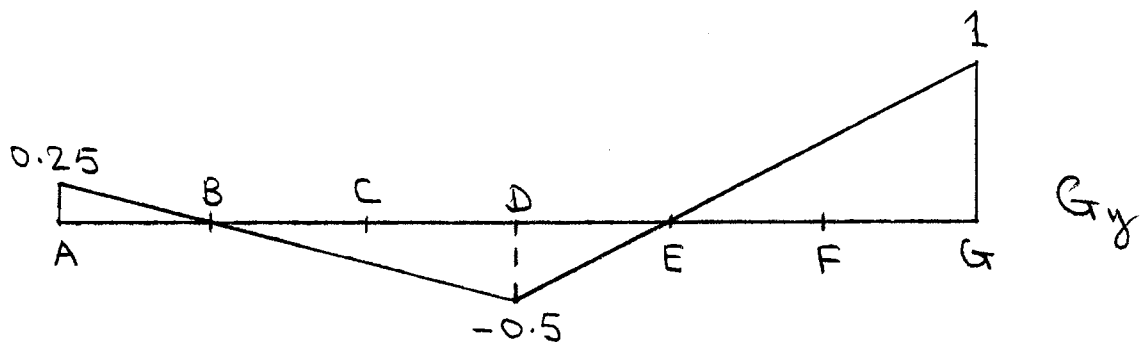
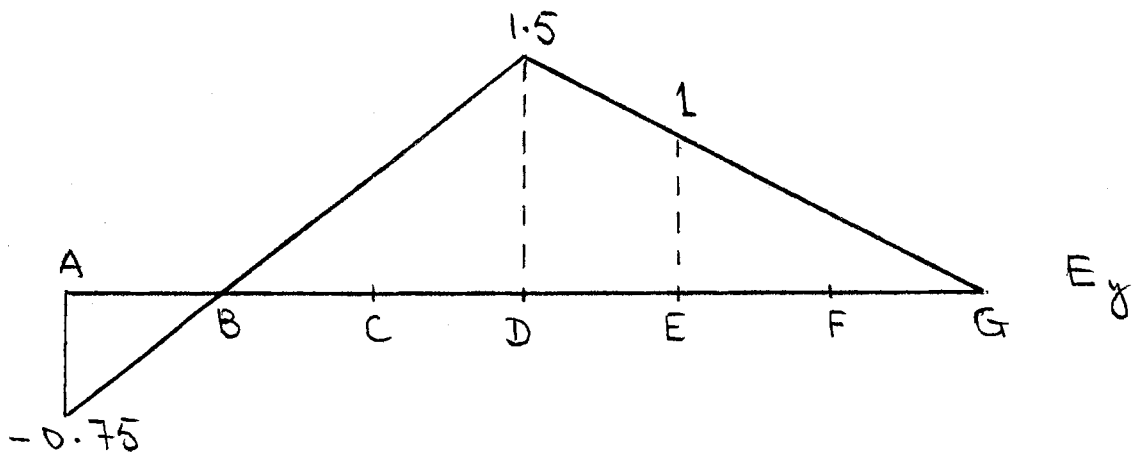
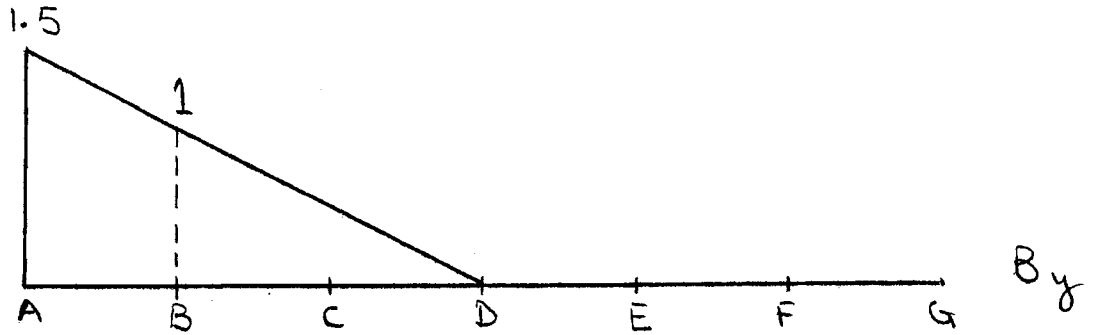


B_y

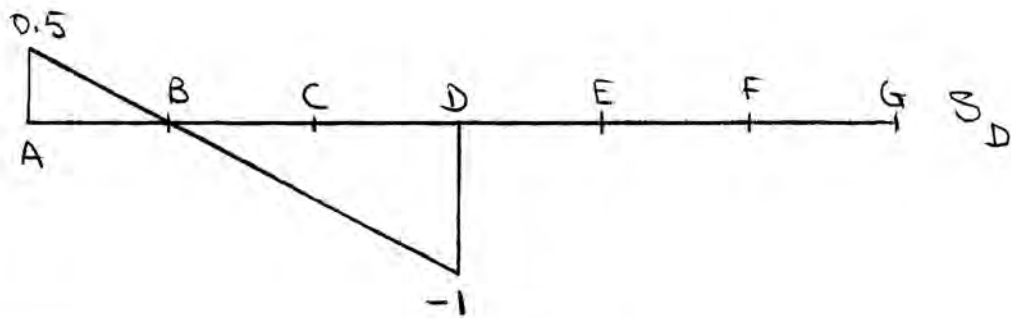
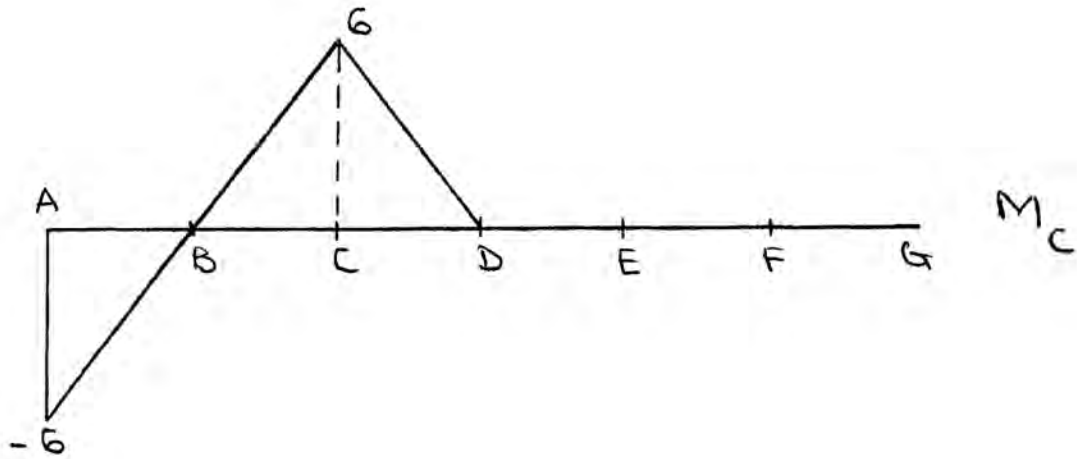
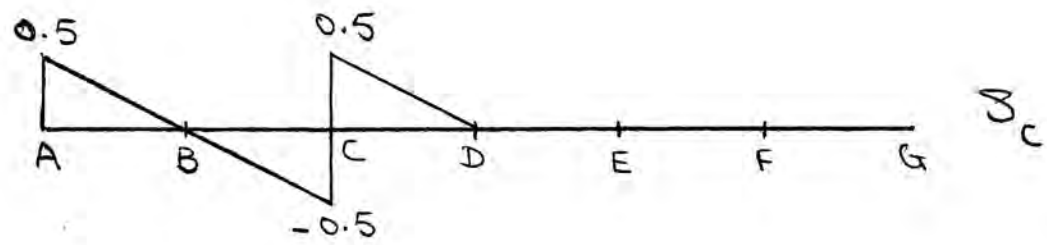


S_D

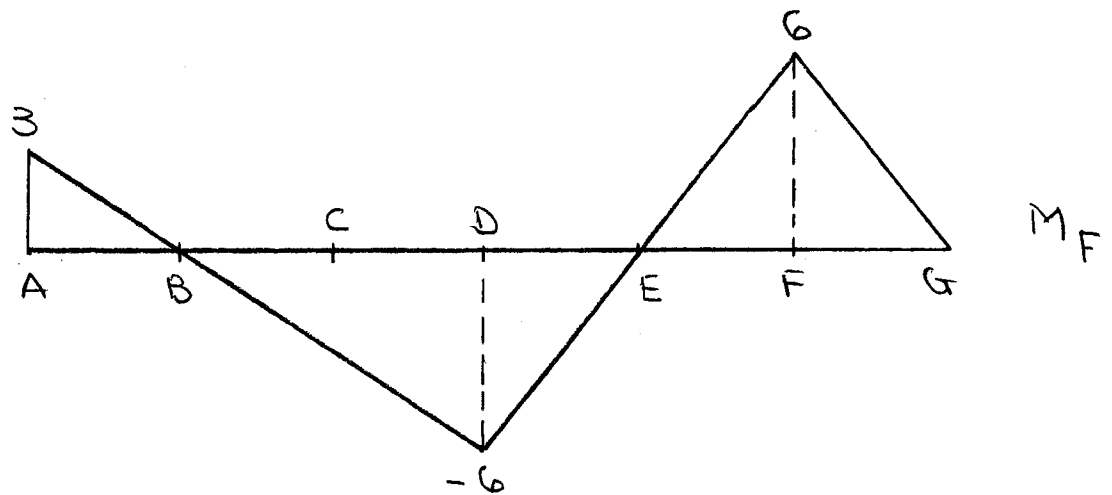
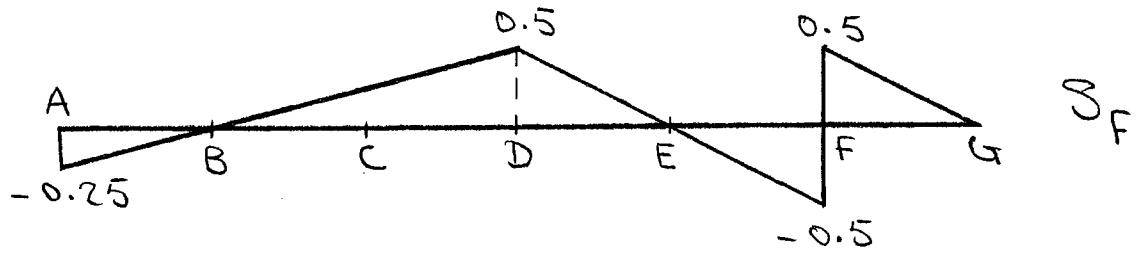
8.20



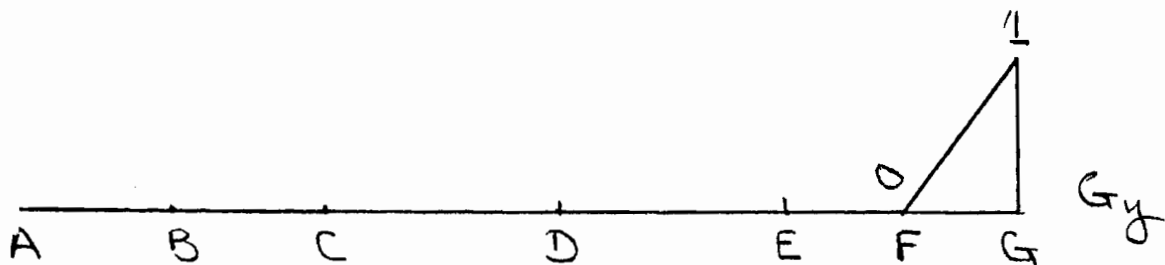
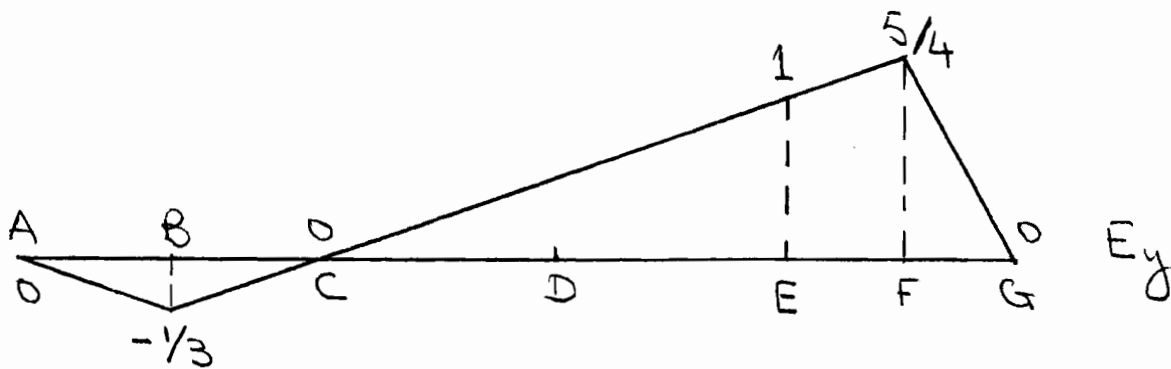
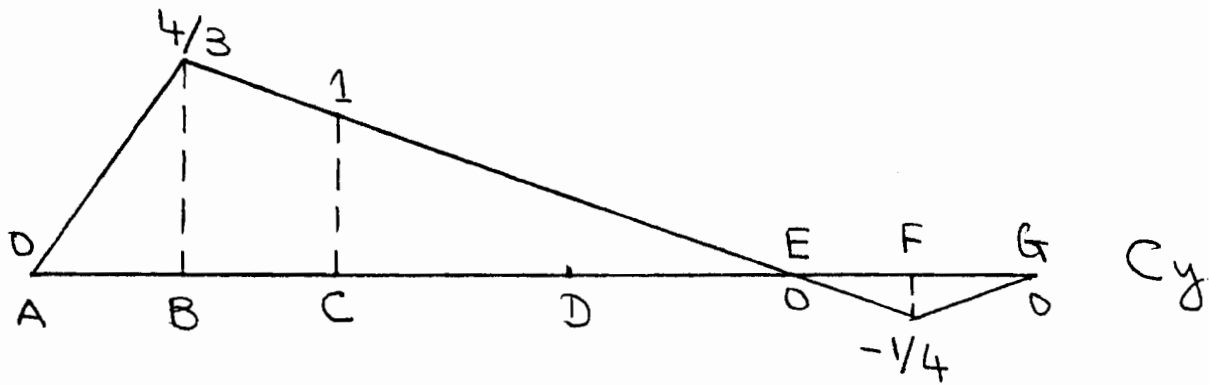
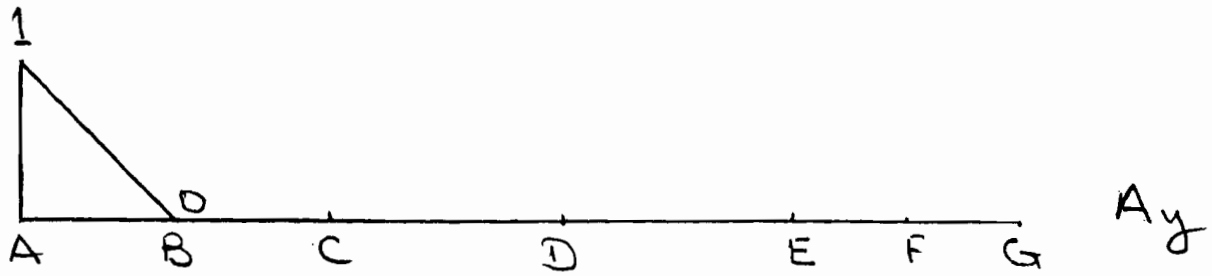
8.21



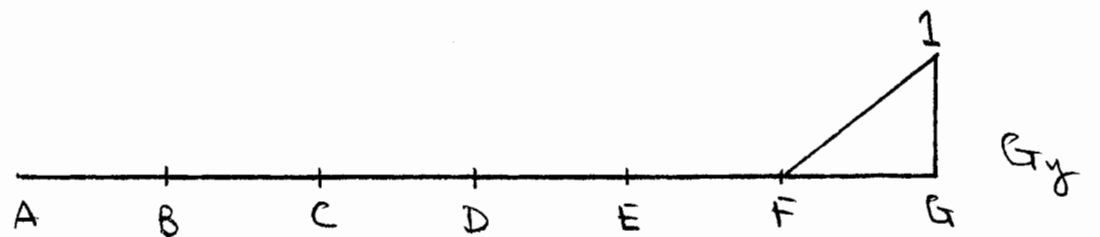
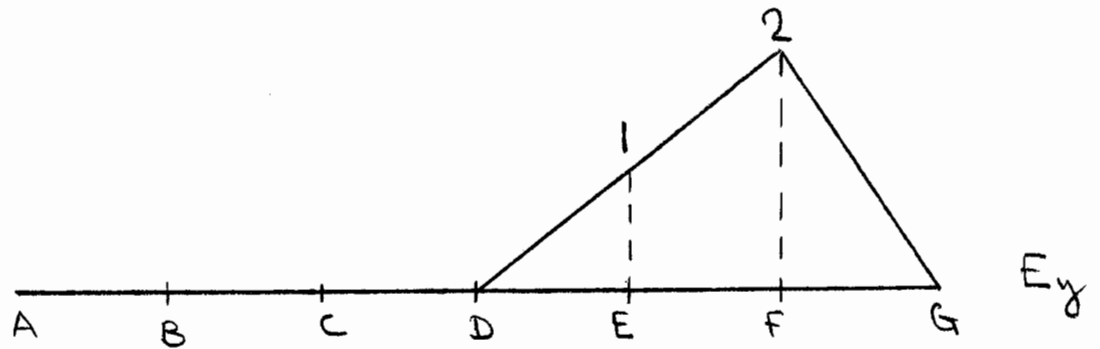
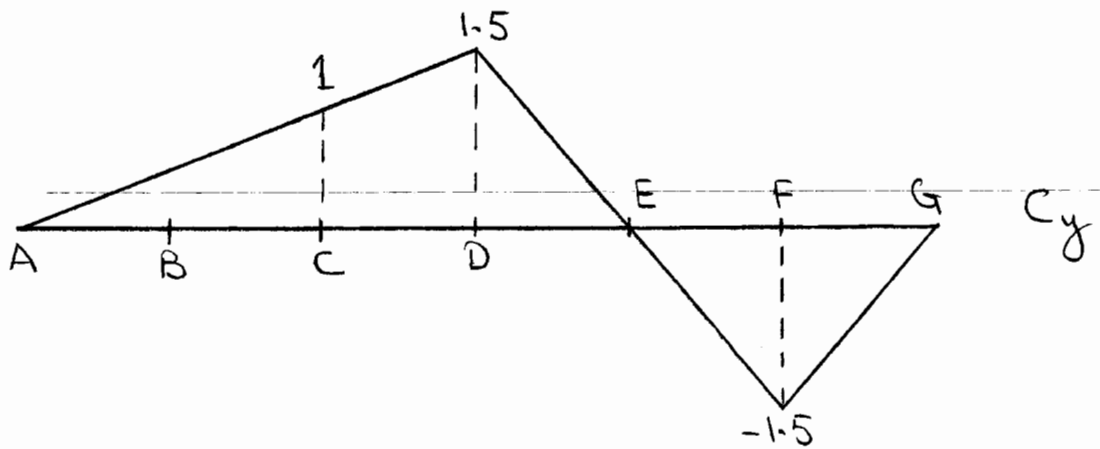
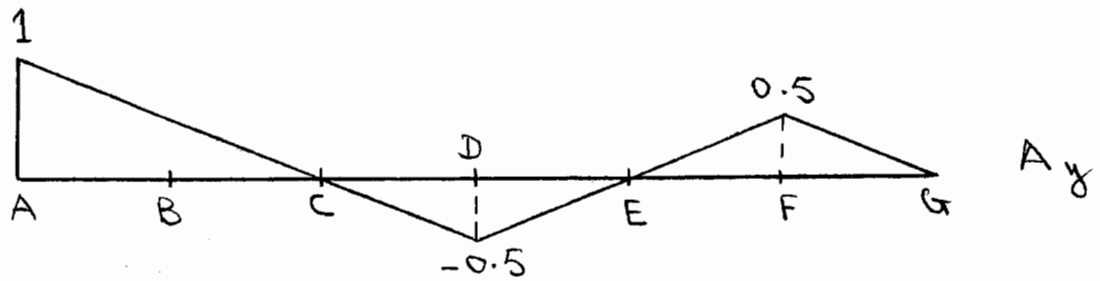
8.22



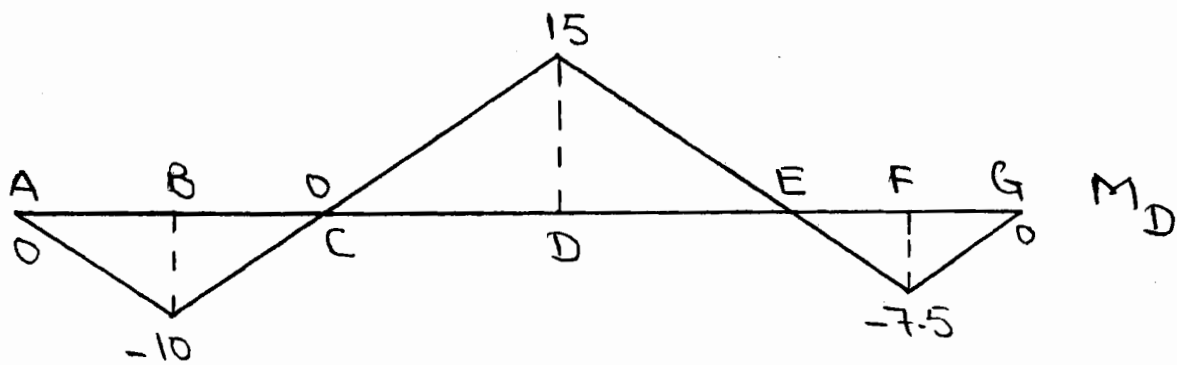
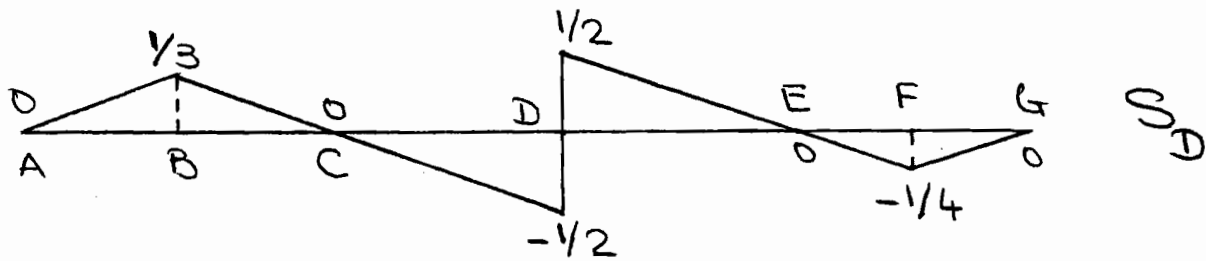
8.23



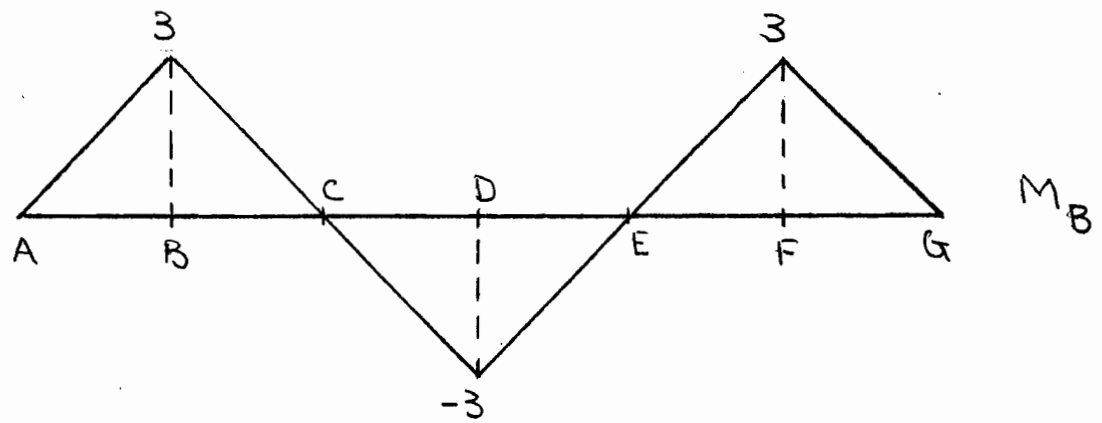
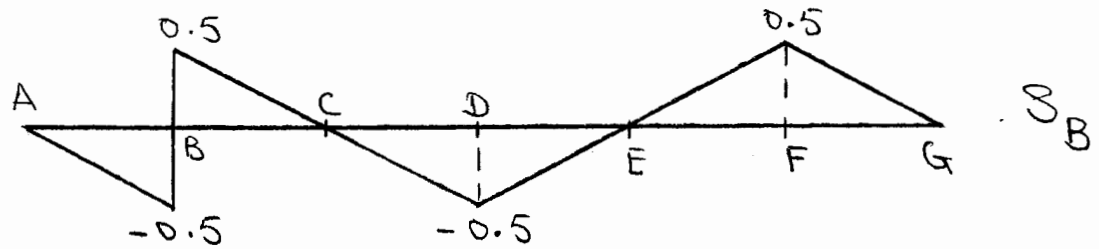
8.24



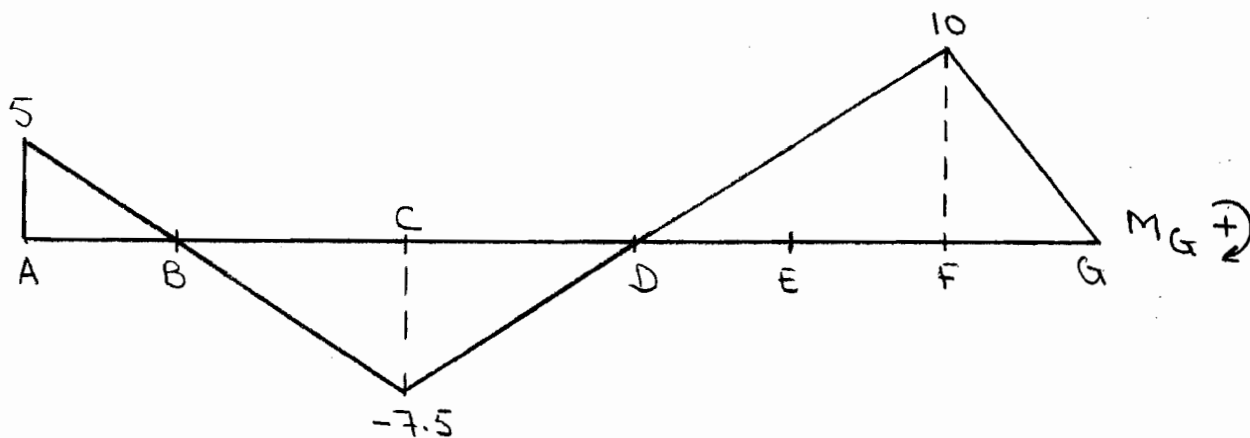
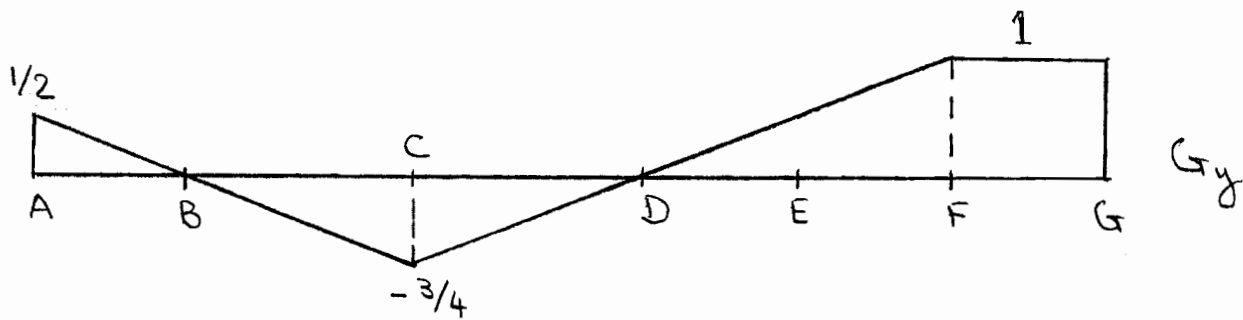
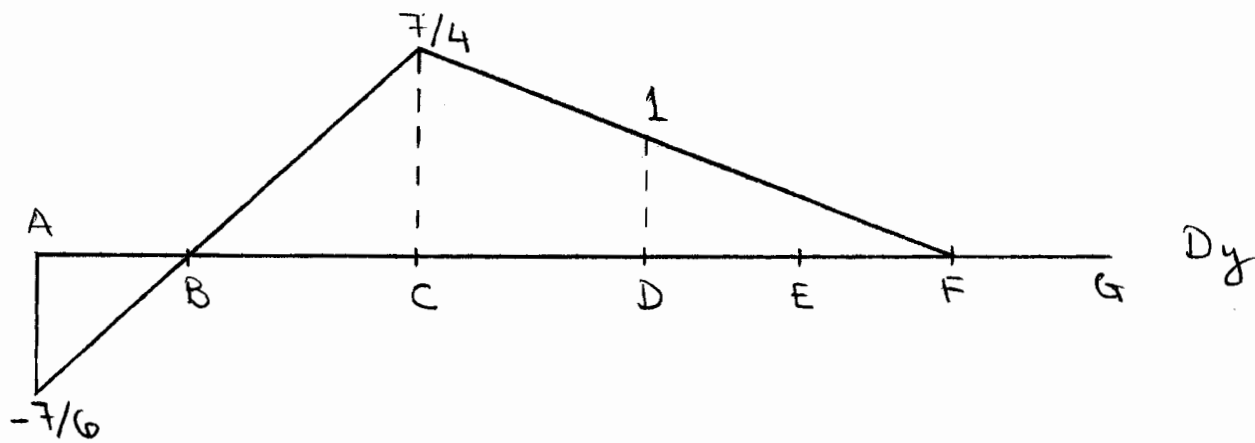
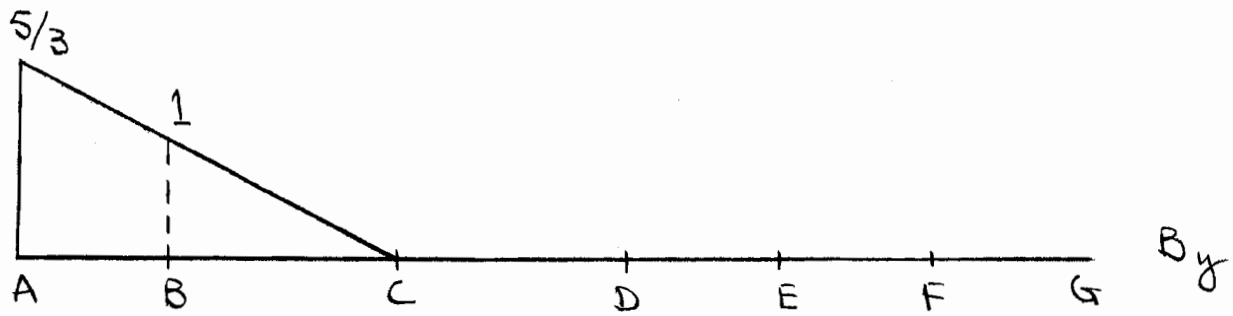
8.25



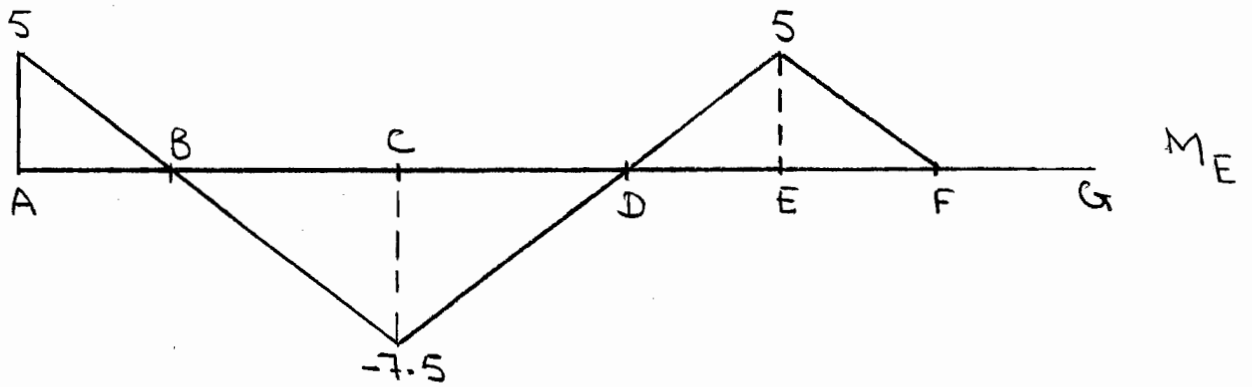
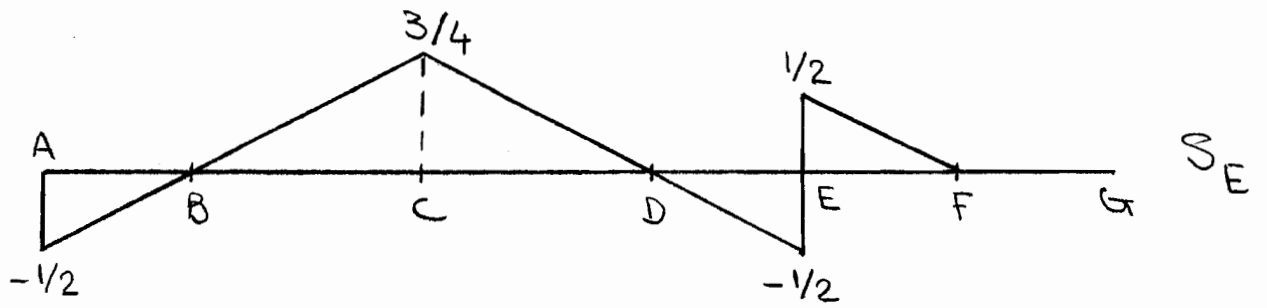
8.26



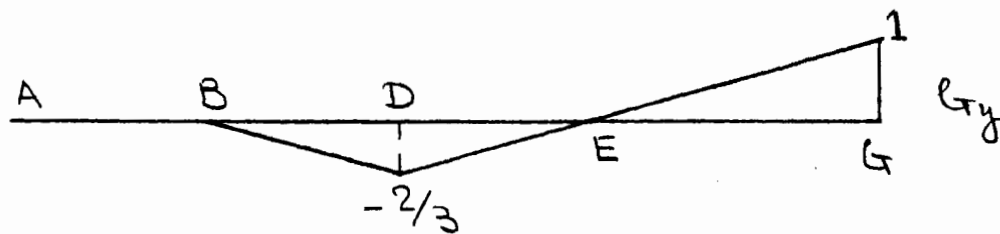
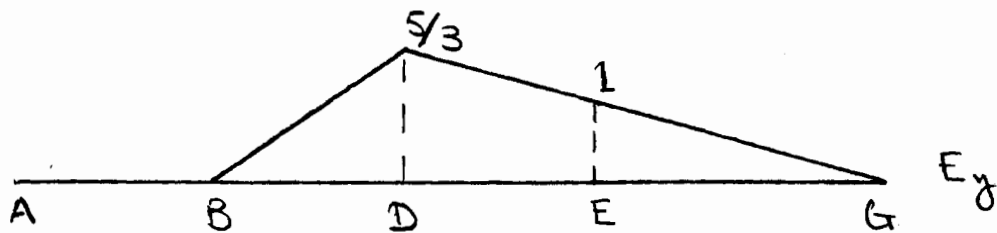
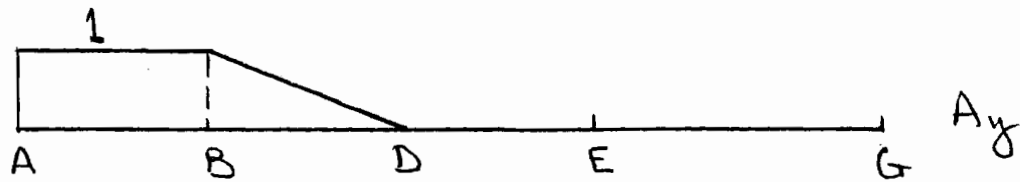
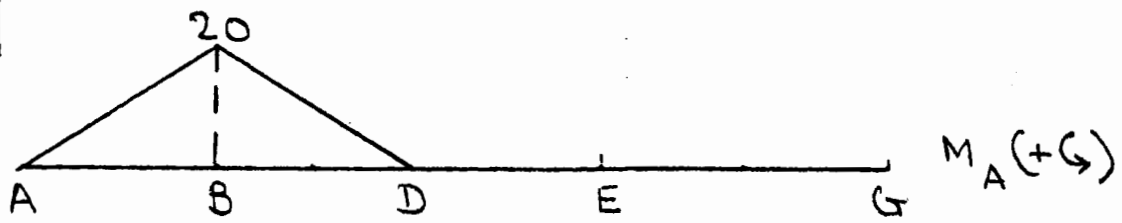
8.27



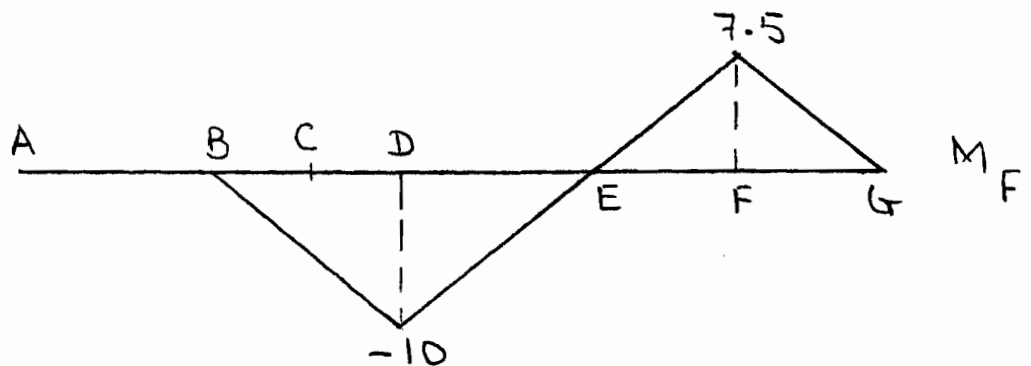
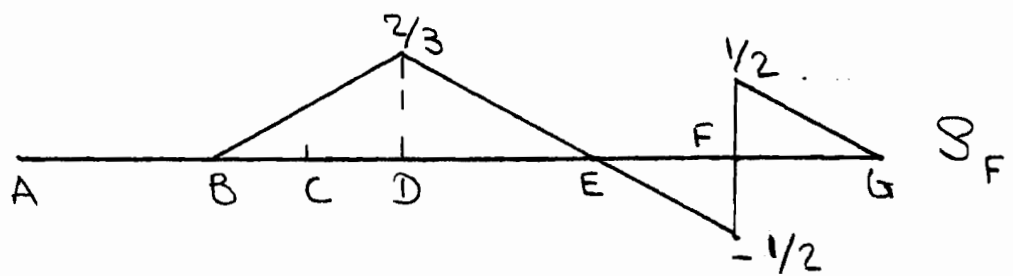
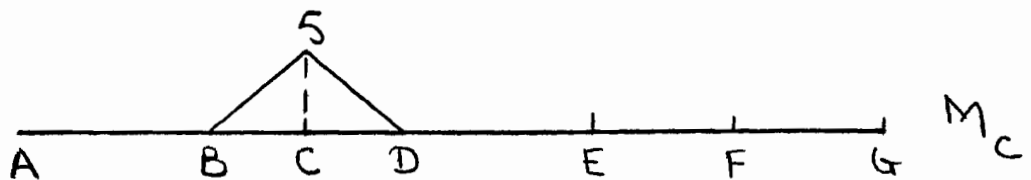
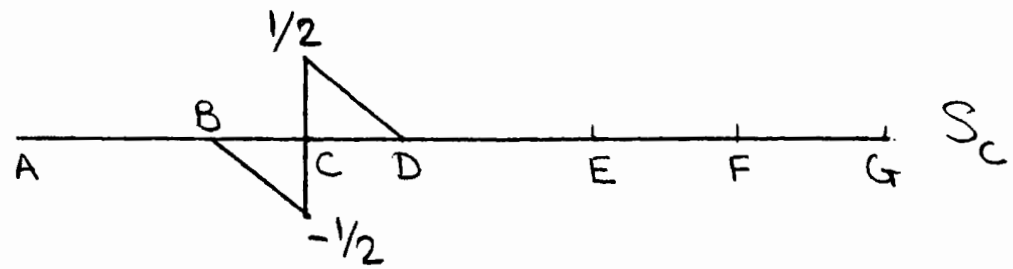
8.28



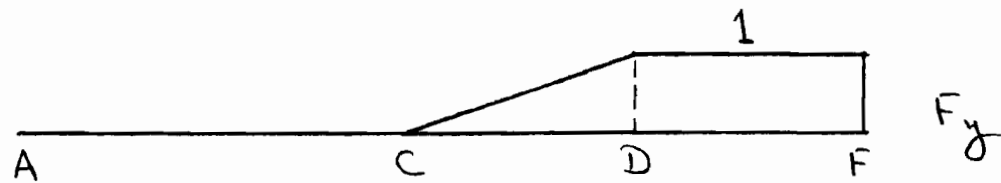
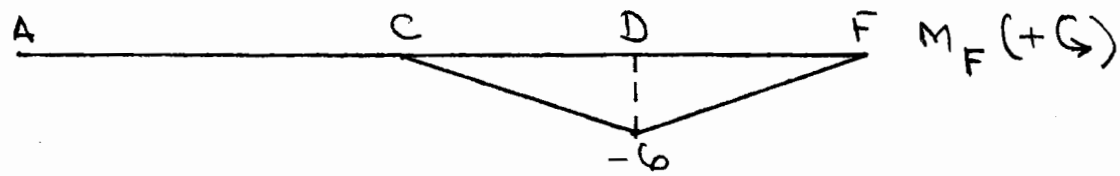
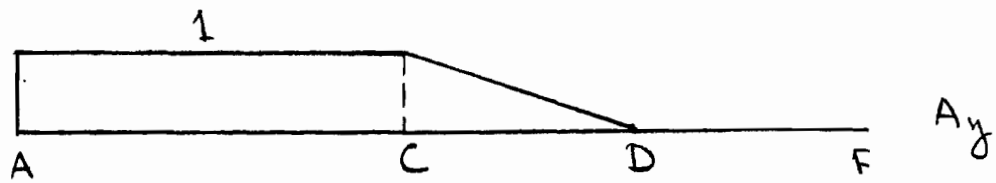
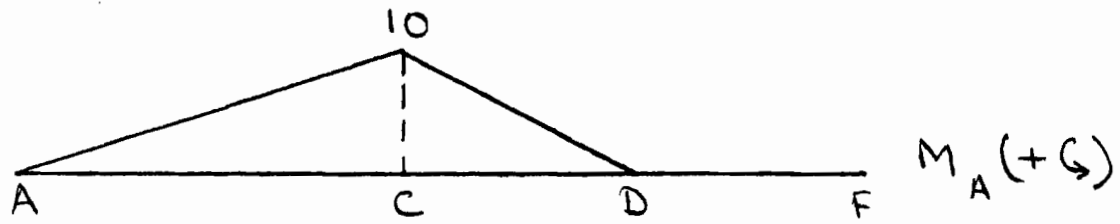
8.29



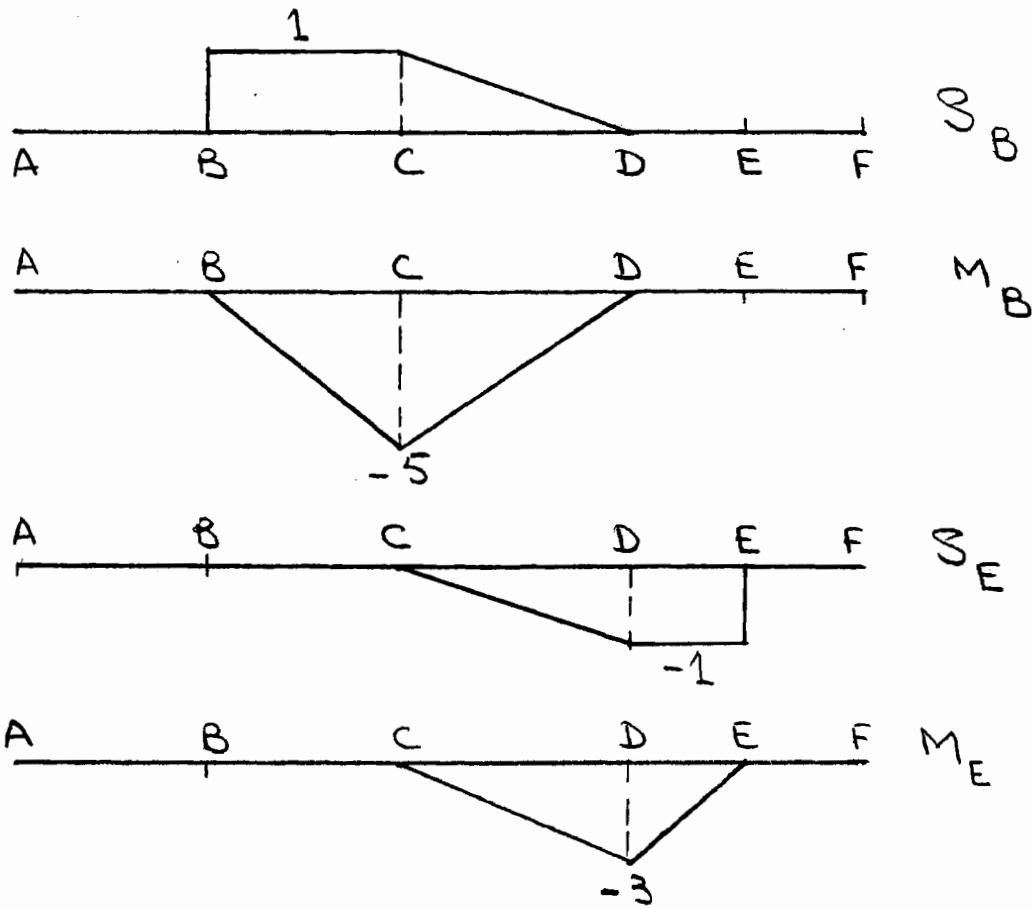
8.30



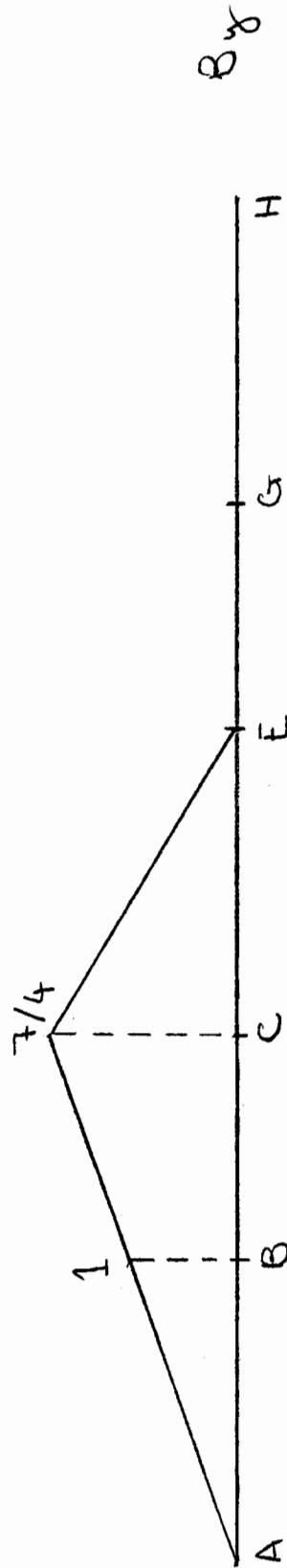
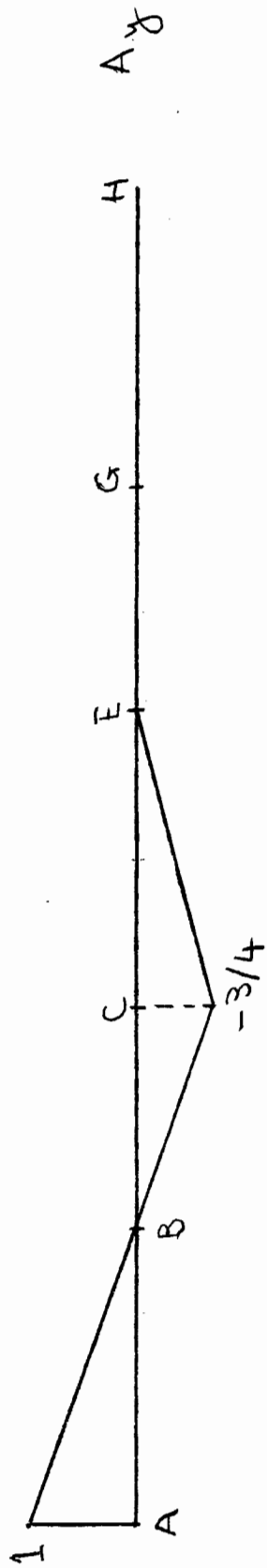
8.31



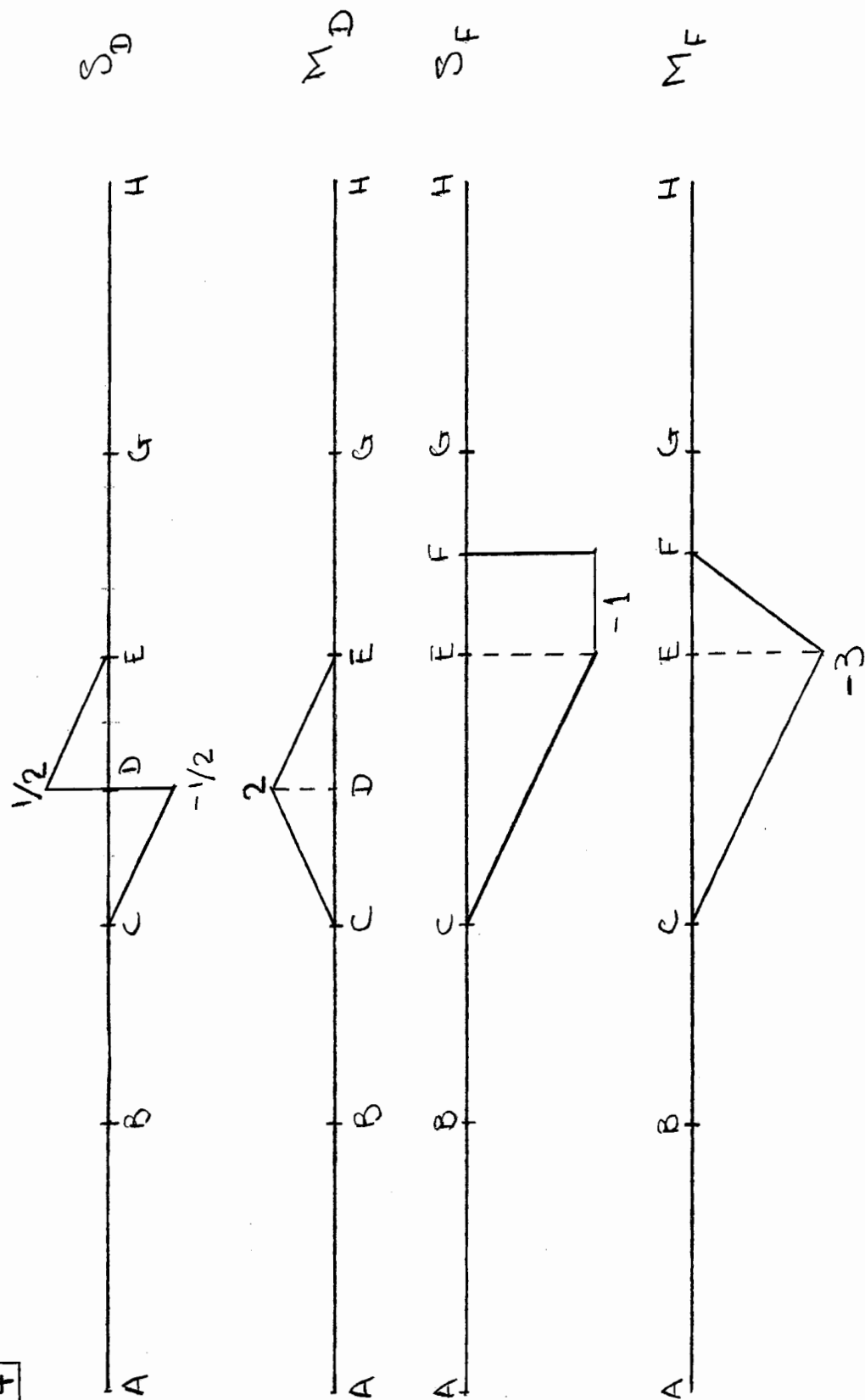
8.32



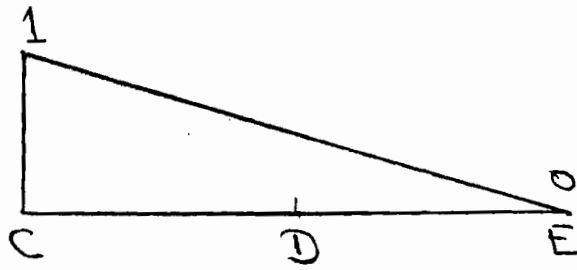
8.33



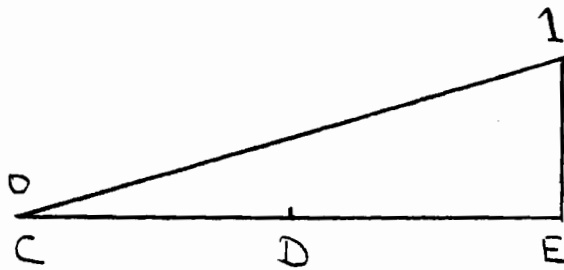
8.34



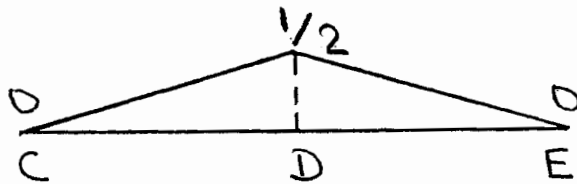
8.35



A_y

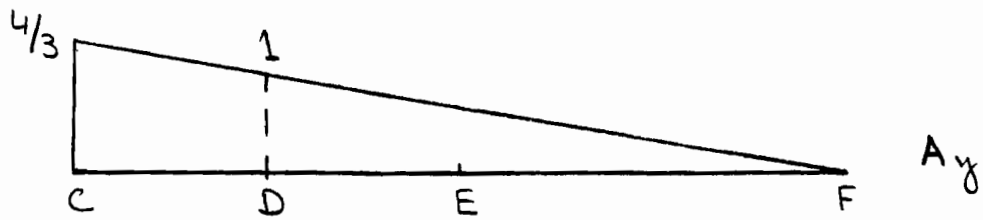


B_y

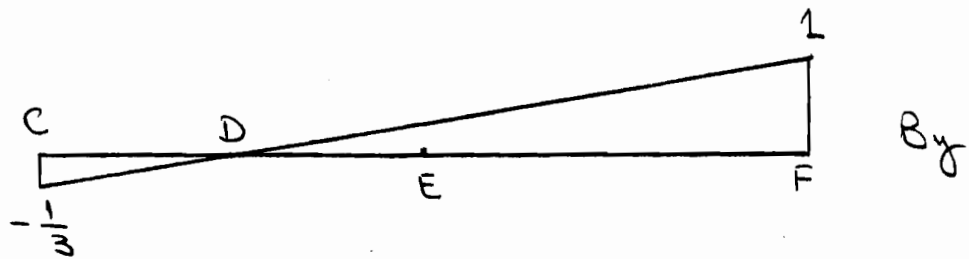


A_x (\rightarrow)
and
 B_x (\leftarrow)

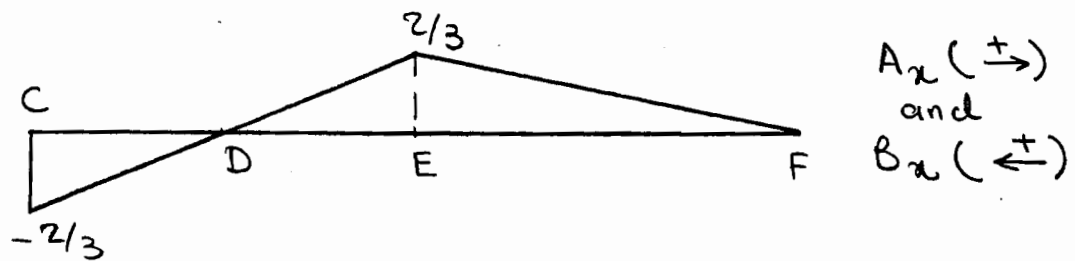
8.36



A_y



B_y

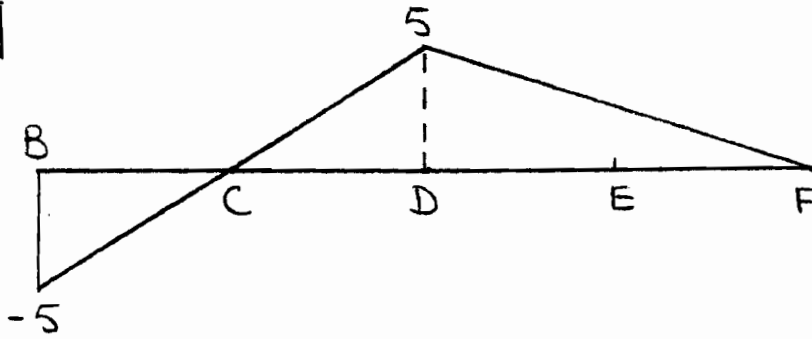


$A_x (\rightarrow)$

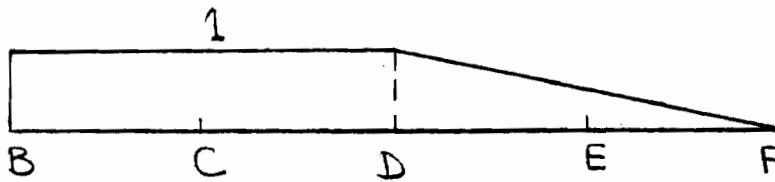
and

$B_x (\leftarrow)$

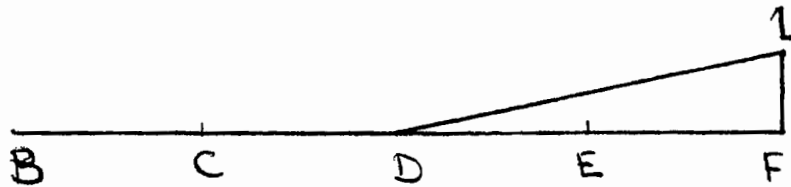
8.37



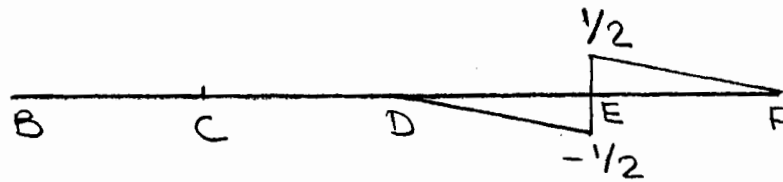
$M_A (+ \curvearrowright)$



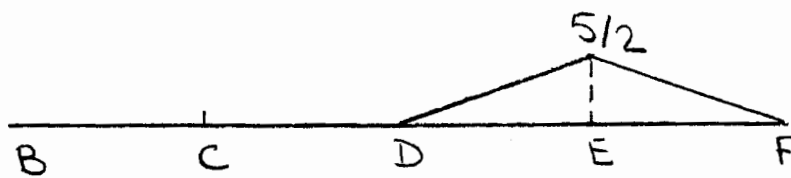
A_y



F_y

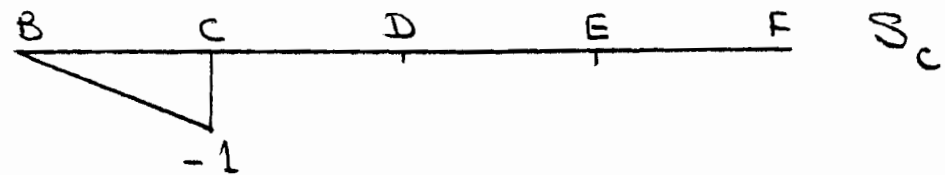
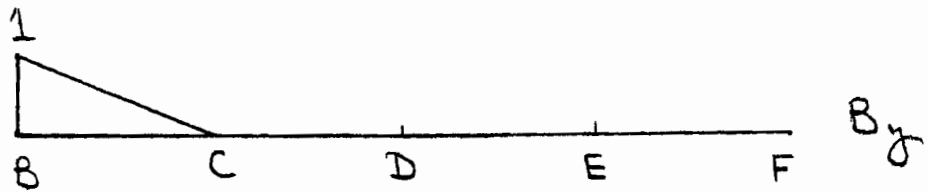
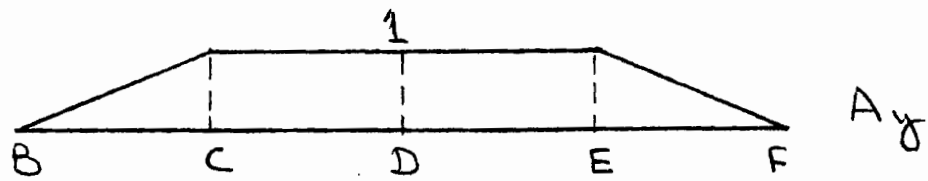
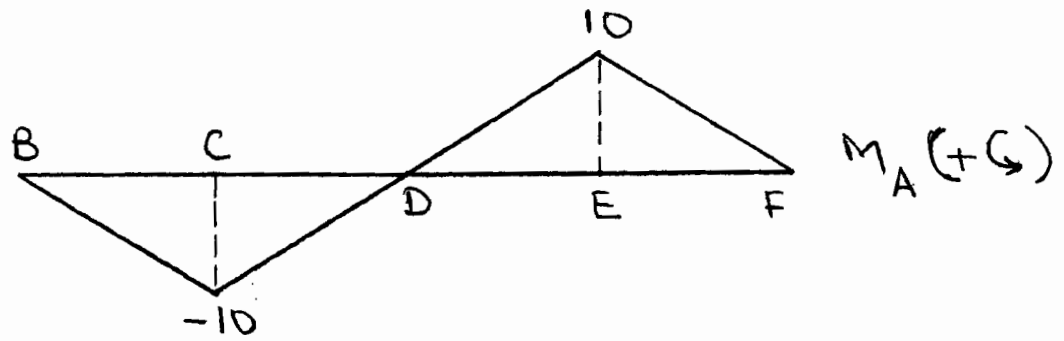


S_E

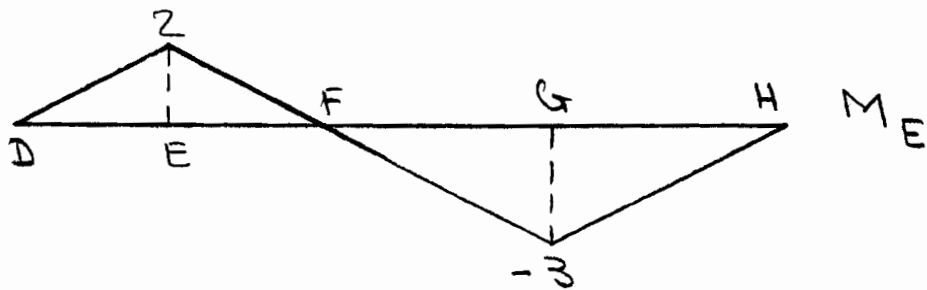
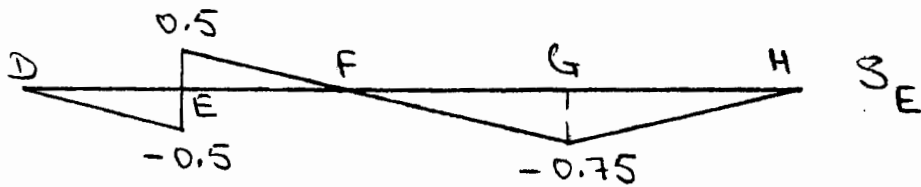
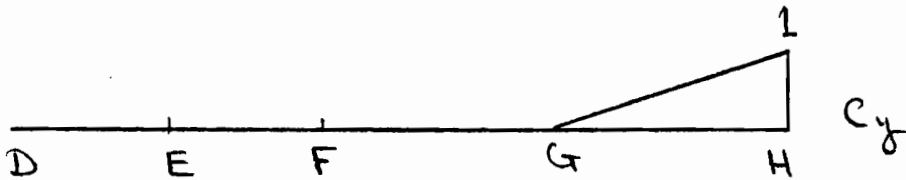
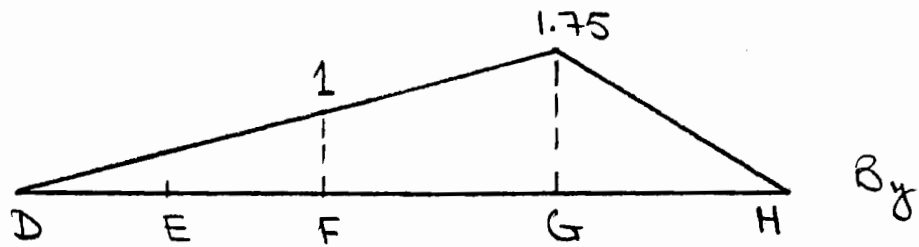
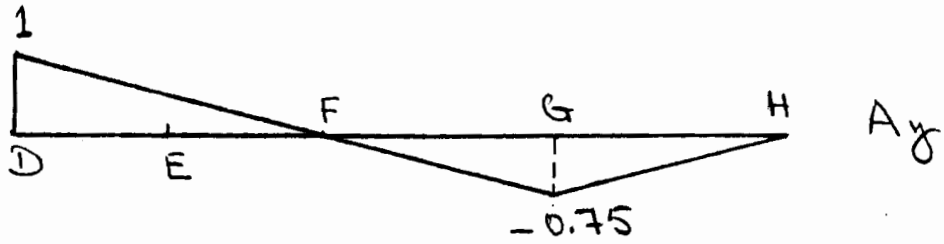


M_E

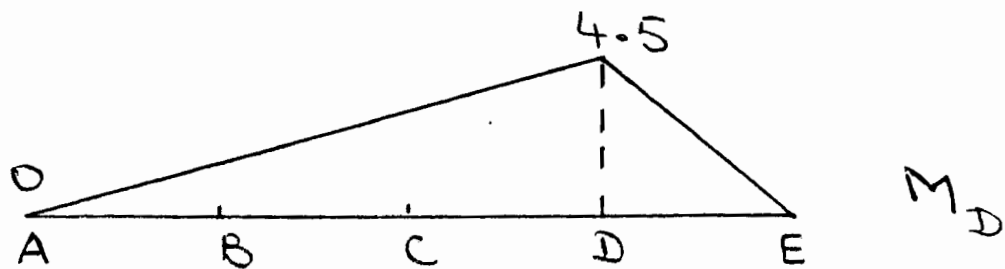
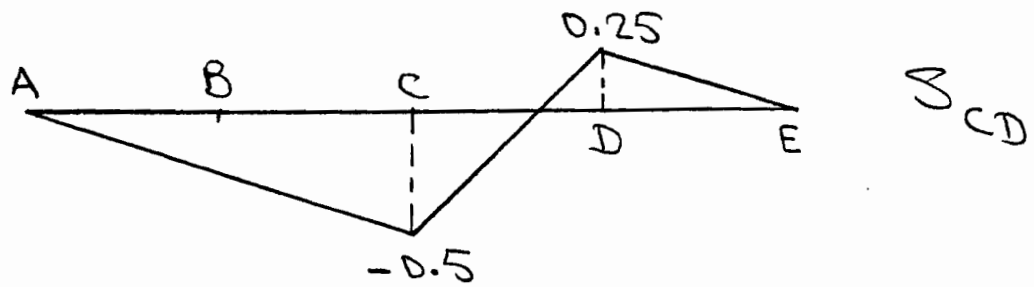
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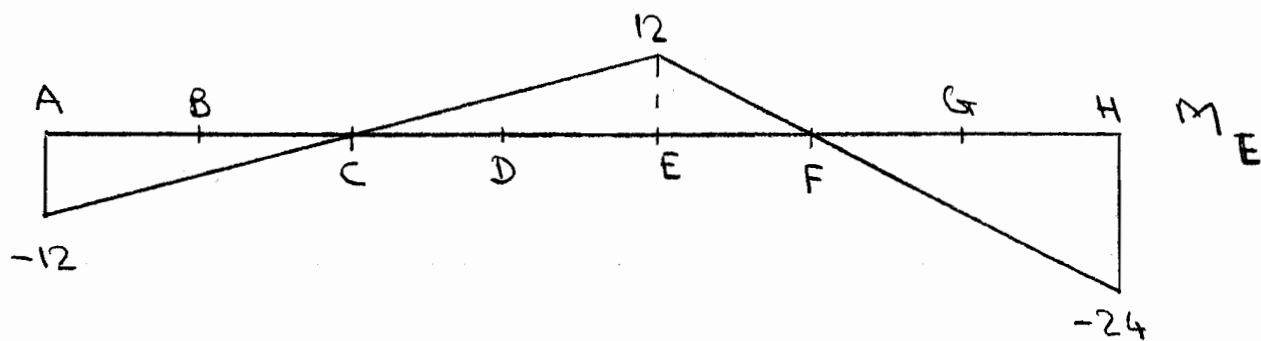
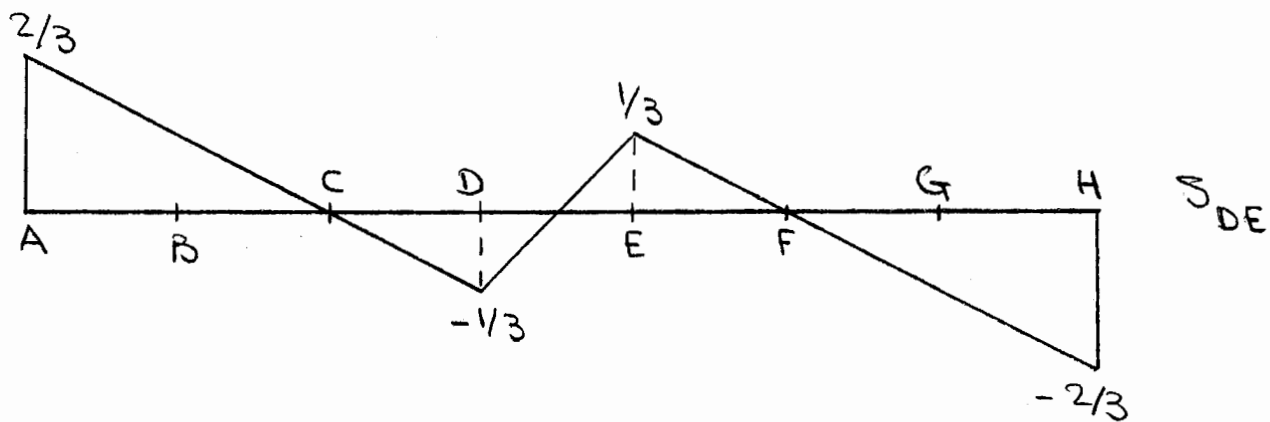
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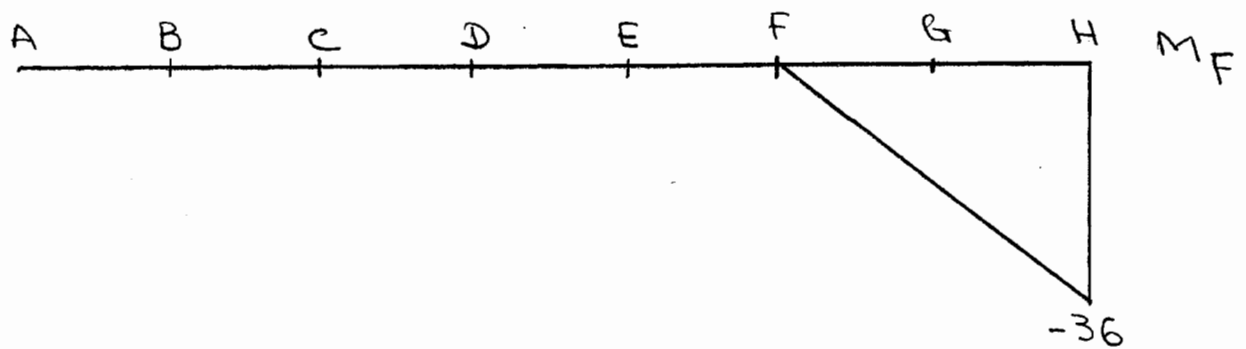
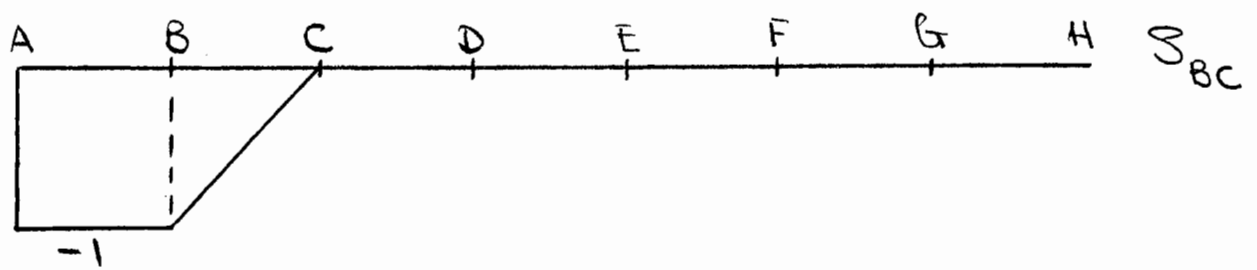
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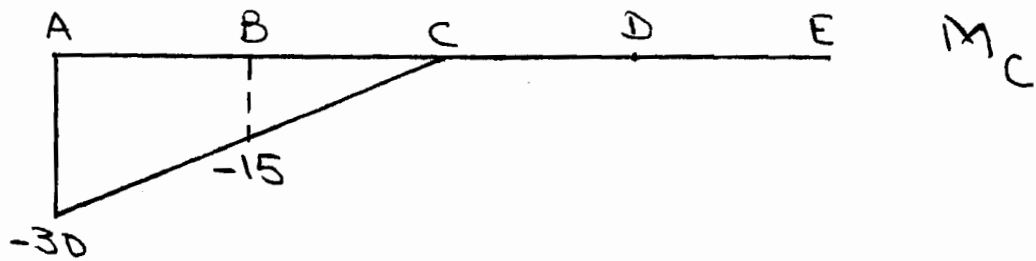
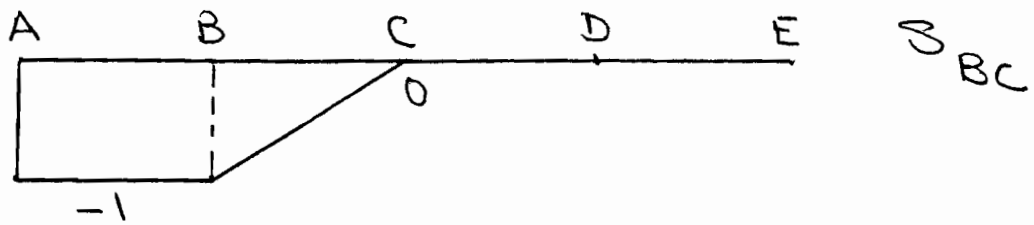
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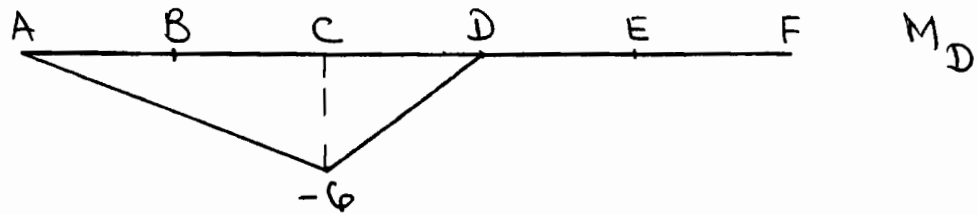
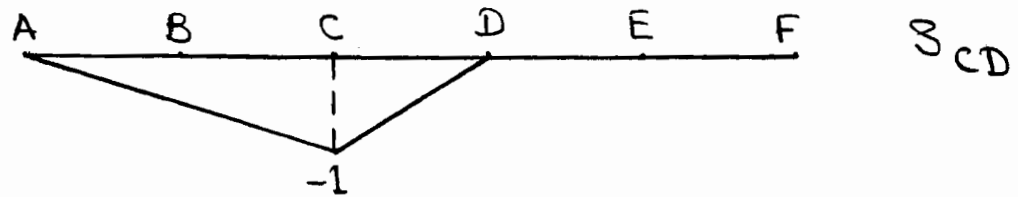
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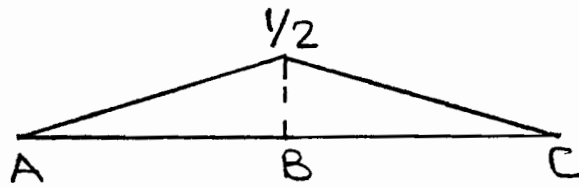
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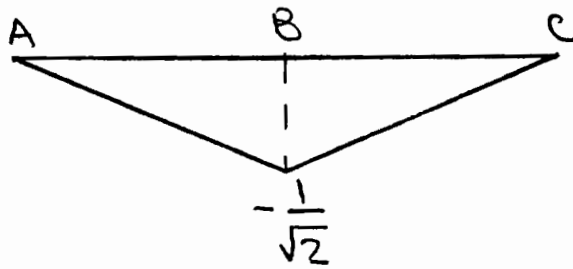
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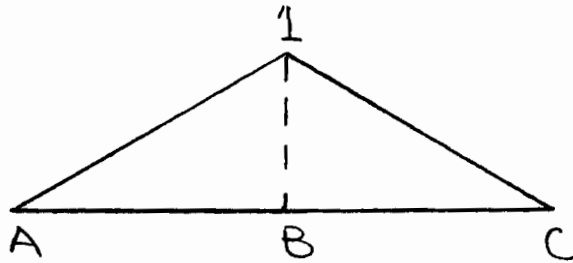
8.45



F_{AB}

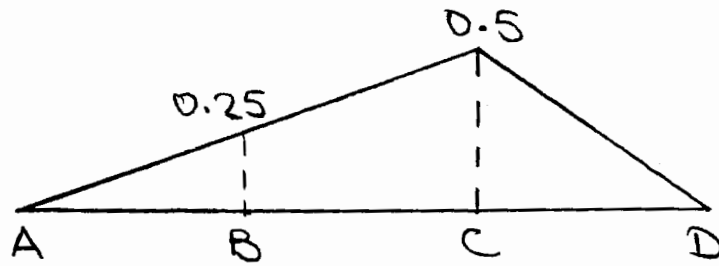


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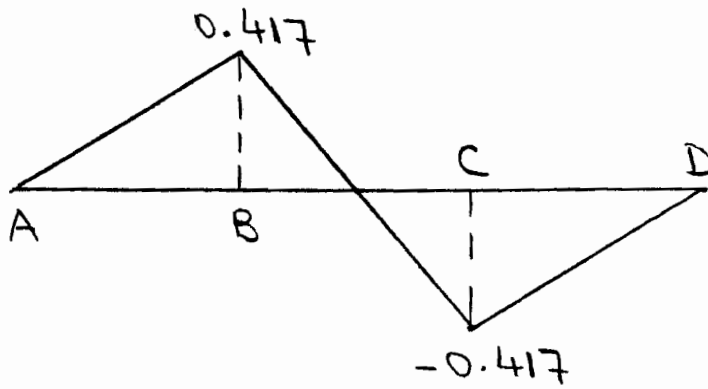


F_{BD}

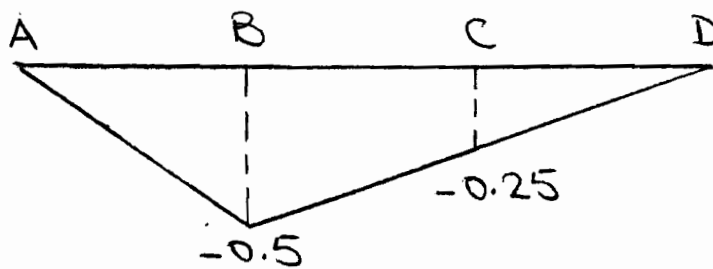
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F_{BC}

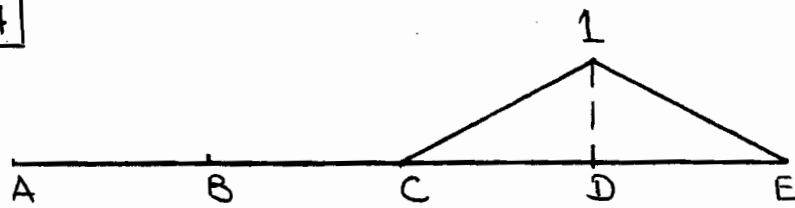


F_{BF}

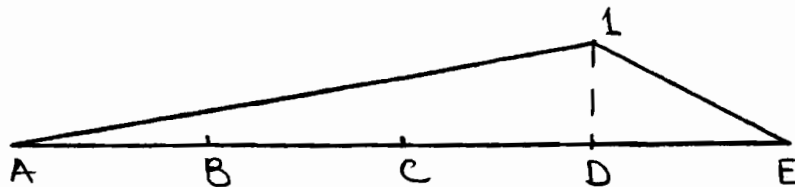


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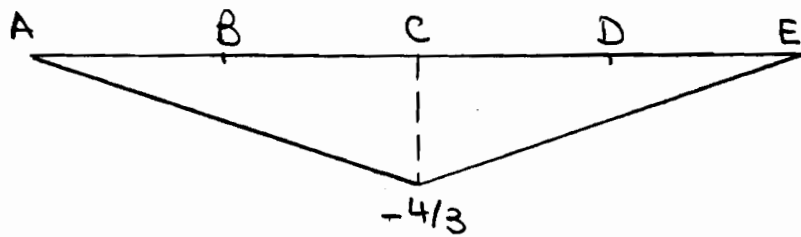
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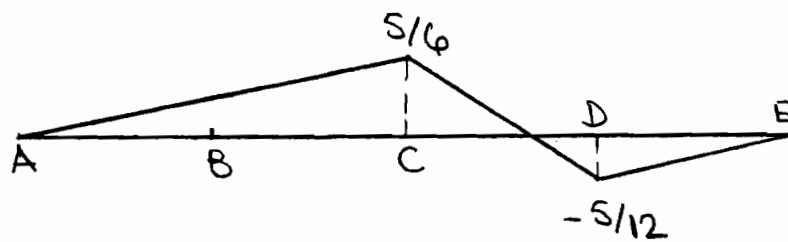
F_{DH}



F_{CD}

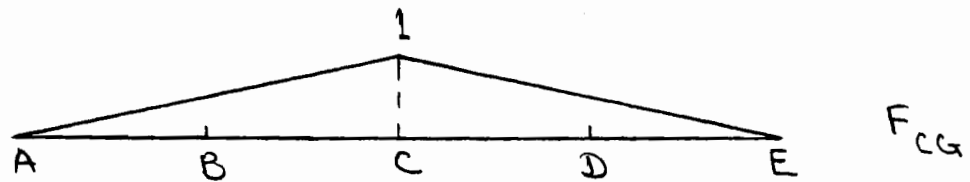
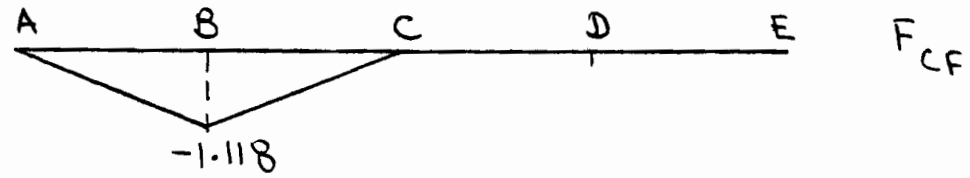
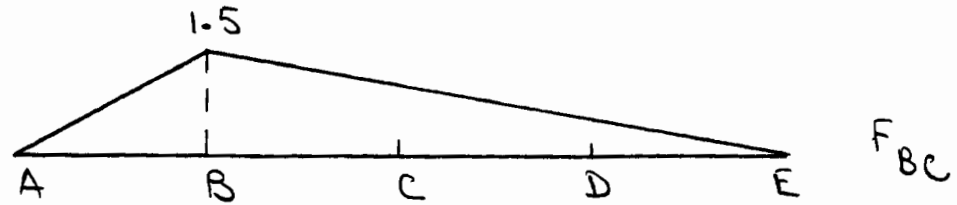
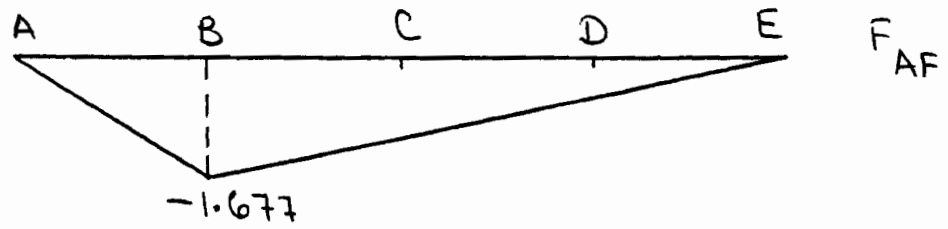


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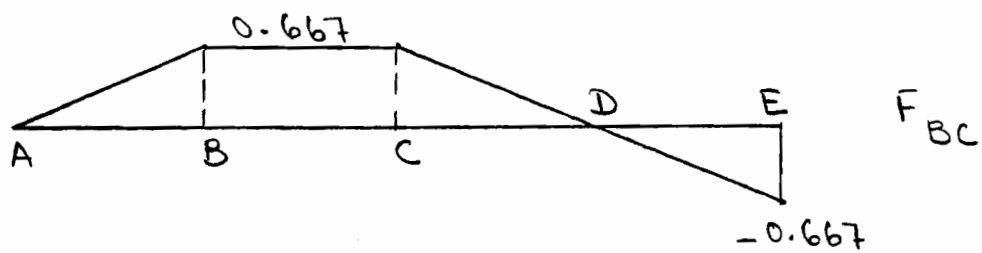
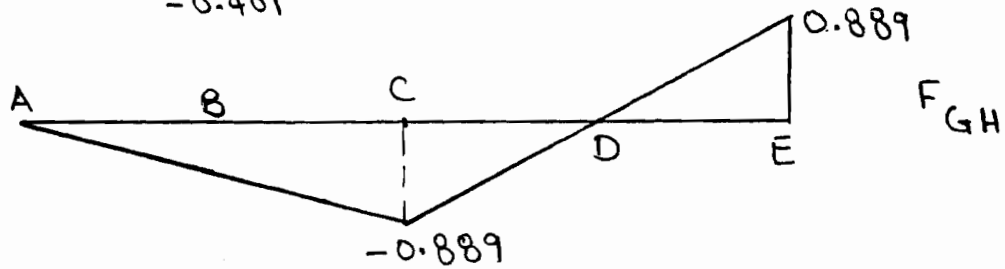
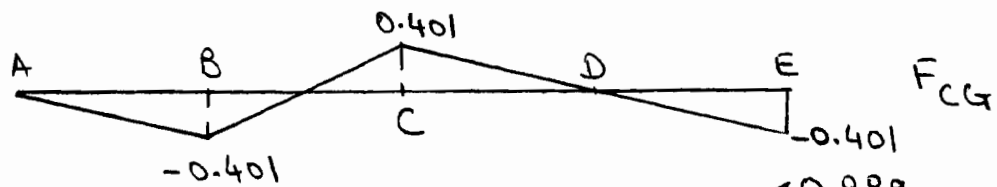
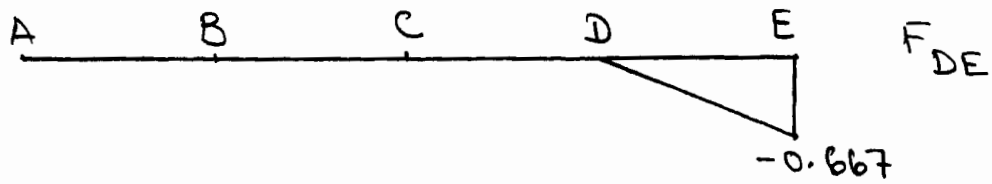


F_{CH}

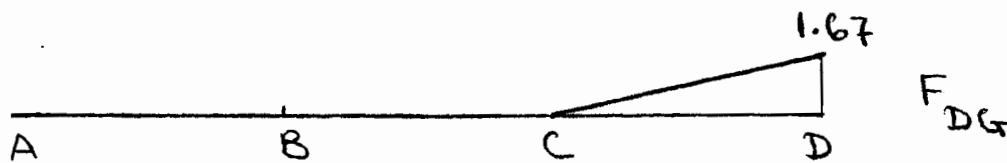
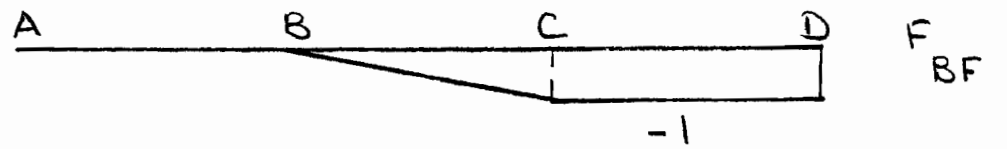
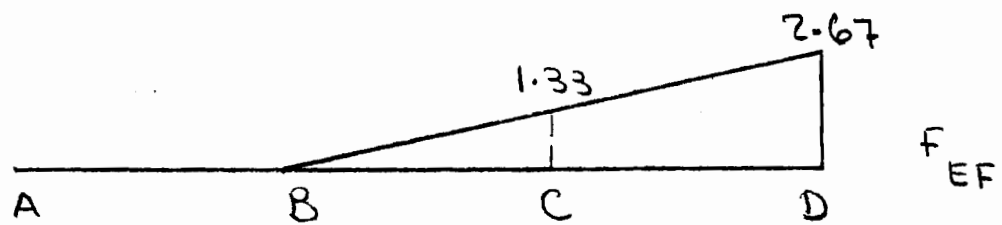
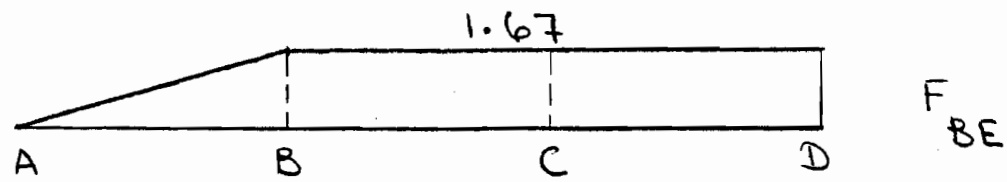
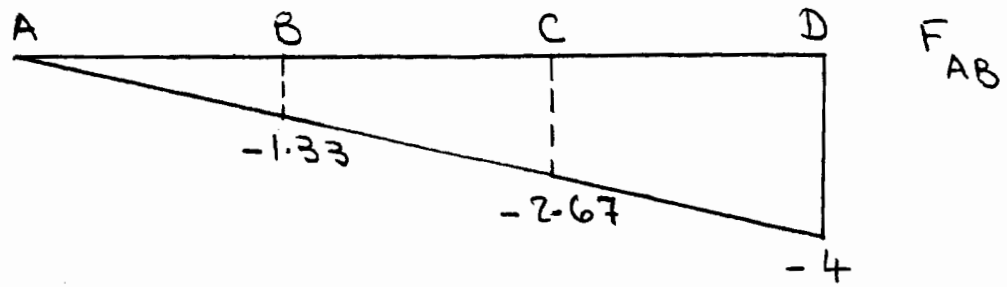
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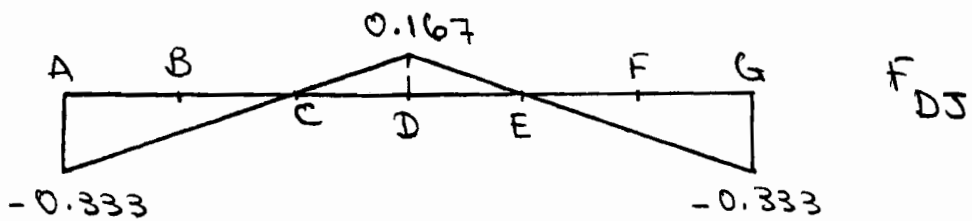
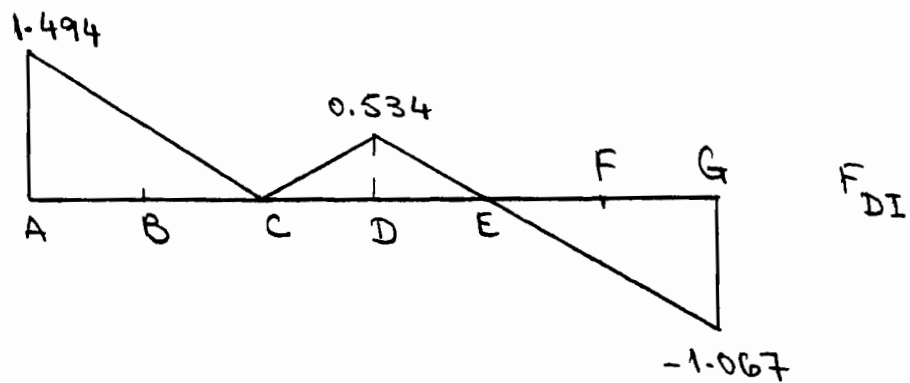
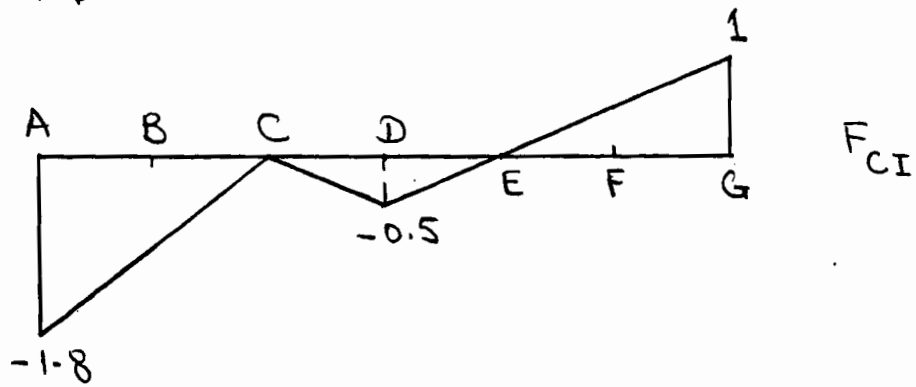
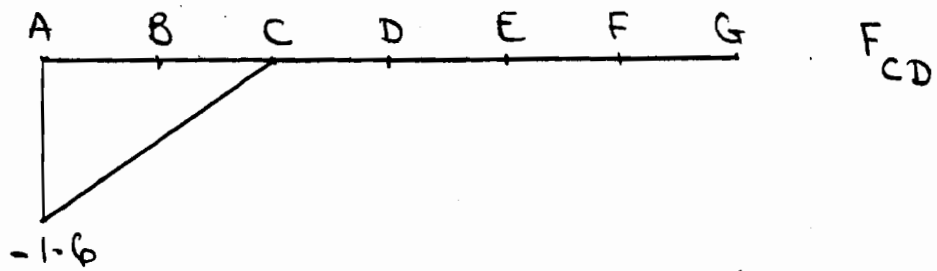
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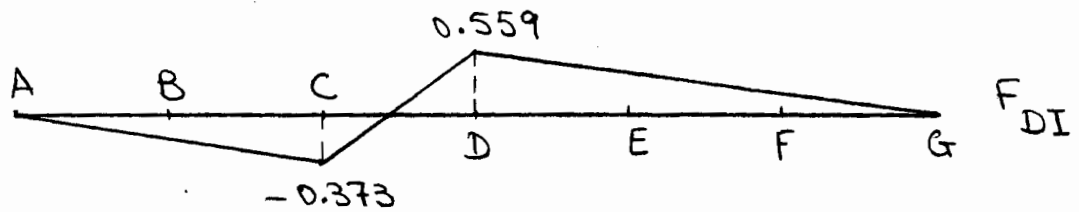
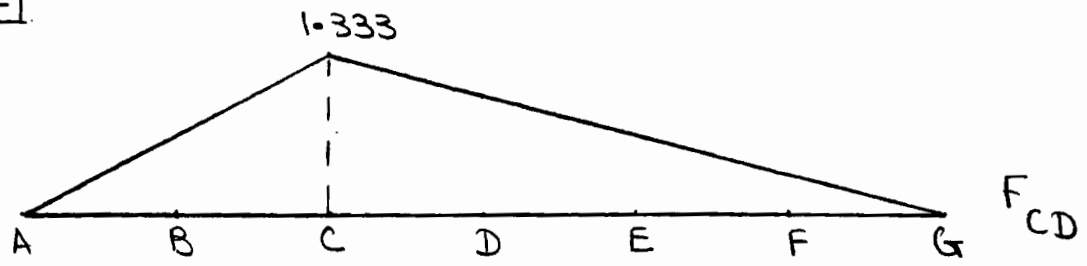
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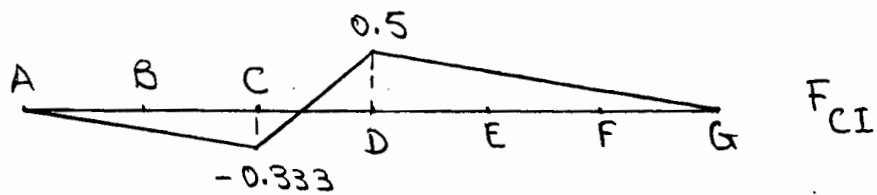
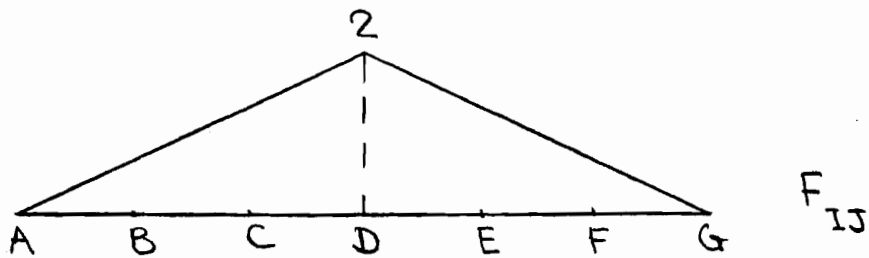
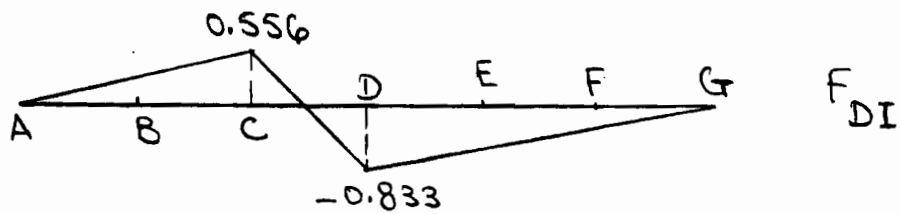
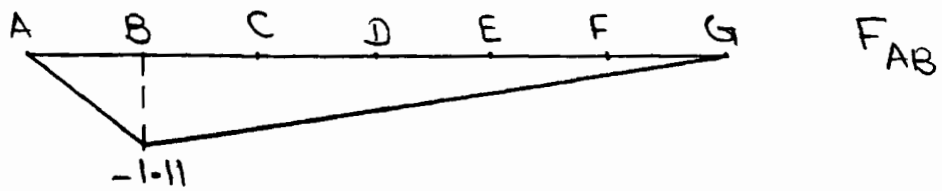
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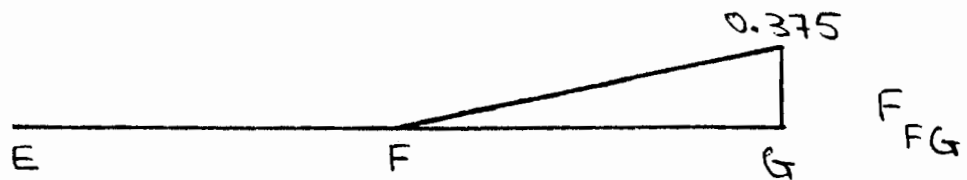
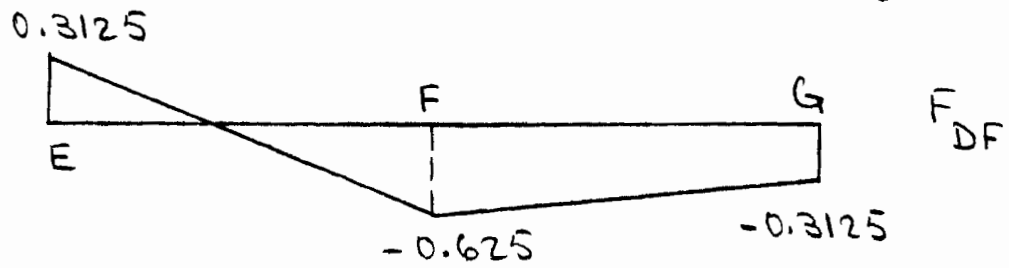
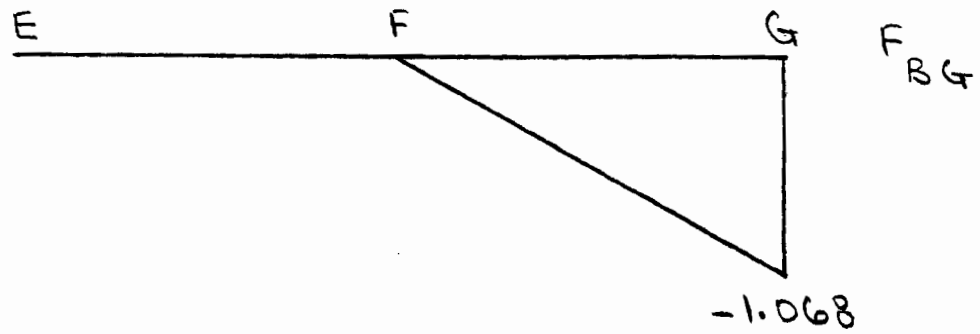
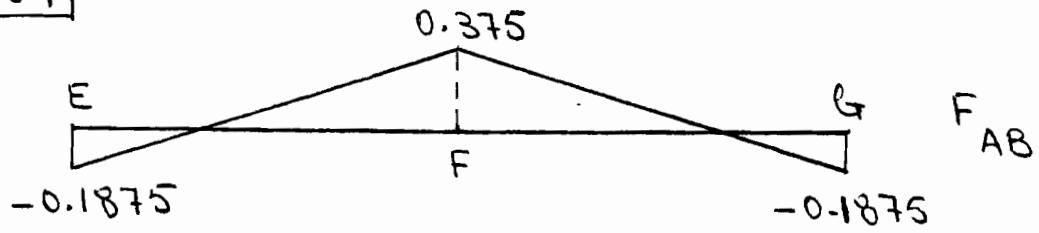
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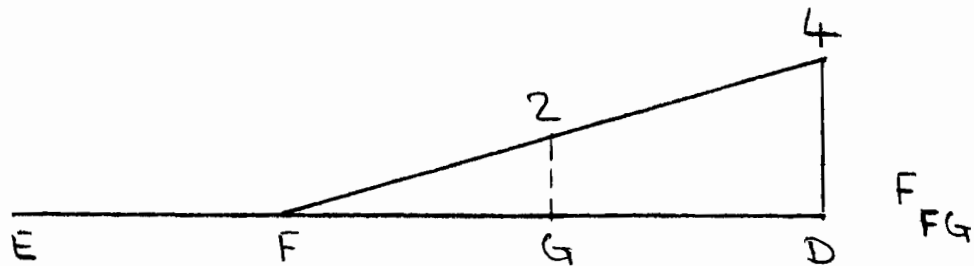
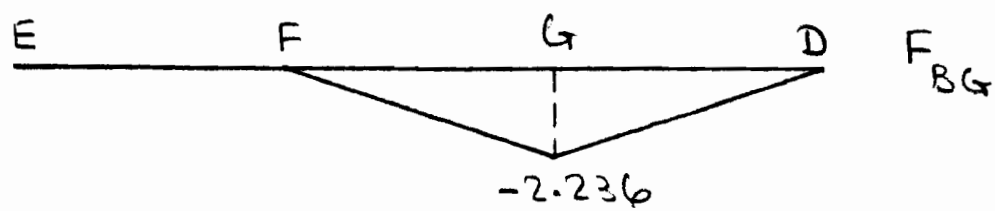
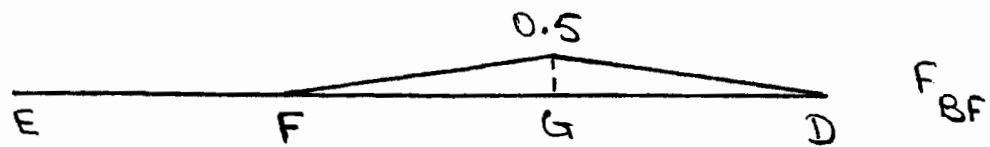
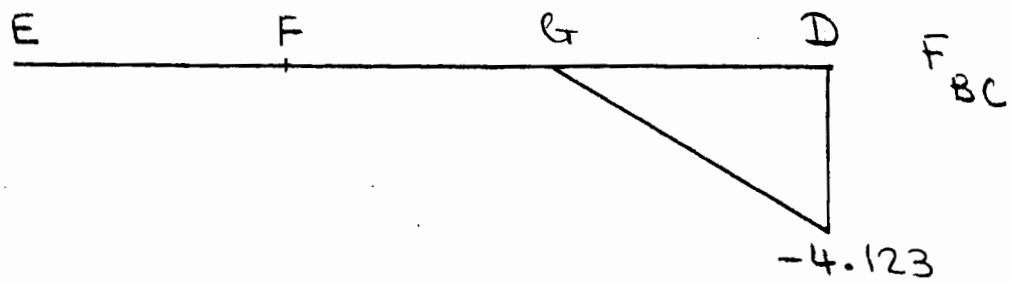
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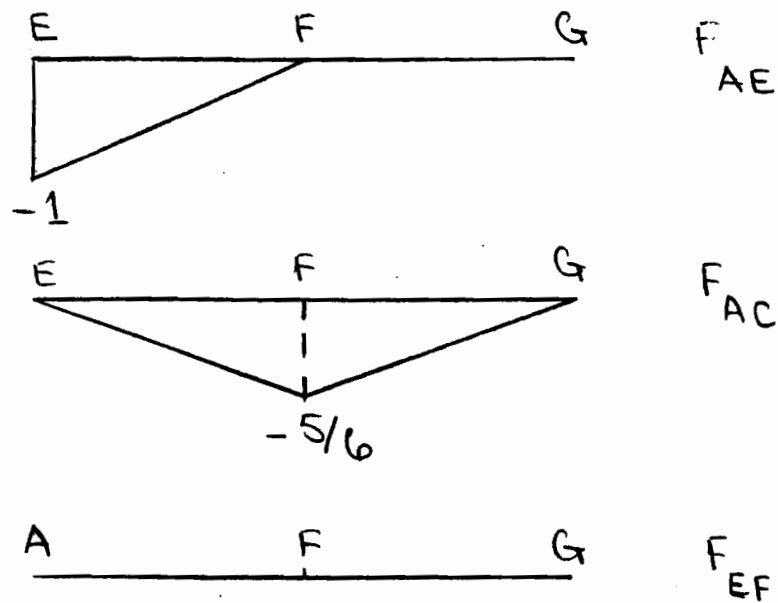
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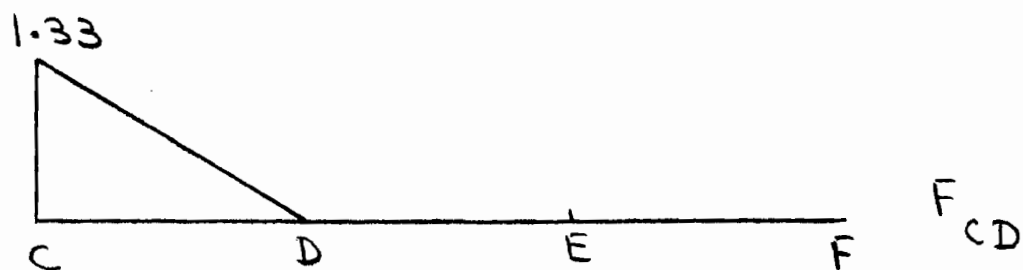
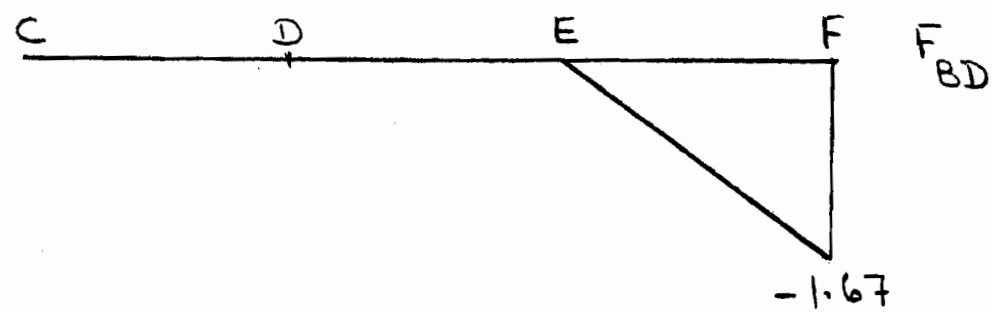
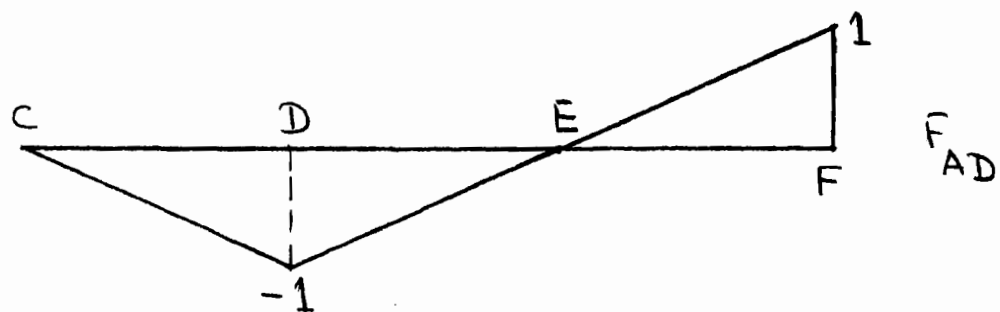
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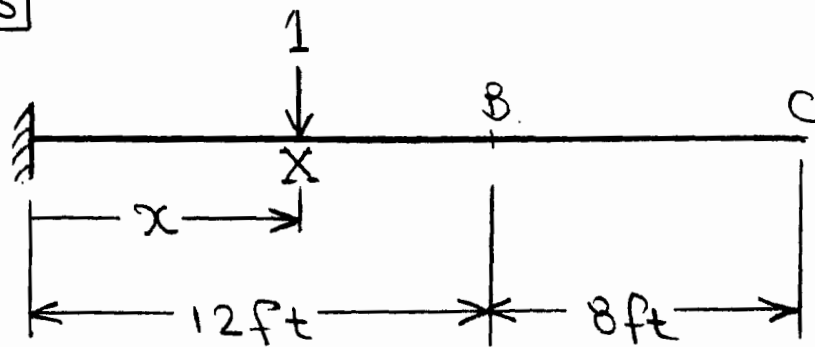
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8.57

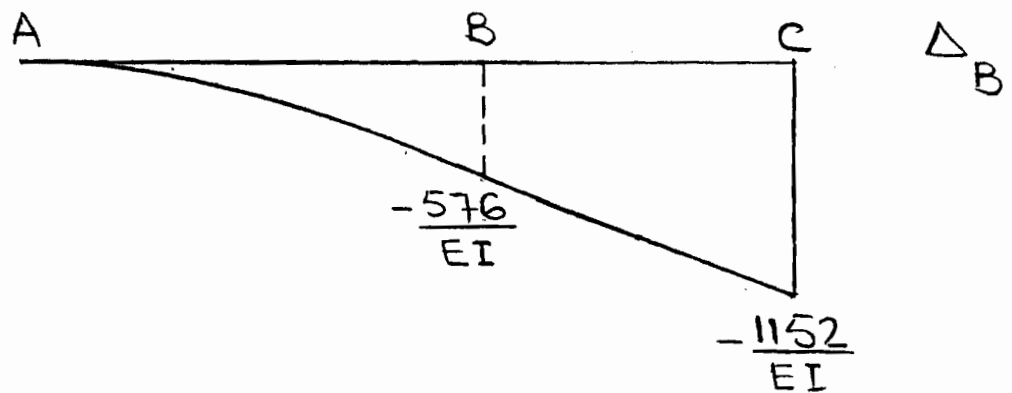


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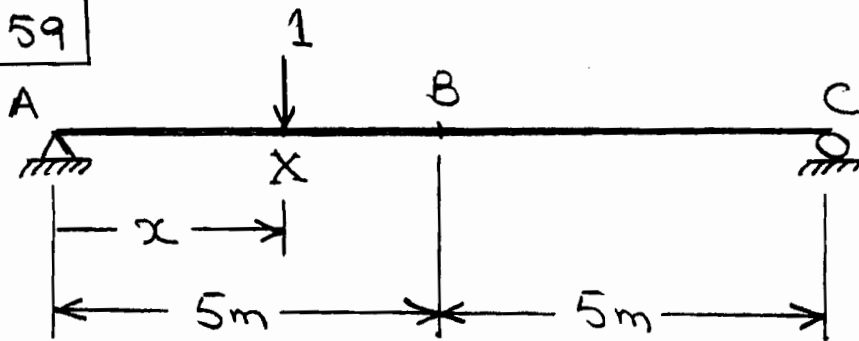


$$0 \leq x \leq 12': \quad f_{BX} = \frac{1}{6EI} (x^3 - 36x^2)$$

$$12' \leq x \leq 20': \quad f_{BX} = \frac{72}{EI} (4 - x)$$

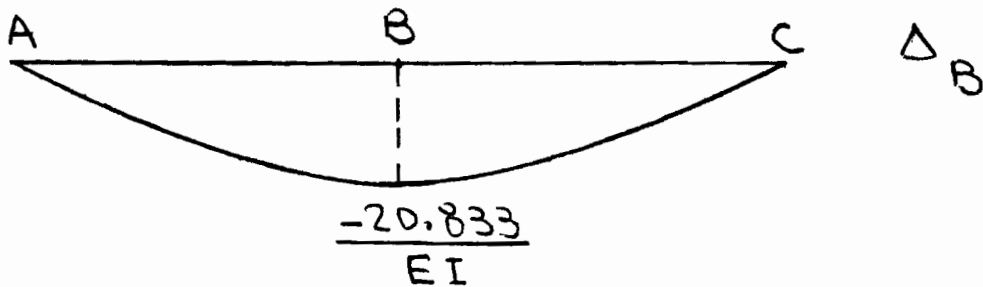


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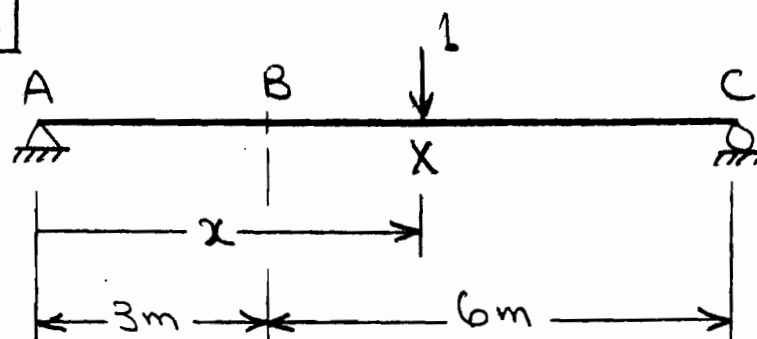


$$0 \leq x \leq 5\text{m}: f_{BX} = \frac{1}{12EI} (x^3 - 75x)$$

$$5\text{m} \leq x \leq 10\text{m}: f_{BX} = \frac{1}{12EI} [(10-x)^3 - 75(10-x)]$$

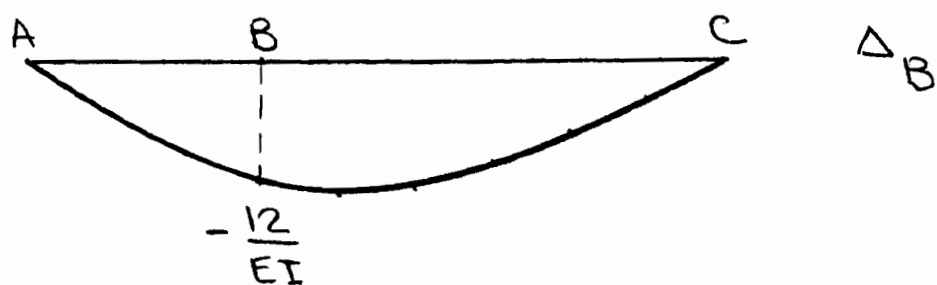


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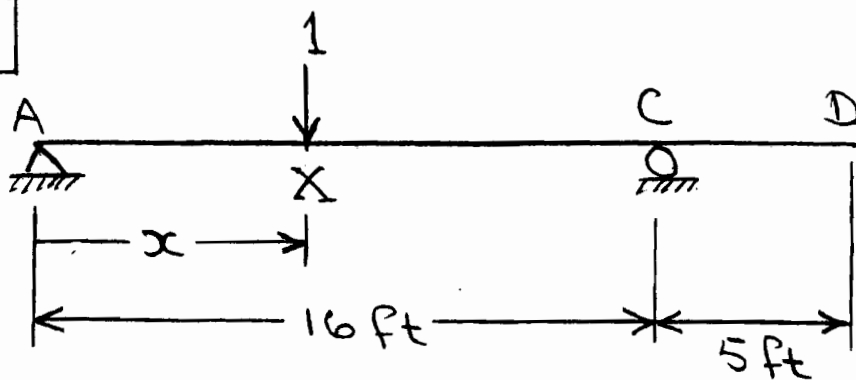


$$0 \leq x \leq 3m \quad f_{BX} = \frac{1}{9EI} (x^3 - 45x)$$

$$3m \leq x \leq 9m \quad f_{BX} = \frac{1}{18EI} (-x^3 + 27x^2 - 171x + 81)$$

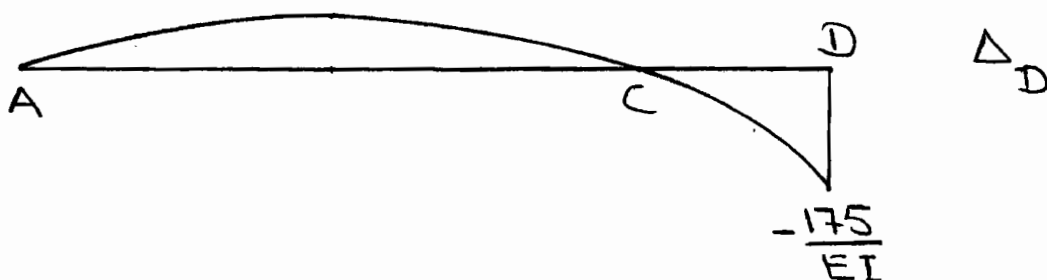


8.61



$$0 \leq x \leq 16': \quad f_{DX} = \frac{5x}{96EI} (256 - x^2)$$

$$16' \leq x \leq 21': \quad f_{DX} = \frac{1}{6EI} (-5376 + 1088x - 63x^2 + x^3)$$

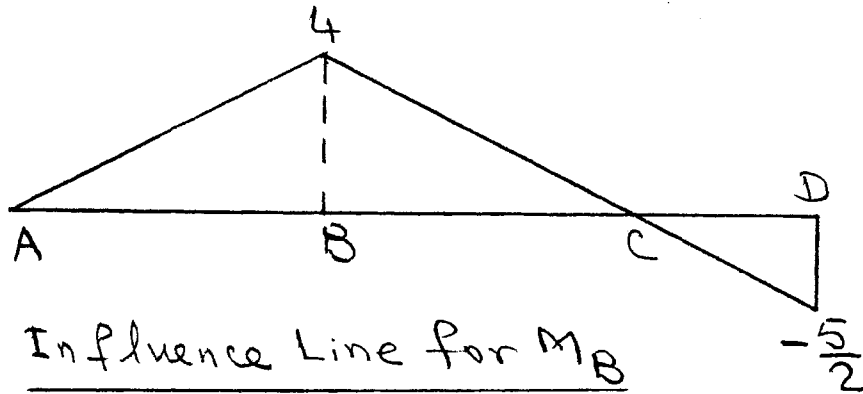


Chapter Nine

Application of Influence Lines

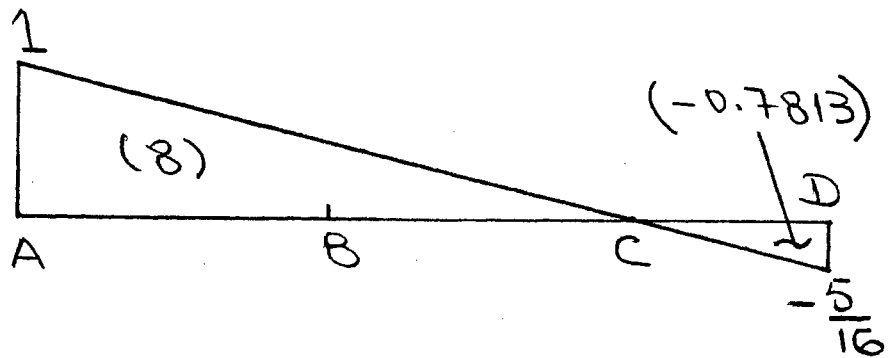
CHAPTER 9

9.1



$$\begin{aligned}\text{Maximum Negative } M_B &= 15\left(-\frac{5}{2}\right) \\ &= \underline{-37.5 \text{ k-ft}}\end{aligned}$$

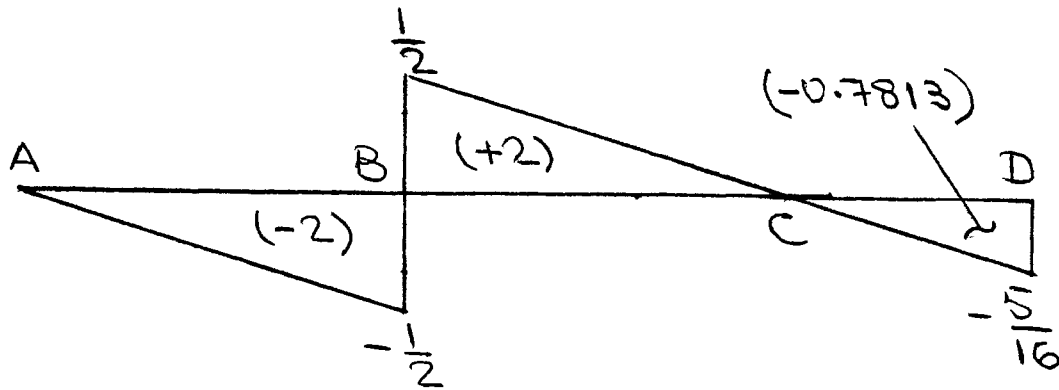
9.2



Influence Line for A_y

Maximum Upward $A_y = 3(8) = \underline{24 \text{ k} \uparrow}$

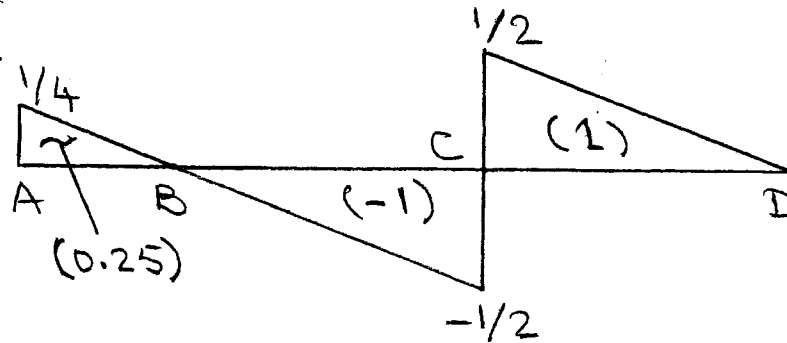
9.3



Influence Line for δ_B

$$\begin{aligned} \text{Maximum Negative } \delta_B &= 3(-2 - 0.7813) \\ &= \underline{-8.344 \text{ k}} \end{aligned}$$

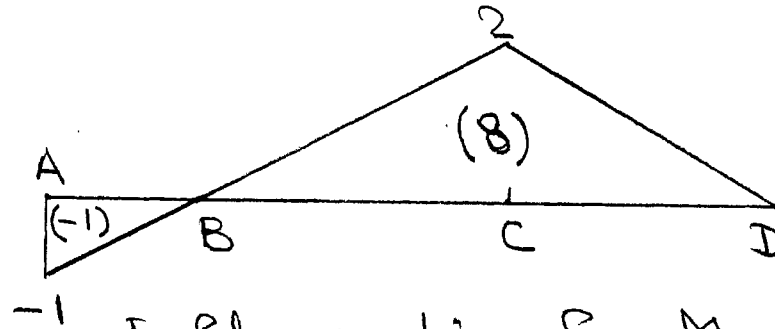
9.4



Influence Line for δ_c

$$\begin{aligned} \text{Maximum Positive } \delta_c &= 100 \left(\frac{1}{2} \right) + 50 (0.25 + 1) \\ &\quad + 20 (0.25 - 1 + 1) \\ &= \underline{117.5 \text{ kN}} \end{aligned}$$

$$\begin{aligned} \text{Maximum Negative } \delta_c &= 100 \left(-\frac{1}{2} \right) + 50 (-1) \\ &\quad + 20 (0.25 - 1 + 1) \\ &= \underline{-95 \text{ kN}} \end{aligned}$$

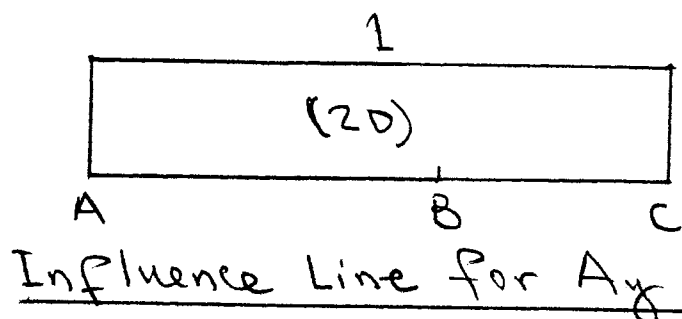


Influence Line for M_c

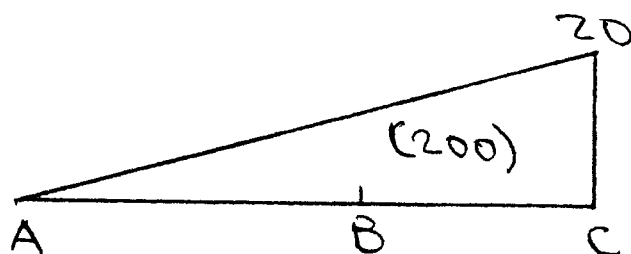
$$\begin{aligned} \text{Maximum Positive } M_c &= 100 (2) + 50 (8) + 20 (7) \\ &= \underline{740 \text{ kN}\cdot\text{m}} \end{aligned}$$

$$\begin{aligned} \text{Maximum Negative } M_c &= 100 (-1) + 50 (-1) + 20 (7) \\ &= \underline{-10 \text{ kN}\cdot\text{m}} \end{aligned}$$

9.5



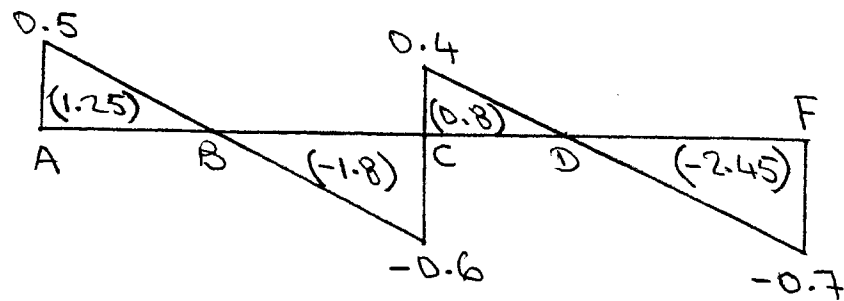
$$\text{Maximum Upward } A_y = 25(1) + 2(20) + 0.5(20) \\ = \underline{75 \text{ k} \uparrow}$$



Influence Line for $M_A (+\curvearrowright)$

$$\text{Maximum Counterclockwise } M_A \\ = 25(20) + 2(200) + 0.5(200) \\ = \underline{1000 \text{ k-ft} \curvearrowleft}$$

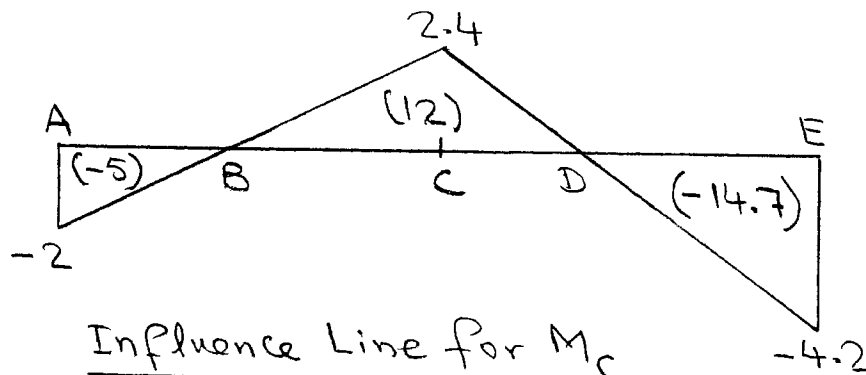
9.6



Influence Line for S_C

$$\begin{aligned}\text{Maximum Positive } S_C &= 150(0.5) + 50(1.25 + 0.8) \\ &\quad + 25(1.25 - 1.8 + 0.8 - 2.45) \\ &= \underline{122.5 \text{ kN}}\end{aligned}$$

$$\begin{aligned}\text{Maximum Negative } S_C &= 150(-0.7) + 50(-1.8 - 2.45) \\ &\quad + 25(1.25 - 1.8 + 0.8 - 2.45) \\ &= \underline{-372.5 \text{ kN}}\end{aligned}$$

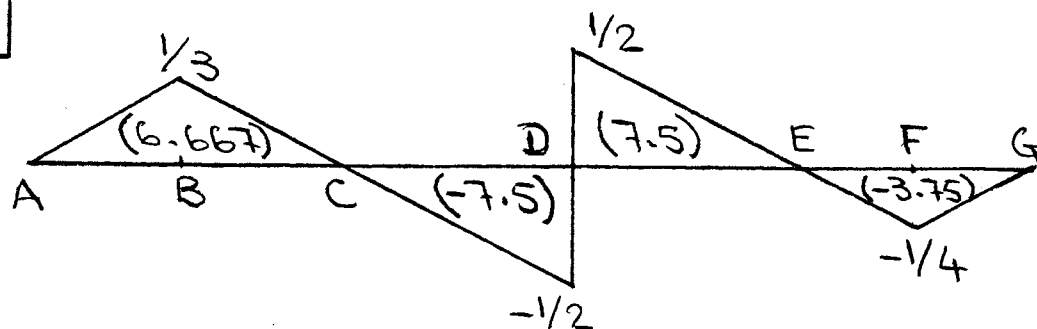


Influence Line for M_C

$$\begin{aligned}\text{Maximum Positive } M_C &= 150(2.4) + 50(12) \\ &\quad + 25(-5 + 12 - 14.7) \\ &= \underline{767.5 \text{ kN.m}}\end{aligned}$$

$$\begin{aligned}\text{Maximum Negative } M_C &= 150(-4.2) + 50(-5 - 14.7) \\ &\quad + 25(-5 + 12 - 14.7) \\ &= \underline{-1807.5 \text{ kN.m}}\end{aligned}$$

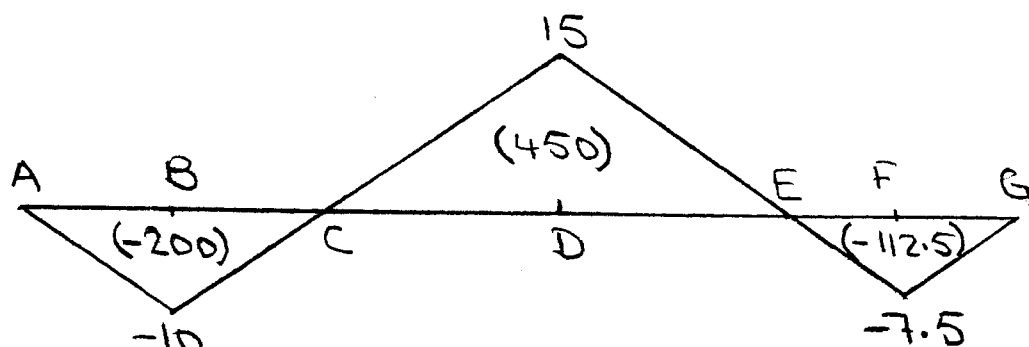
9.7



Influence Line for S_D

$$\begin{aligned}\text{Maximum Positive } S_D &= 30\left(\frac{1}{2}\right) + 3(6.667 + 7.5) \\ &\quad + 1(6.667 - 7.5 + 7.5 - 3.75) \\ &= \underline{60.417 \text{ k}}\end{aligned}$$

$$\begin{aligned}\text{Maximum Negative } S_D &= 30\left(-\frac{1}{2}\right) + 3(-7.5 - 3.75) \\ &\quad + 1(6.667 - 7.5 + 7.5 - 3.75) \\ &= \underline{-45.833 \text{ k}}\end{aligned}$$

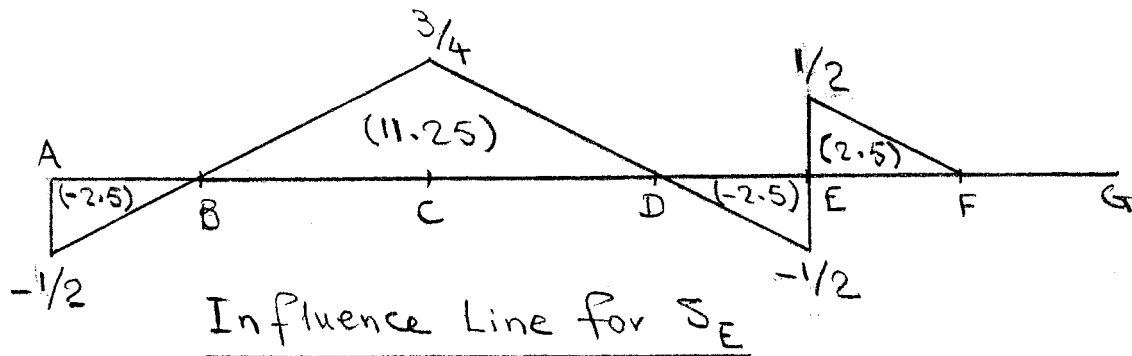


Influence Line for M_D

$$\begin{aligned}\text{Maximum Positive } M_D &= 30(15) + 3(450) \\ &\quad + 1(-200 + 450 - 112.5) \\ &= \underline{1937.5 \text{ k-ft}}\end{aligned}$$

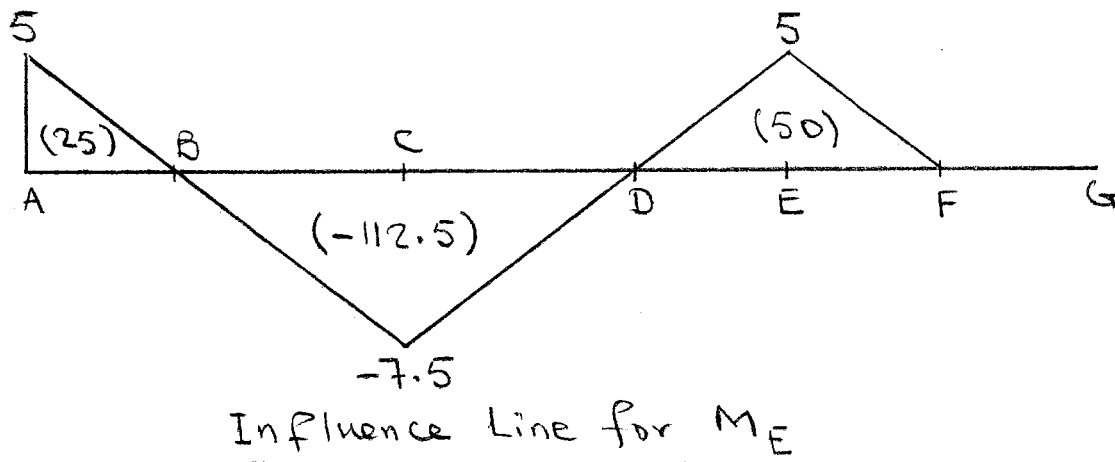
$$\begin{aligned}\text{Maximum Negative } M_D &= 30(-10) + 3(-200 - 112.5) \\ &\quad + 1(-200 + 450 - 112.5) \\ &= \underline{-1100 \text{ k-ft}}\end{aligned}$$

9.8



$$\begin{aligned} \text{Maximum Positive } S_E &= 40\left(\frac{3}{4}\right) + 2(11.25 + 2.5) + 1(-2.5 + 11.25) \\ &= \underline{66.25 \text{ k}} \end{aligned}$$

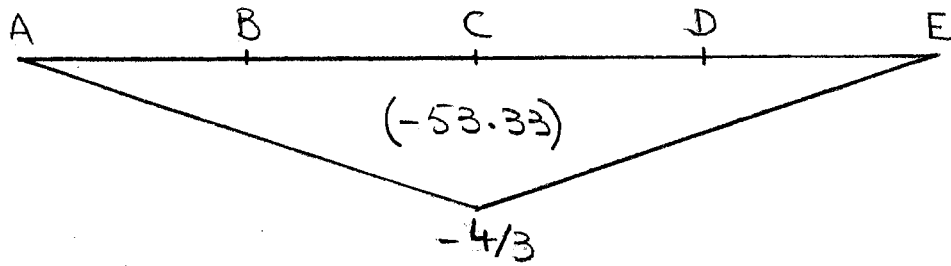
$$\begin{aligned} \text{Maximum Negative } S_E &= 40\left(-\frac{1}{2}\right) + 2(-2.5 - 2.5) + 1(-2.5 + 11.25) \\ &= \underline{-21.25 \text{ k}} \end{aligned}$$



$$\begin{aligned} \text{Maximum Positive } M_E &= 40(5) + 2(25 + 50) + 1(25 - 112.5 + 50) \\ &= \underline{312.5 \text{ k-ft}} \end{aligned}$$

$$\begin{aligned} \text{Maximum Negative } M_E &= 40(-7.5) + 2(-112.5) + 1(25 - 112.5 + 50) \\ &= \underline{-562.5 \text{ k-ft}} \end{aligned}$$

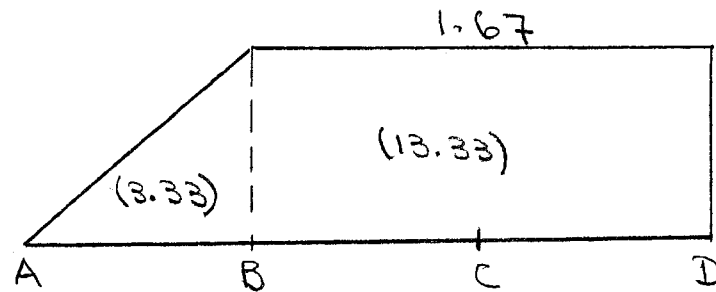
9.9



Influence Line for F_{GH}

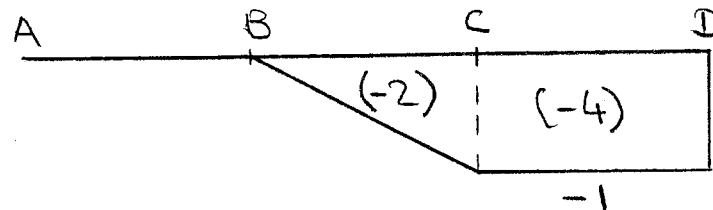
$$\begin{aligned}\text{Maximum Compressive } F_{GH} &= 30\left(-\frac{4}{3}\right) + 3(-53.33) \\ &= -200 \text{ k} = \underline{200 \text{ k (C)}}\end{aligned}$$

9.10



Influence Line for F_{BE}

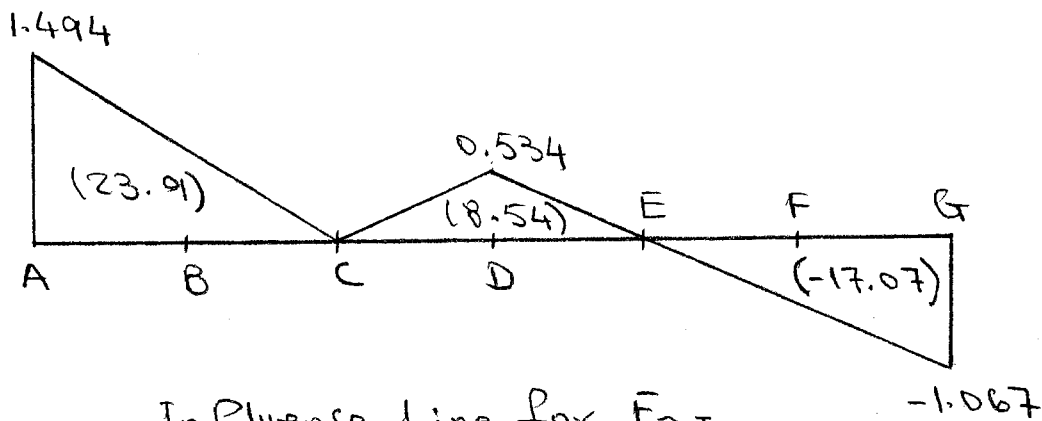
$$\begin{aligned} \text{Maximum Tensile } F_{BE} &= 120(1.67) + 60(3.33 + 13.33) \\ &= \underline{1200 \text{ kN (T)}} \end{aligned}$$



Influence Line for F_{BF}

$$\begin{aligned} \text{Maximum Compressive } F_{BF} &= 120(-1) + 60(-2 - 4) \\ &= -480 \text{ kN} \\ &= \underline{480 \text{ kN (C)}} \end{aligned}$$

9.11

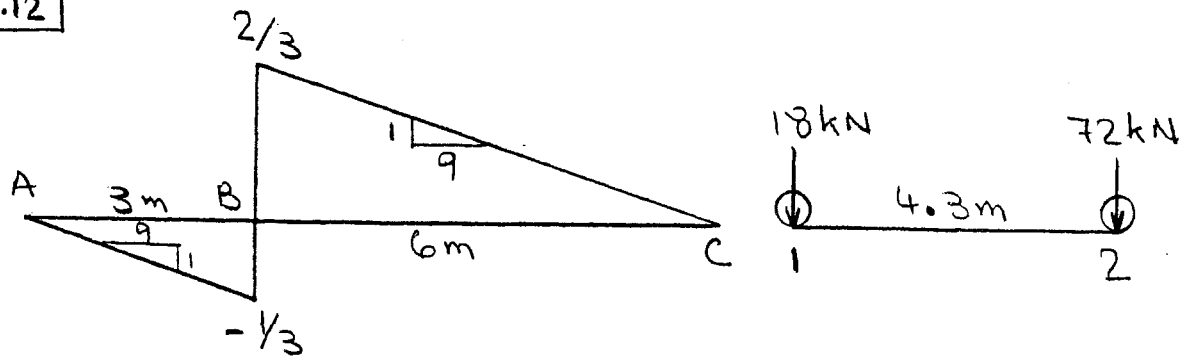


Influence Line for F_{DI}

$$\begin{aligned} \text{Maximum Tensile } F_{DI} &= 40(1.494) + 4(23.9 + 8.54) \\ &\quad + 2(23.9 + 8.54 - 17.07) = \underline{220.3 \text{ k (T)}} \end{aligned}$$

$$\begin{aligned} \text{Maximum Compressive } F_{DI} &= 40(-1.067) + 4(-17.07) \\ &\quad + 2(23.9 + 8.54 - 17.07) = -80.2 \text{ k} \\ &= \underline{80.2 \text{ k (C)}} \end{aligned}$$

9.12



Influence Line for S_B

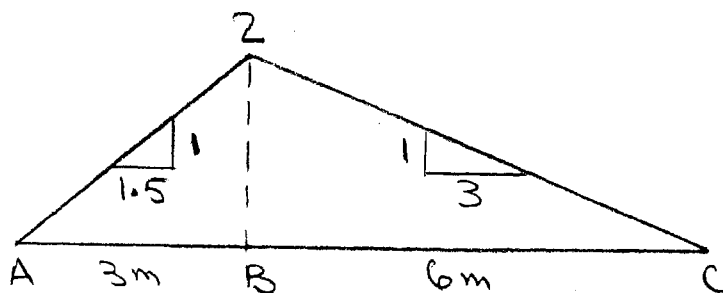
Loading position 1:

$$S_B = 18 \left(\frac{2}{3} \right) + 72 \left(\frac{1.7}{9} \right) = 25.6 \text{ kN}$$

Loading position 2:

$$S_B = 72 \left(\frac{2}{3} \right) = 48 \text{ kN}$$

$$\text{Max. Positive } S_B = \underline{48 \text{ kN}}$$



Influence Line for M_B

Loading position 1:

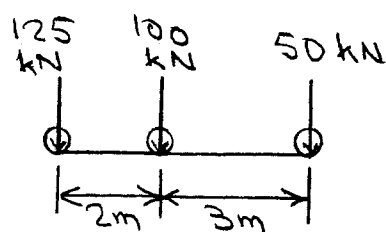
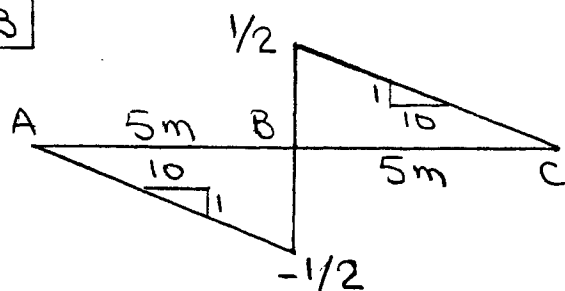
$$M_B = 18(2) + 72 \left(\frac{1.7}{3} \right) = 76.8 \text{ kN}\cdot\text{m}$$

Loading position 2:

$$M_B = 72(2) = 144 \text{ kN}\cdot\text{m}$$

$$\text{Max. Positive } M_B = \underline{144 \text{ kN}\cdot\text{m}}$$

9.13



Influence Line for S_B

Loading position 1:

$$S_B = 125\left(\frac{1}{2}\right) + 100\left(\frac{3}{10}\right) = 92.5 \text{ kN}$$

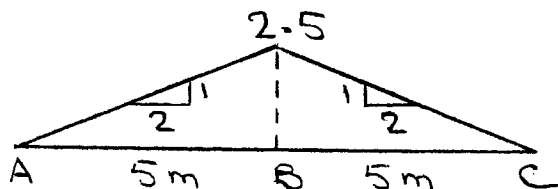
Loading position 2:

$$S_B = 125\left(-\frac{3}{10}\right) + 100\left(\frac{1}{2}\right) + 50\left(\frac{2}{10}\right) = 22.5 \text{ kN}$$

Loading position 3:

$$S_B = 100\left(-\frac{2}{10}\right) + 50\left(\frac{1}{2}\right) = 5 \text{ kN}$$

$$\text{Max. Positive } S_B = \underline{92.5 \text{ kN}}$$



Influence Line for M_B

Loading position 1:

$$M_B = 125(2.5) + 100(1.5) = 462.5 \text{ kN.m}$$

Loading position 2:

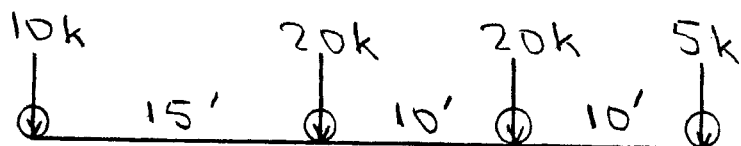
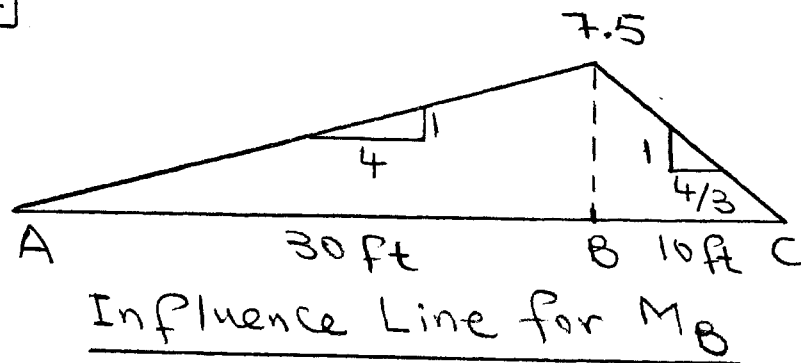
$$M_B = 125(1.5) + 100(2.5) + 50(1) = 487.5 \text{ kN.m}$$

Loading position 3:

$$M_B = 100(1) + 50(2.5) = 225 \text{ kN.m}$$

$$\text{Max. Positive } M_B = \underline{487.5 \text{ kN.m}}$$

9.14



Loading position 1:

$$M_B = 10(7.5) = 75 \text{ k-ft}$$

Loading position 2:

$$M_B = 10\left(\frac{15}{4}\right) + 20(7.5) = 187.5 \text{ k-ft}$$

Loading position 3:

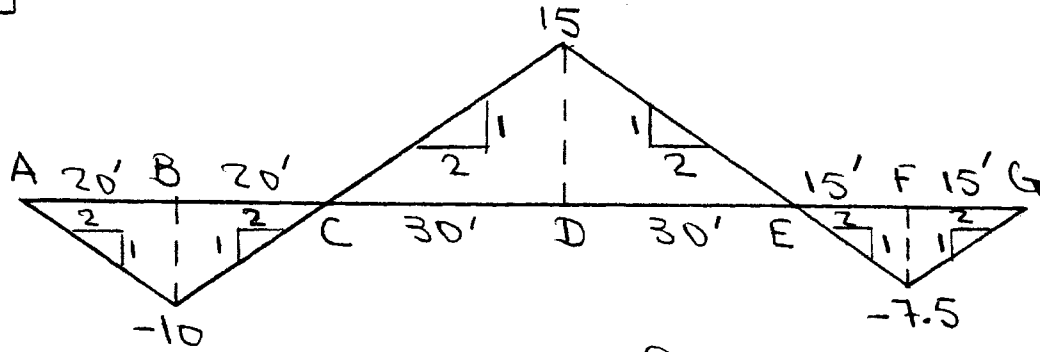
$$M_B = 10\left(\frac{5}{4}\right) + 20(5) + 20(7.5) = 262.5 \text{ k-ft}$$

Loading position 4:

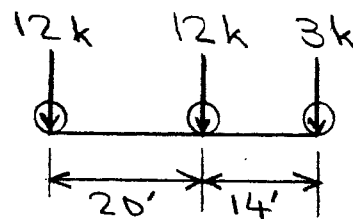
$$M_B = 20\left(\frac{10}{4}\right) + 20(5) + 5(7.5) = 187.5 \text{ k-ft}$$

$$\text{Max. Positive } M_B = \underline{262.5 \text{ k-ft}}$$

9.15



Influence Line for M_D



Loading position 1:

$$M_D = 12(15) + 12(5) + 3\left(-\frac{4}{2}\right) = 234 \text{ k-ft}$$

Loading position 2:

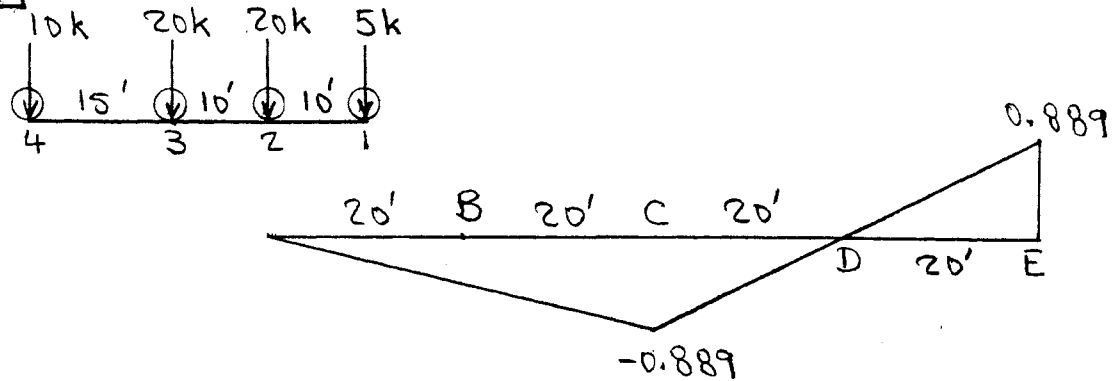
$$M_D = 12(5) + 12(15) + 3\left(\frac{16}{2}\right) = 264 \text{ k-ft}$$

Loading position 3:

$$M_D = 12\left(-\frac{4}{2}\right) + 12\left(\frac{16}{2}\right) + 3(15) = 117 \text{ k-ft}$$

Max. Positive $M_D = 264 \text{ k-ft}$

9.16



Influence Line for F_{GH}

Loading position 1:

$$F_{GH} = [5(40) + 20(30 + 20) + 10(5)] \left(\frac{-0.889}{40} \right) = -27.78 \text{ k}$$

Loading position 2:

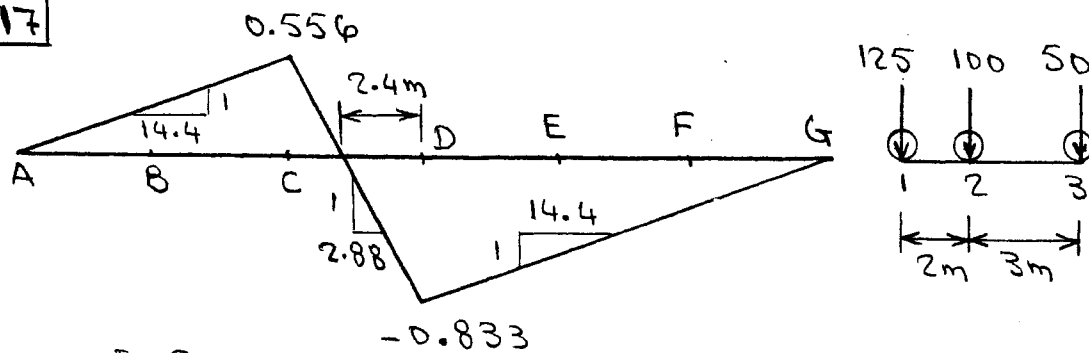
$$F_{GH} = 5(10) \left(\frac{-0.889}{20} \right) + [20(40 + 30) + 10(15)] \left(\frac{-0.889}{40} \right) \\ = -36.67 \text{ k}$$

Loading position 3:

$$F_{GH} = [5(0) + 20(10)] \left(\frac{-0.889}{20} \right) + [20(40) + 10(25)] \left(\frac{-0.889}{40} \right) \\ = -32.23 \text{ k}$$

Maximum compressive $F_{GH} = -36.67 \text{ k} = \underline{36.67 \text{ k (C)}}$

9.17



Influence Line for F_{DI}

Loading position 1:

$$F_{DI} = 125 (0.556) - 100 \left(\frac{0.4}{2.88} \right) - 50 \left(\frac{11}{14.4} \right) = 17.42 \text{ kN}$$

Loading position 2:

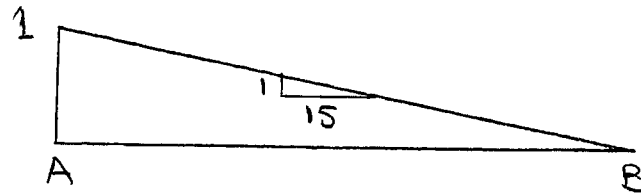
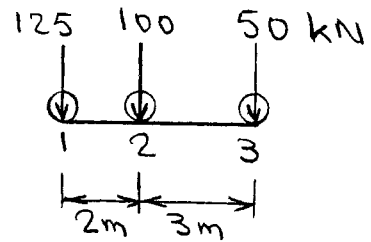
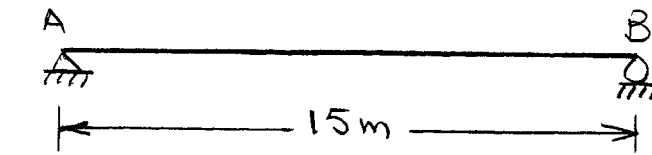
$$F_{DI} = 125 \left(\frac{6}{14.4} \right) + 100 (0.556) - 50 \left(\frac{1.4}{2.88} \right) = 83.38 \text{ kN}$$

Loading position 3:

$$F_{DI} = 125 \left(\frac{3}{14.4} \right) + 100 \left(\frac{5}{14.4} \right) + 50 (0.556) = 88.56 \text{ kN}$$

Maximum Tensile $F_{DI} = \underline{88.56 \text{ kN (T)}}$

9.18



Influence Line $S_{A,R}$

Loading position 1:

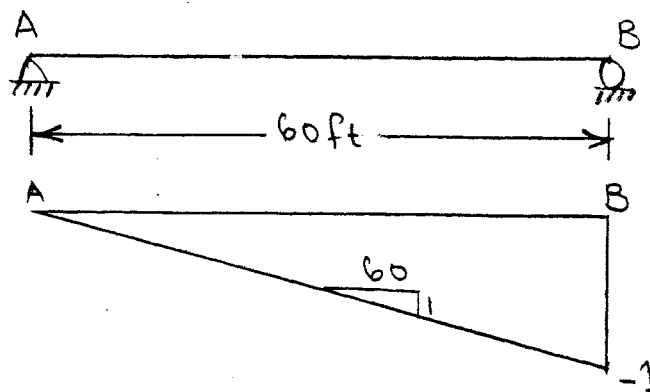
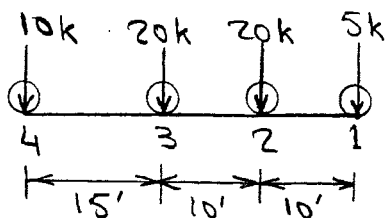
$$S_{A,R} = [125(15) + 100(13) + 50(10)] \frac{1}{15} = 245 \text{ kN}$$

Loading position 2:

$$S_{A,R} = [100(15) + 50(12)] \frac{1}{15} = 140 \text{ kN}$$

Absolute maximum shear = 245 kN

9.19



Influence Line for $S_{B,L}$

Loading position 1:

$$S_{B,L} = [5(60) + 20(50+40) + 10(25)] \left(-\frac{1}{60}\right) = -39.17 \text{ k}$$

Loading position 2:

$$S_{B,L} = [20(60+50) + 10(35)] \left(-\frac{1}{60}\right) = -42.5 \text{ k}$$

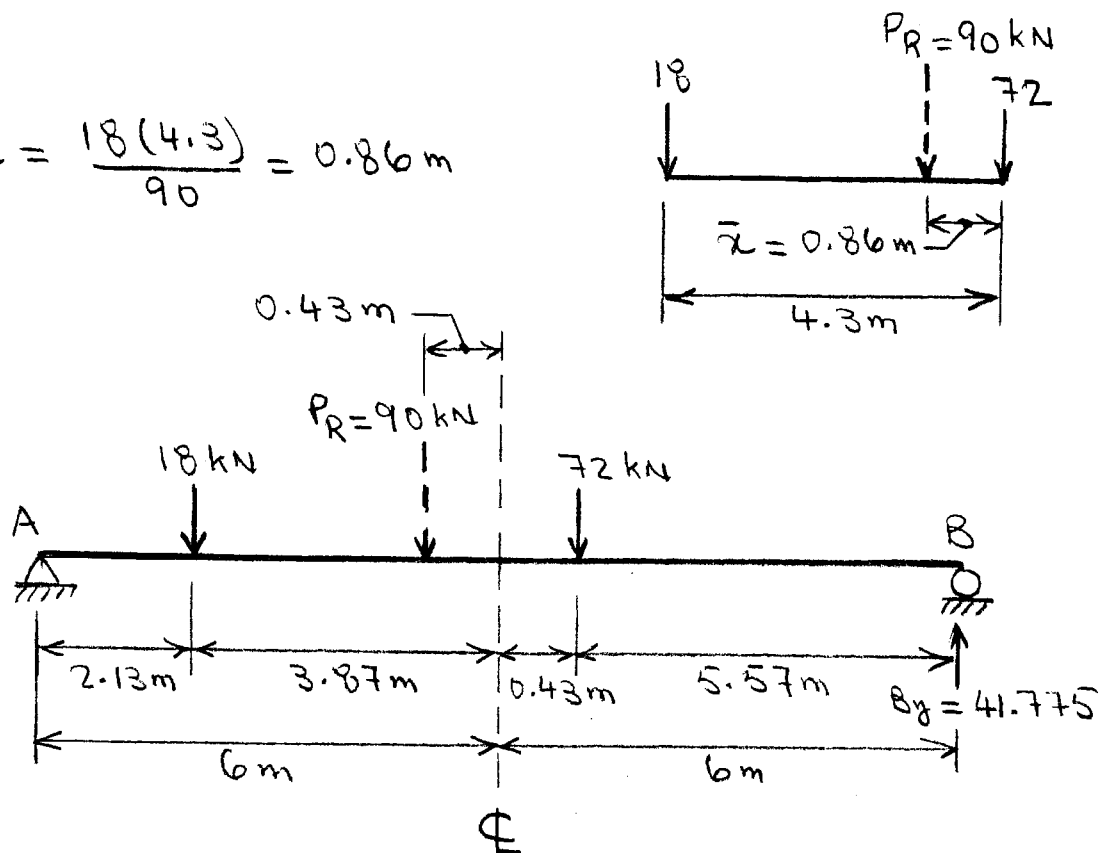
Loading position 3:

$$S_{B,L} = [20(60) + 10(45)] \left(-\frac{1}{60}\right) = -27.5 \text{ k}$$

Absolute maximum shear = 42.5 k

9.20

$$\bar{x} = \frac{18(4.3)}{90} = 0.86 \text{ m}$$



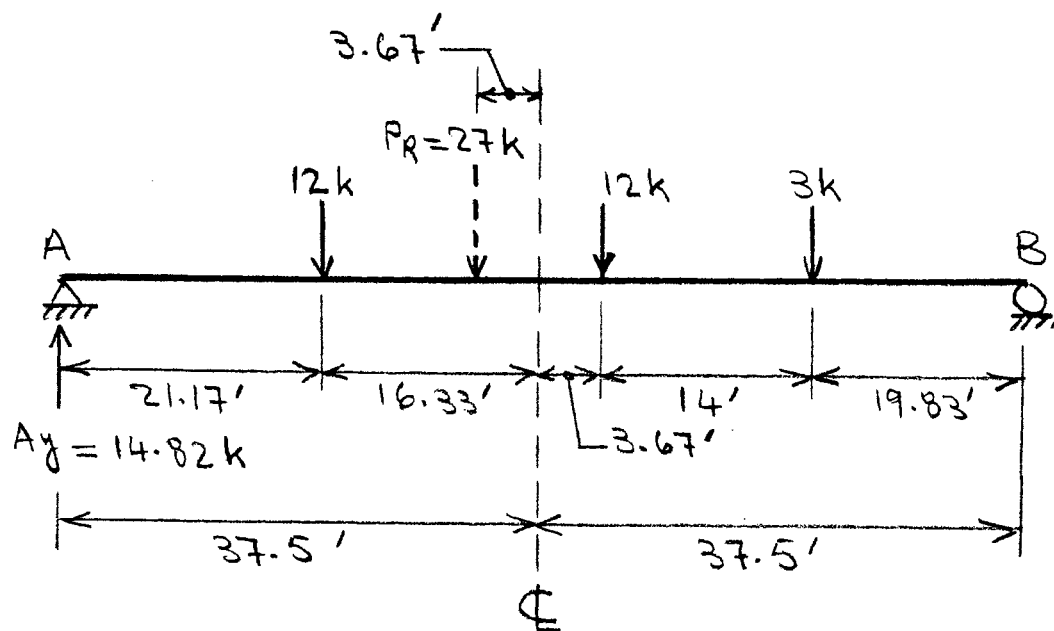
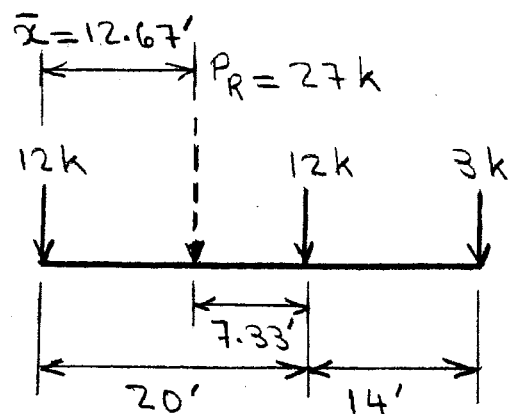
$$B_y = \frac{90(6 - 0.43)}{12} = 41.775 \text{ kN}$$

$$M_{\max} = 41.775 (5.57) = \underline{232.7 \text{ kN.m}}$$

9.21

$$\bar{x} = \frac{12(20) + 3(34)}{27}$$

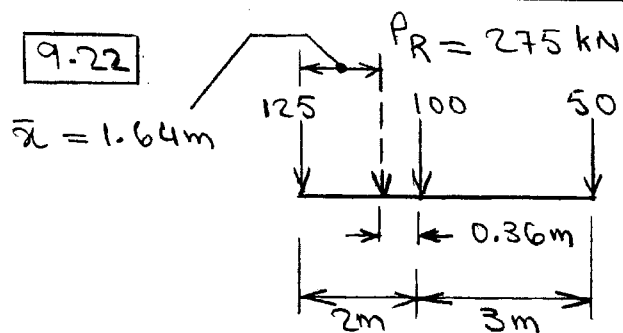
$$= 12.67 \text{ ft}$$



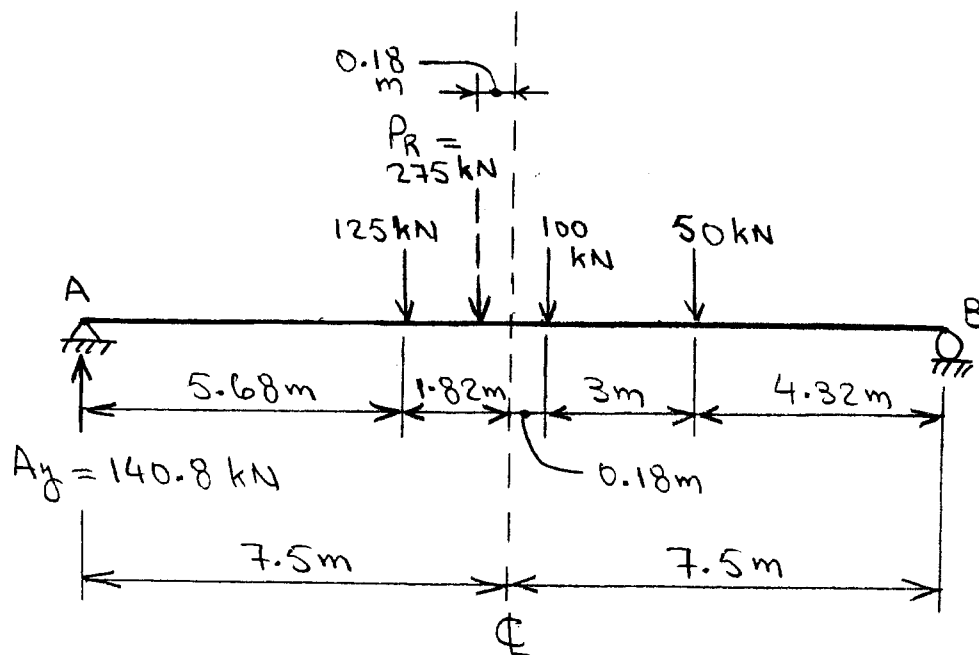
$$A_y = \frac{27(37.5 + 3.67)}{75} = 14.82 \text{ k}$$

$$M_{\max} = 14.82(37.5 + 3.67) - 12(20) = \underline{370.1 \text{ k-ft}}$$

9.22



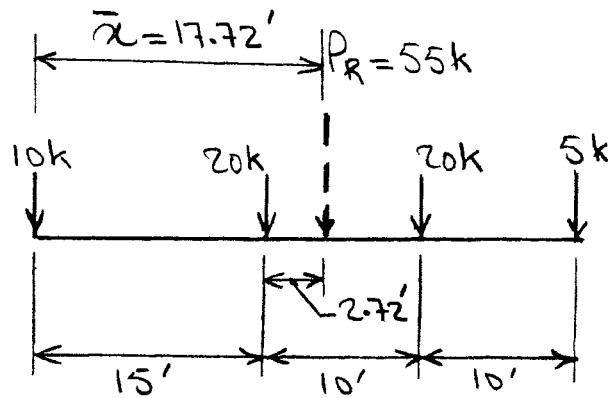
$$\bar{x} = \frac{100(2) + 50(5)}{275} = 1.64 \text{ m}$$



$$A_y = 275 \left(\frac{0.18 + 7.5}{15} \right) = 140.8 \text{ kN}$$

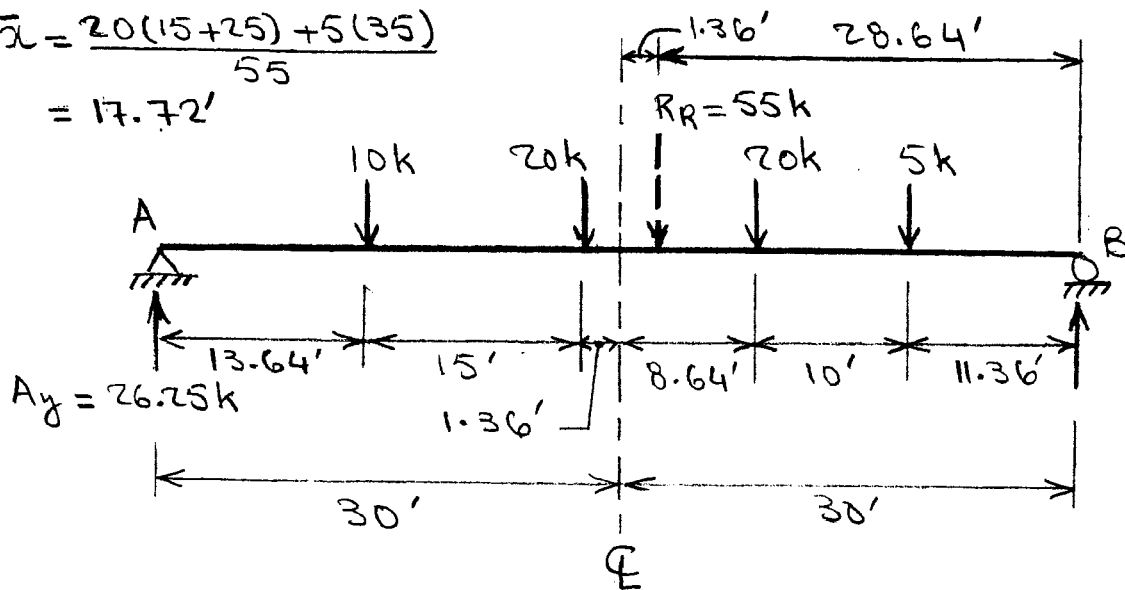
$$M_{\max} = 140.8 (7.5 + 0.18) - 125(2) = \underline{831.3 \text{ kN}\cdot\text{m}}$$

9.23



$$\bar{x} = \frac{20(15+25) + 5(35)}{55}$$

$$= 17.72'$$



$$A_y = 55(28.64/60) = 26.25k$$

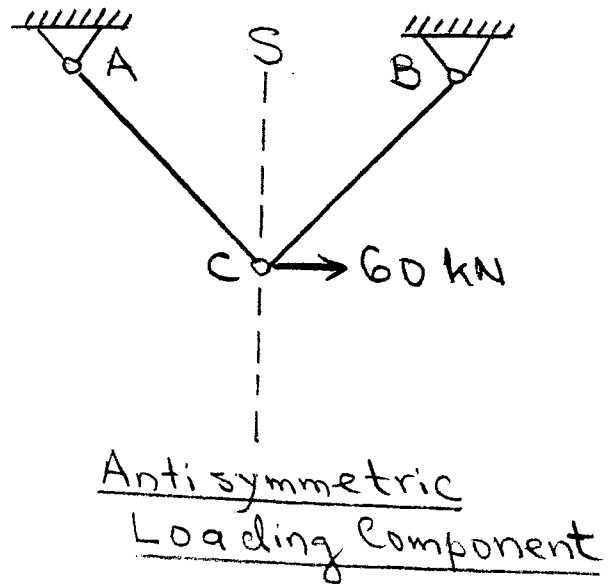
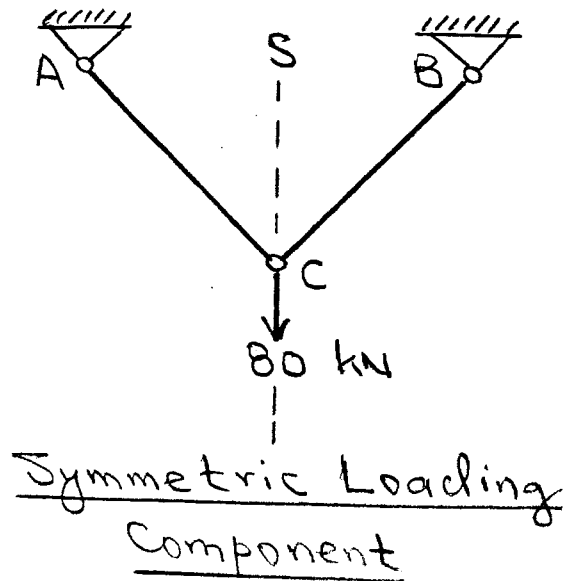
$$M_{max} = 26.25(28.64) - 10(15) = \underline{601.8 \text{ k-ft}}$$

Chapter Ten

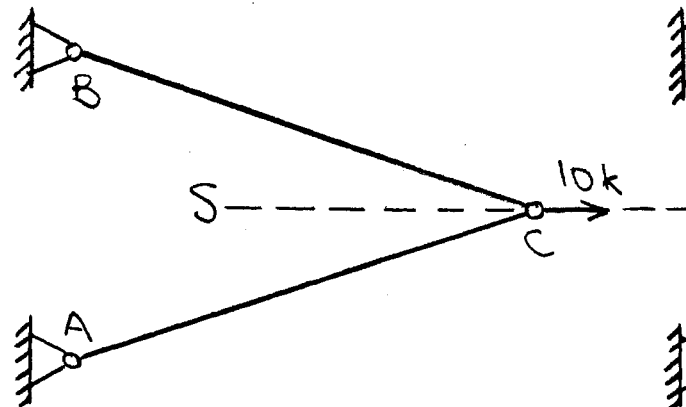
Analysis of Symmetric Structures

CHAPTER 10

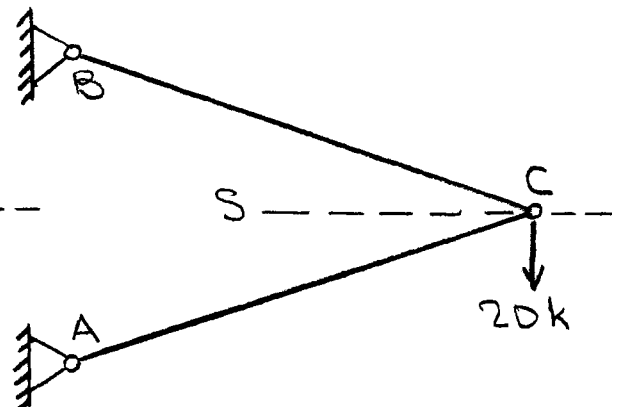
10.1



10.2

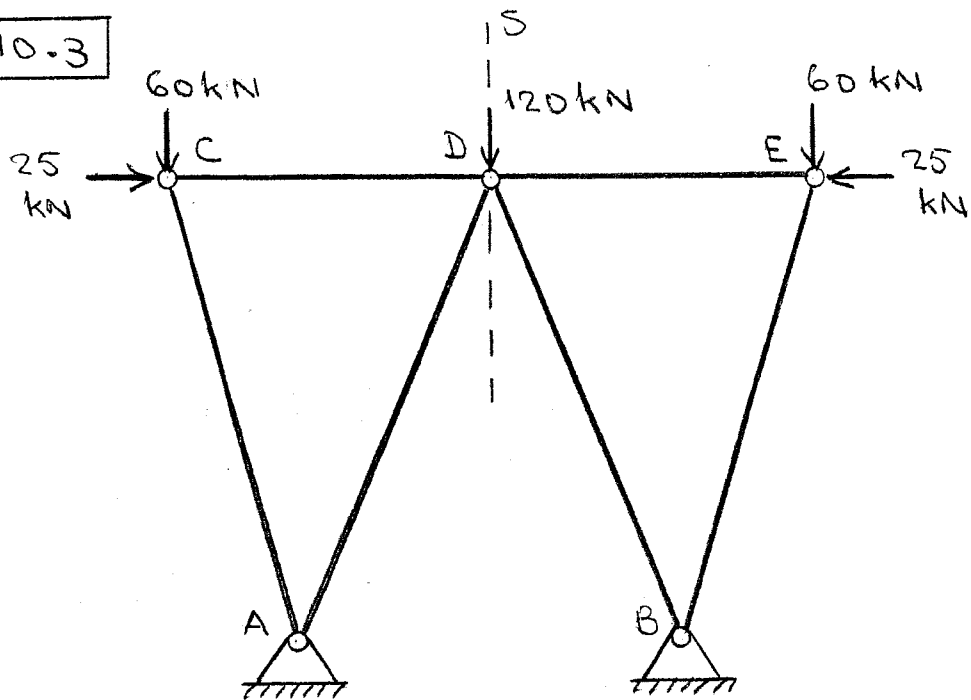


Symmetric Loading
component

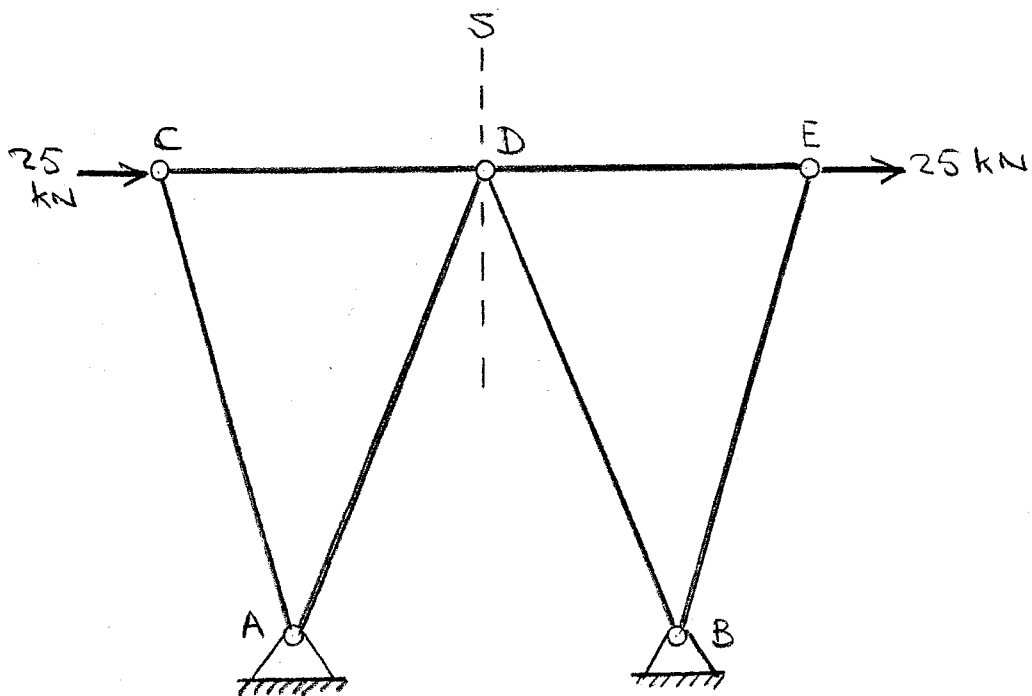


Antisymmetric Loading
component

10.3

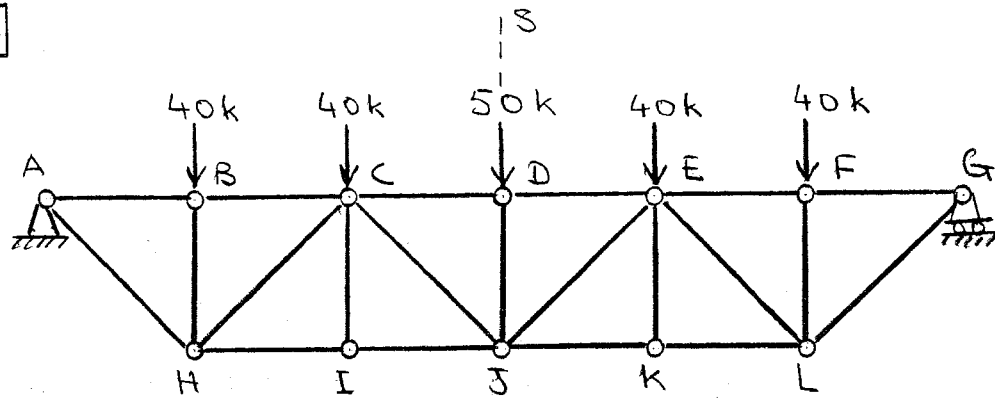


Symmetric Loading Component

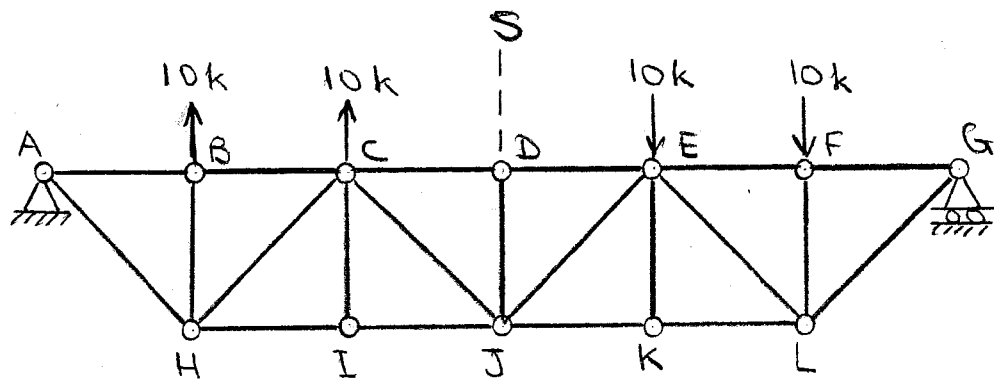


Anti symmetric Loading Component

10.4

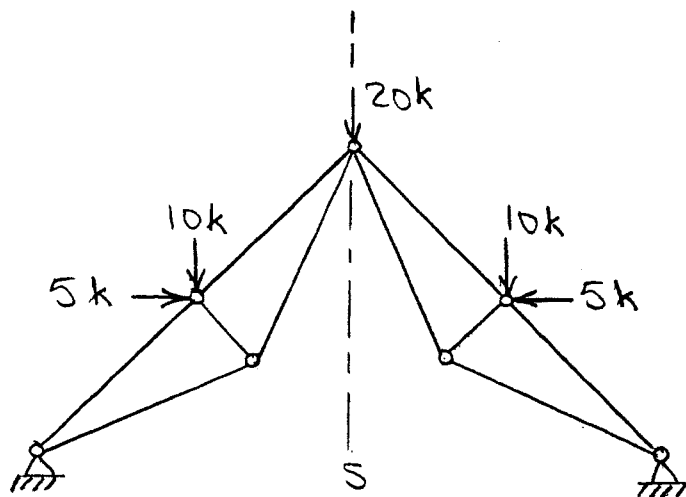


Symmetric Loading Component

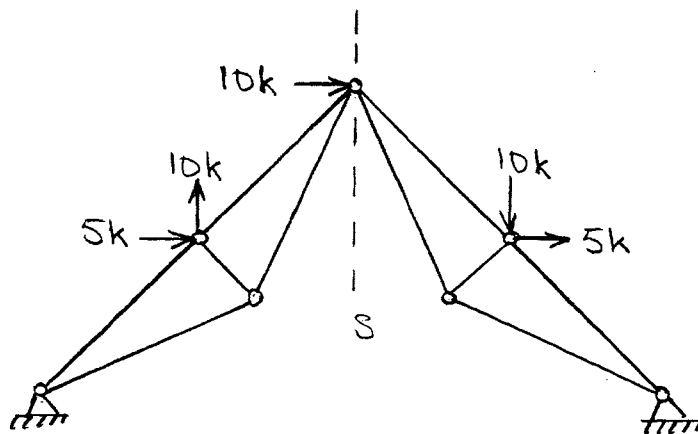


Antisymmetric Loading Component

10.5

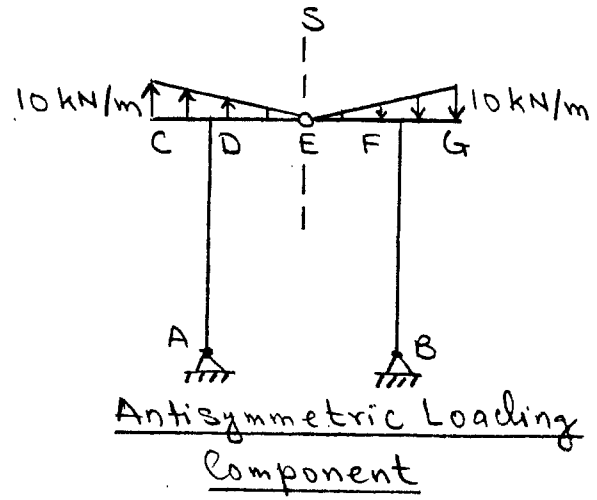
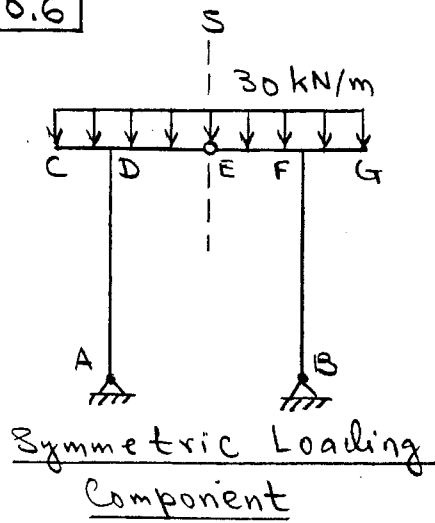


Symmetric Loading Component

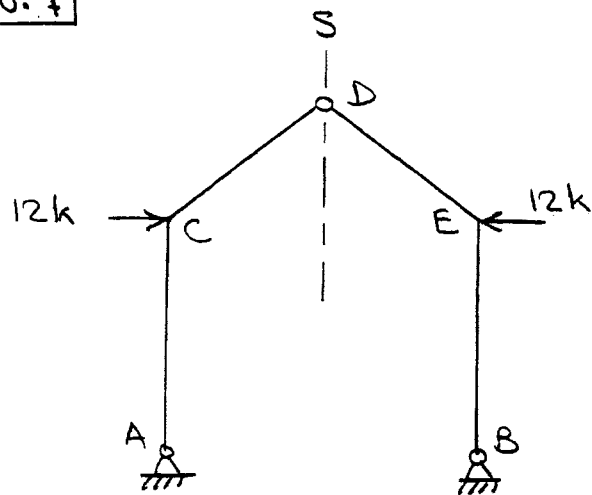


Antisymmetric Loading Component

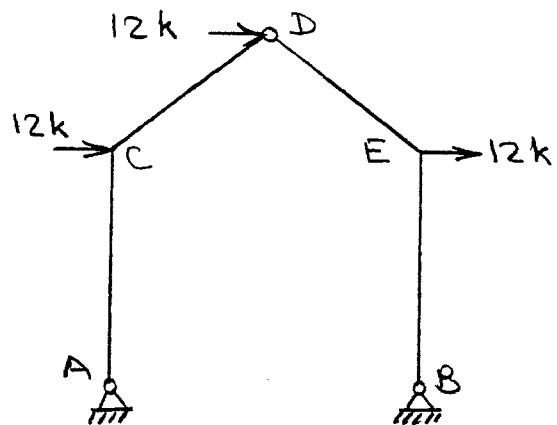
10.6



10.7

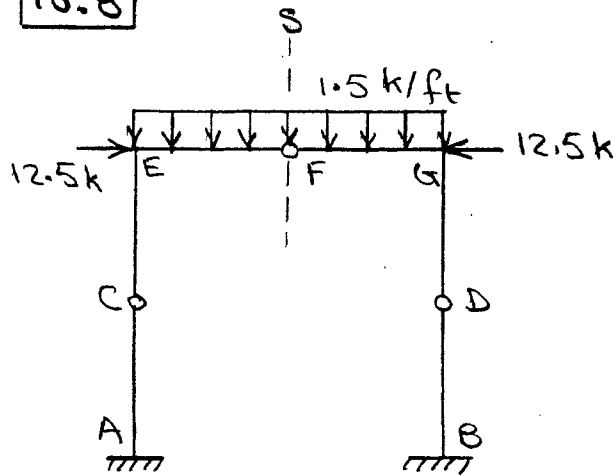


Symmetric Loading
Component

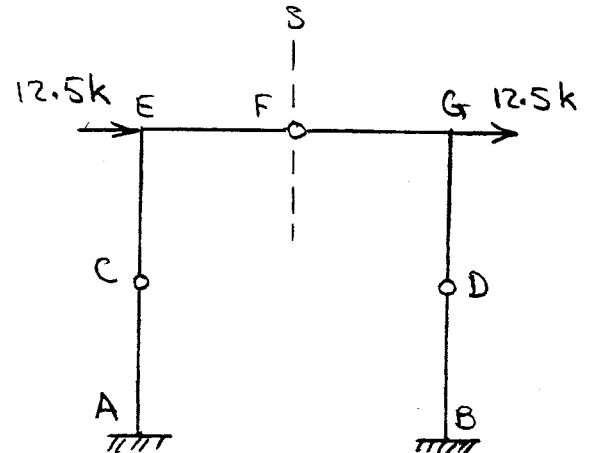


Antisymmetric Loading
Component

10.8

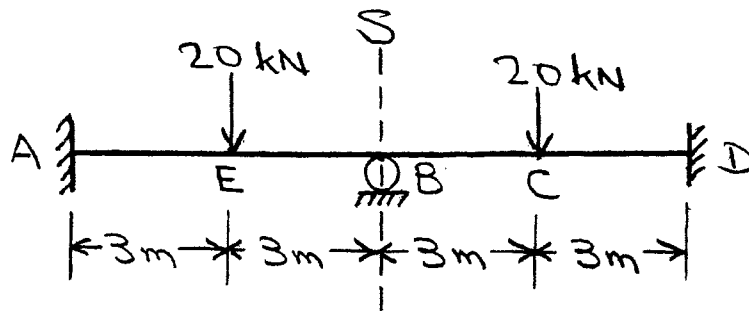


Symmetric Loading
Component

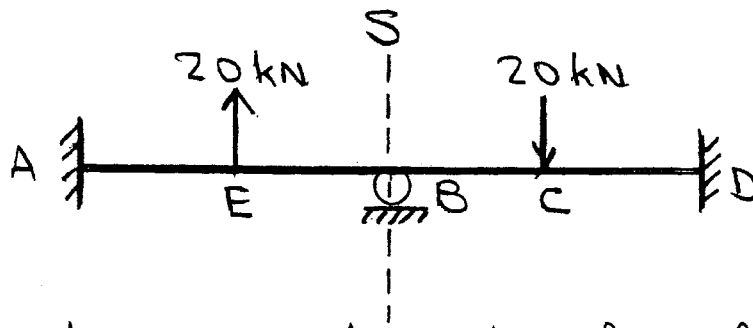


Antisymmetric Loading
Component

10.9

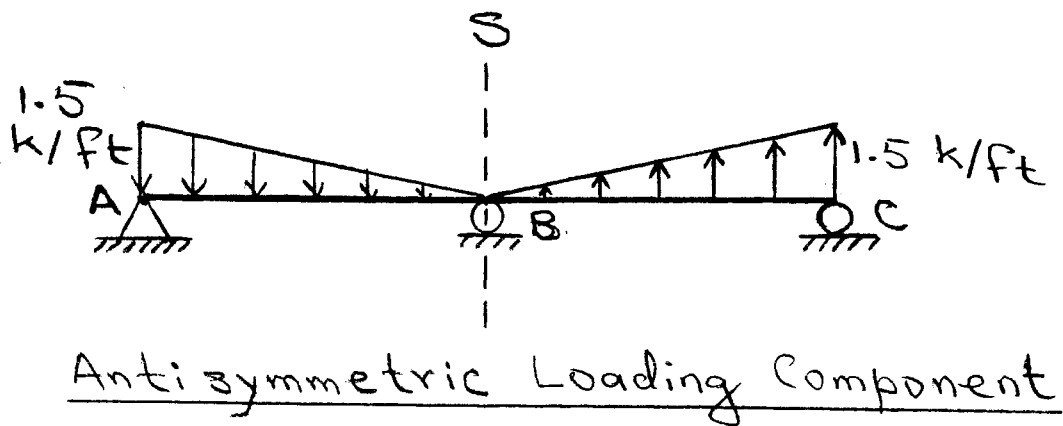
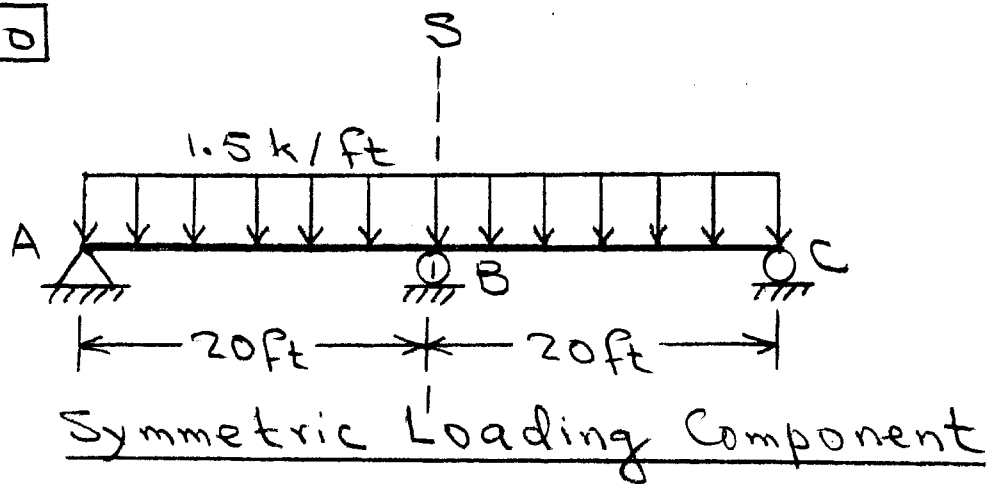


Symmetric Loading Component

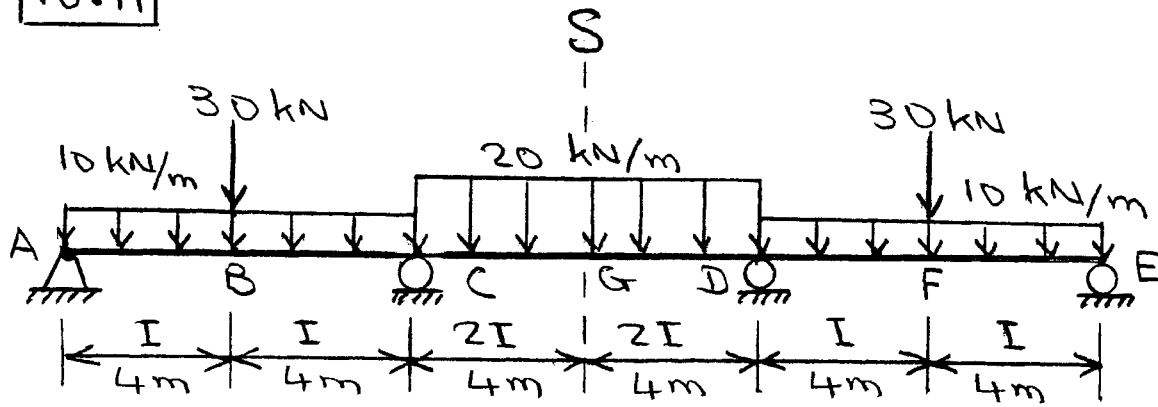


Antisymmetric Loading Component

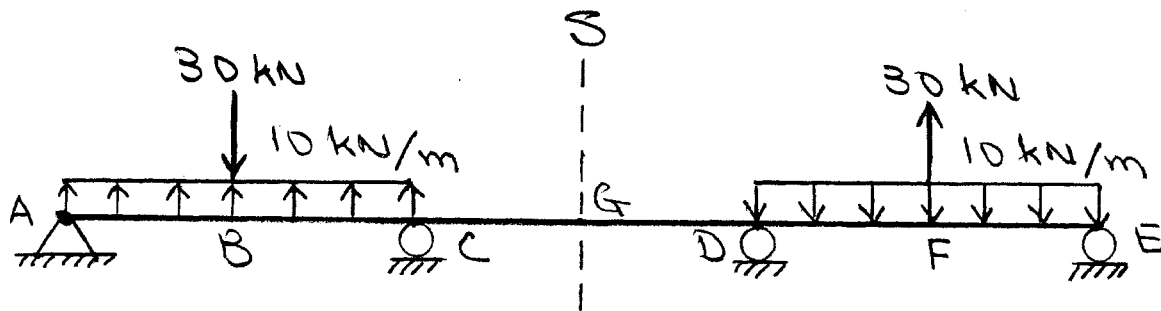
10.10



10.11

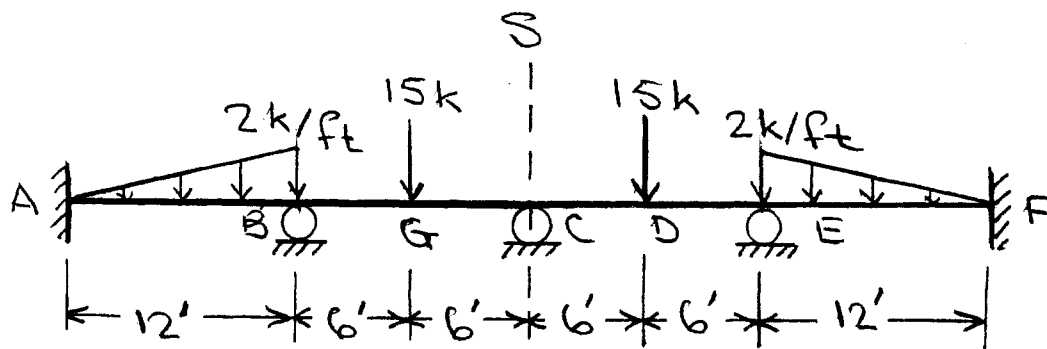


Symmetric Loading Component

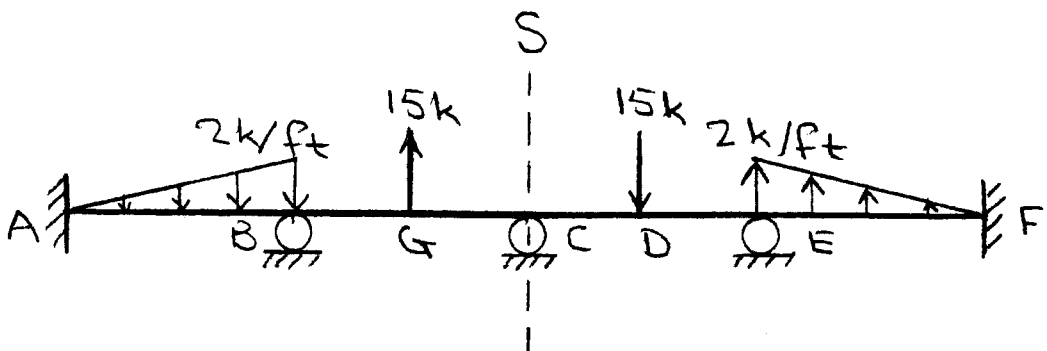


Antisymmetric Loading Component

10.12

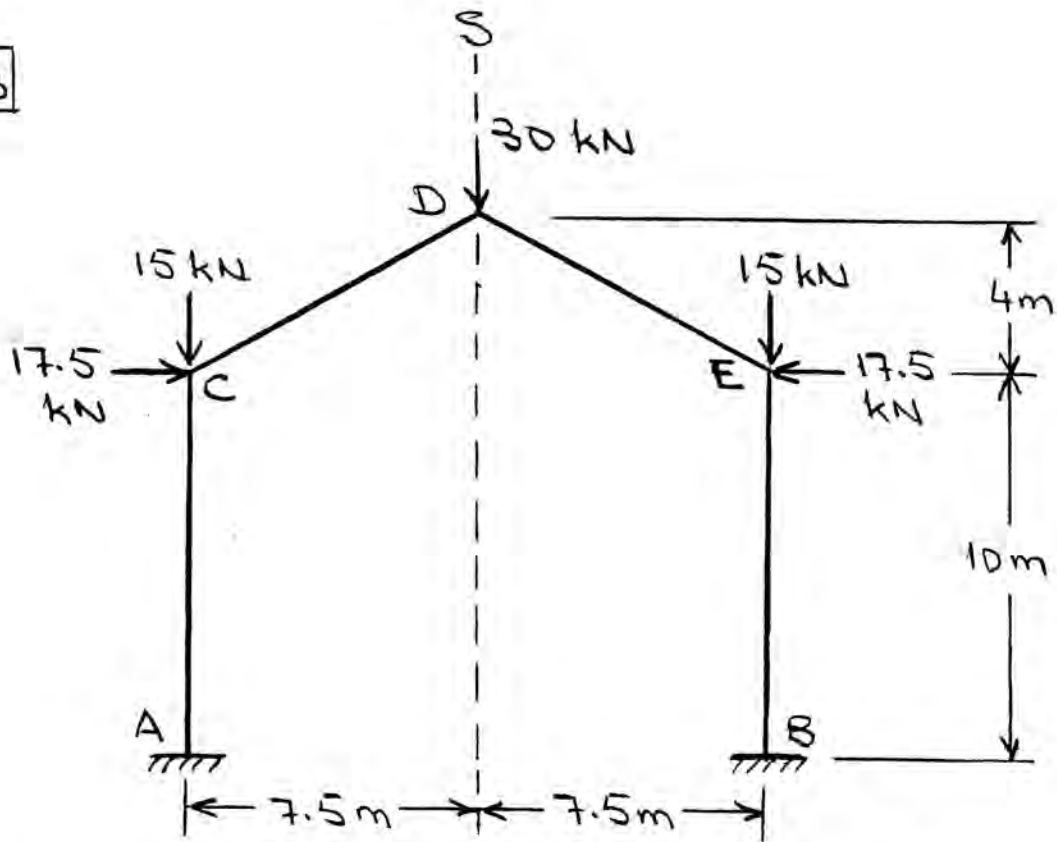


Symmetric Loading Component

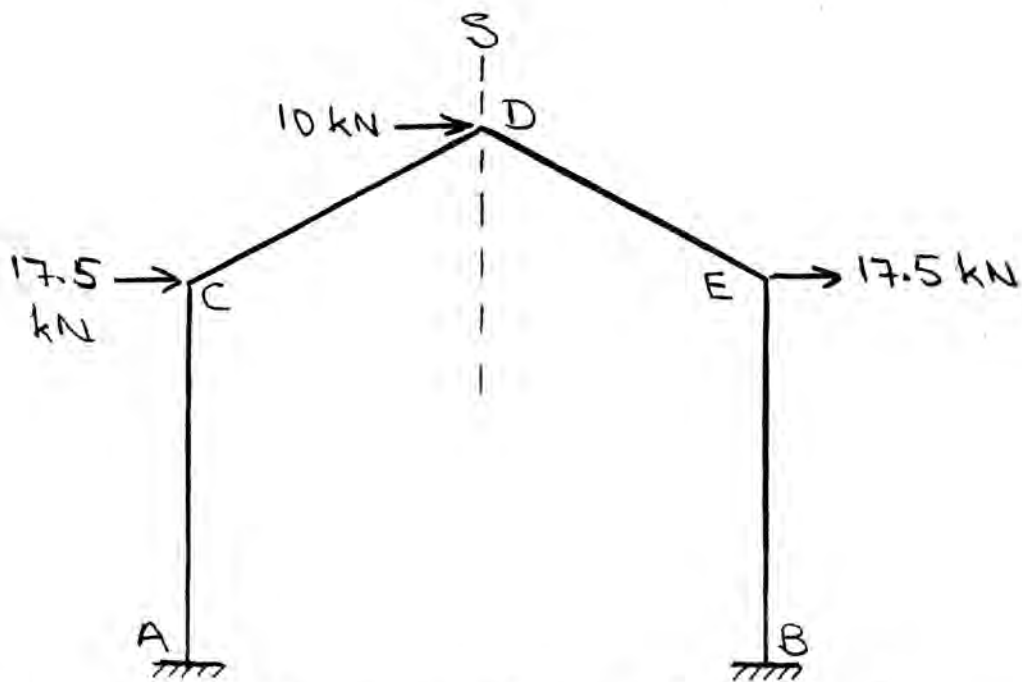


Antisymmetric Loading Component

10.13

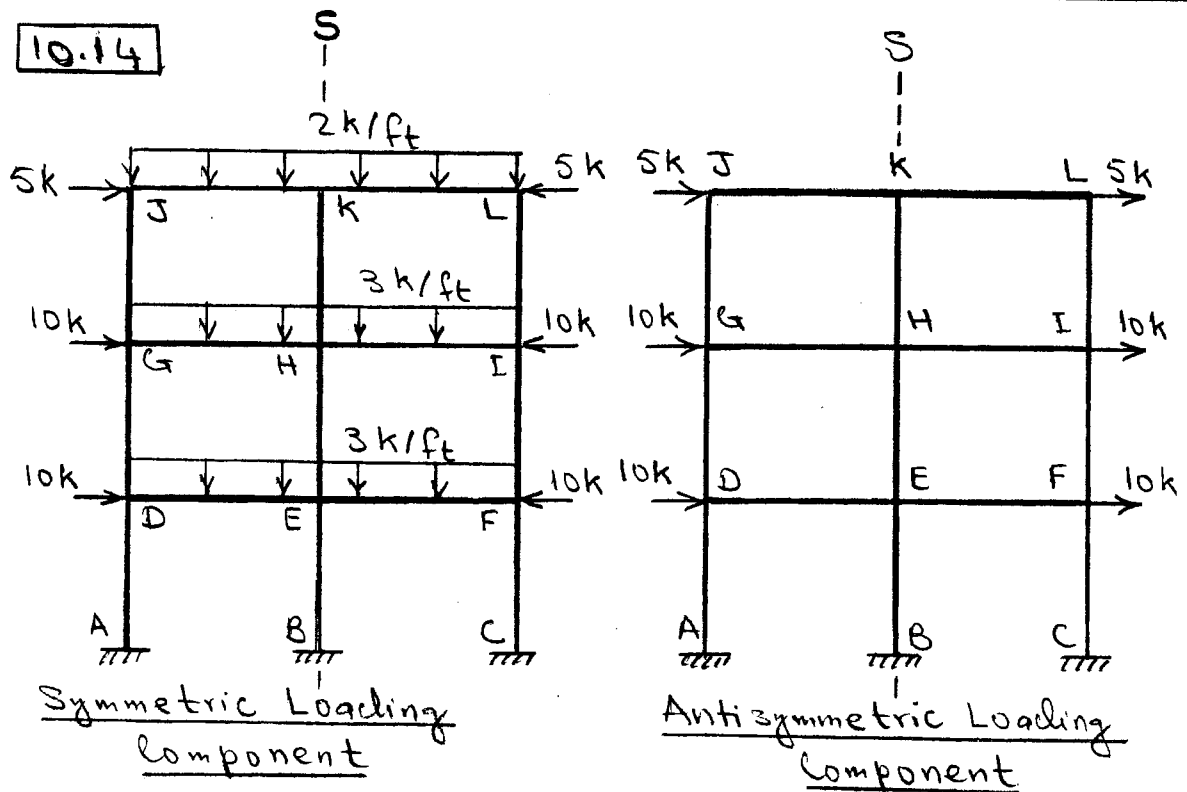


Symmetric Loading Component

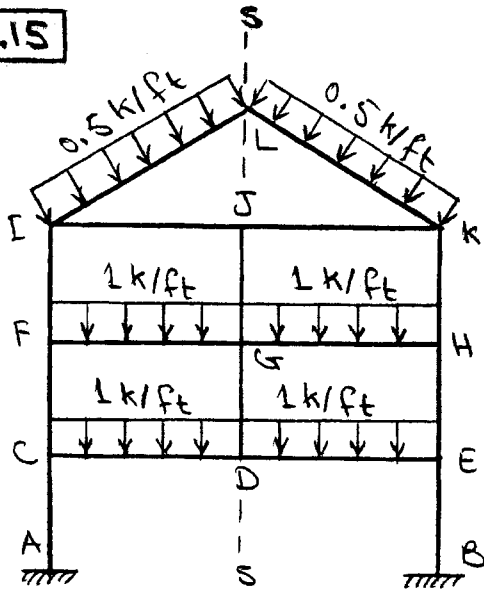


Antisymmetric Loading Component

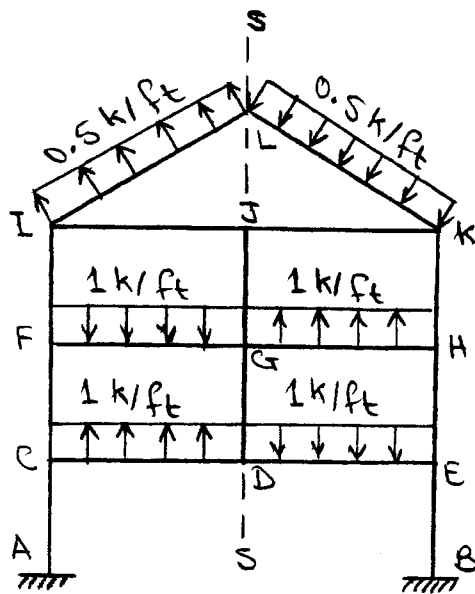
10.14



10.15

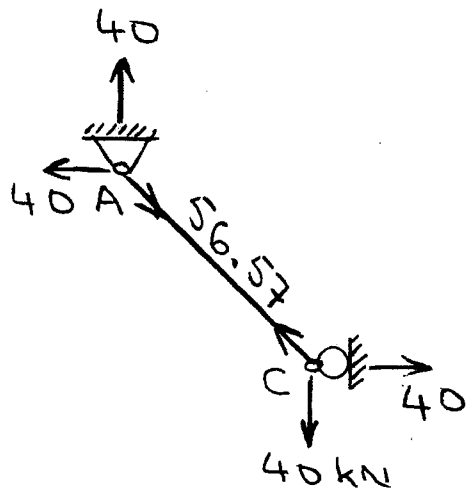


Symmetric Loading Component

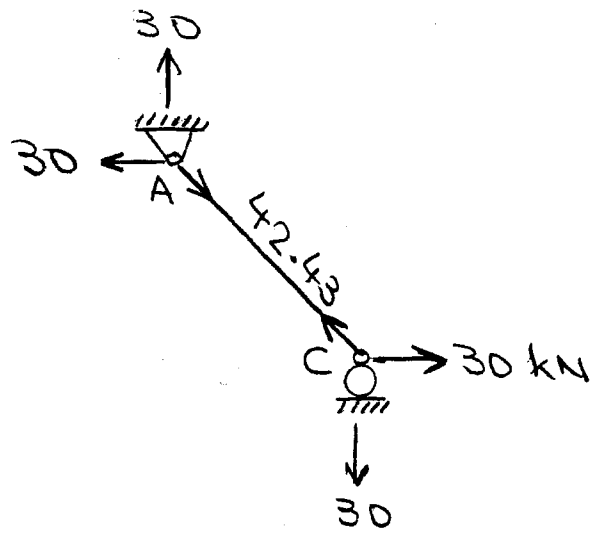


Antisymmetric Loading Component

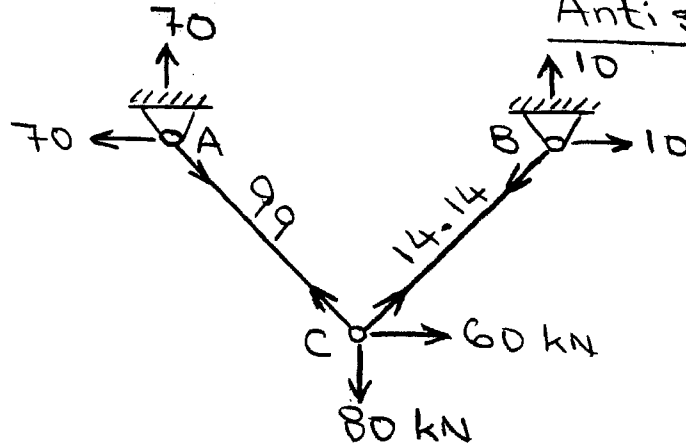
10.16



Symmetric

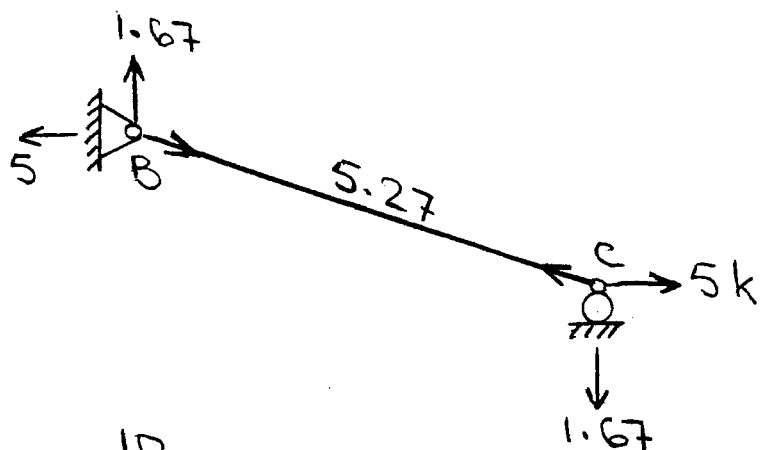


Antisymmetric

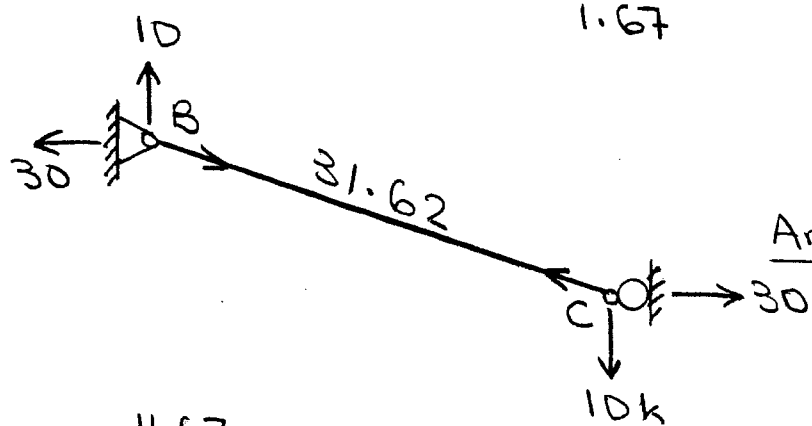


Member forces

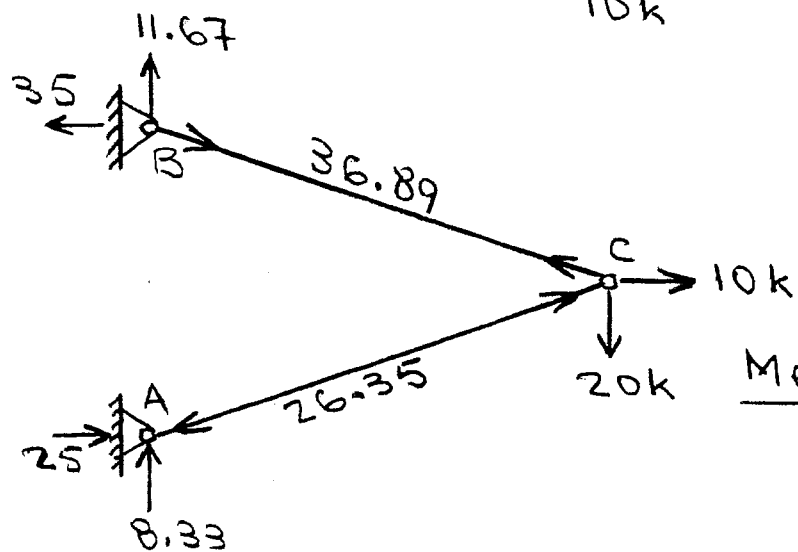
10.17



Symmetric

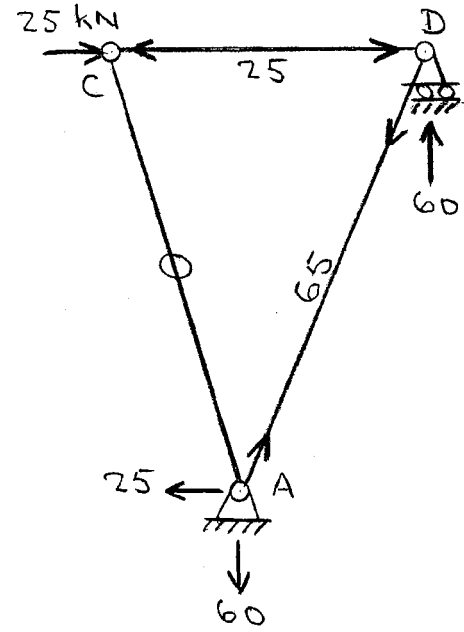


Antisymmetric

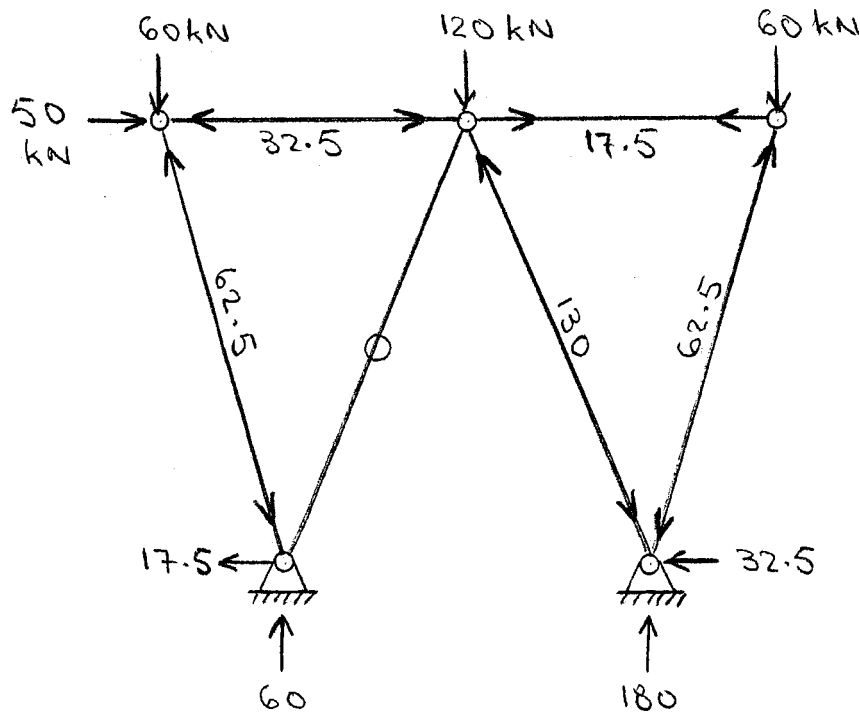


Member Forces

See solution of Problem 10.3

Symmetric

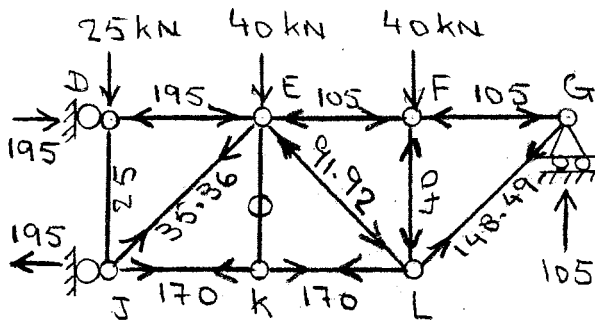
Antisymmetric



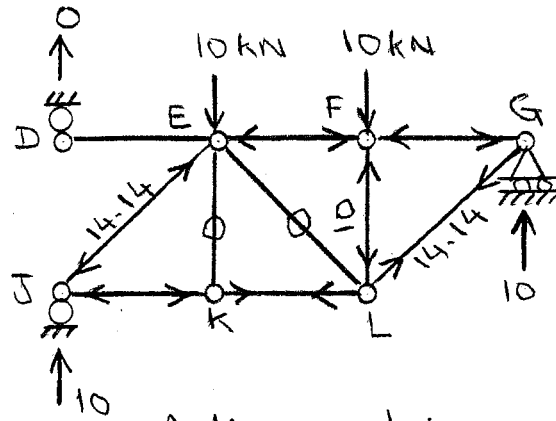
Member Forces

10.19

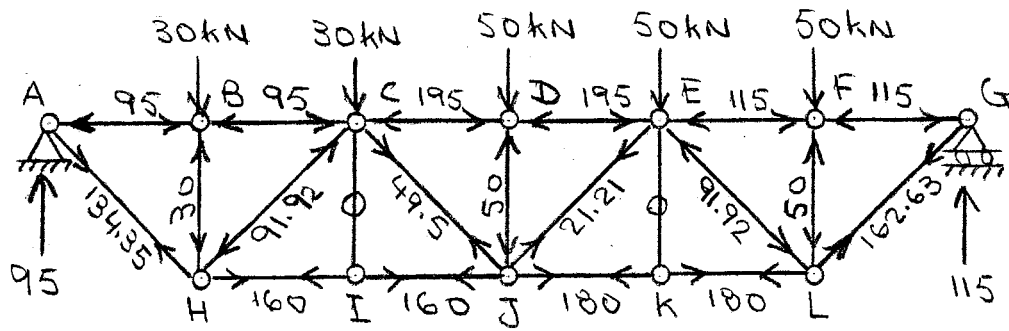
See solution of Problem 10.4.



Symmetric

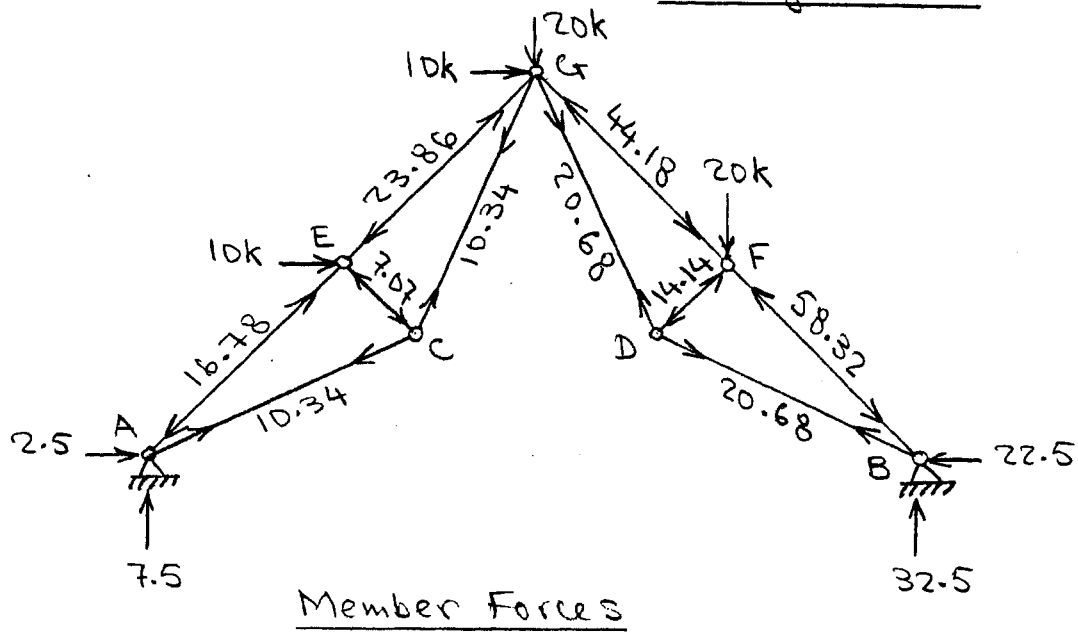
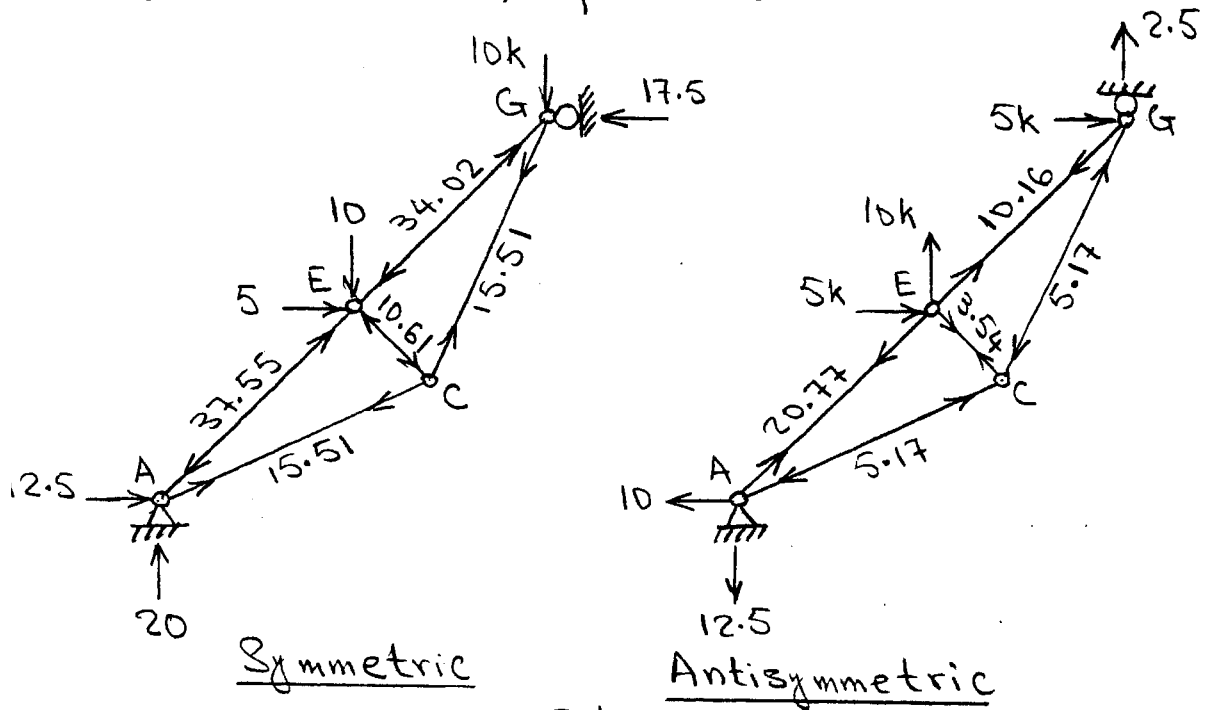


Antisymmetric

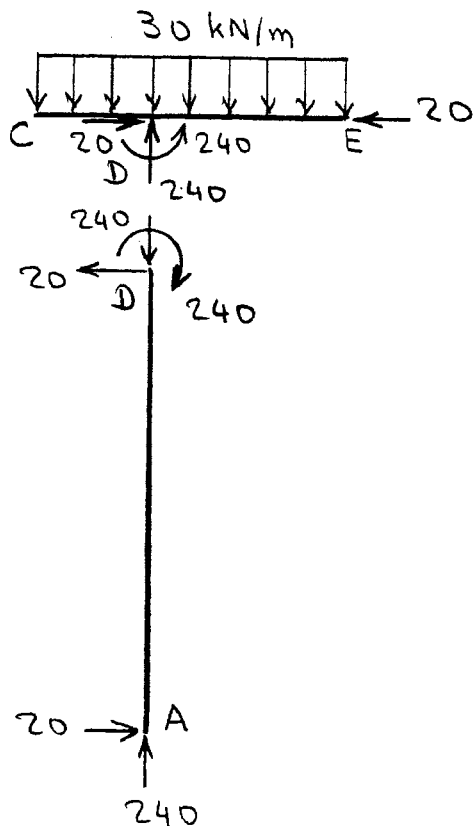
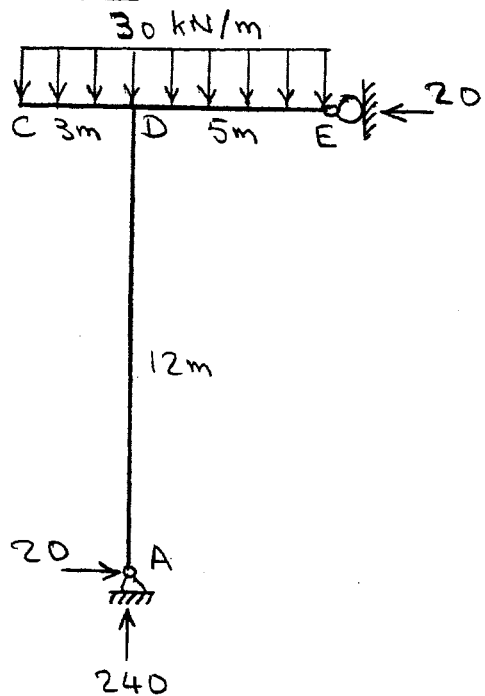


Member Forces

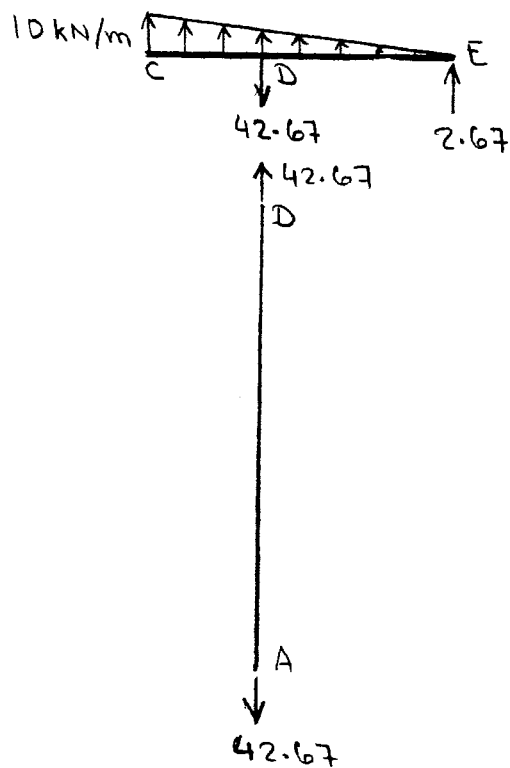
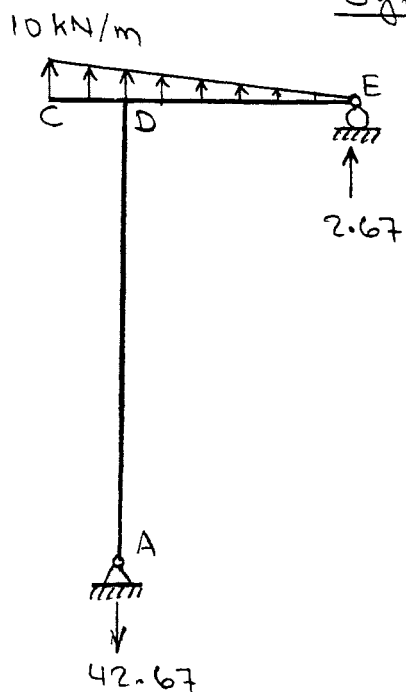
10.20 See solution of Problem 10.5.



10.21 See solution of Problem 10.6.

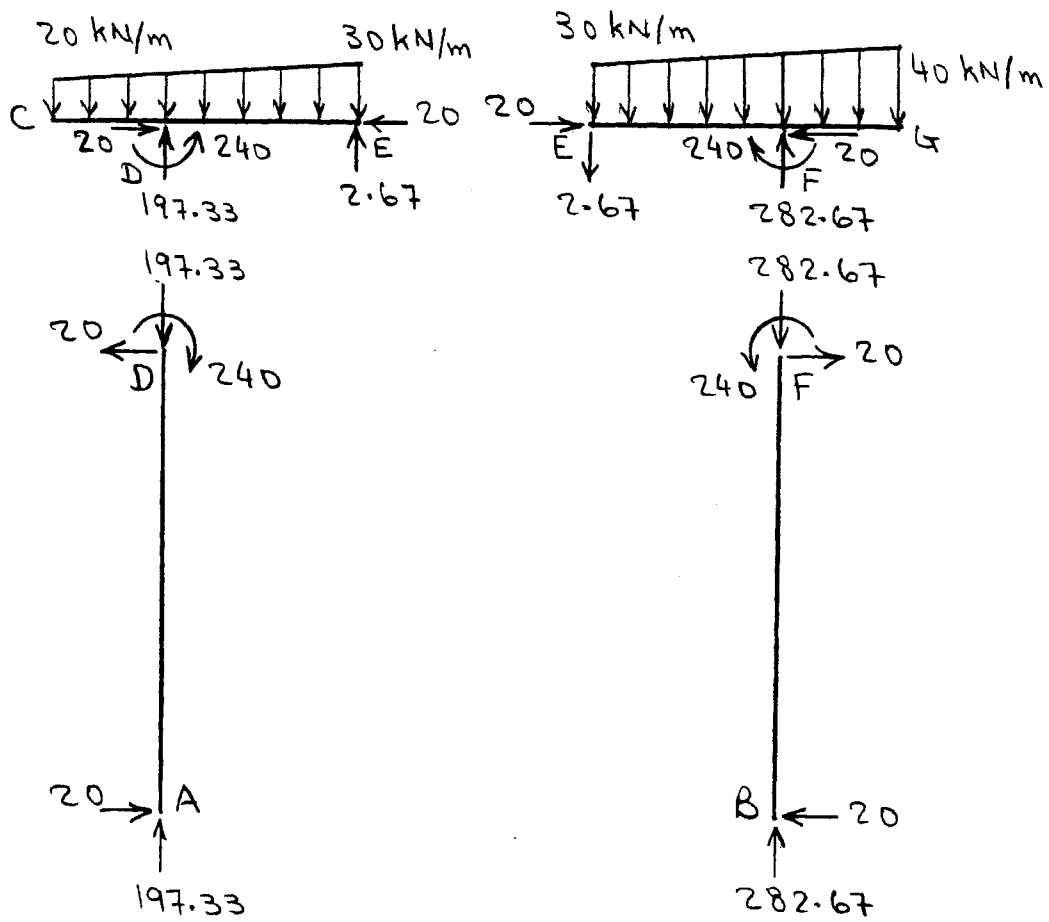


Symmetric



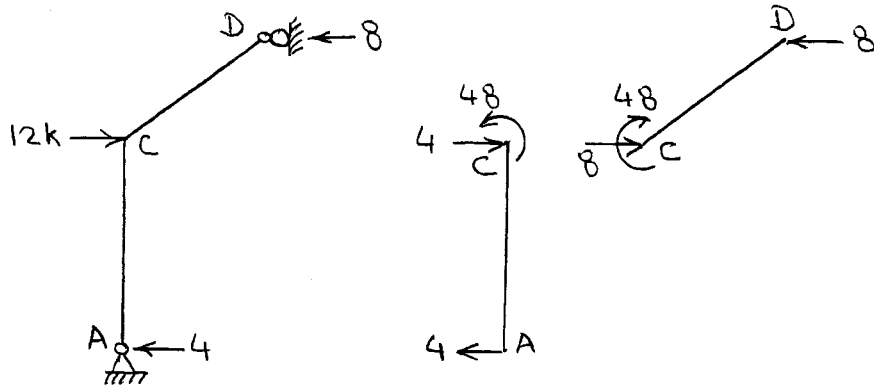
Anti symmetric

10.21 (Contd.)

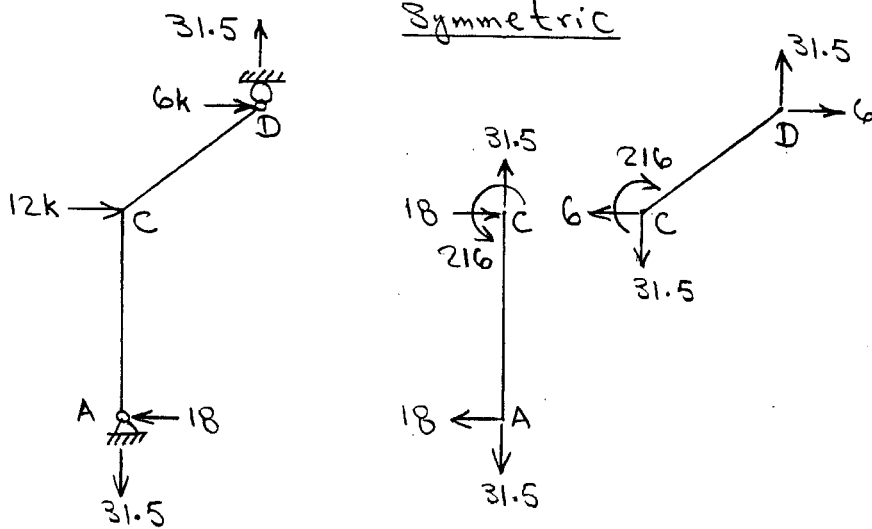


Member End Forces

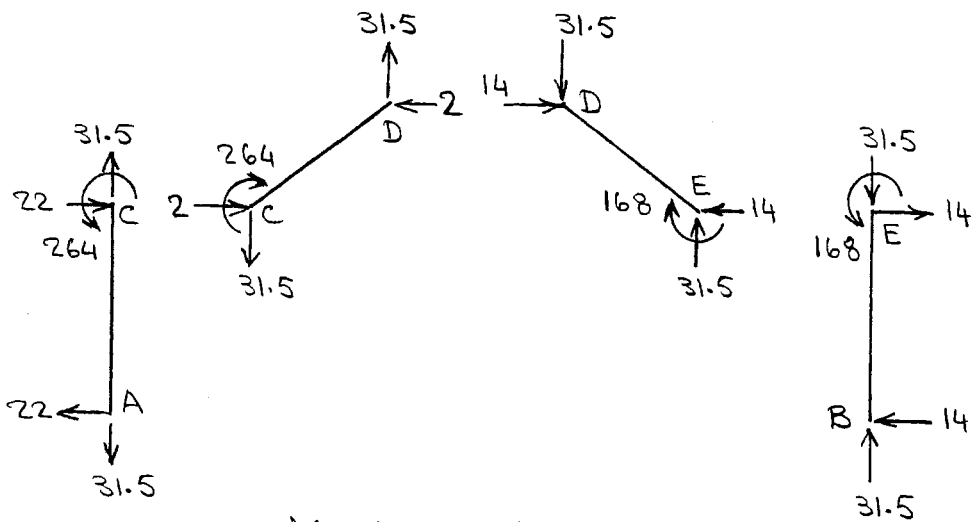
10.22 See solution of Problem 10.7.



Symmetric

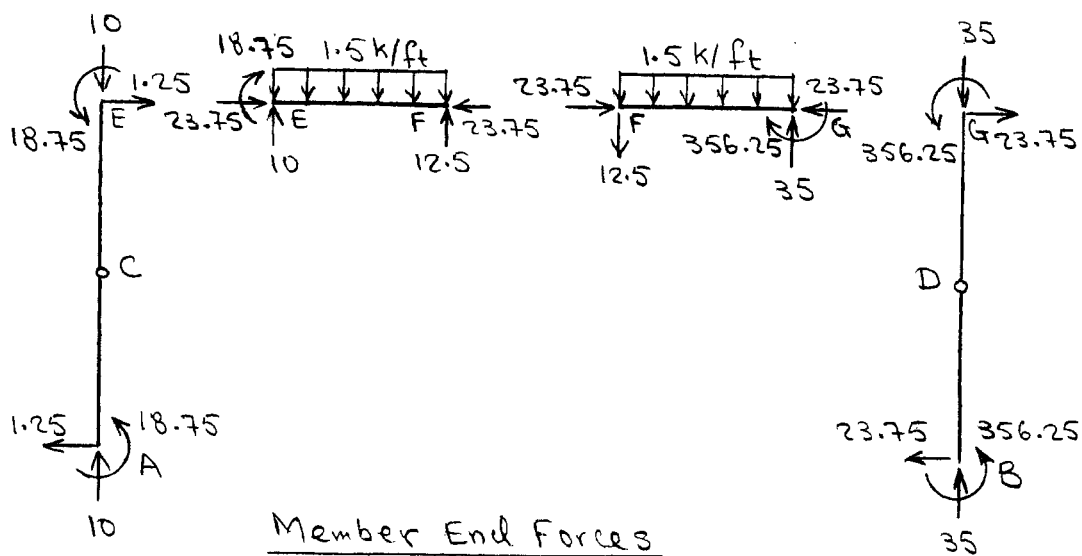
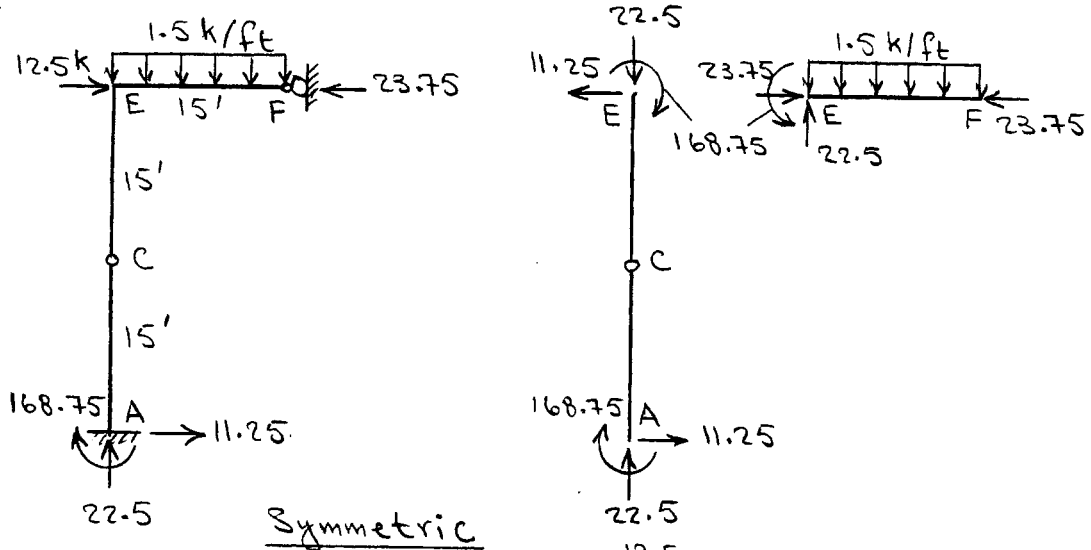


Antisymmetric

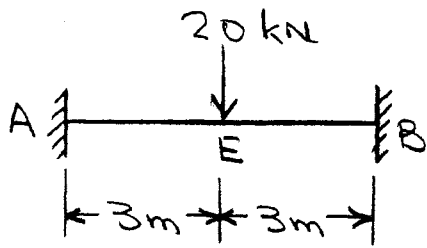


Member End Forces

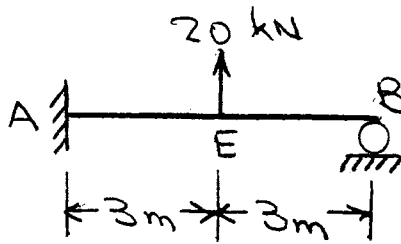
10.23 See solution of Problem 10.8.



10.24 See solution of Problem 10.9.

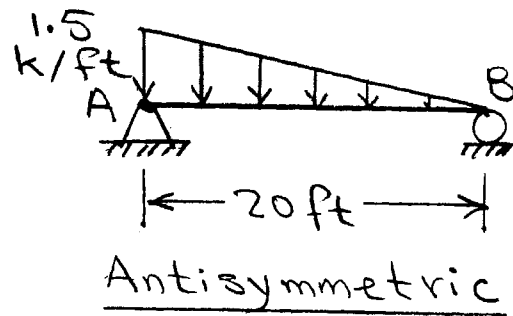
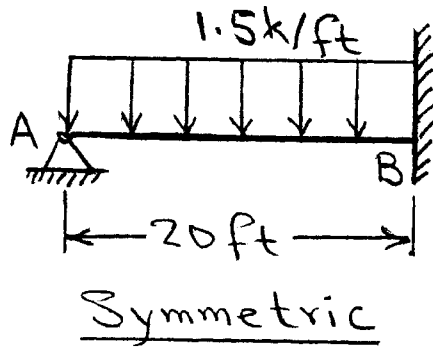


Symmetric



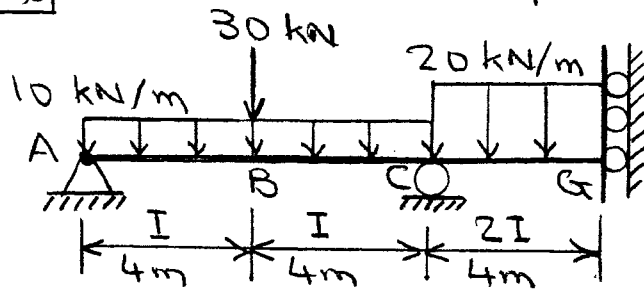
Antisymmetric

10.25 See solution of Problem 10.10.

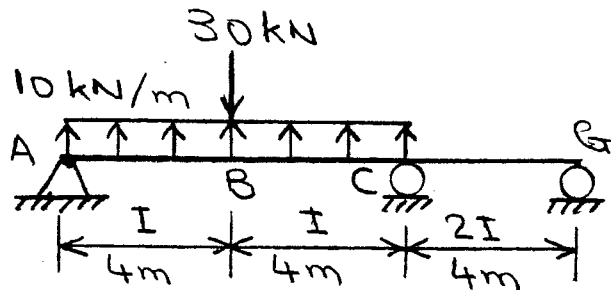


10.26

See solution of Problem 10.11



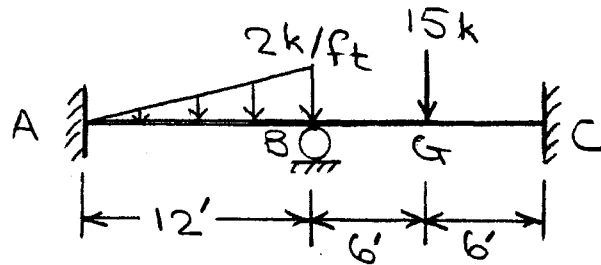
Symmetric



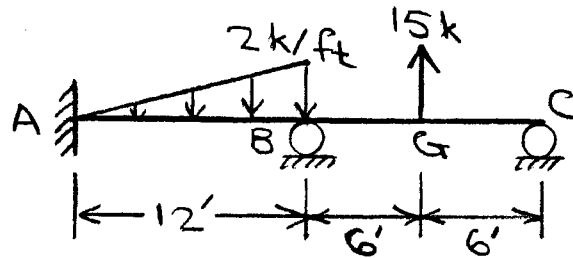
Antisymmetric

10.27

See solution of Problem 10.12.

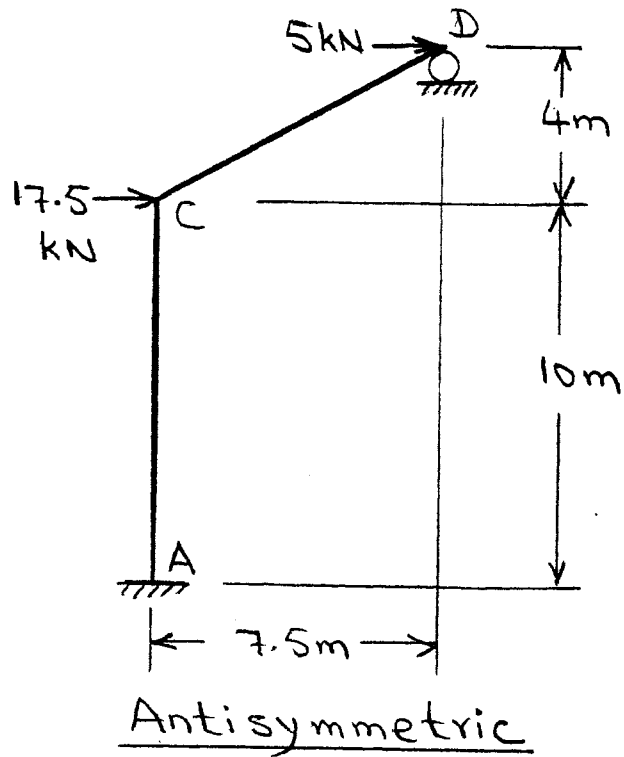
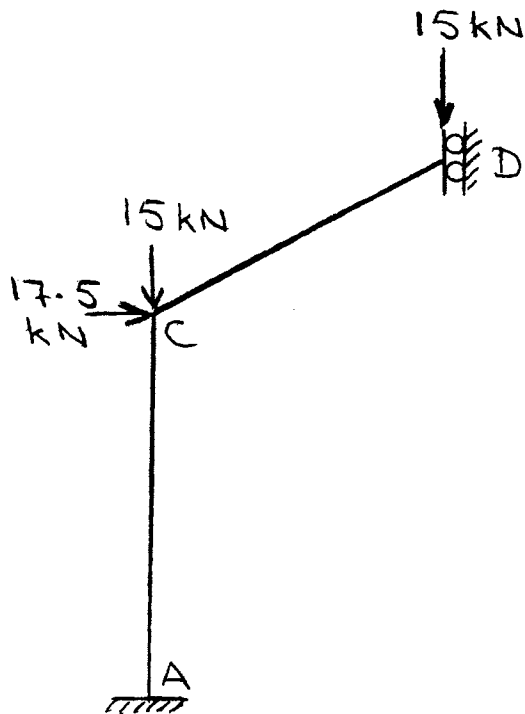


Symmetric

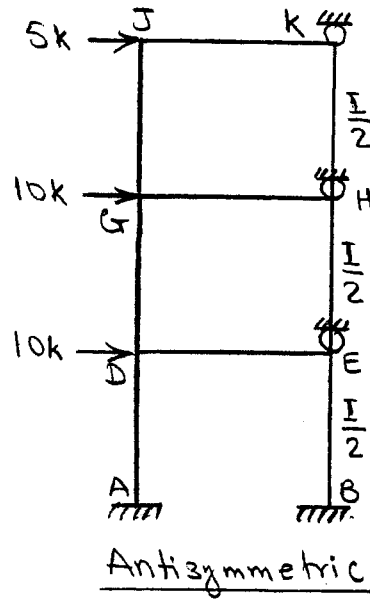
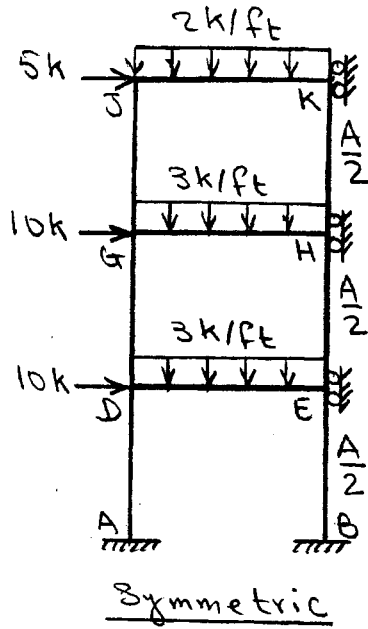


Antisymmetric

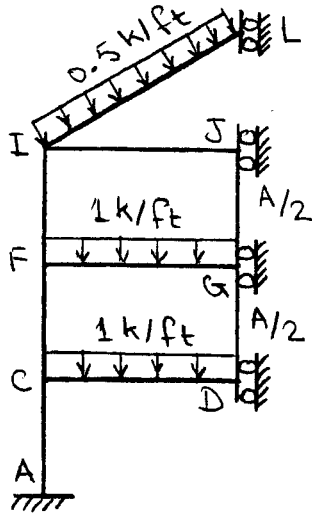
10.28 See solution of Problem 10.13.



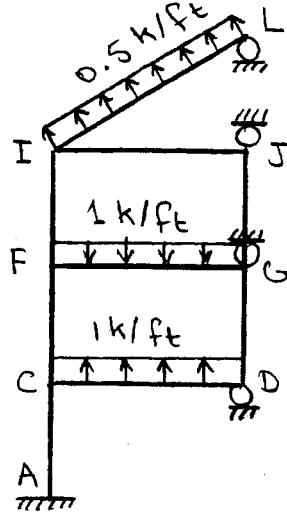
10.29 See solution of Problem 10.14.



10.30 See solution of Problem 10.15.



Symmetric



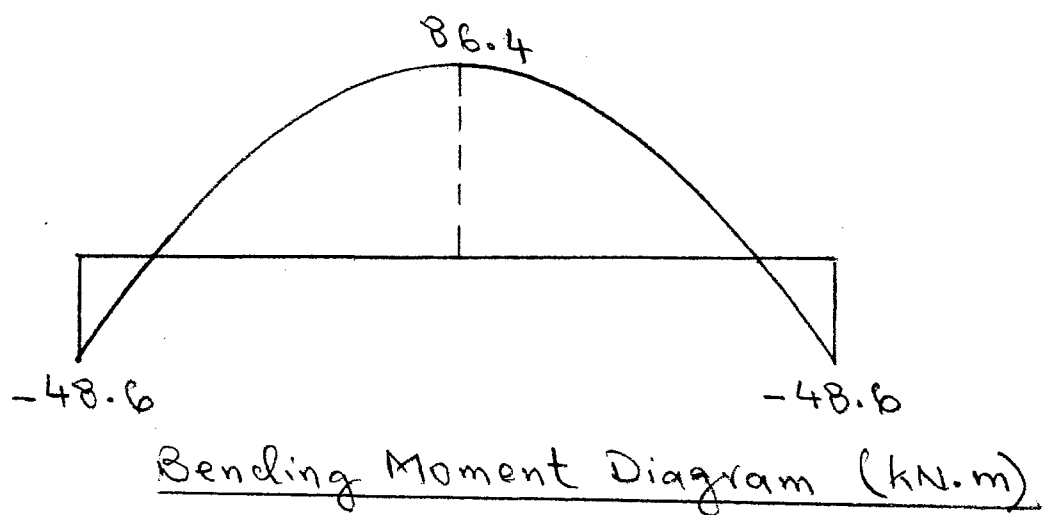
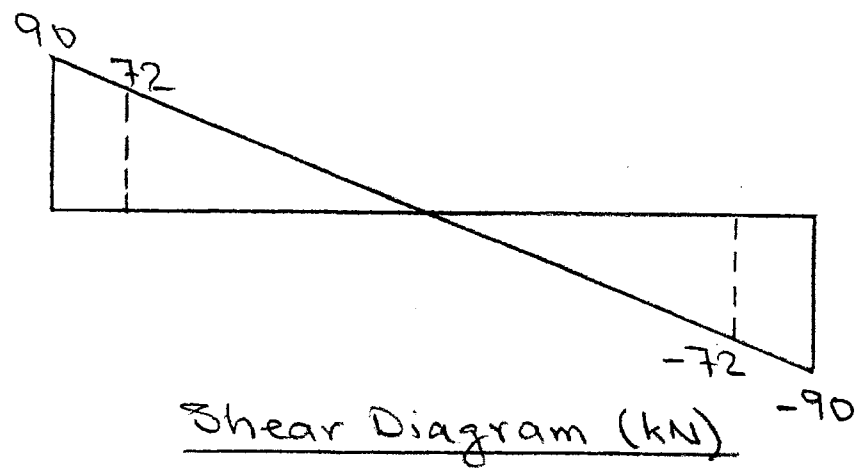
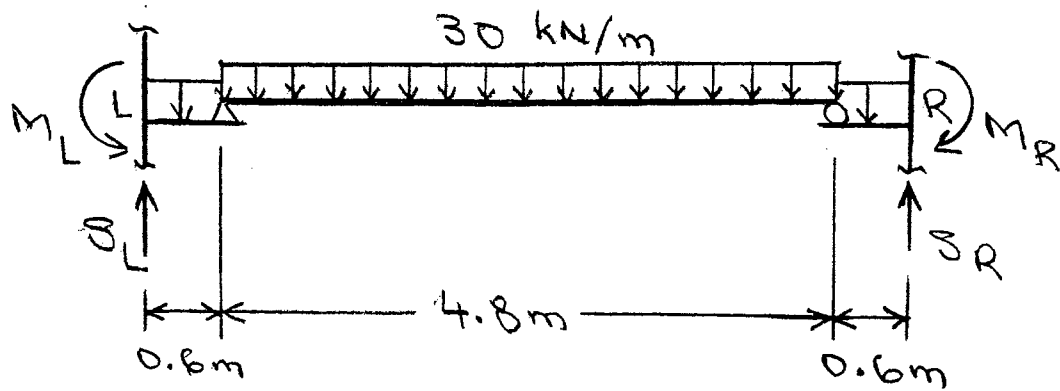
Antisymmetric

Chapter Twelve

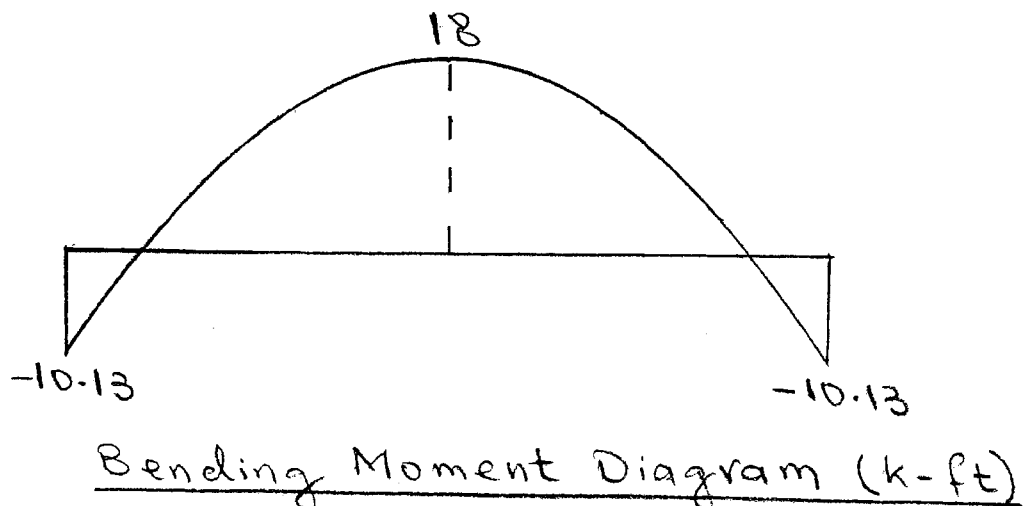
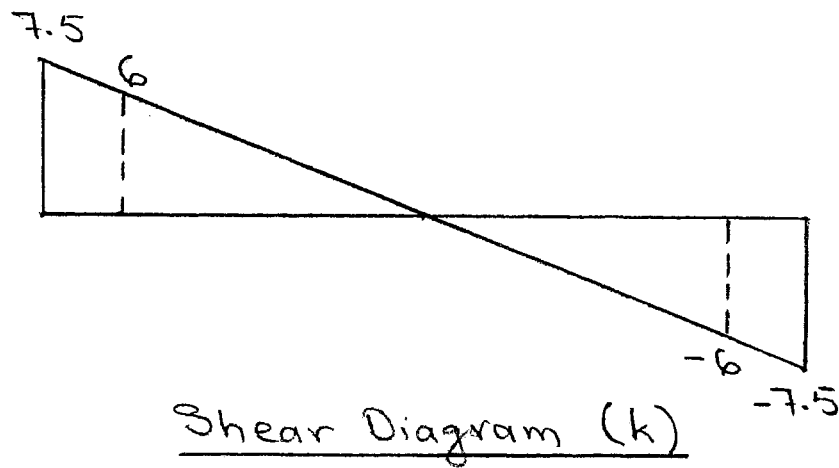
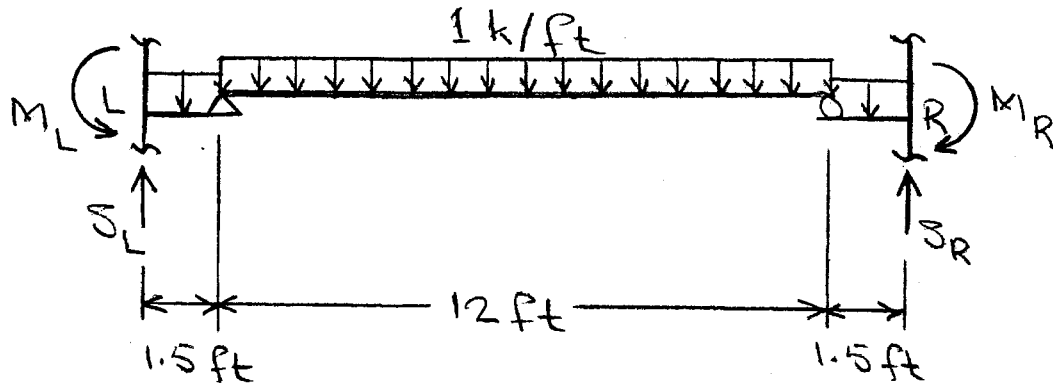
Approximate Analysis of Rectangular Building Frames

CHAPTER 12

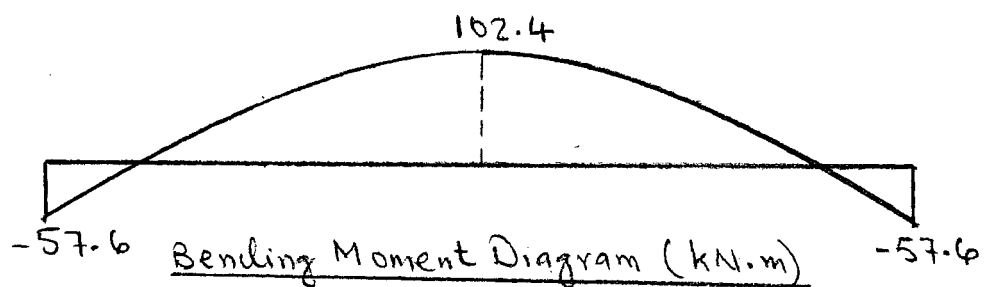
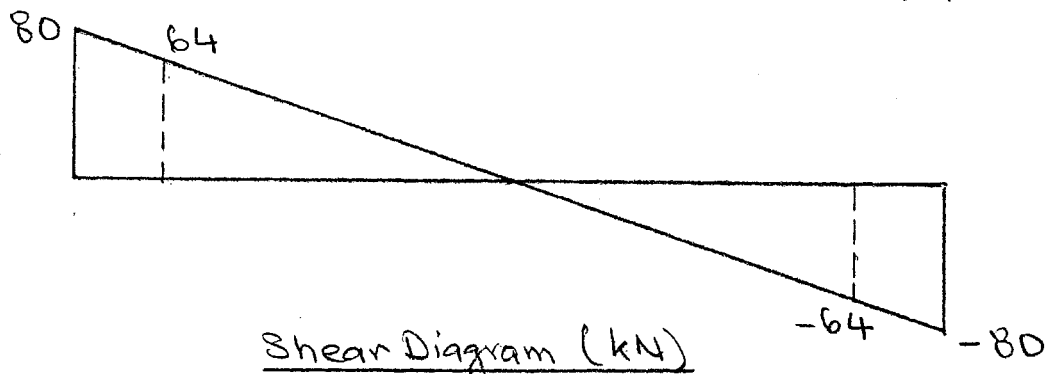
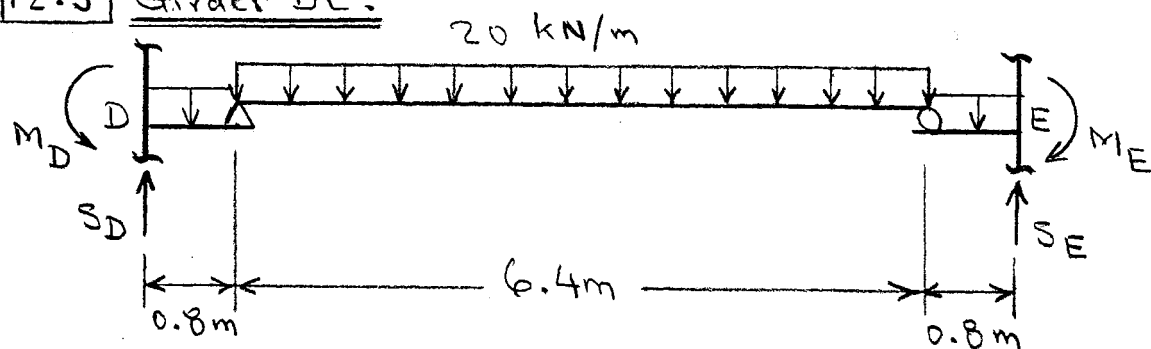
12.1



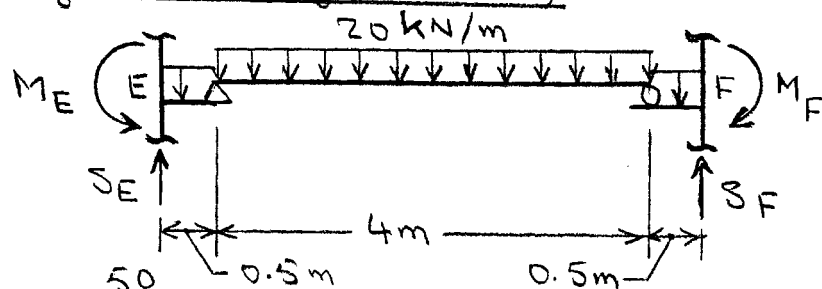
12.2



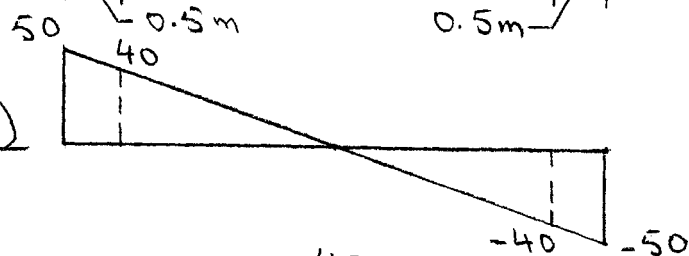
12.3 Girder DE:



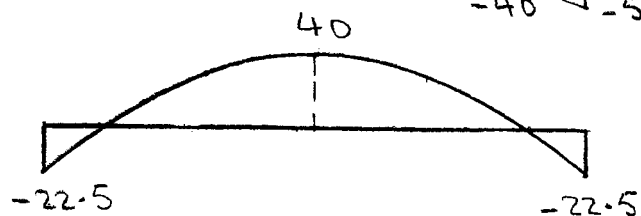
Girder EF:



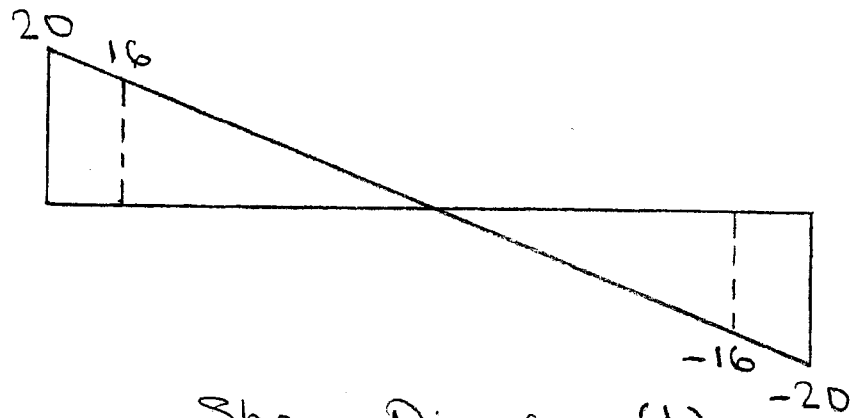
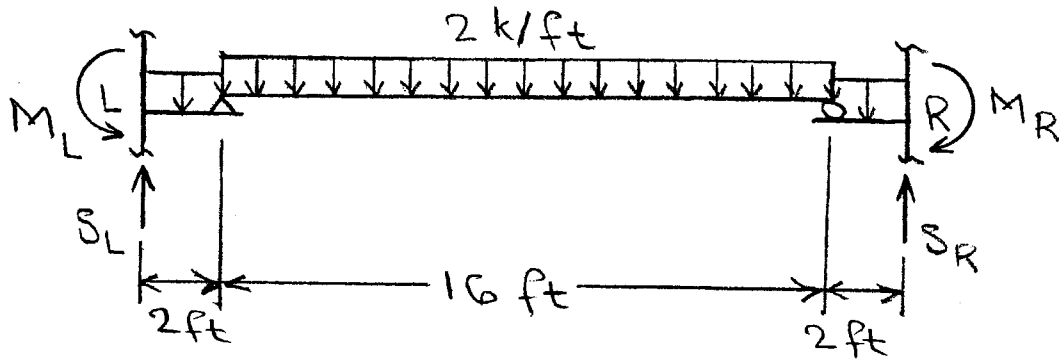
Shear Diagram (kN)



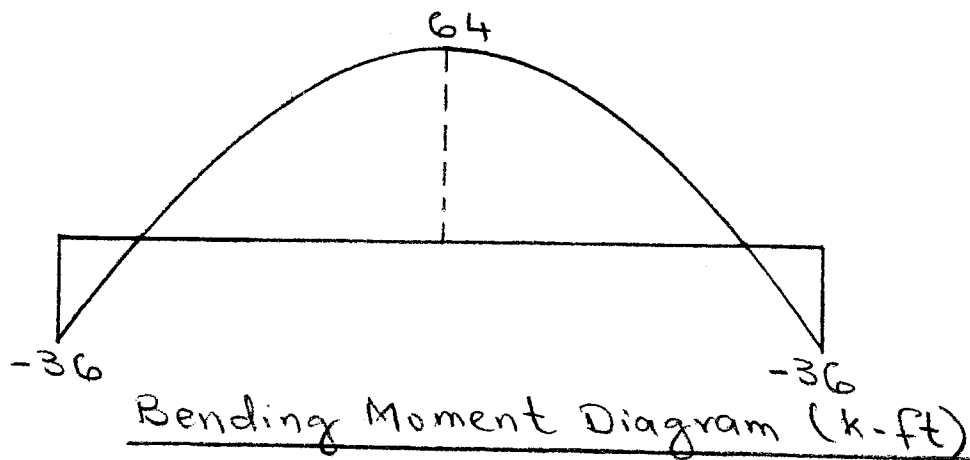
Bending Moment Diagram (kN.m)



12.4

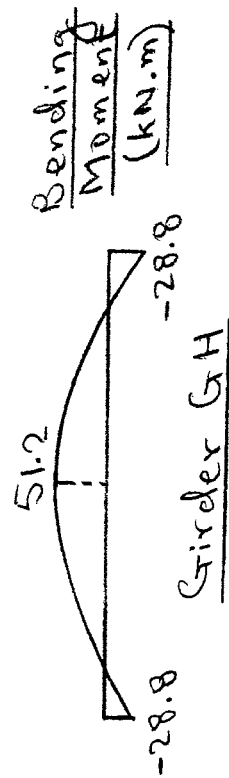
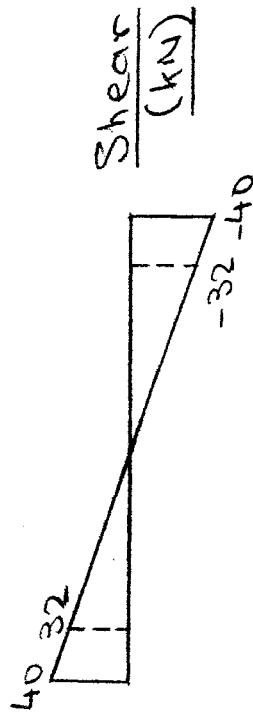
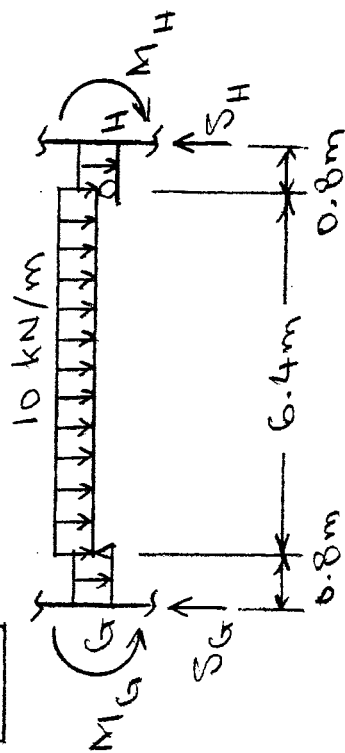


Shear Diagram (k)



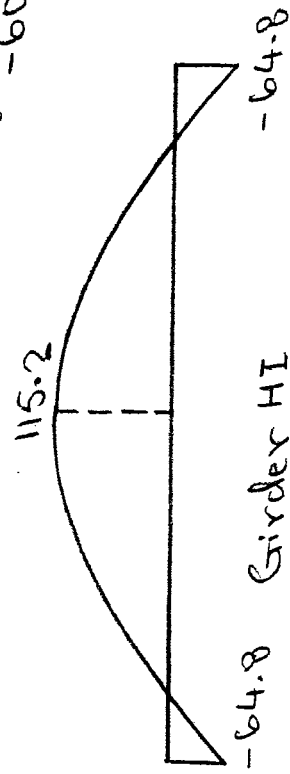
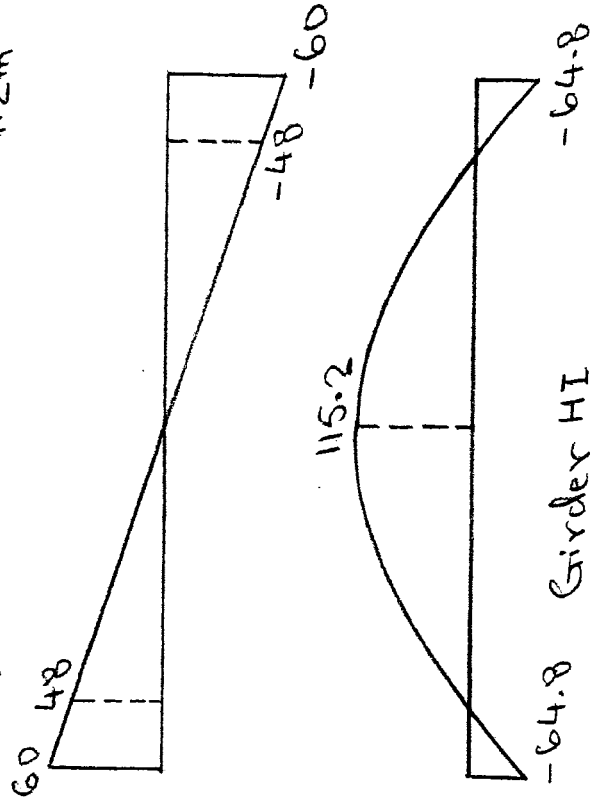
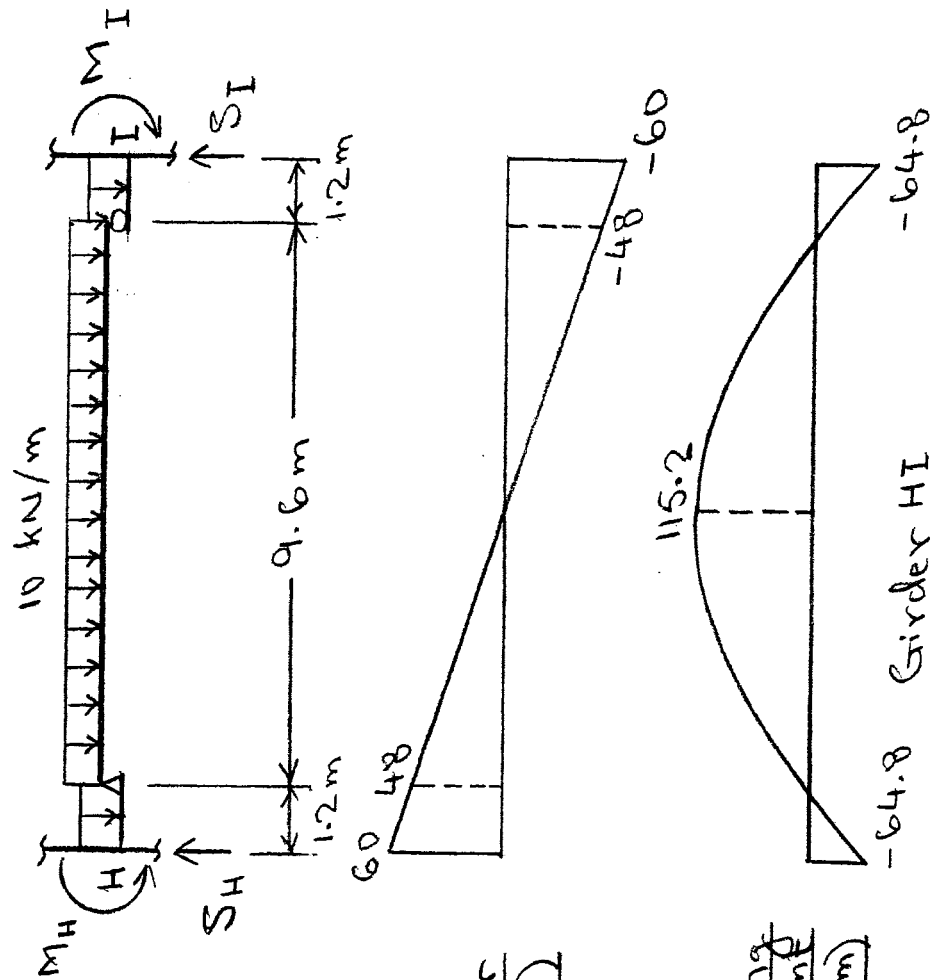
Bending Moment Diagram (k-ft)

12-5

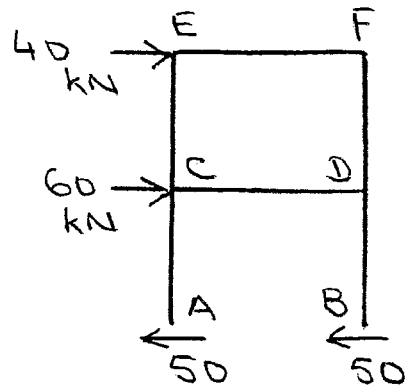
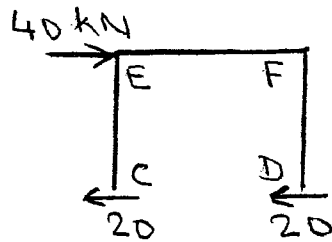


Shear in Girder DE = 2 x Shear in Girder GH.
 Moment in Girder DE = 2 x Moment in Girder GH.

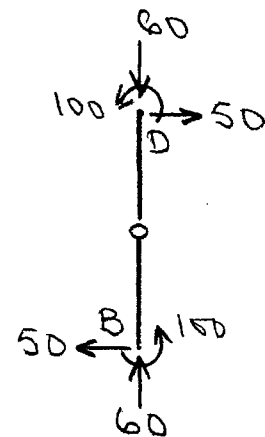
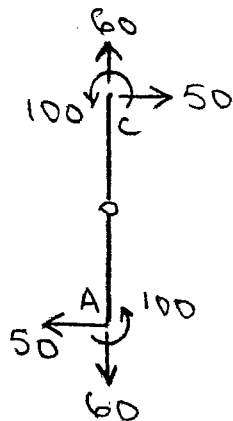
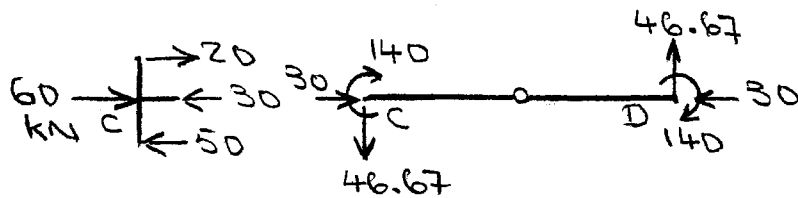
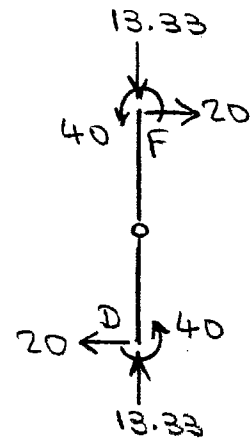
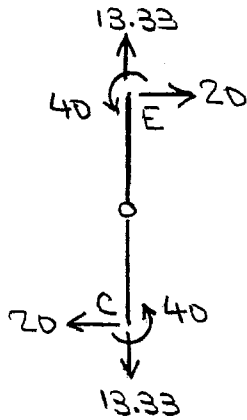
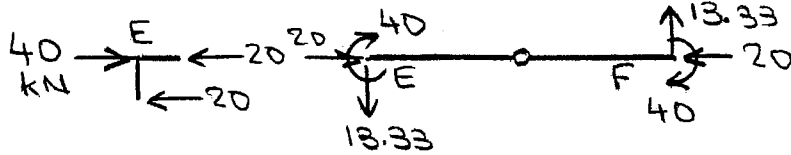
Shear in Girder EF = 2 x Shear in Girder HI.
 Moment in Girder EF = 2 x Moment in Girder HI.



12.6

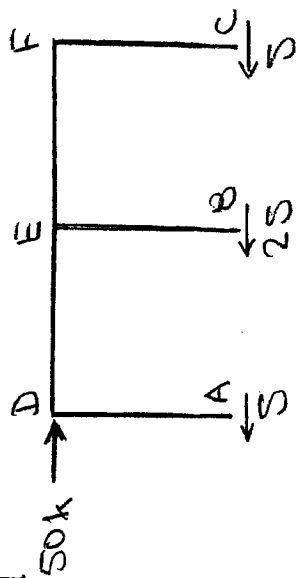


Column Shears



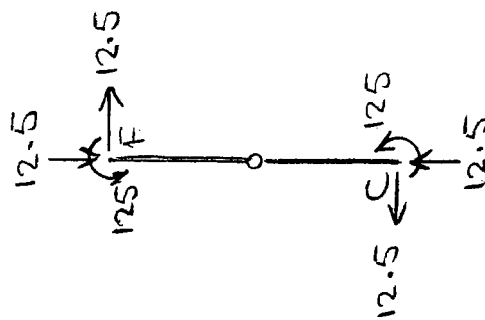
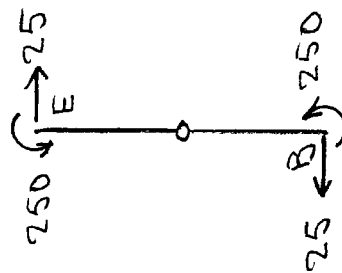
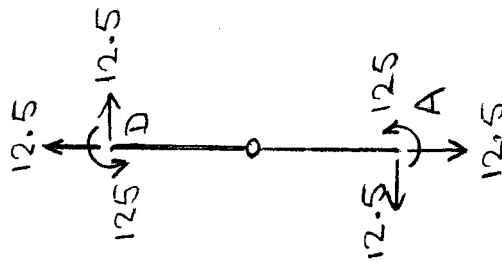
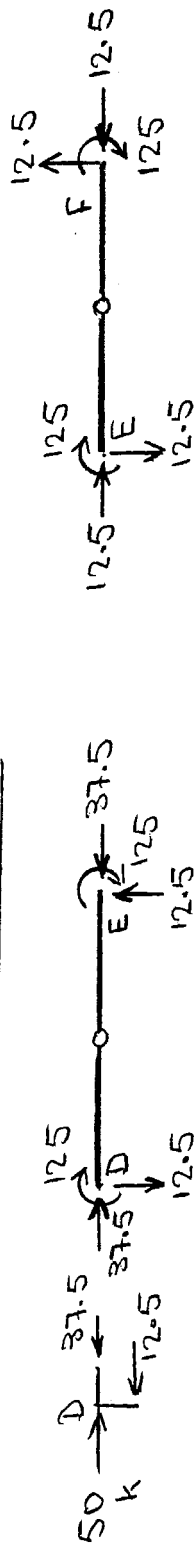
Member Axial Forces, Shears, and Moments

12.7



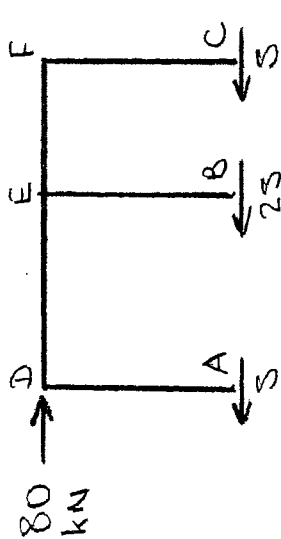
$$4S = 50 \quad S = 12.5 \text{ k}$$

Column Shears



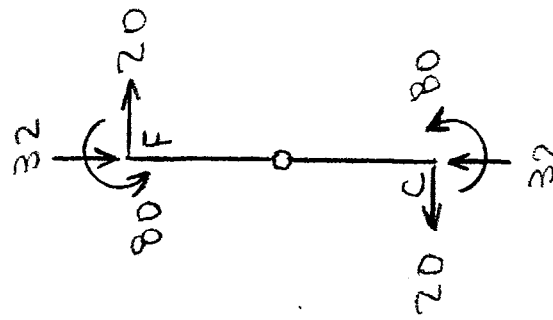
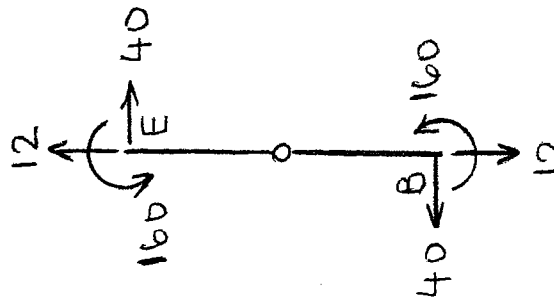
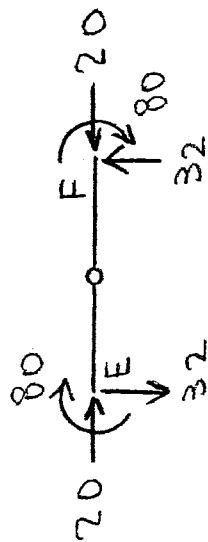
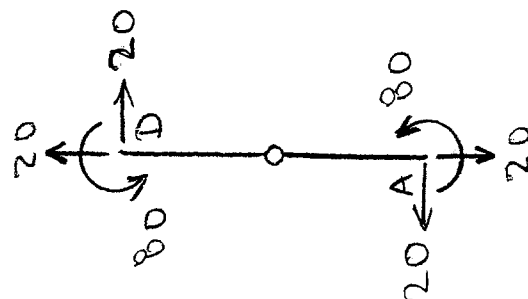
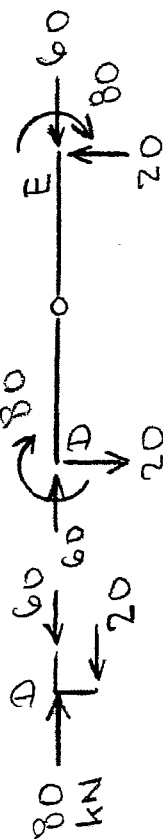
Member Axial Forces, Shears, and Moments

12.8

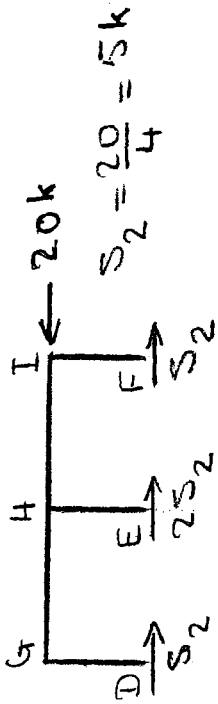


$$4S = 80 \quad S = 20 \text{ kN}$$

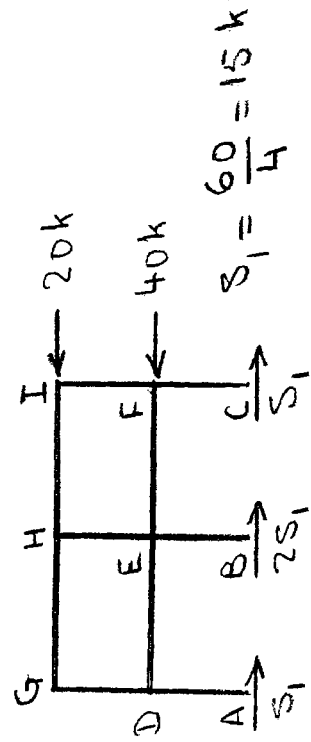
Column Shears



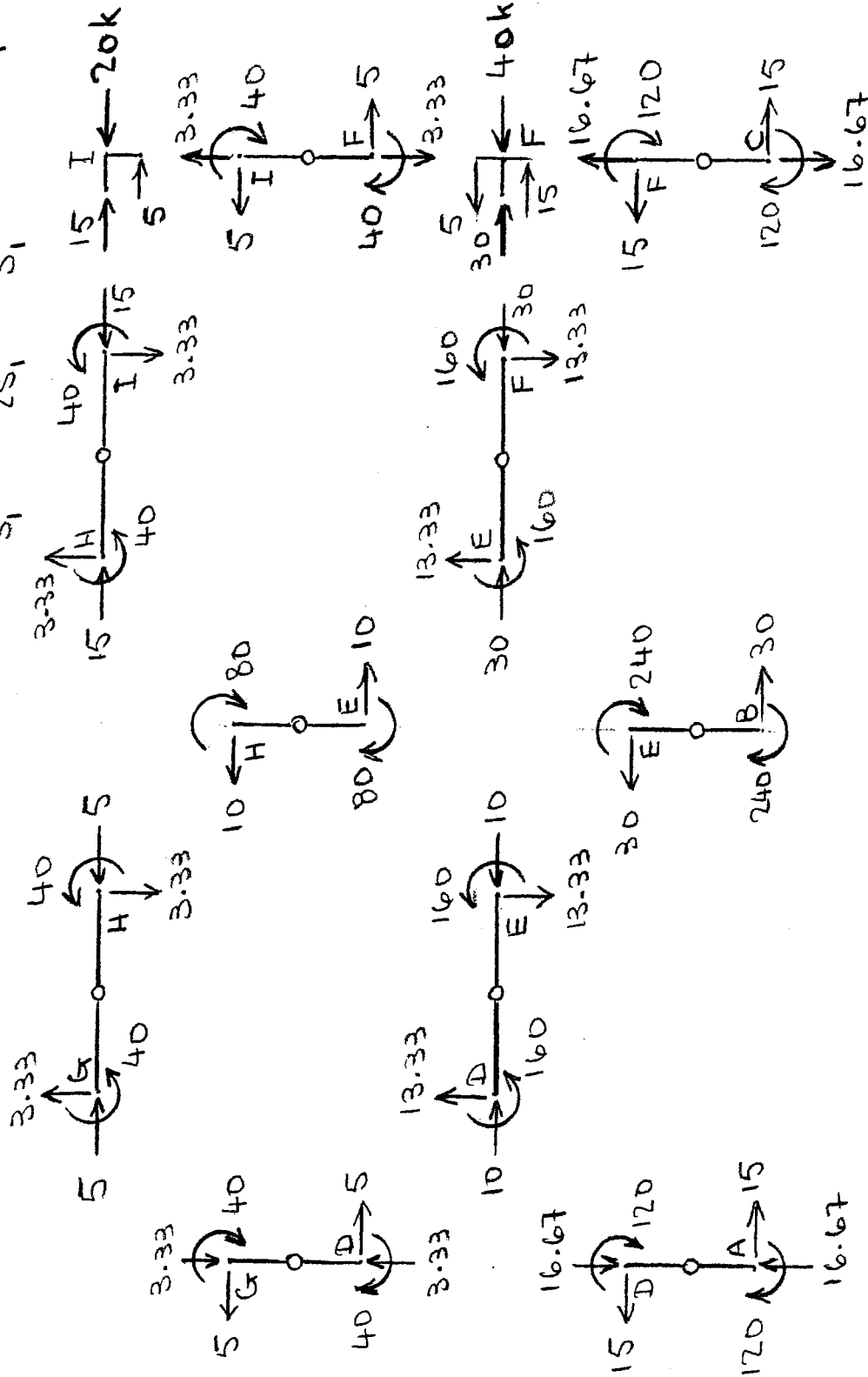
12.9



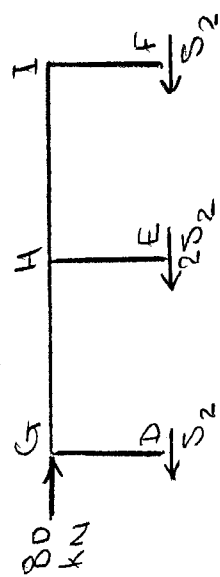
$$S_2 = \frac{20}{4} = 5k$$



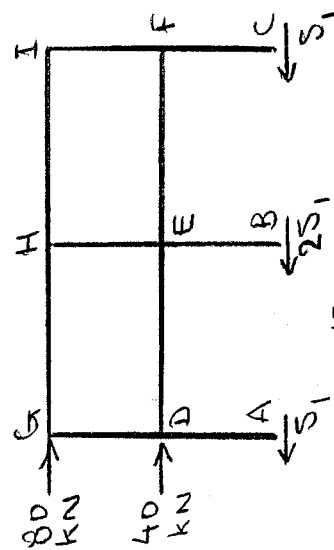
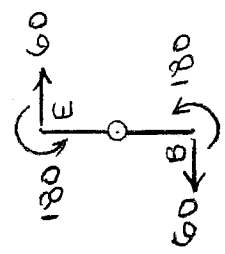
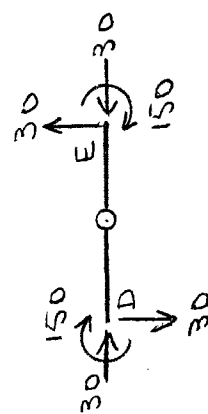
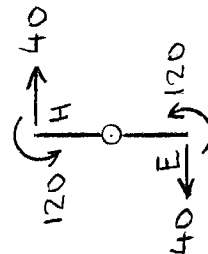
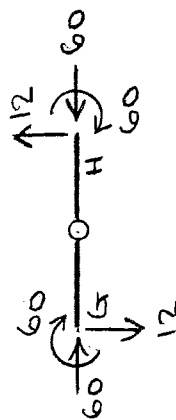
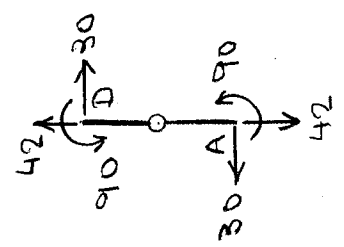
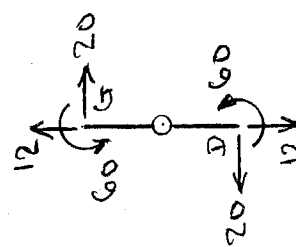
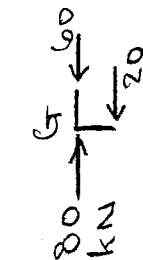
$$S_1 = \frac{60}{4} = 15k$$



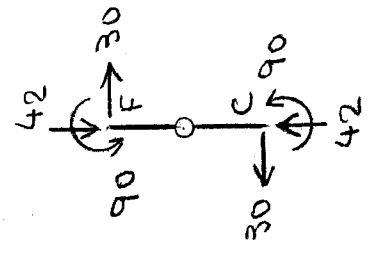
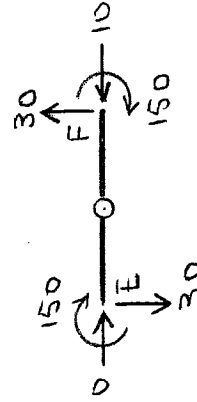
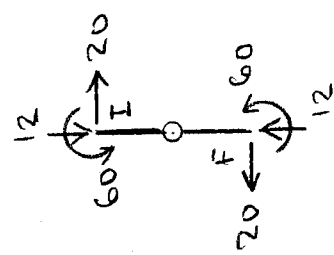
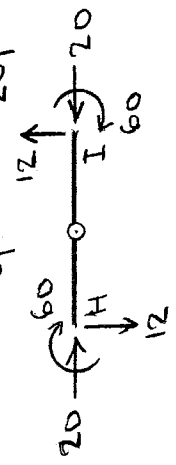
12.10

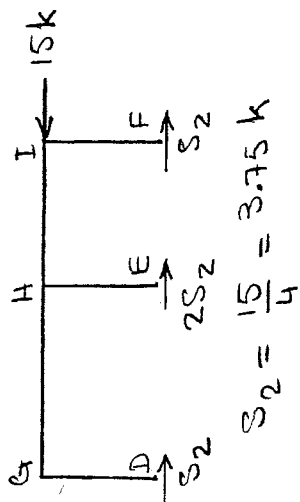


$$S_2 = \frac{80}{4} = 20 \text{ kN}$$



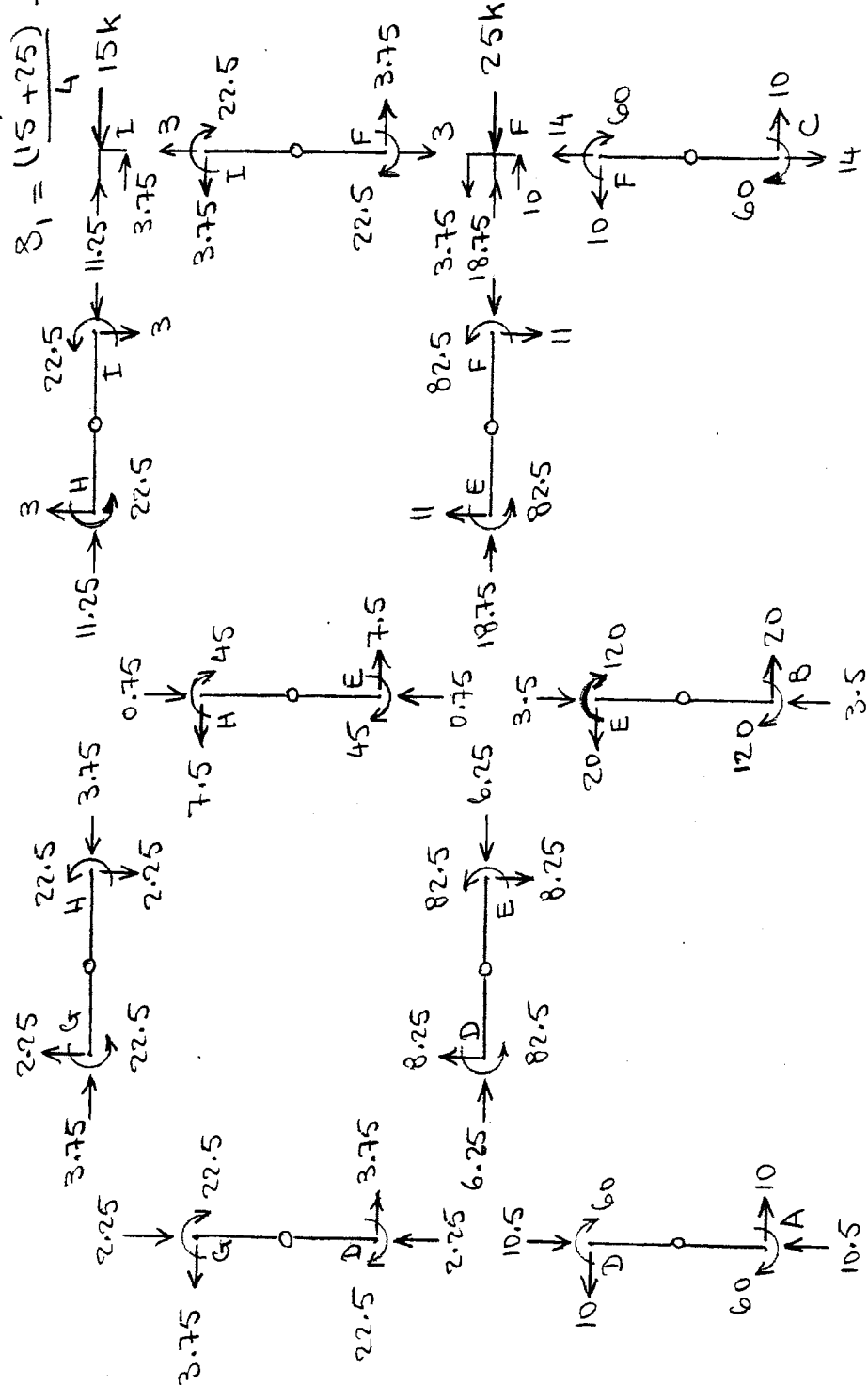
$$S_1 = \frac{(80+40)}{4} = 30 \text{ kN}$$



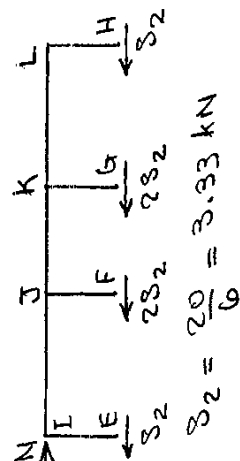


$$s_2 = \frac{5}{5} = 3.75 \text{ k}$$

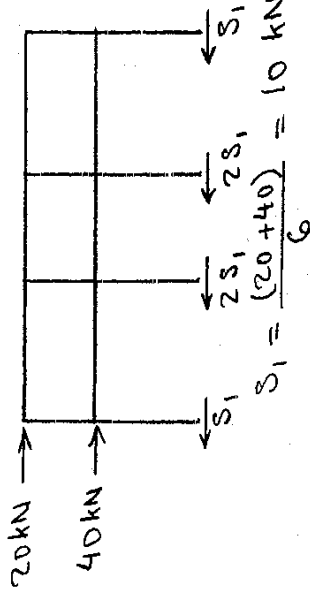
$$S_1 = \frac{(15+25)}{2} = 10k$$



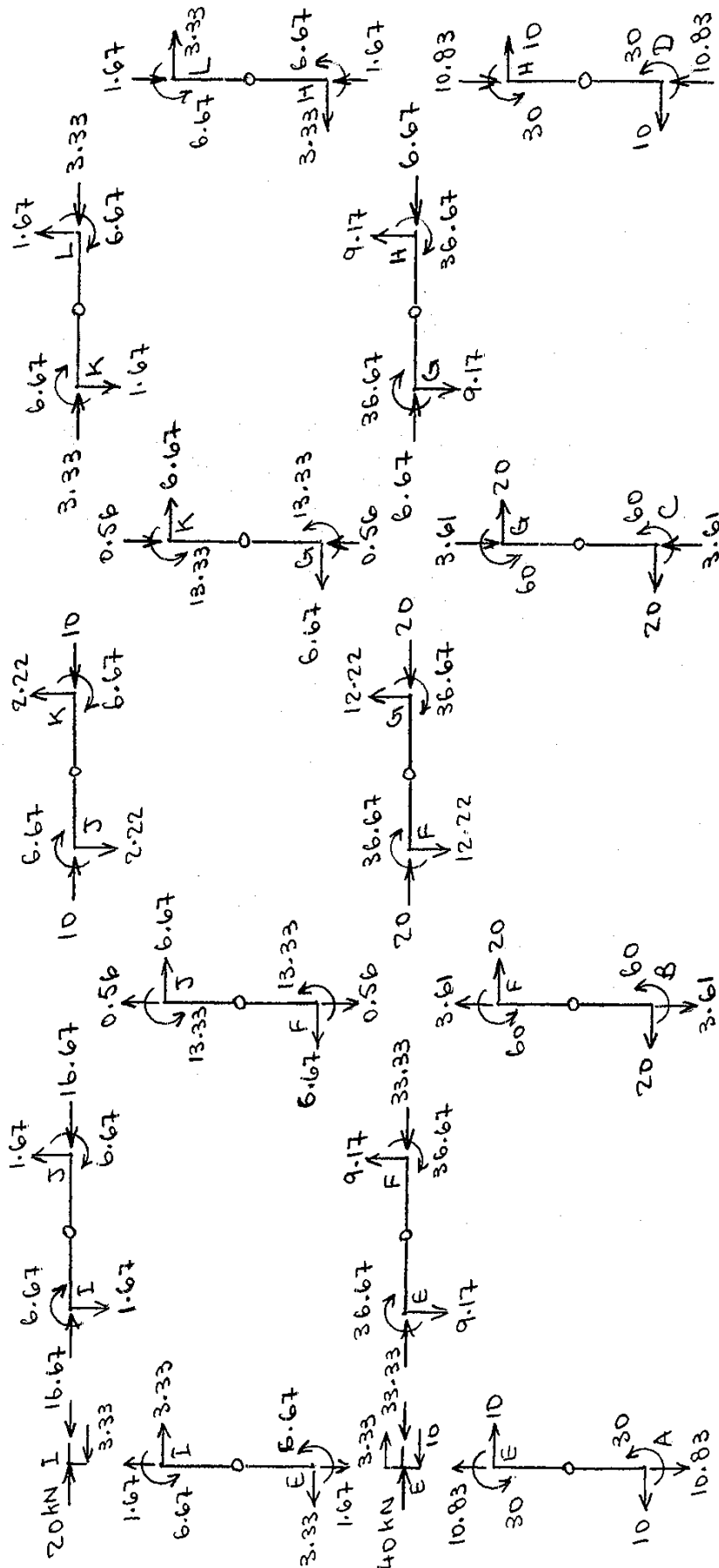
12-12



$$S_2 = \frac{20}{6} = 3.33 \text{ kN}$$

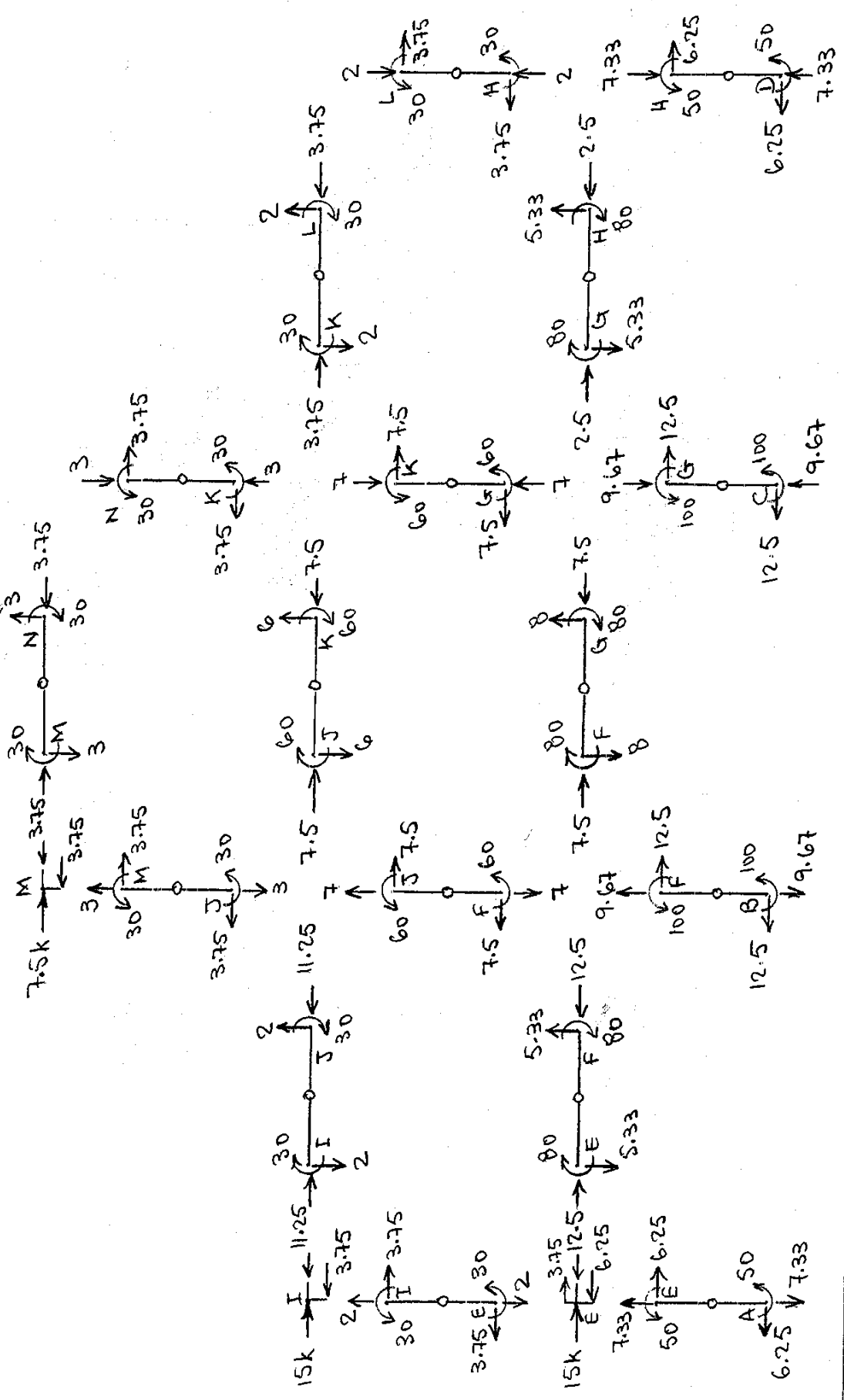


$$S_1 = \frac{(20+40)}{6} = 10 \text{ kN}$$

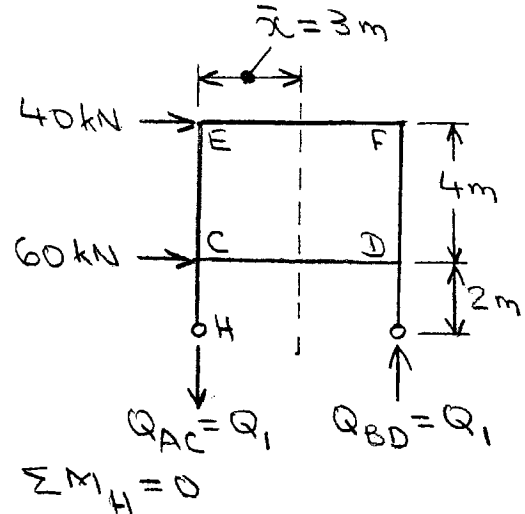
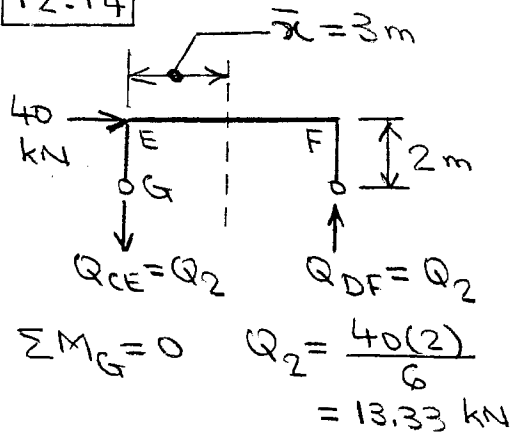


12.13

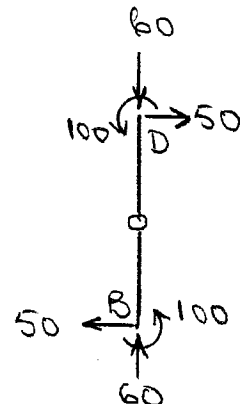
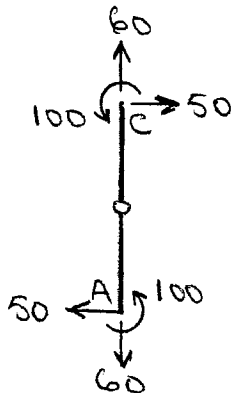
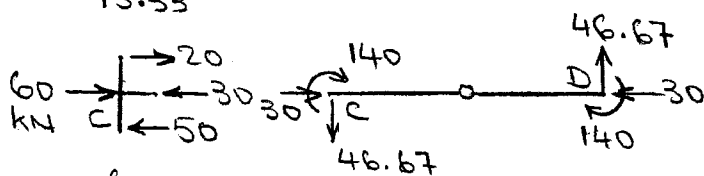
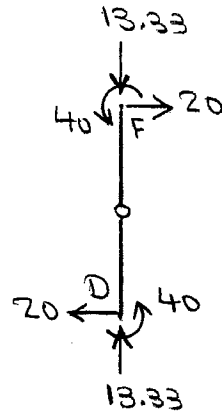
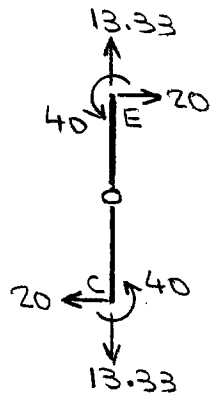
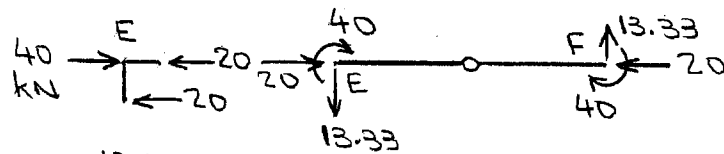
$$S_3 = \frac{7.5}{2} = 3.75 \text{ k}; \quad S_2 = \frac{(7.5+15)}{6} = 3.75 \text{ k}; \quad S_1 = \frac{(7.5+15+15)}{6} = 6.25 \text{ k}$$

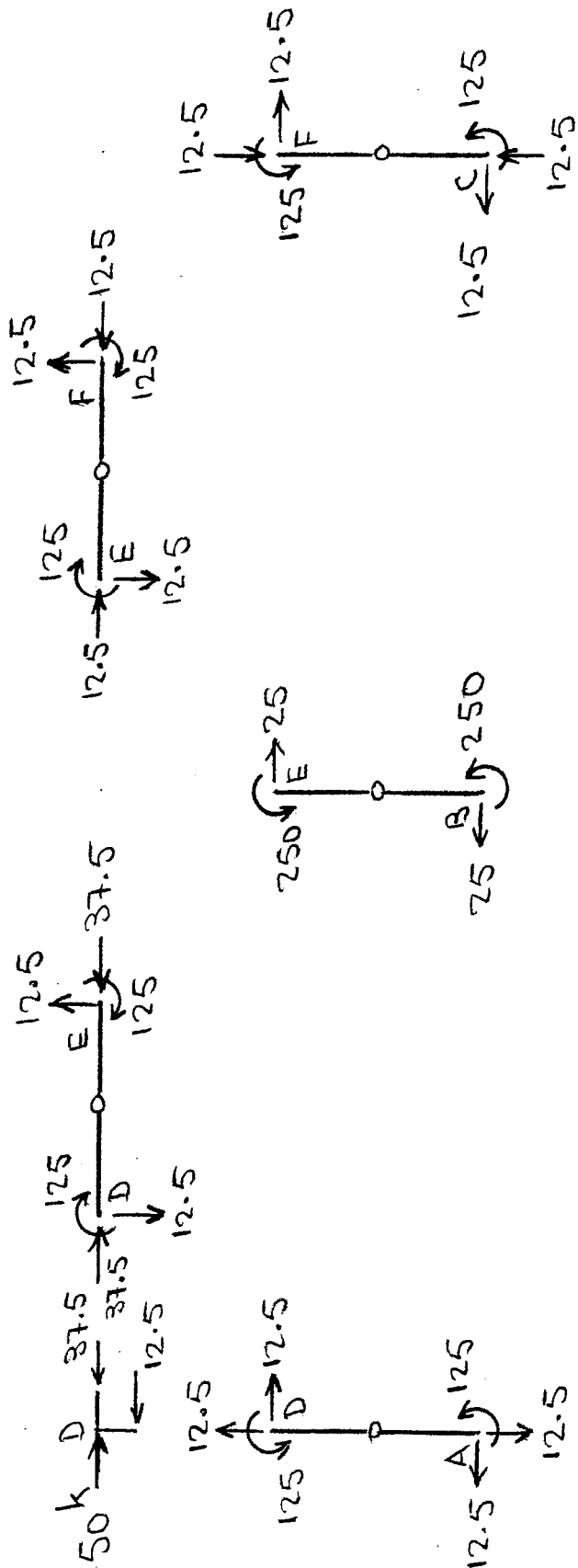
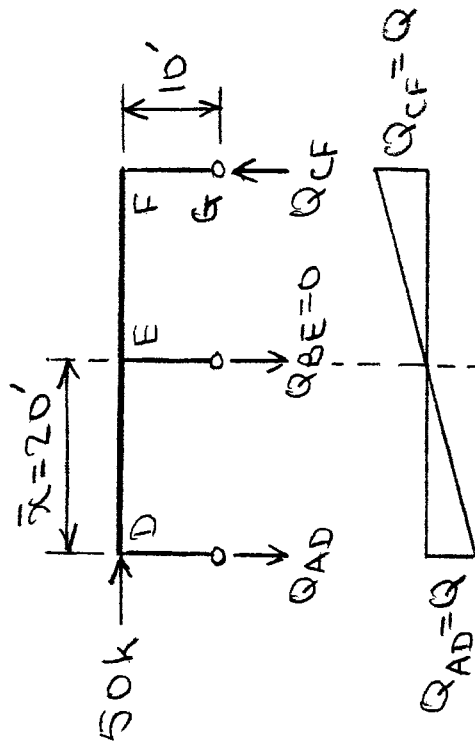


12.14

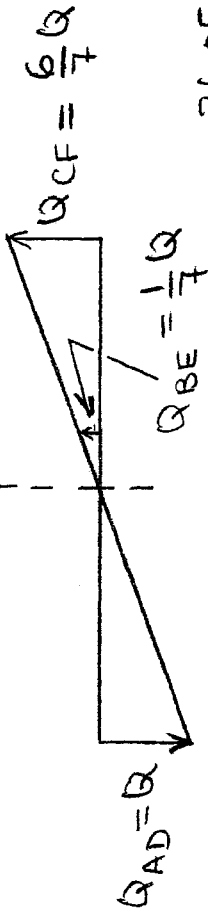
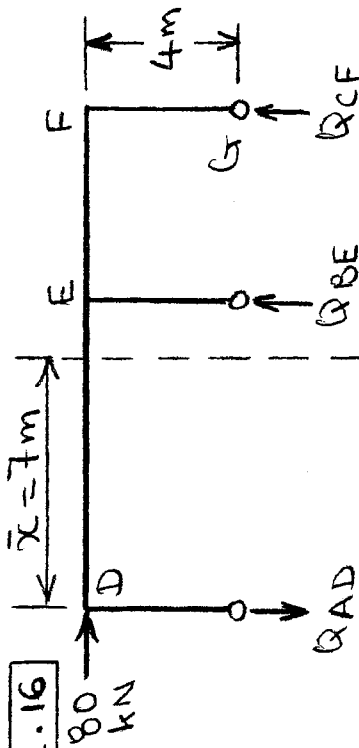


$$Q_1 = \frac{40(6) + 60(2)}{6} = 60 \text{ kN}$$



$$\begin{aligned} \rightarrow \Sigma M_G &= 0 \\ -50(10) + Q(40) &= 0 \\ Q &= 12.5 \text{ k} \end{aligned}$$


12.16



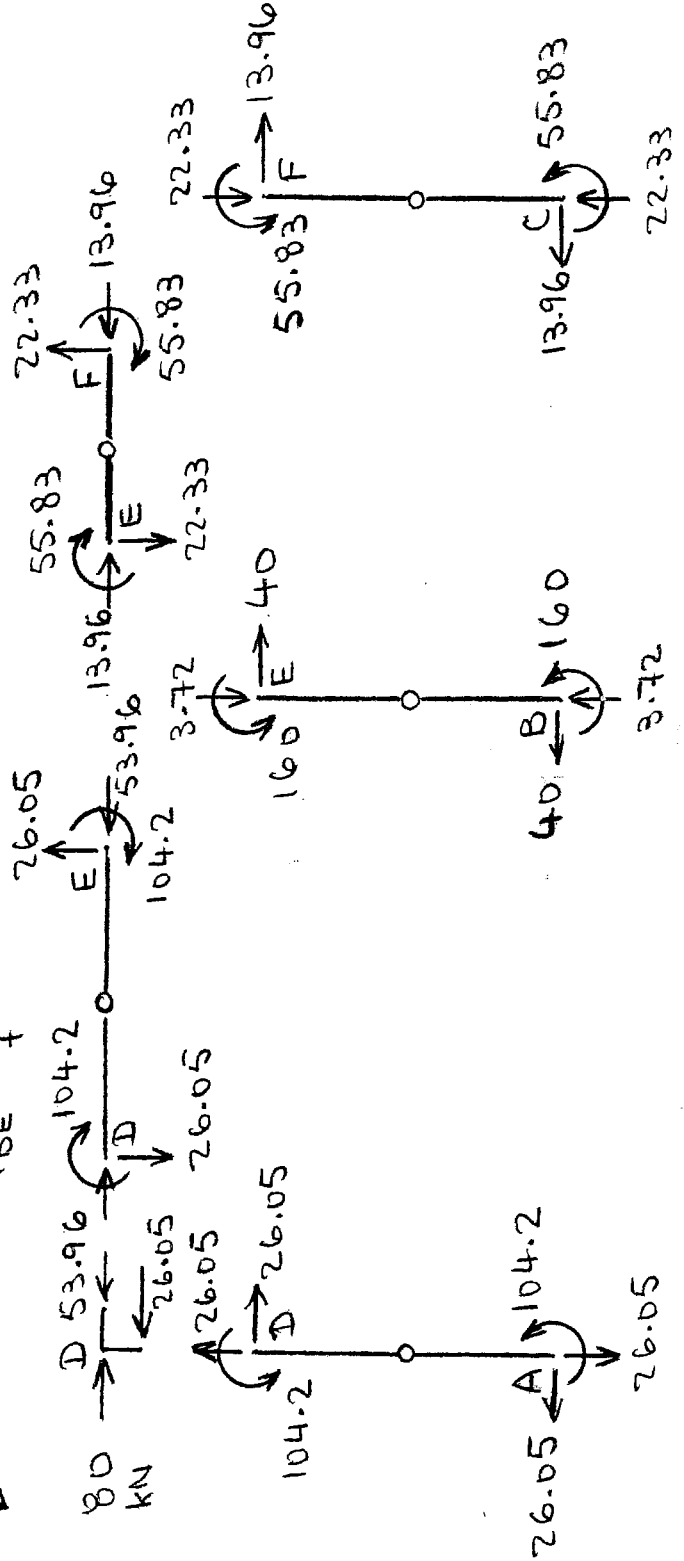
$$+\circlearrowleft \sum M_G = 0$$

$$-80(4) + Q(13) - \frac{1}{7}Q(5) = 0$$

$$Q = 26.05 \text{ kN}$$

$$Q_{AD} = 26.05 \text{ kN}, \quad Q_{BE} = 3.72 \text{ kN}$$

$$Q_{CF} = 22.33 \text{ kN}$$



12.17

$\bar{x} = 24'$ | $\bar{y} = 24'$ | $\bar{z} = 24'$

$\sum M_J = 0$
 $Q_2 = \frac{20(8)}{48} = 3.33 \text{ k}$
 $Q_{DG} = Q_2$
 $Q_{FI} = Q_2$

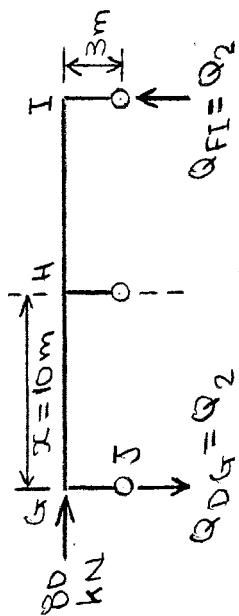
$\sum M_K = 0$
 $Q_1 = \frac{20(24) + 40(8)}{48} = 16.67 \text{ k}$
 $Q_{AD} = Q_1$
 $Q_{CF} = Q_1$

Diagram showing the frame structure with dimensions and loads. The frame consists of a central vertical column and two horizontal beams. The central column has a height of 24' and is subjected to a horizontal load of 20 k at the top. The two horizontal beams are 24' long and are subjected to a horizontal load of 40 k at the end. The frame is supported by a fixed support at the base of the central column and roller supports at the ends of the horizontal beams. The diagram also shows the internal forces and moments at various points in the frame.

Internal forces and moments are labeled as follows:

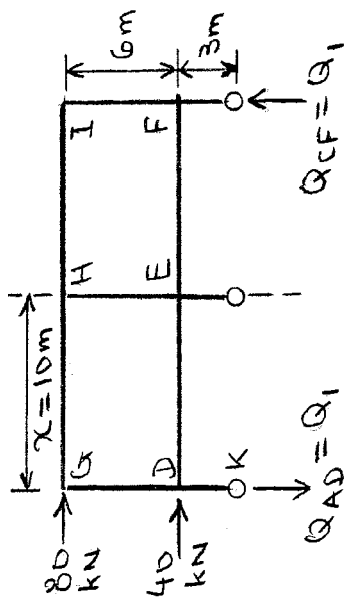
- At the base of the central column: $H = 10$, $V = 80$, $M = 800$
- At the top of the central column: $H = 10$, $V = 80$, $M = 800$
- At the ends of the horizontal beams: $H = 10$, $V = 80$, $M = 800$
- At the joints: $H = 10$, $V = 80$, $M = 800$

12.18



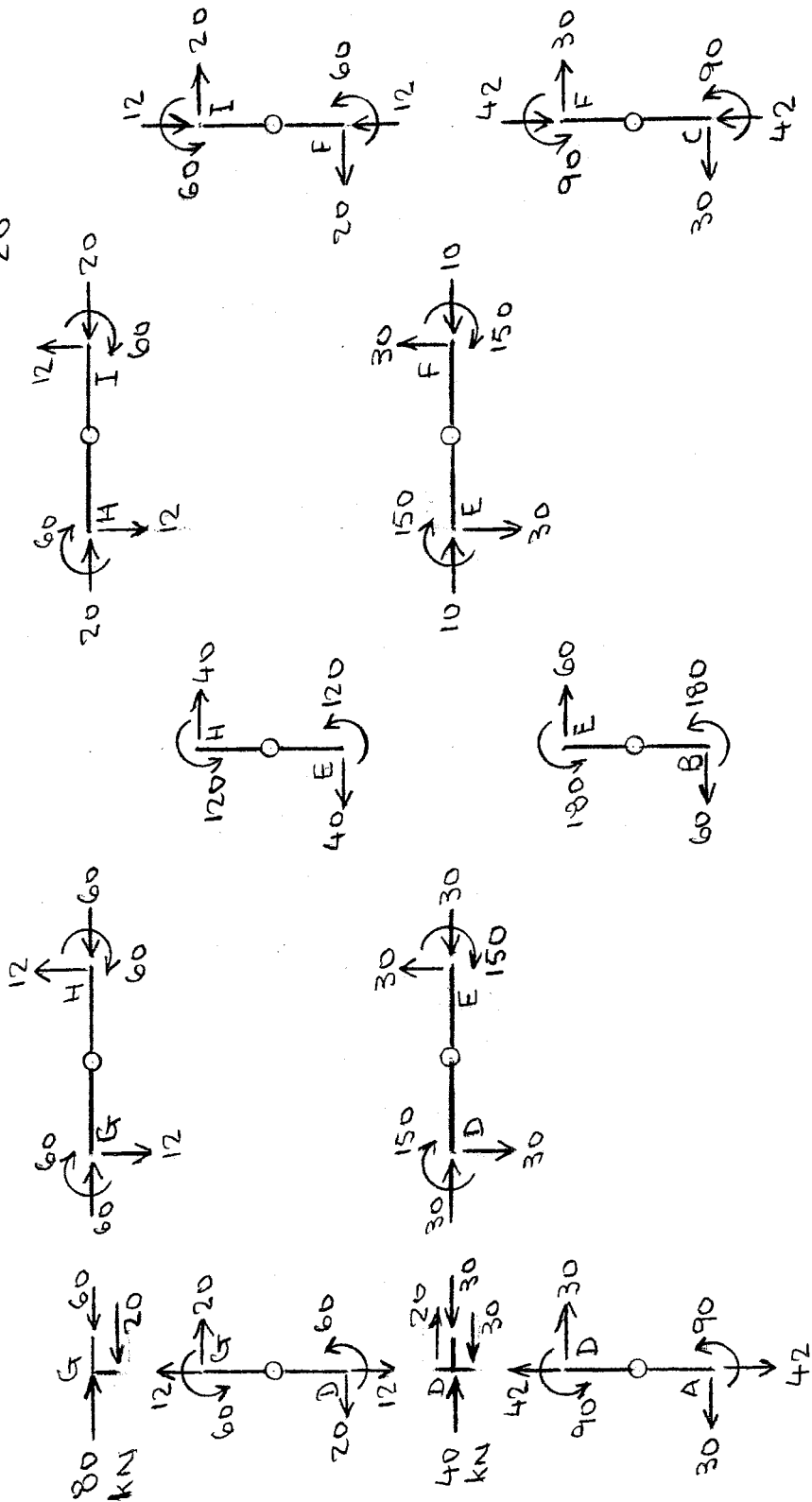
$$\sum M_J = 0 \quad Q_2 = \frac{80(3)}{20} = 12 \text{ kN}$$

$$Q_{DG} = Q_2 \quad Q_{FI} = Q_2$$

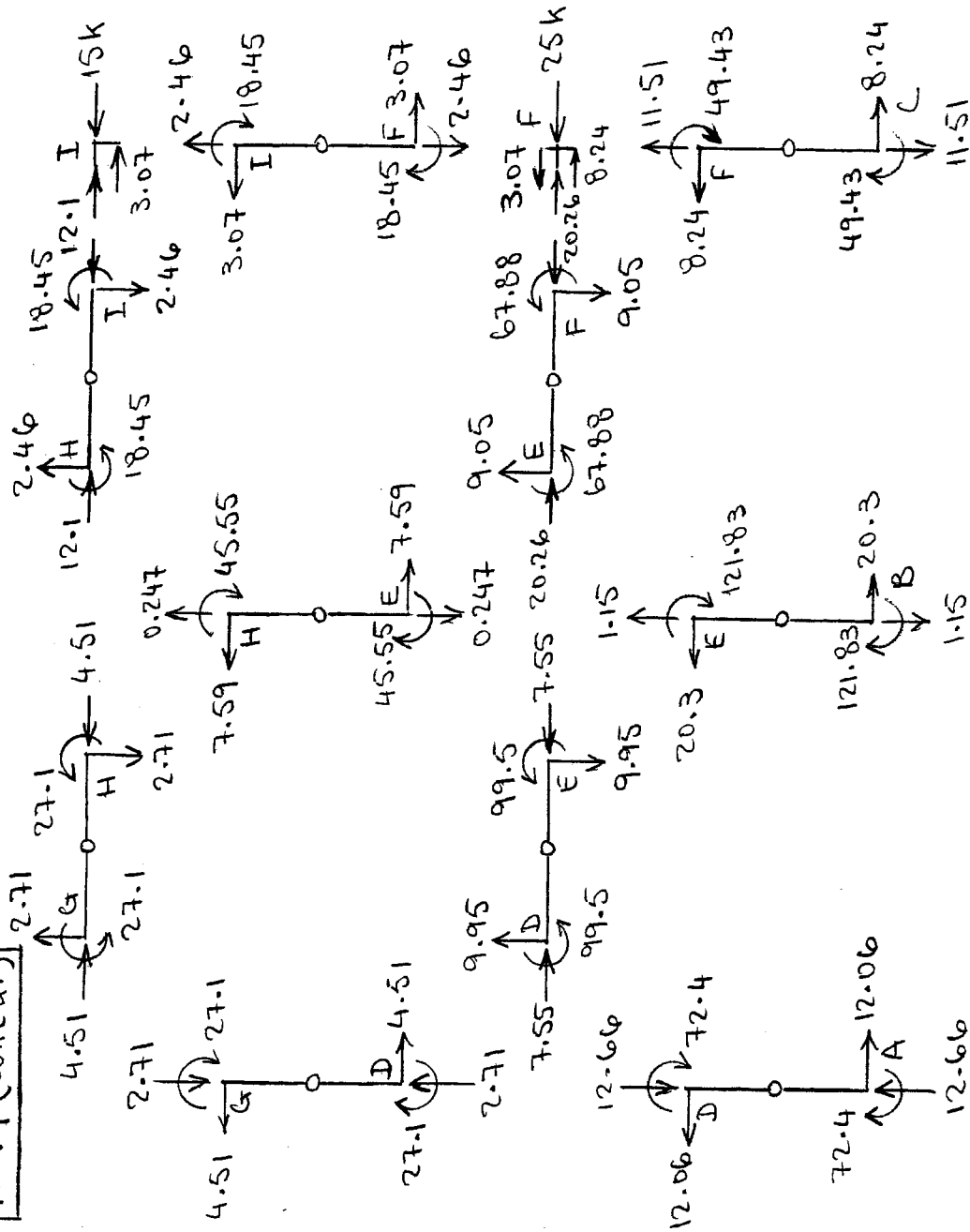


$$Q_{AD} = Q_1 \quad Q_{CF} = Q_1$$

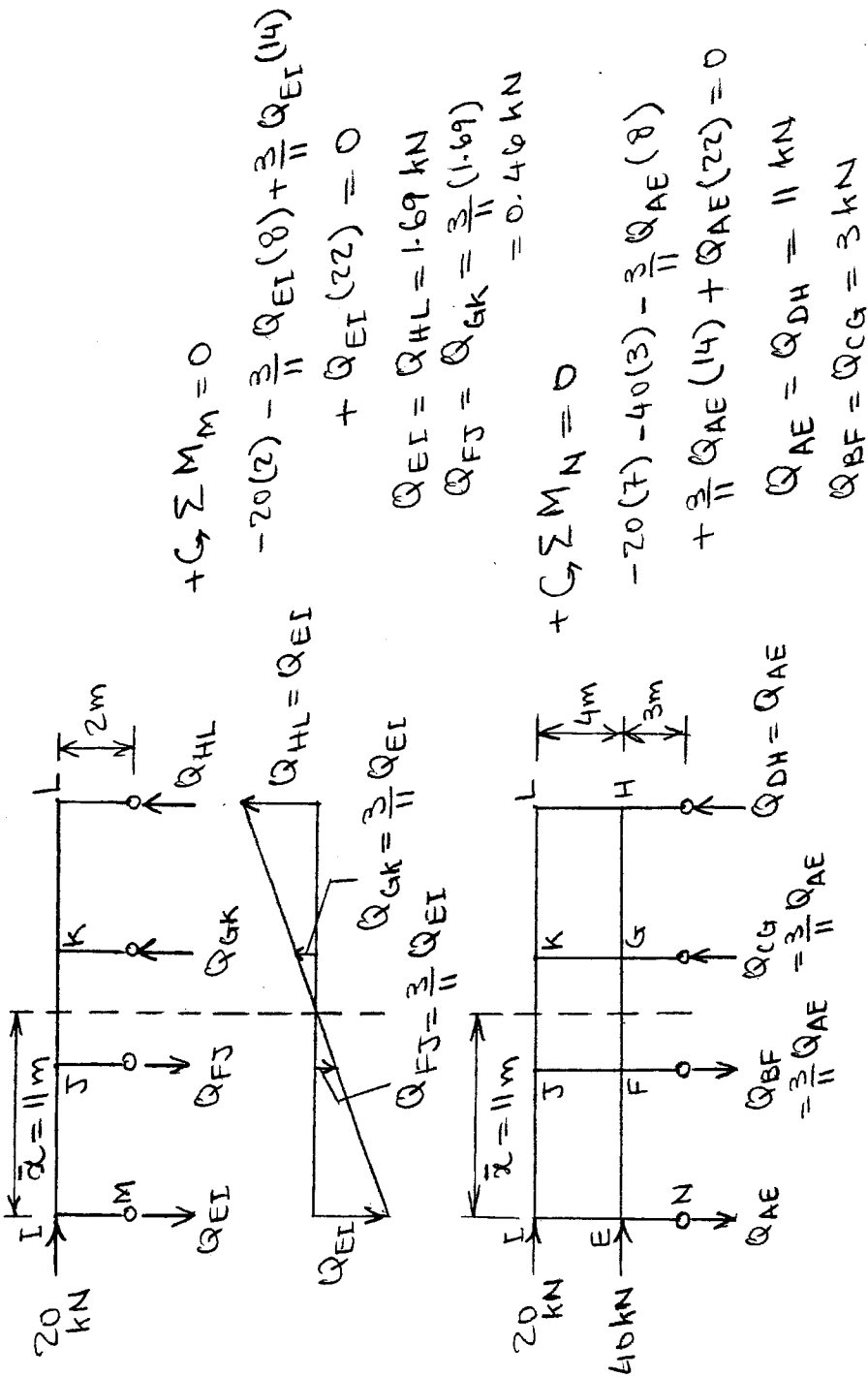
$$\sum M_K = 0 \quad Q_1 = \frac{80(9) + 40(3)}{20} = 42 \text{ kN}$$



12.19 (contd.)

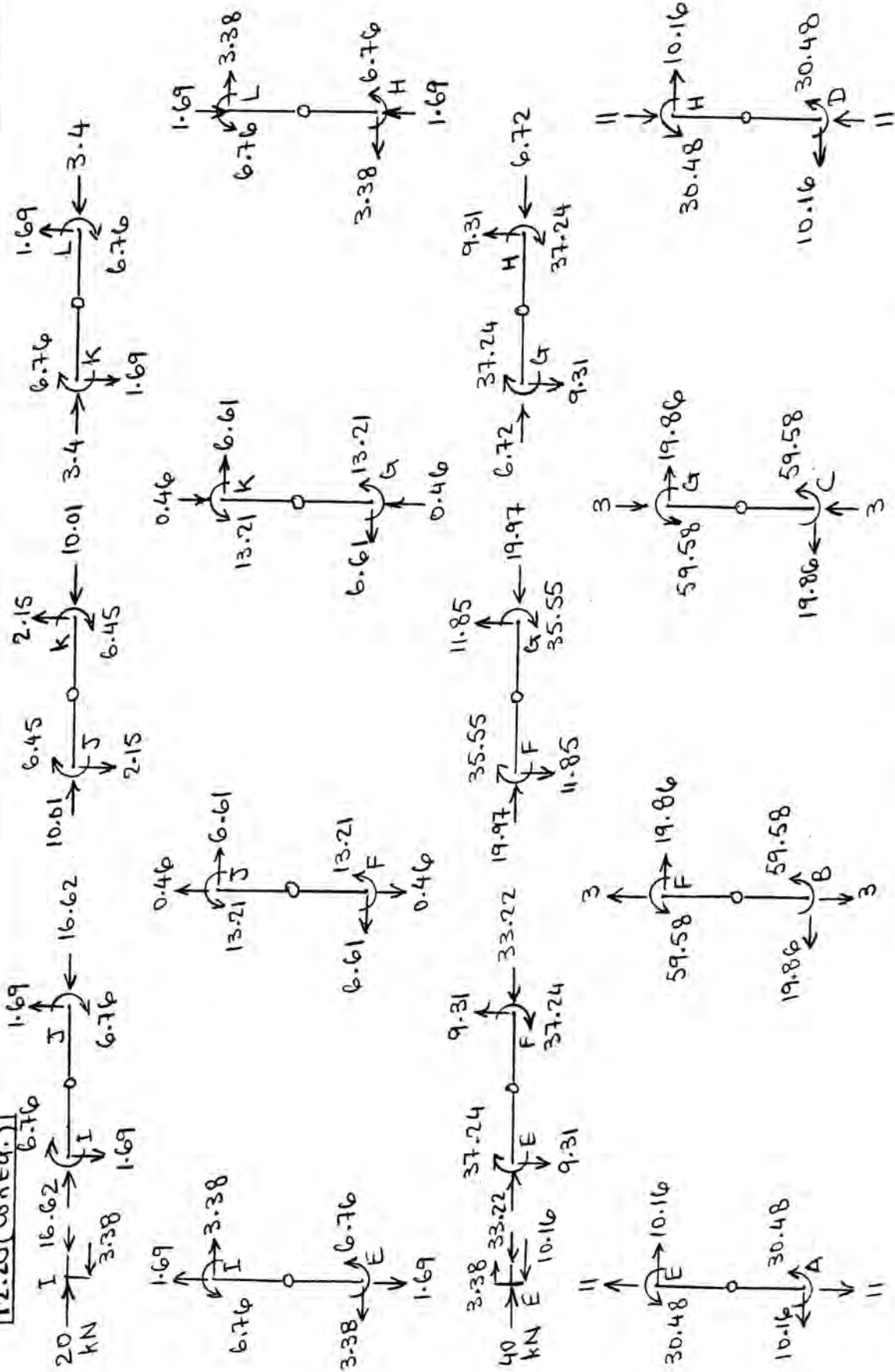


12.20



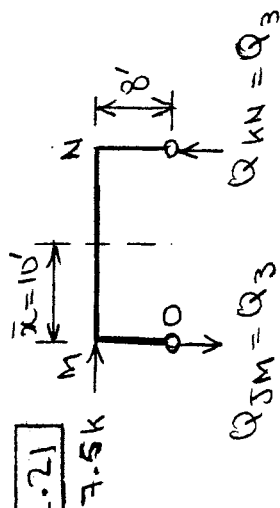
Column Axial Forces

12.20 (Contd.)



Member Axial Forces, Shears, and Moments

12.21

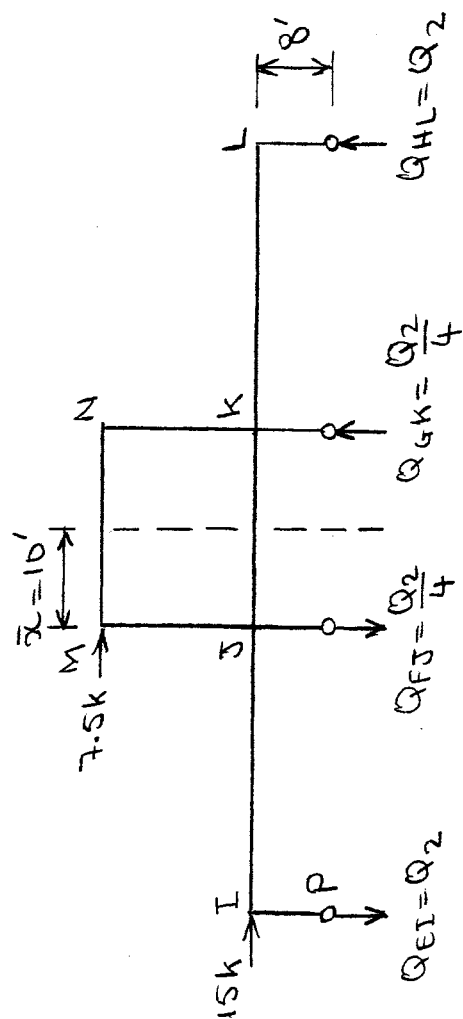


$$+\circlearrowleft \sum M_I = 0$$

$$-7.5(8) + Q_3(20) = 0$$

$$Q_3 = 3 \text{ k}$$

$$Q_{3M} = Q_{3N} = 3 \text{ k}$$

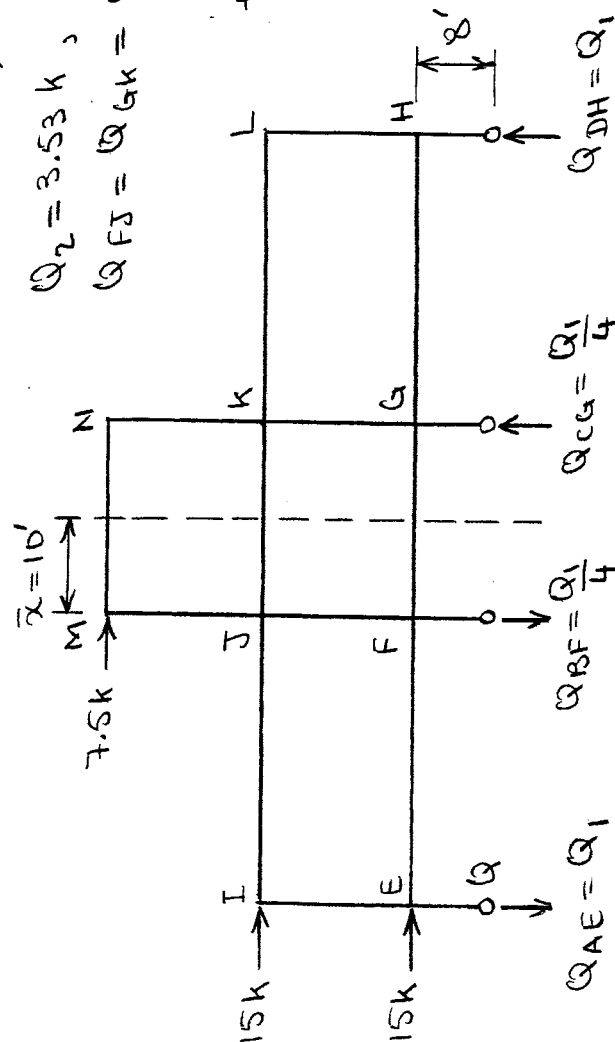


$$+\circlearrowleft \sum M_I = 0$$

$$-7.5(24) - 15(8) - \frac{Q_2(30)}{4} + \frac{Q_2(50)}{4} + Q_2(80) = 0$$

$$Q_2 = 3.53 \text{ k}, \quad Q_{2I} = Q_{2L} = 3.53 \text{ k}$$

$$Q_{2J} = Q_{2K} = 0.882 \text{ k}$$



$$+\circlearrowleft \sum M_I = 0$$

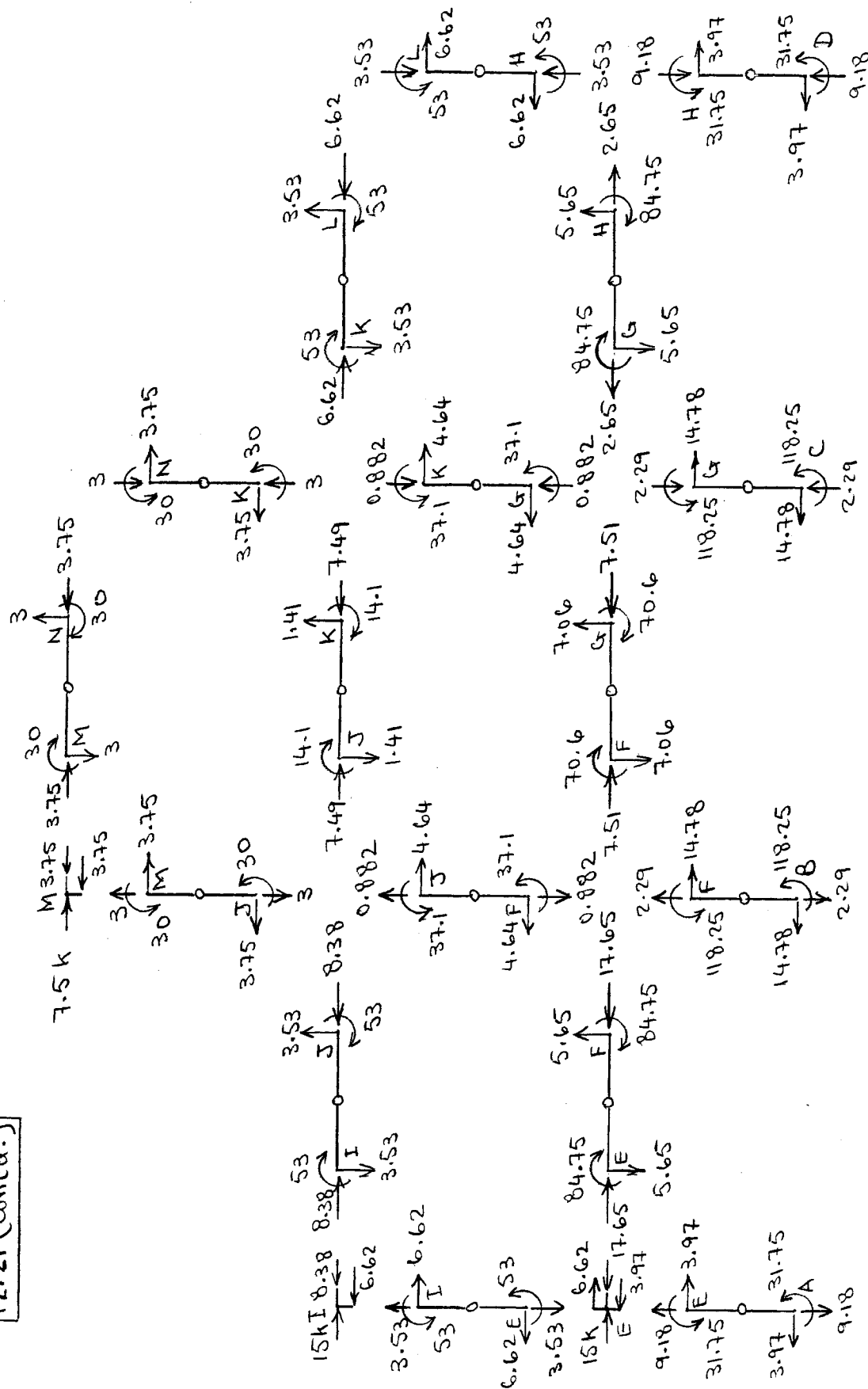
$$-7.5(40) - 15(24) - 15(8) - \frac{Q_1(30)}{4} + \frac{Q_1(50)}{4} + Q_1(80) = 0$$

$$Q_1 = 9.18 \text{ k}$$

$$Q_{1E} = Q_{1H} = 9.18 \text{ k}$$

$$Q_{1F} = Q_{1G} = 2.29 \text{ k}$$

12.21 (contd.)

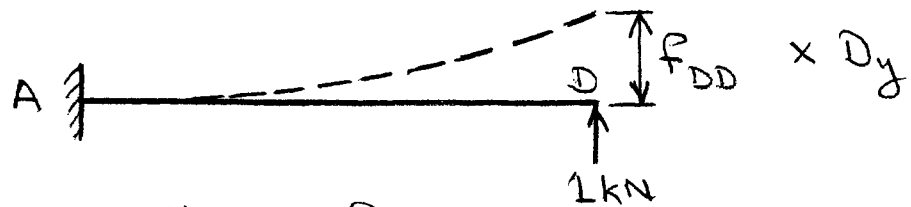
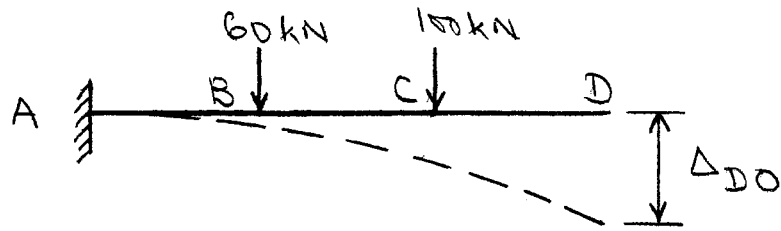


Chapter Thirteen

Method of Consistent Deformations – Force Method

CHAPTER 13

13.1



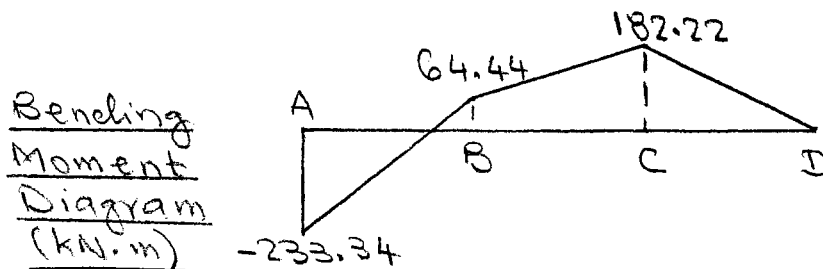
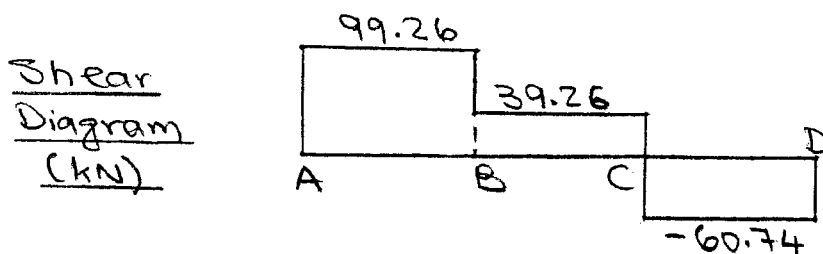
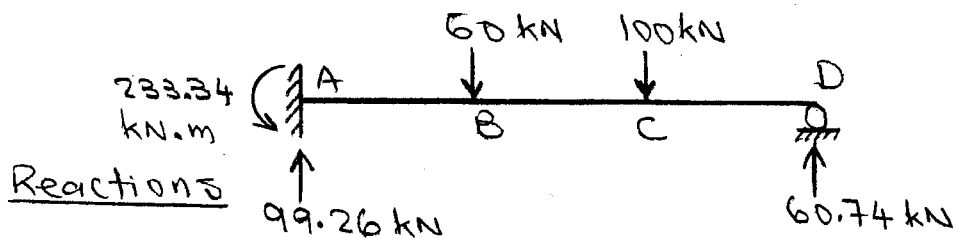
Using beam deflection formulas:

$$\Delta_{D0} = -\frac{14760 \text{ kN}\cdot\text{m}^3}{EI}$$

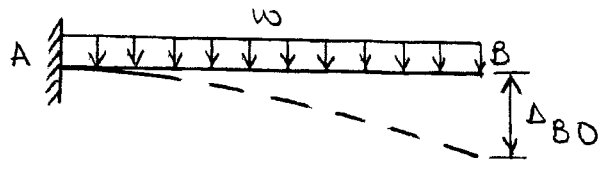
$$F_{DD} = \frac{243 \text{ kN}\cdot\text{m}^3/\text{kN}}{EI}$$

Compatibility Equation:

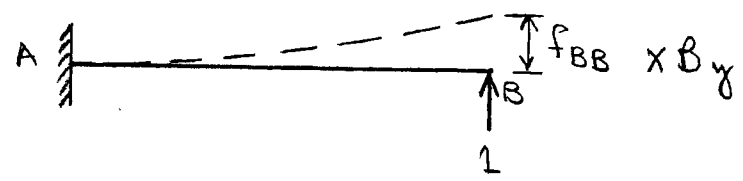
$$\Delta_{D0} + F_{DD} D_y = 0 \quad D_y = \frac{14760}{243} = 60.74 \text{ kN} \uparrow$$



13.2

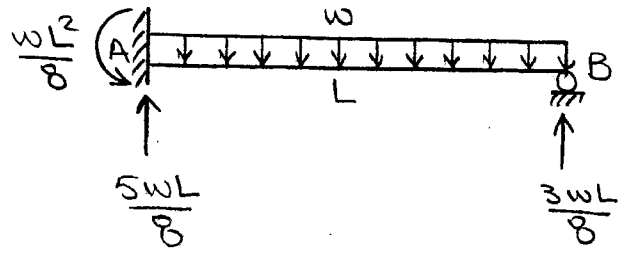


$$\Delta_{B0} = - \frac{wL^4}{8EI}$$

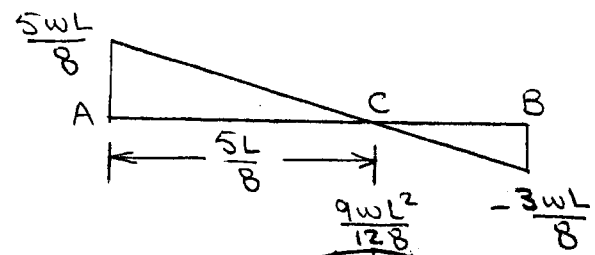


$$f_{BB} = \frac{L^3}{3EI}$$

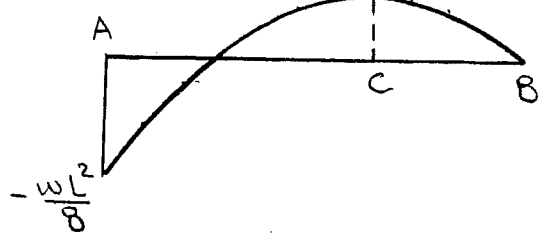
$$\Delta_{B0} + P_{BB} B_y = 0 \quad B_y = \left(\frac{wL^4}{8EI} \right) \frac{3EI}{L^3} = \frac{3wL}{8} \uparrow$$



Reactions



Shear Diagram

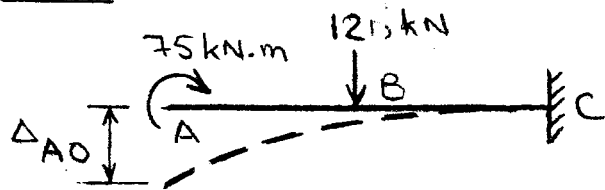


Bending Moment Diagram

13.3

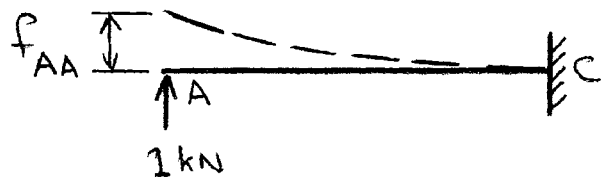
Using beam deflection

Formulas:



$$\Delta_{AO} = \frac{5400}{EI} - \frac{21600}{EI}$$

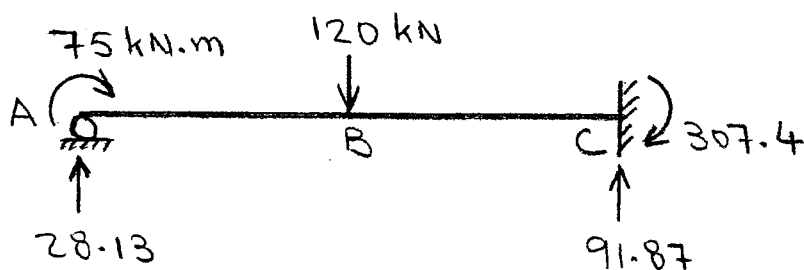
$$= - \frac{16200 \text{ kN.m}^3}{EI}$$



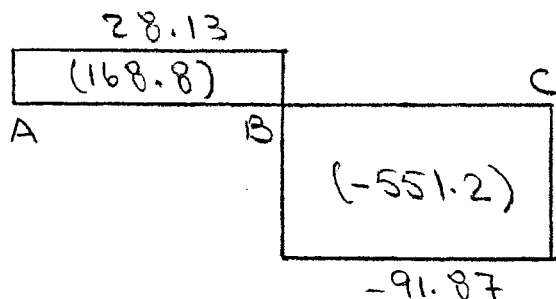
$$f_{AA} = \frac{576 \text{ kN.m}^3/\text{kN}}{EI}$$

$$\Delta_{AO} + f_{AA} A_y = 0 \quad A_y = \frac{16200}{576} = 28.13 \text{ kN} \uparrow$$

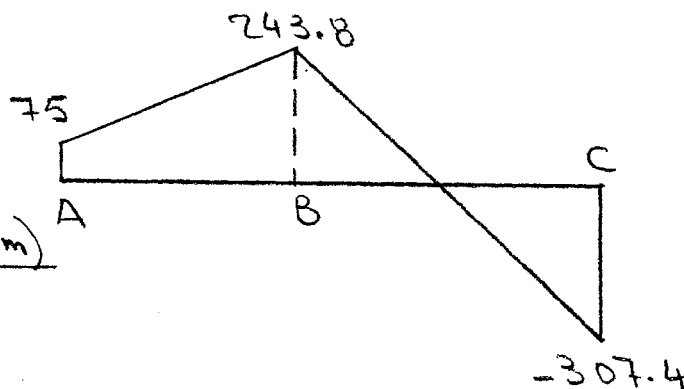
Reactions



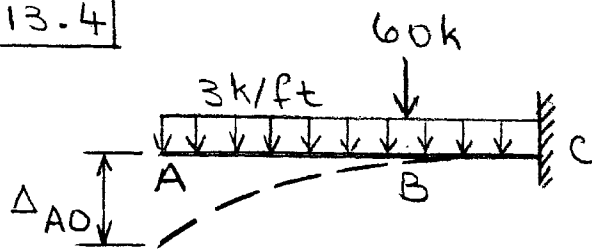
Shear
Diagram
(kN)



Bending
Moment
Diagram (kN.m)

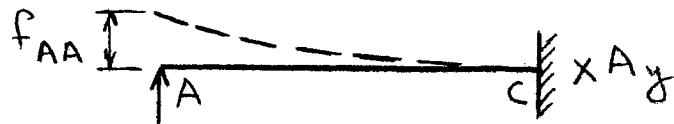


13.4



Using beam deflection formulas:

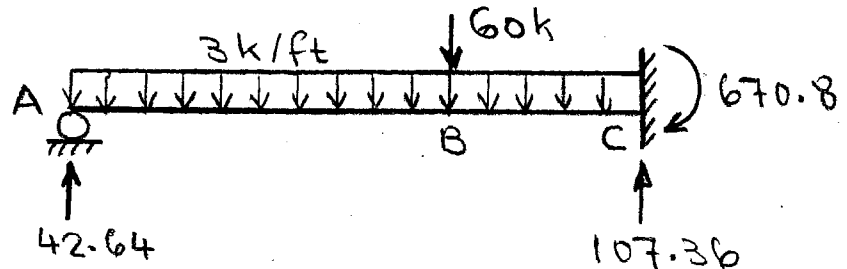
$$\Delta_{AO} = -\frac{303750}{EI} - \frac{80000}{EI} = -\frac{383750 \text{ k-ft}^3}{EI}$$



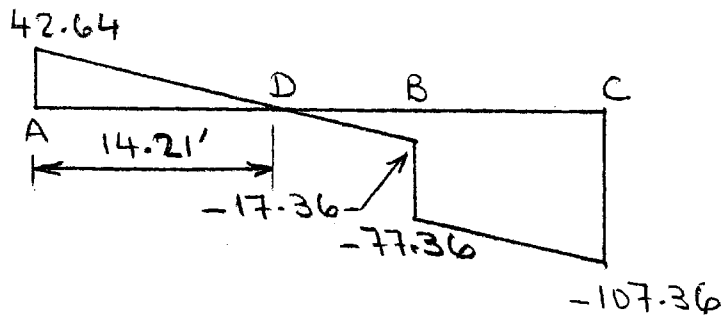
$$f_{AA} = \frac{9000 \text{ k-ft}^3/\text{k}}{EI}$$

$$\Delta_{AO} + f_{AA} A_y = 0 \quad A_y = \frac{383750}{9000} = 42.64 \text{ k} \uparrow$$

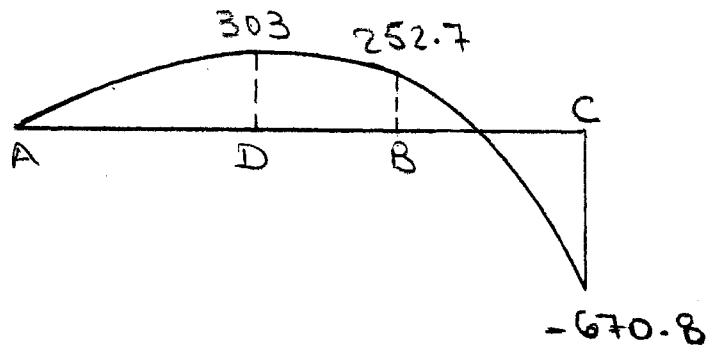
Reactions



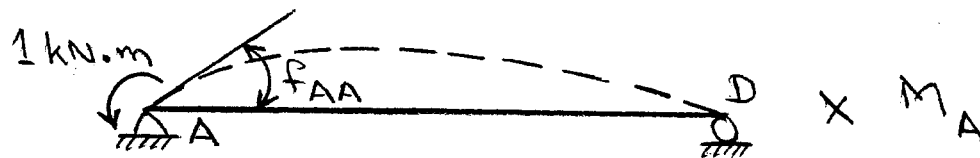
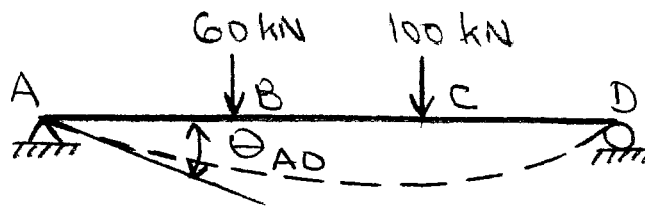
Shear Diagram (k)



Bending Moment Diagram (k-ft)



13.5



Using beam deflection formulas:

$$\theta_{AD} = -\frac{700 \text{ kN}\cdot\text{m}^2}{EI}$$

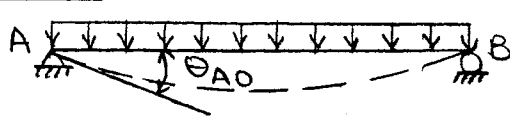
$$P_{AA} = \frac{3 \text{ kN}\cdot\text{m}^2/\text{kN}\cdot\text{m}}{EI}$$

Compatibility Equations:

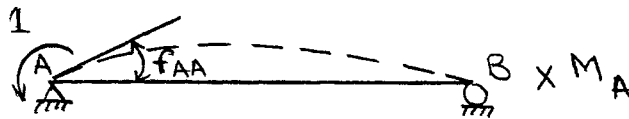
$$\theta_{AD} + P_{AA} M_A = 0 \quad M_A = \frac{700}{3} = \underline{233.33 \text{ kN}\cdot\text{m}}$$

For reactions, and shear and moment diagrams, see solution of Problem 13.1.

13.6



$$\theta_{A0} = -\frac{wL^3}{24EI}$$



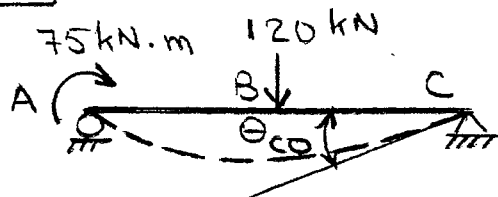
$$P_{AA} = \frac{L}{3EI}$$

$$\theta_{A0} + P_{AA} M_A = 0$$

$$M_A = \frac{wL^3}{24EI} \left(\frac{3EI}{L} \right) = \frac{wL^2}{8}$$

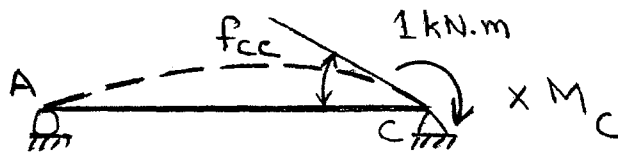
For reactions, and shear and moment diagrams, see solution of Problem 13.2.

13.7



Using beam deflection formulas:

$$\theta_{CO} = -\frac{150}{EI} - \frac{1080}{EI} = -\frac{1230 \text{ kN.m}^2}{EI}$$

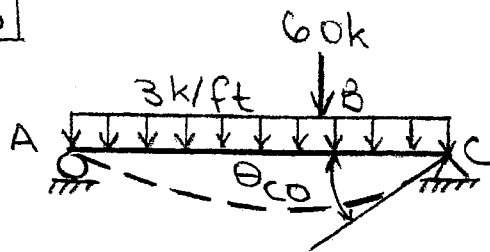


$$f_{CC} = \frac{4 \text{ kN.m}^2/\text{kN.m}}{EI}$$

$$\theta_{CO} + f_{CC} M_C = 0 \quad M_C = \frac{1230}{4} = \underline{307.5 \text{ kN.m}}$$

For reactions, and shear and moment diagrams, see solution of Problem 13.3.

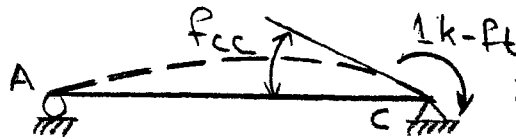
13.8



Using beam deflection formula:

$$\theta_{C0} = -\frac{3375}{EI} - \frac{3333}{EI}$$

$$= -\frac{6708 \text{ k-ft}^2}{EI}$$

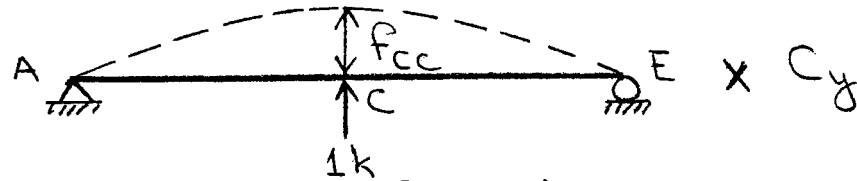
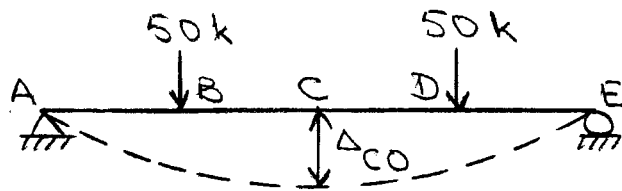


$$f_{cc} = \frac{10 \text{ k-ft}^2 / \text{k-ft}}{EI}$$

$$\theta_{C0} + f_{cc} M_C = 0 \quad M_C = \frac{6708}{10} = 670.8 \text{ k-ft}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.4.

13.9

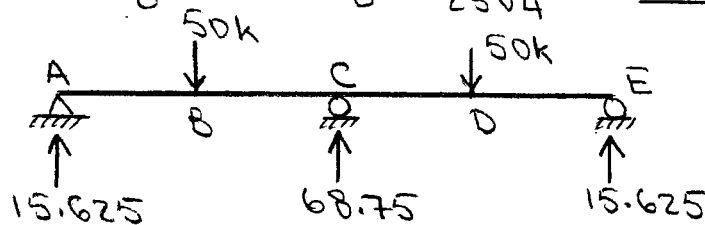


Using beam deflection formulas:

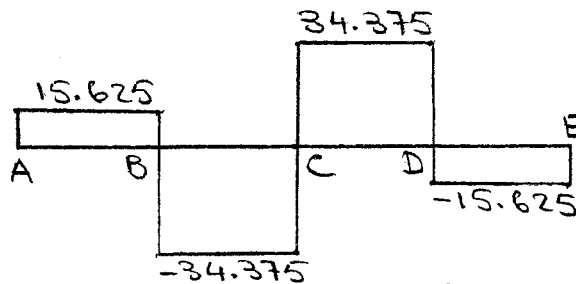
$$\Delta_{CO} = -\frac{158400 \text{ k-ft}^3}{EI} \quad f_{CC} = \frac{2304 \text{ k-ft}^3/\text{k}}{EI}$$

Compatibility Equation:

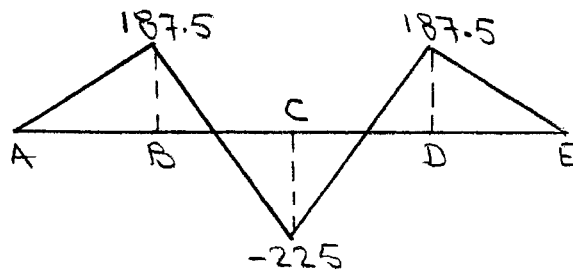
$$\Delta_{CO} + f_{CC} C_y = 0 \quad C_y = \frac{158400}{2304} = 68.75 \text{ k} \uparrow$$



Reactions

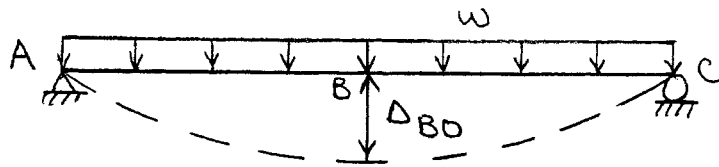


Shear Diagram (k)

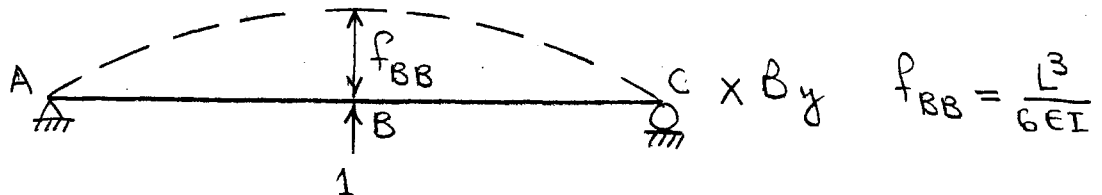


Bending Moment Diagram (k-ft)

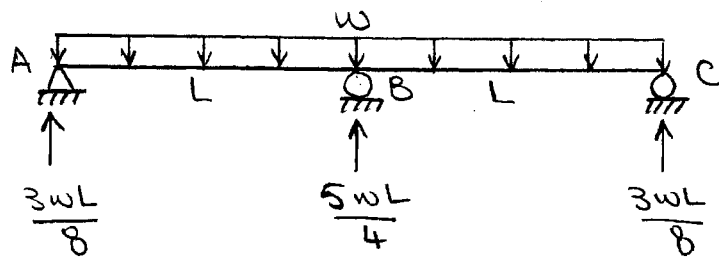
13.10



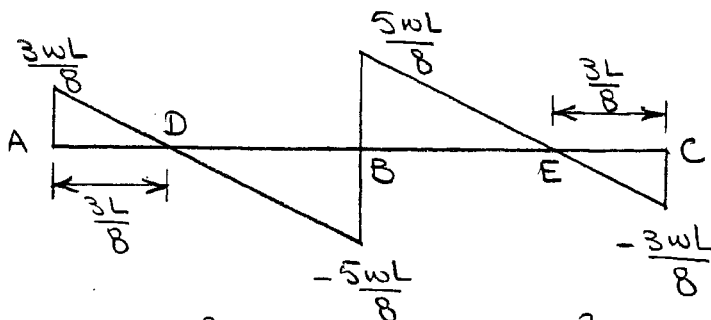
$$\Delta_{B0} = -\frac{5wL^4}{24EI}$$



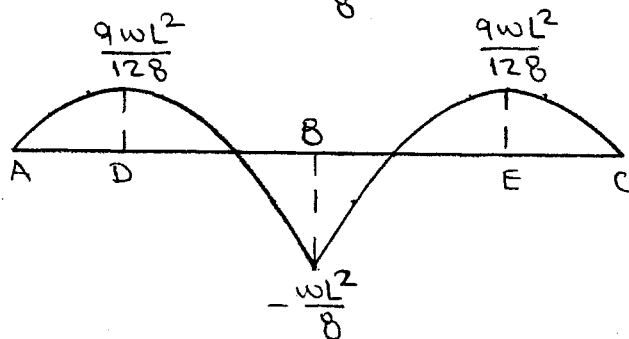
$$\Delta_{B0} + P_{BB} B_y = 0 \quad B_y = \frac{5wL^4}{24EI} \left(\frac{6EI}{L^3} \right) = \frac{5wL}{4} \uparrow$$



Reactions

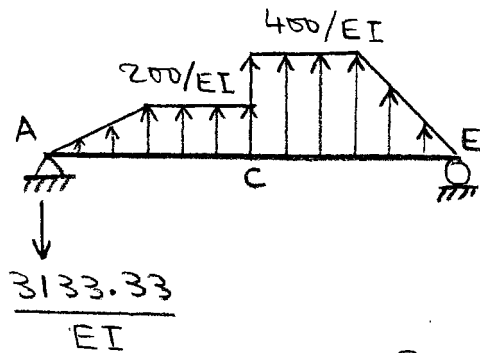
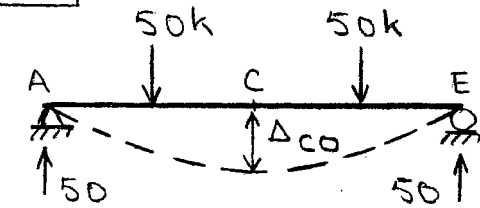


Shear Diagram

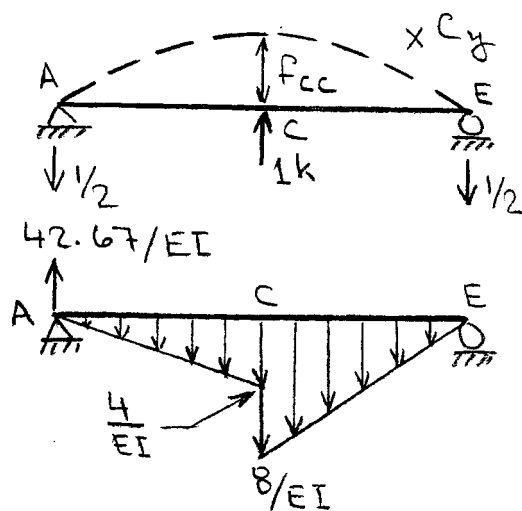


Bending Moment Diagram

13.11



Conjugate Beam for
External Loading



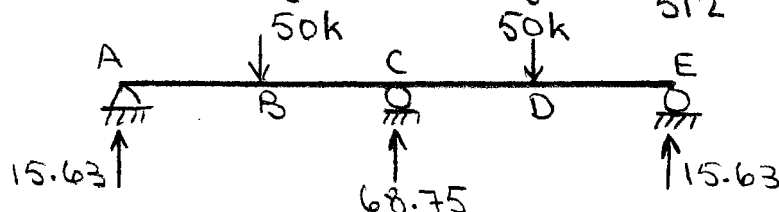
Conjugate Beam for
Unit Value of
Redundant C_y

$$\Delta_{C0} = -\frac{3133.33}{EI}(16) + \frac{1}{2}(16)\left(\frac{200}{EI}\right)\left(\frac{8}{3} + 8\right) + \left(\frac{200}{EI}\right)8(4)$$

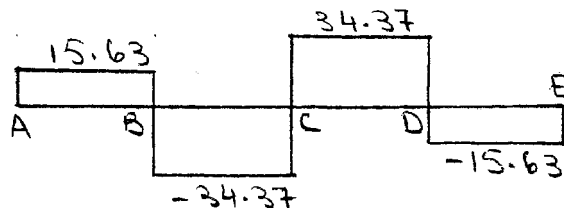
$$= -\frac{35200}{EI}$$

$$P_{CC} = \frac{42.67}{EI}(16) - \frac{1}{2}(16)\left(\frac{4}{EI}\right)\left(\frac{16}{3}\right) = \frac{512}{EI}$$

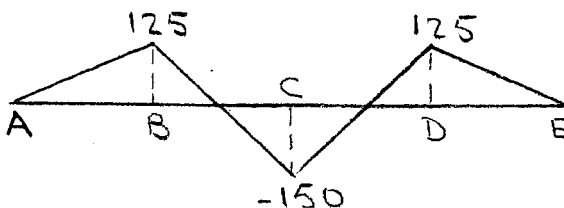
$$\Delta_{C0} + P_{CC} C_y = 0 \quad C_y = \frac{35200}{512} = 68.75 \text{ k} \uparrow$$



Reactions

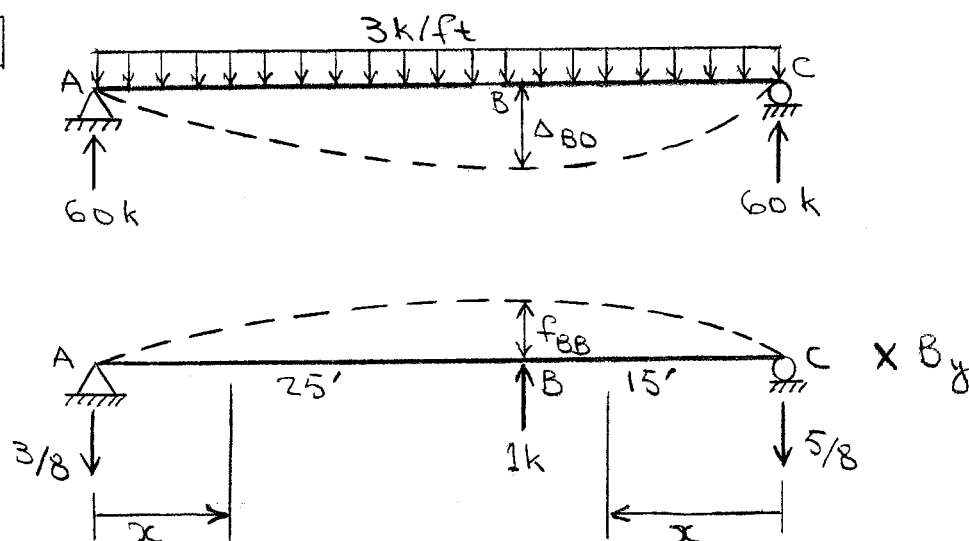


Shear Diagram (k)



Bending Moment
Diagram (k-ft)

13.12



Using the virtual work method:

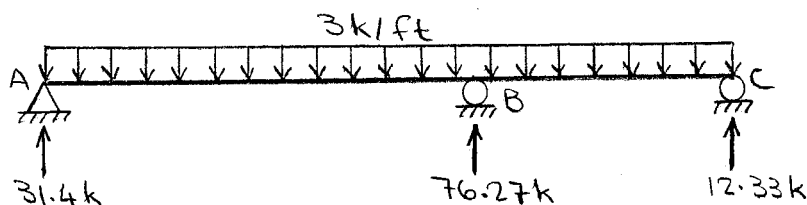
$$\Delta_{BO} = \frac{1}{EI} \left[\frac{1}{2} \int_0^{25} (60x - \frac{3x^2}{2}) (-\frac{3x}{8}) dx + \int_0^{15} (60x - \frac{3x^2}{2}) (-\frac{5x}{8}) dx \right]$$

$$= -\frac{61450.2 \text{ k-ft}^3}{EI}$$

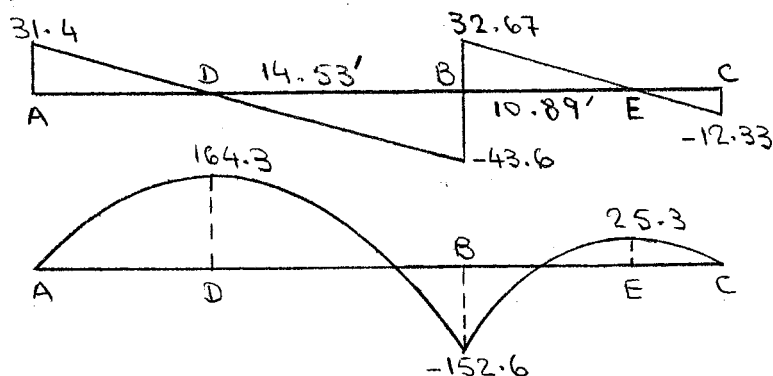
$$f_{BB} = \frac{1}{EI} \left[\frac{1}{2} \int_0^{25} (-\frac{3x}{8})^2 dx + \int_0^{15} (-\frac{5x}{8})^2 dx \right] = \frac{805.66 \text{ k-ft}^3/\text{k}}{EI}$$

Compatibility Equation:

$$\Delta_{BO} + f_{BB} B_y = 0 \quad B_y = \frac{61450.2}{805.66} = 76.27 \text{ k} \uparrow$$



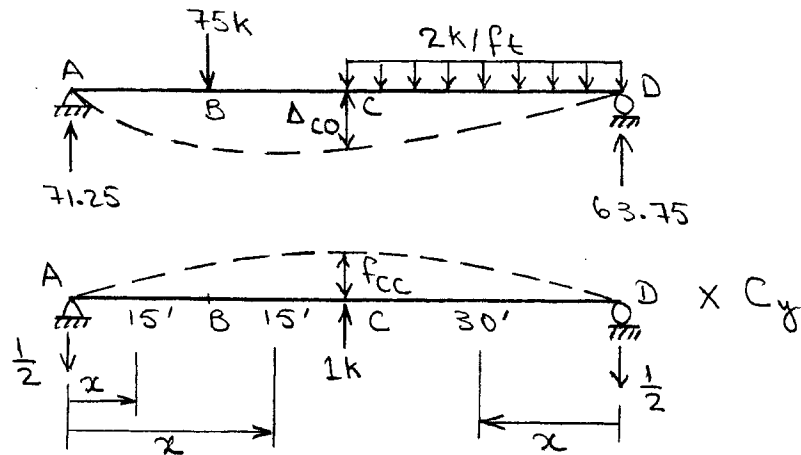
Reactions



Shear Diagram
(k)

Bending Moment
Diagram (k-ft)

13.13



Using the method of virtual work:

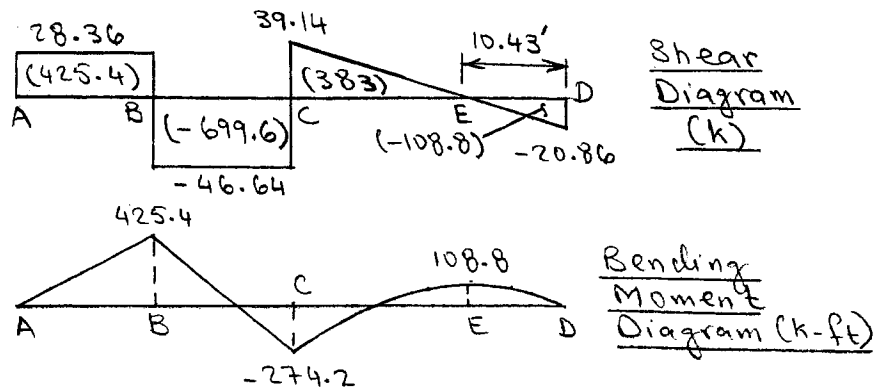
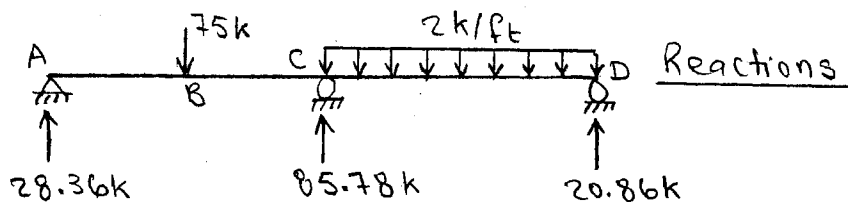
$$\Delta_{co} = \frac{1}{EI} \left[\frac{1}{3} \int_0^{15} (71.25x) \left(-\frac{x}{2}\right) dx + \frac{1}{3} \int_{15}^{30} \{71.25x - 75(x-15)\} \left(-\frac{x}{2}\right) dx + \int_0^{30} (63.75x - x^2) \left(-\frac{x}{2}\right) dx \right]$$

$$= -\frac{257343.75 \text{ k-ft}^3}{EI}$$

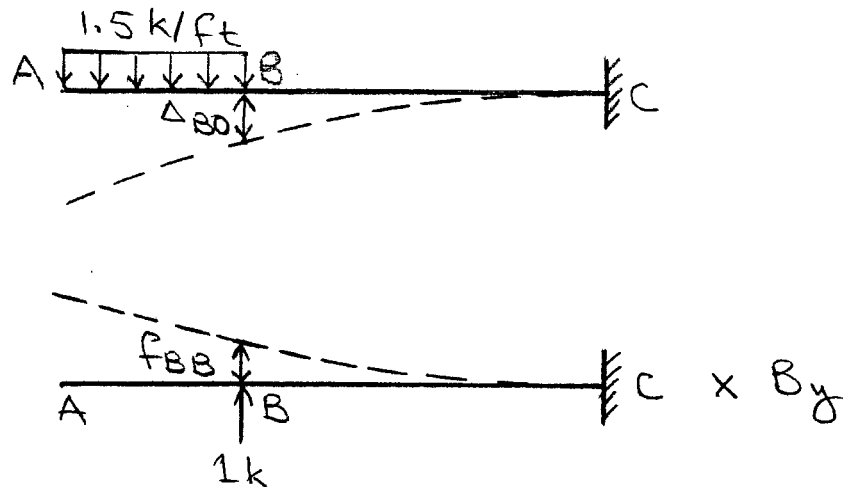
$$f_{cc} = \frac{1}{EI} \left[\frac{1}{3} \int_0^{30} \left(-\frac{x}{2}\right)^2 dx + \int_0^{30} \left(-\frac{x}{2}\right)^2 dx \right] = \frac{3000 \text{ k-ft}^3/\text{k}}{EI}$$

Compatibility Equation: $\Delta_{co} + f_{cc} C_y = 0$

$$C_y = \frac{257343.75}{3000} = 85.78 \text{ k} \uparrow$$



13.14



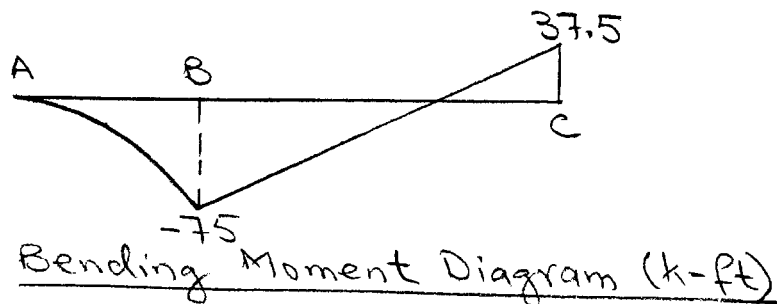
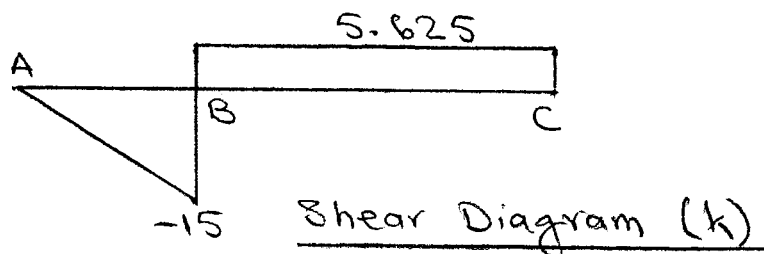
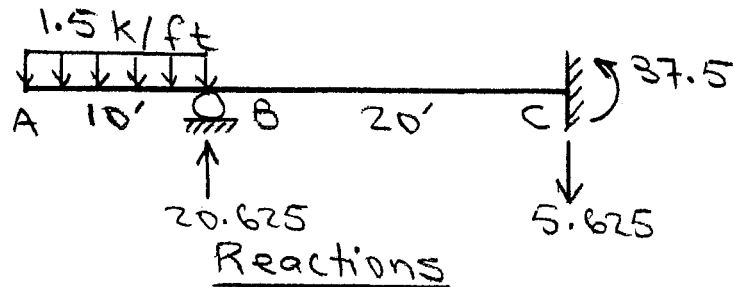
Using beam deflection formulas:

$$\Delta_{B0} = -\frac{55000 \text{ k-ft}^3}{EI}$$

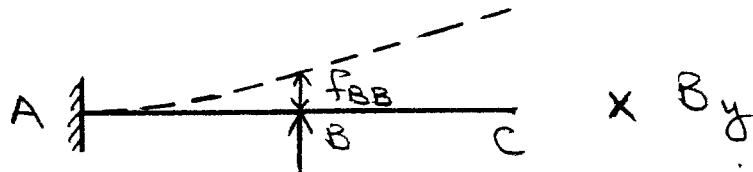
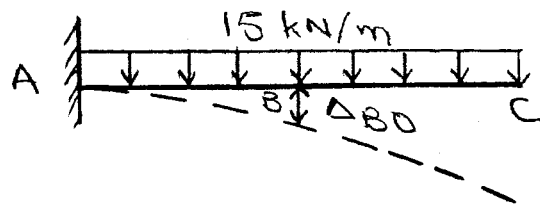
$$F_{BB} = \frac{2666.67 \text{ k-ft}^3/\text{k}}{EI}$$

Compatibility Equation:

$$\Delta_{B0} + F_{BB} B_y = 0 \quad B_y = \frac{55000}{2666.67} = 20.625 \text{ k} \uparrow$$



13.15

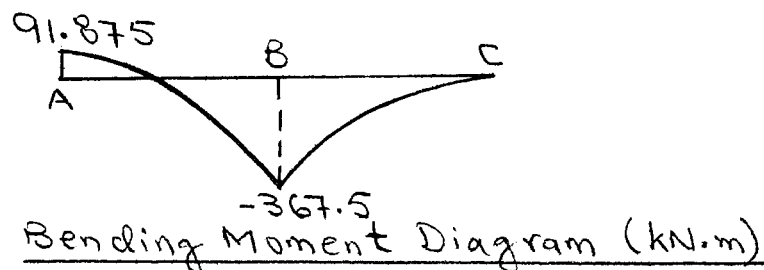
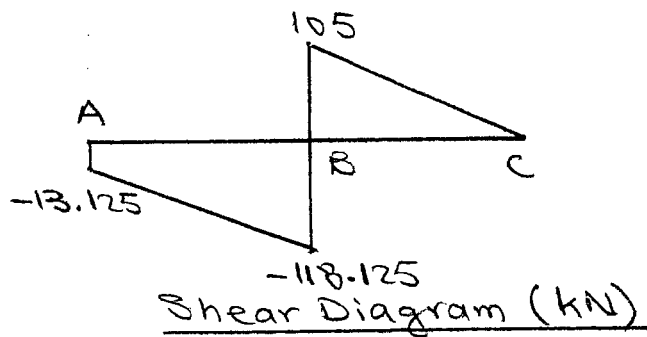
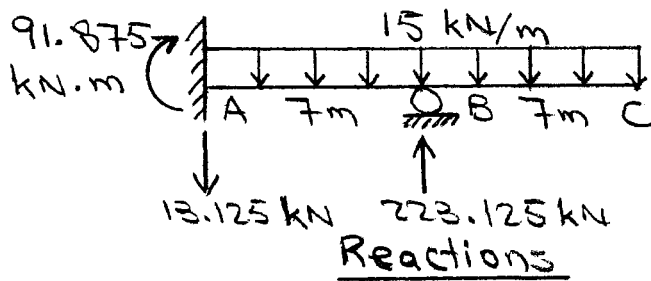


Using beam deflection formulas:

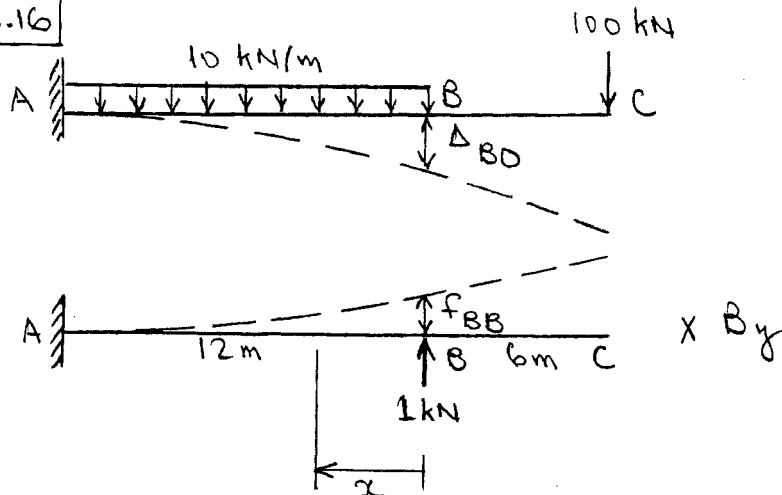
$$\Delta_{B0} = -\frac{25510.625 \text{ kN}\cdot\text{m}^3}{EI} \quad f_{BB} = \frac{114.333 \text{ kN}\cdot\text{m}^3/\text{kN}}{EI}$$

Compatibility Equation: $\Delta_{B0} + f_{BB} B_y = 0$

$$B_y = \frac{25510.625}{114.333} = 223.125 \text{ kN} \uparrow$$



B.16



Using the virtual work method:

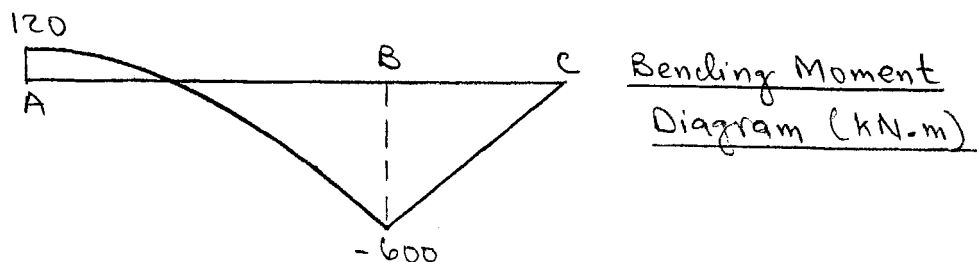
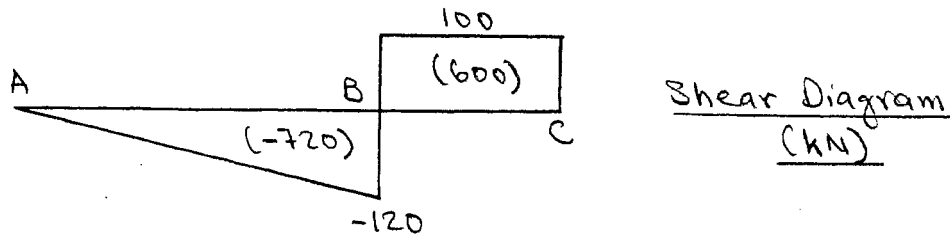
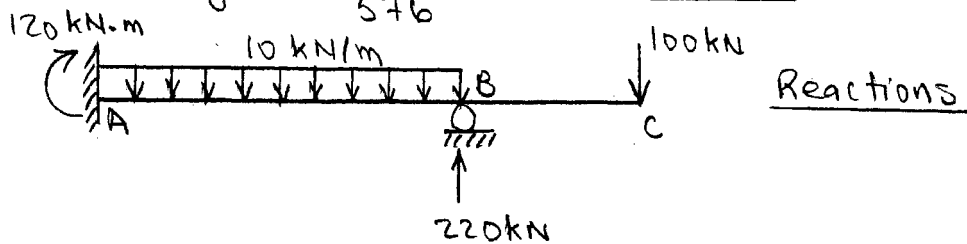
$$\Delta_{BO} = \frac{1}{EI} \left[\int_0^{12} \{-100(6+x) - 5x^2\} x dx \right]$$

$$= - \frac{126720 \text{ kN-m}^3}{EI}$$

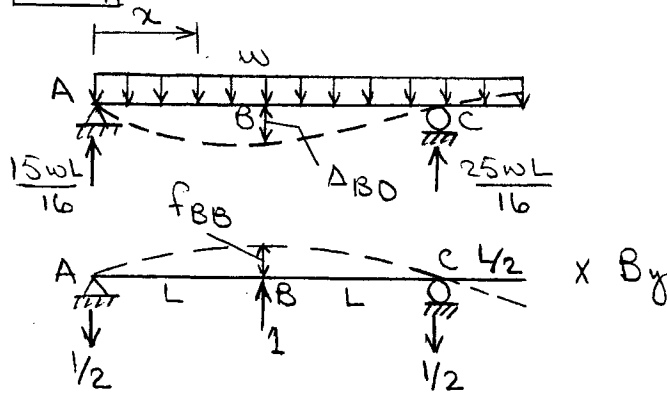
$$f_{BB} = \frac{1}{EI} \left[\int_0^{12} x^2 dx \right] = \frac{576 \text{ kN-m}^3/\text{kN}}{EI}$$

Compatibility Equation: $\Delta_{BO} + f_{BB} B_y = 0$

$$B_y = \frac{126720}{576} = 220 \text{ kN} \uparrow$$



13.17



Using the virtual work method:

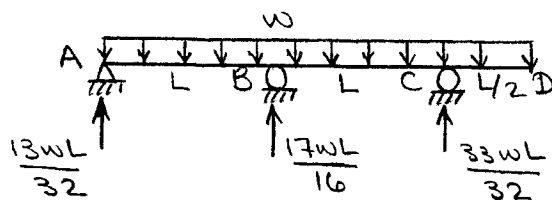
$$\Delta_{BO} = \frac{1}{EI} \left[\int_0^L \left(\frac{15wL}{16}x - \frac{wx^2}{2} \right) \left(-\frac{x}{2} \right) dx + \int_L^{2L} \left(\frac{15wL}{16}x - \frac{wx^2}{2} \right) \left\{ -\frac{x}{2} + 1(x-L) \right\} dx \right]$$

$$= -\frac{17wL^4}{96EI}$$

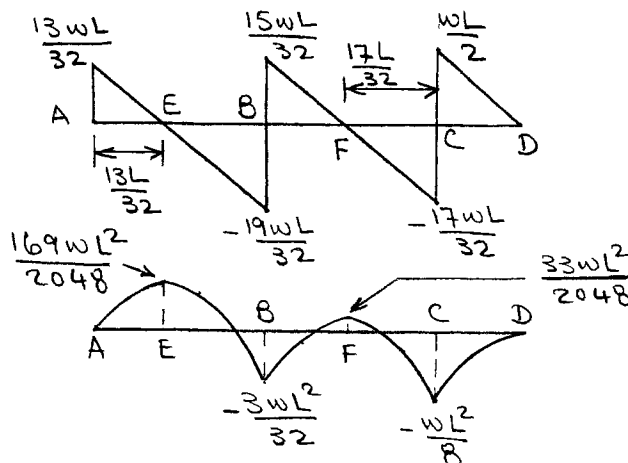
$$f_{BB} = \frac{2}{EI} \int_0^L \left(-\frac{x}{2} \right)^2 dx = \frac{L^3}{6EI}$$

Compatibility Equation: $\Delta_{BO} + f_{BB} B_y = 0$

$$B_y = \frac{17wL^4}{96EI} \left(\frac{6EI}{L^3} \right) = \frac{17wL}{16} \uparrow$$



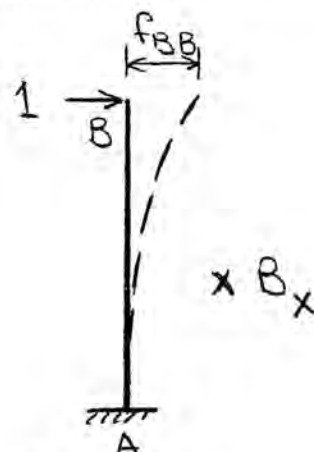
Reactions



Shear Diagram

Bending Moment Diagram

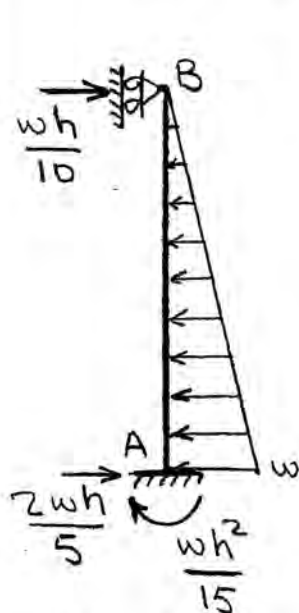
13.18



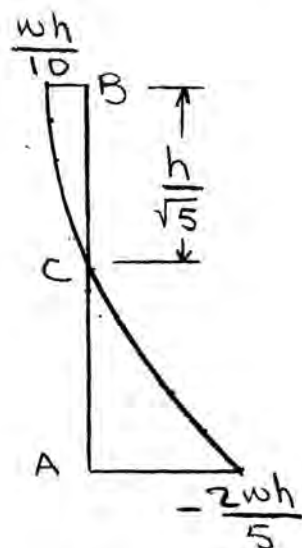
Using beam deflection formulas:

$$\Delta_{B0} = -\frac{wh^4}{30EI}; \quad f_{BB} = \frac{h^3}{3EI}$$

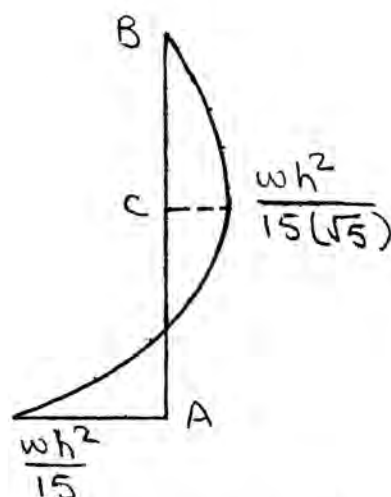
$$\Delta_{B0} + f_{BB} B_x = 0 \quad B_x = \left(\frac{wh^4}{30EI}\right) \left(\frac{3EI}{h^3}\right) = \frac{wh}{10} \rightarrow$$



Reactions

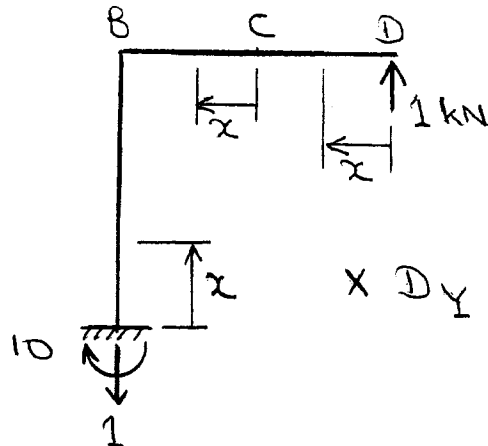
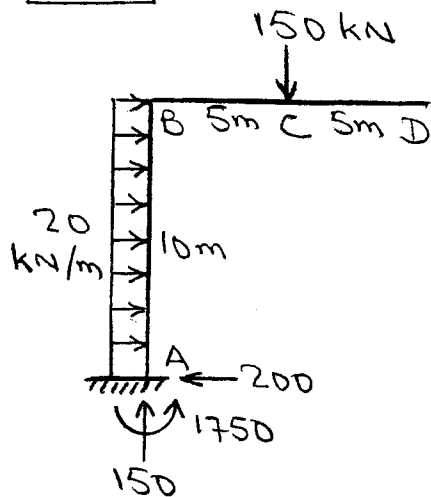


Shear Diagram



Bending Moment Diagram

13.19



Using the virtual work method:

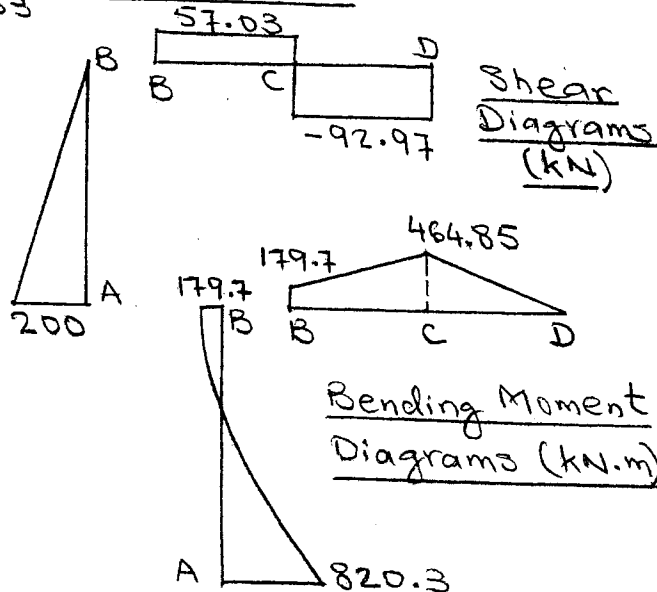
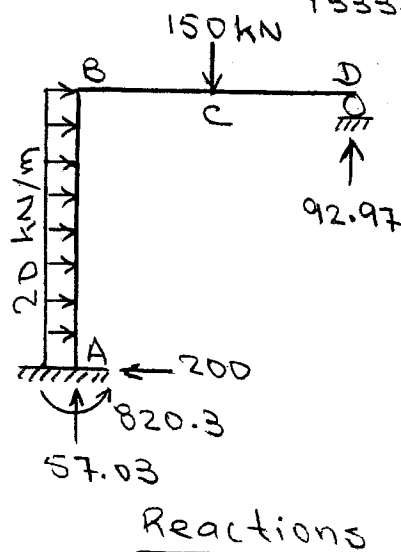
$$\Delta_{D0} = \frac{1}{EI} \left[\int_0^{10} (-1750 + 200x - 10x^2)(10) dx + \int_0^5 (-150x)1(x+5) dx \right]$$

$$= - \frac{123958.33 \text{ kN}\cdot\text{m}^3}{EI}$$

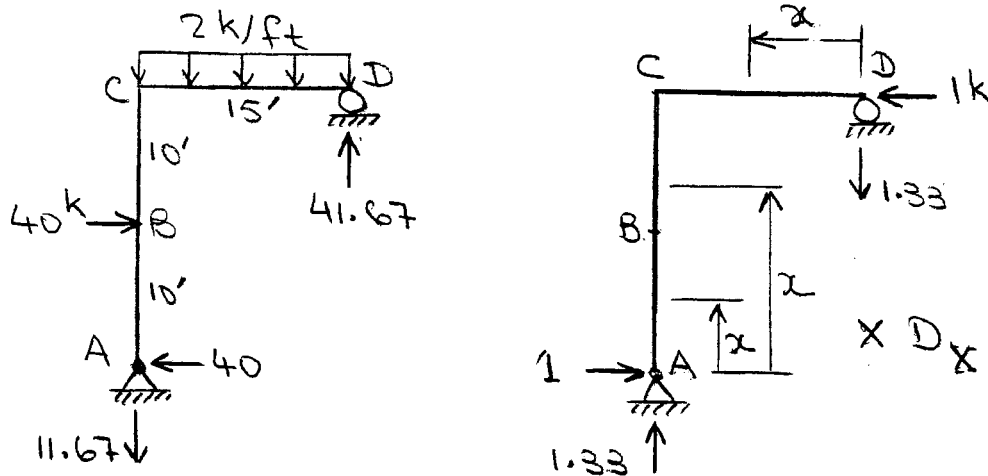
$$f_{DD} = \frac{1}{EI} \left[\int_0^{10} (10)^2 dx + \int_0^5 (x)^2 dx \right] = \frac{1333.33 \text{ kN}\cdot\text{m}^3/\text{kN}}{EI}$$

Compatibility Equation: $\Delta_{D0} + f_{DD} D_Y = 0$

$$D_Y = \frac{123958.33}{1333.33} = 92.97 \text{ kN} \uparrow$$



13.20



Using the virtual work method:

$$\Delta_{D0} = \frac{1}{EI} \left[\int_0^{10} (40x)(-x) dx + \int_{10}^{20} (400)(-x) dx + \int_0^{15} (41.67x - x^2)(-1.33x) dx \right]$$

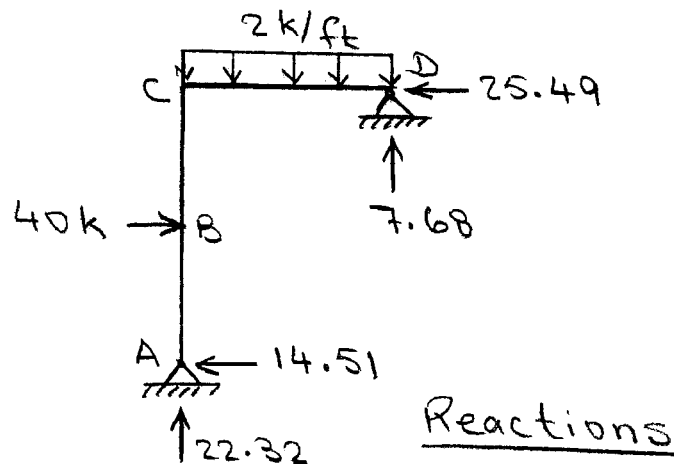
$$= - \frac{118958.33 \text{ k-ft}^3}{EI}$$

$$f_{DD} = \frac{1}{EI} \left[\int_0^{20} (-x)^2 dx + \int_0^{15} (-1.33x)^2 dx \right]$$

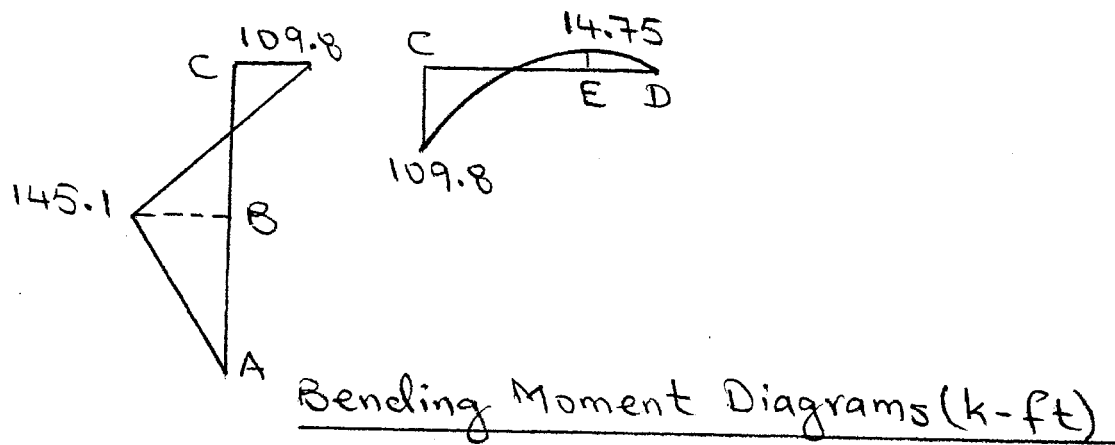
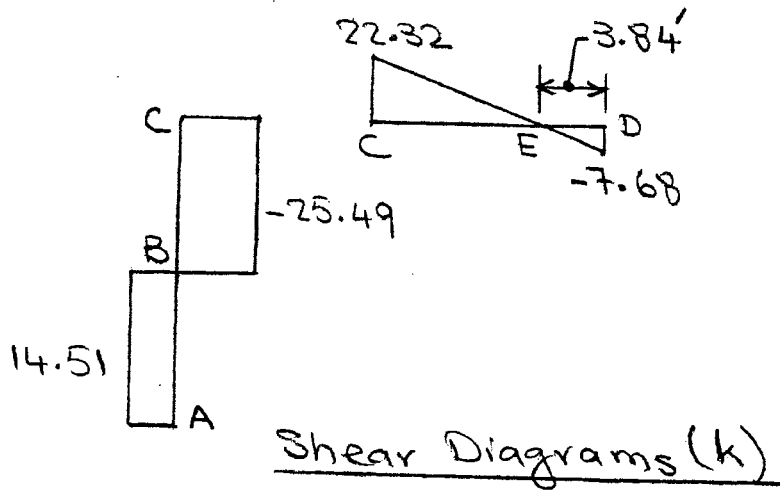
$$= \frac{4666.67 \text{ k-ft}^3/\text{k}}{EI}$$

Compatibility Equation: $\Delta_{D0} + f_{DD} D_X = 0$

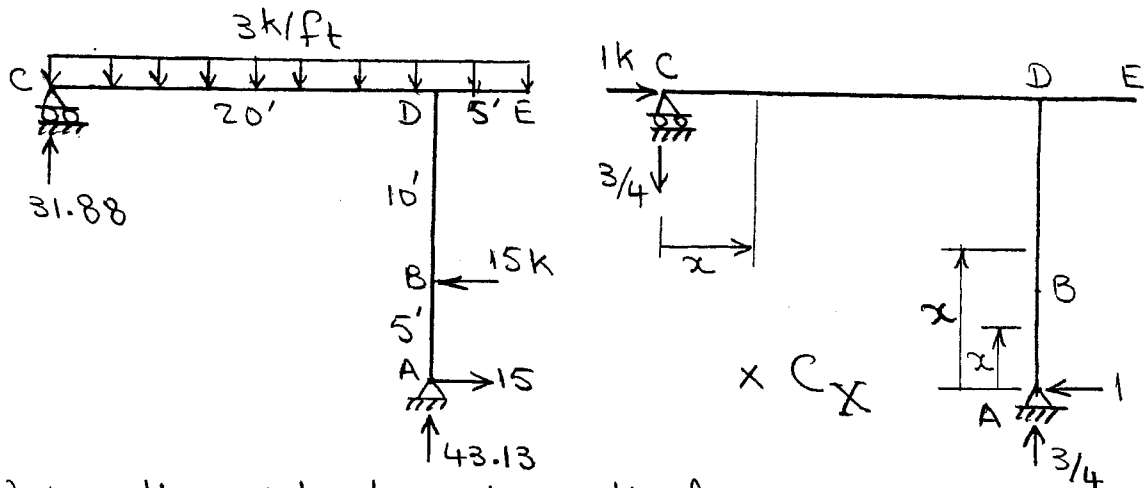
$$D_X = \frac{118958.33}{4666.67} = 25.49 \text{ k} \leftarrow$$



13.20 (contd.)



13.21



Using the virtual work method:

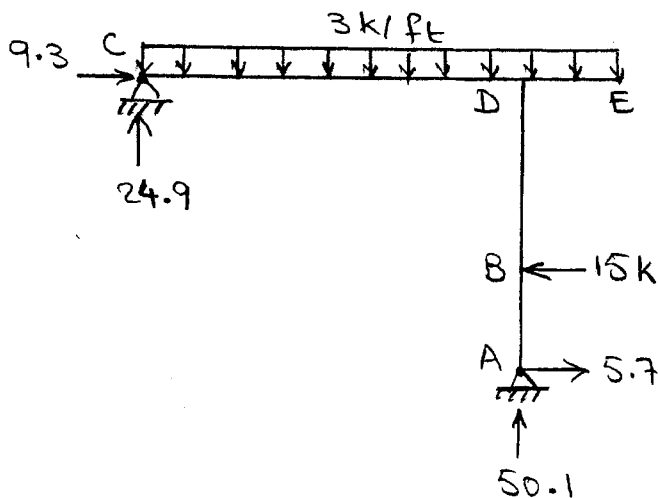
$$\Delta_{CO} = \frac{1}{EI} \left[\frac{1}{2} \int_0^{20} (31.88x - 1.5x^2) \left(-\frac{3}{4}x\right) dx + \int_0^5 (-15x)x dx + \int_5^{15} \{-15x + 15(x-5)\} x dx \right]$$

$$= -\frac{17505 \text{ k-ft}^3}{EI}$$

$$f_{CC} = \frac{1}{EI} \left[\frac{1}{2} \int_0^{20} \left(-\frac{3}{4}x\right)^2 dx + \int_0^{15} (x)^2 dx \right] = \frac{1875 \text{ k-ft}^3/\text{k}}{EI}$$

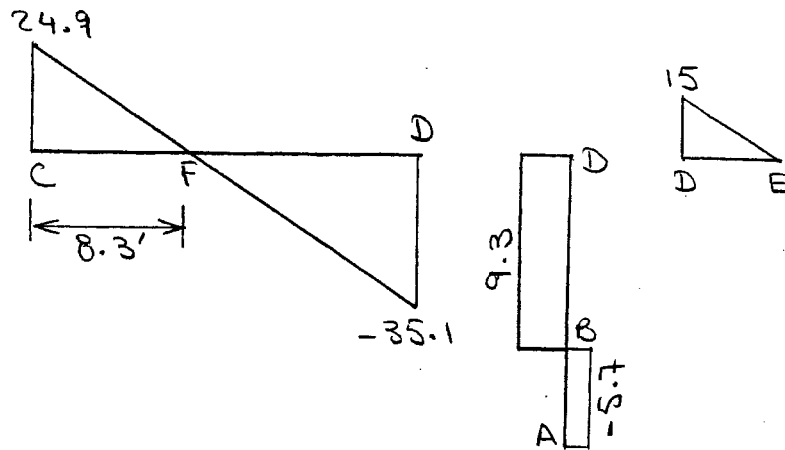
Compatibility Equation: $\Delta_{CO} + f_{CC} C_X = 0$

$$C_X = \frac{17505}{1875} = 9.3 \text{ k} \rightarrow$$

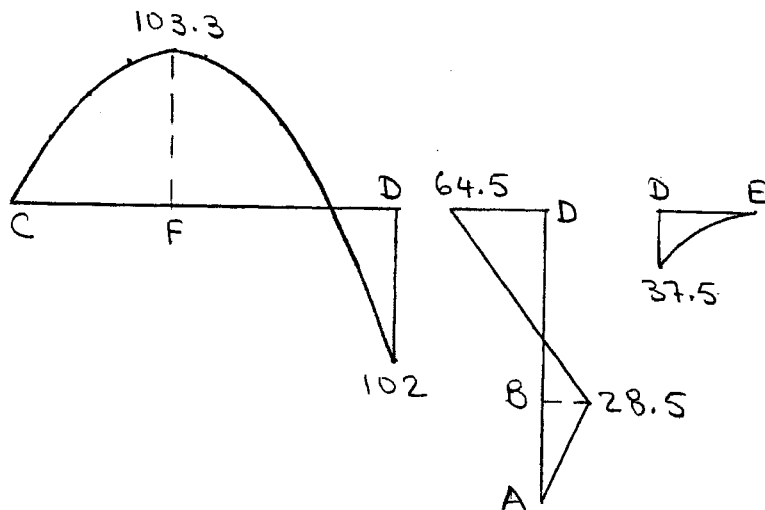


Reactions

13.21 (Contd.)

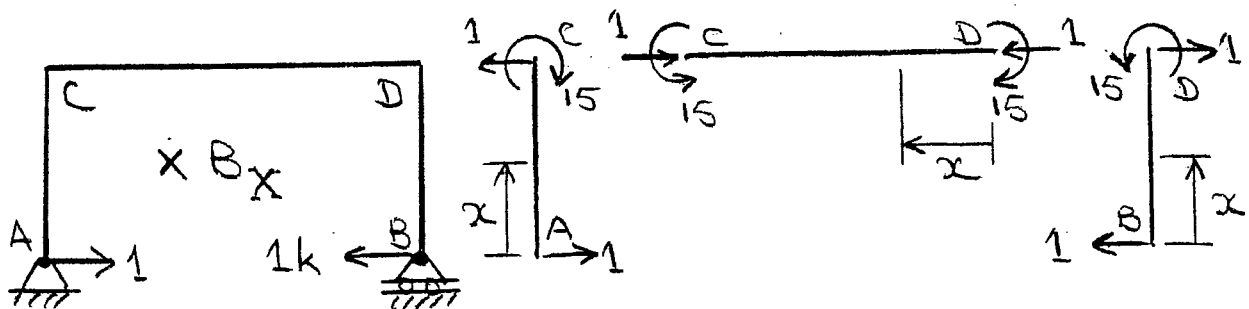
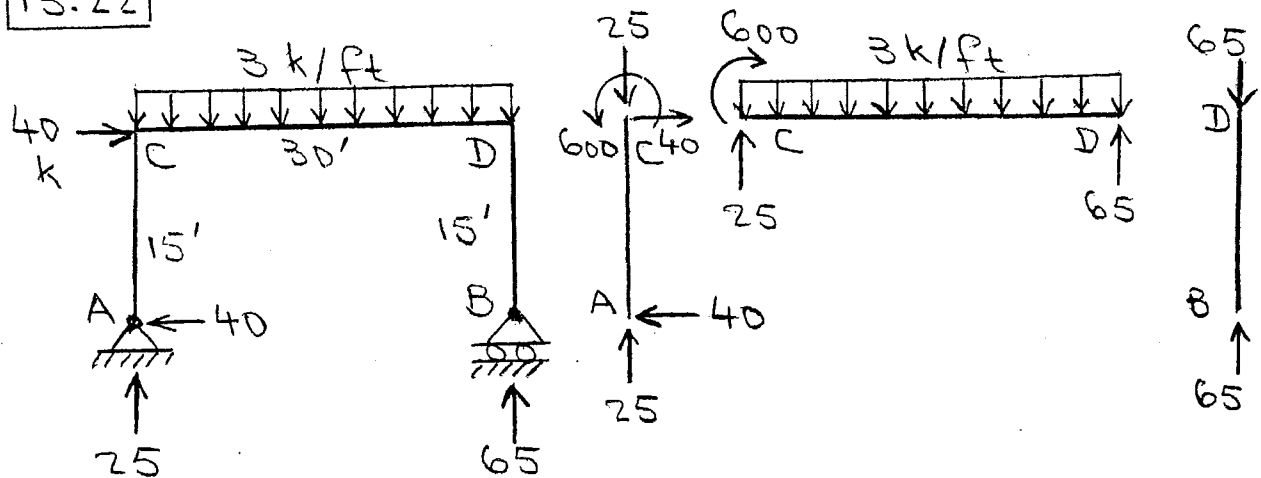


Shear Diagrams (k)



Bending Moment Diagrams (k-ft)

13.22



Using the virtual work method:

$$\Delta_{B0} = \frac{1}{EI} \left[\int_0^{15} (40x)(-x) dx + \int_0^{30} (65x - 1.5x^2)(-15) dx \right]$$

$$= - \frac{281250 \text{ k-ft}^3}{EI}$$

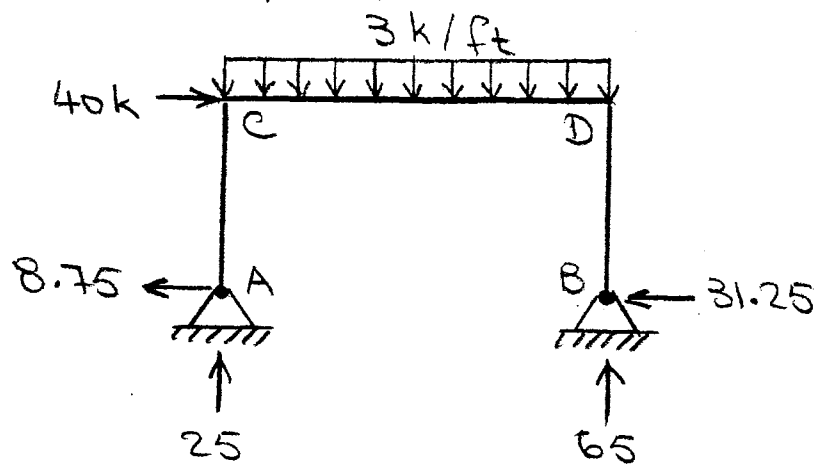
$$f_{BB} = \frac{1}{EI} \left[\int_0^{15} (-x)^2 dx + \int_0^{30} (-15)^2 dx + \int_0^{15} x^2 dx \right]$$

$$= \frac{9000 \text{ k-ft}^3/\text{k}}{EI}$$

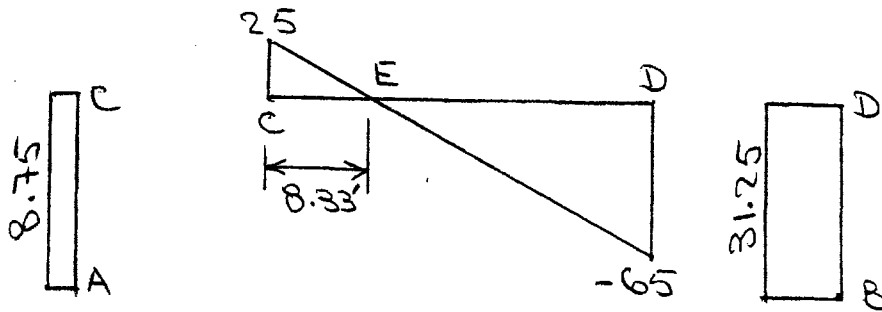
Compatibility Equation: $\Delta_{B0} + f_{BB} B_x = 0$

$$B_x = \frac{281250}{9000} = \underline{31.25 \text{ k} \leftarrow}$$

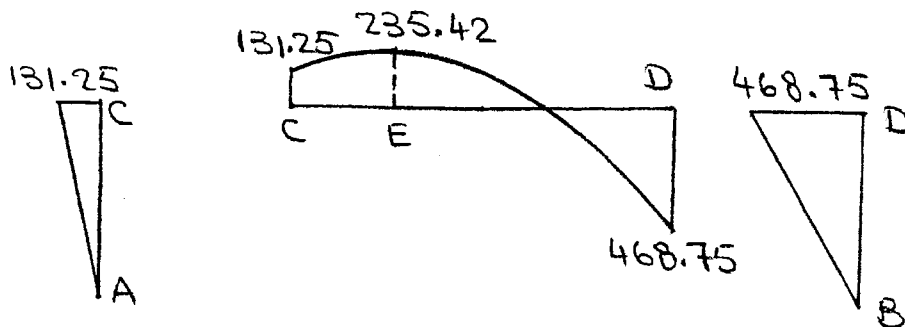
13.22 (Contd.)



Reactions

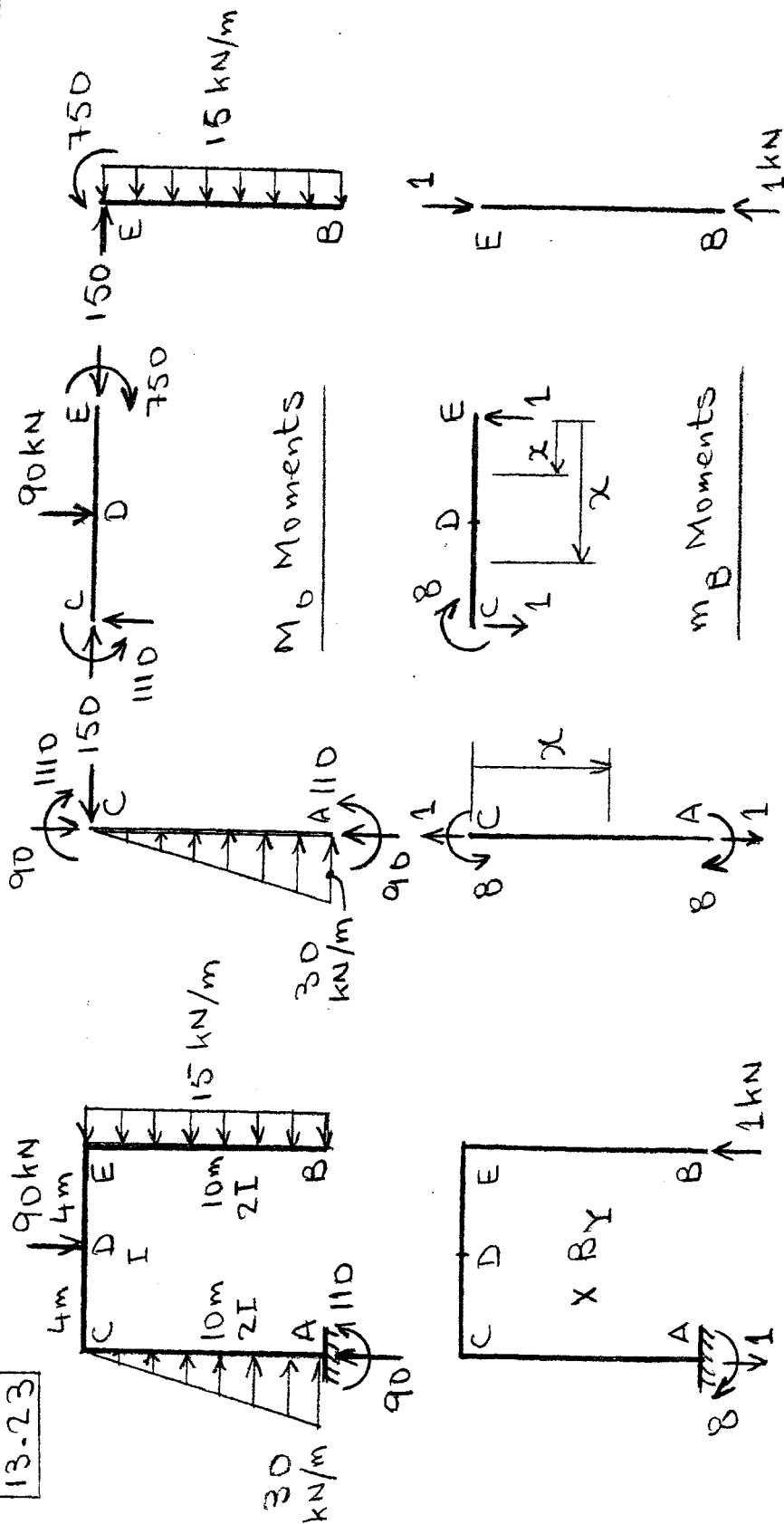


Shear Diagrams (k)



Bending Moment Diagrams (k-ft)

13.23



Segment	x coordinate		M ₀ (kN.m)	m _B (kN.m/kN)
	Origin	Limits (m)		
CA	C	0-10	150x - 1110 - $\frac{1}{2}(x)3x(\frac{x}{3})$	8
ED	E	0-4	-750	1x
DC	E	4-8	-750 - 90(x-4)	1x

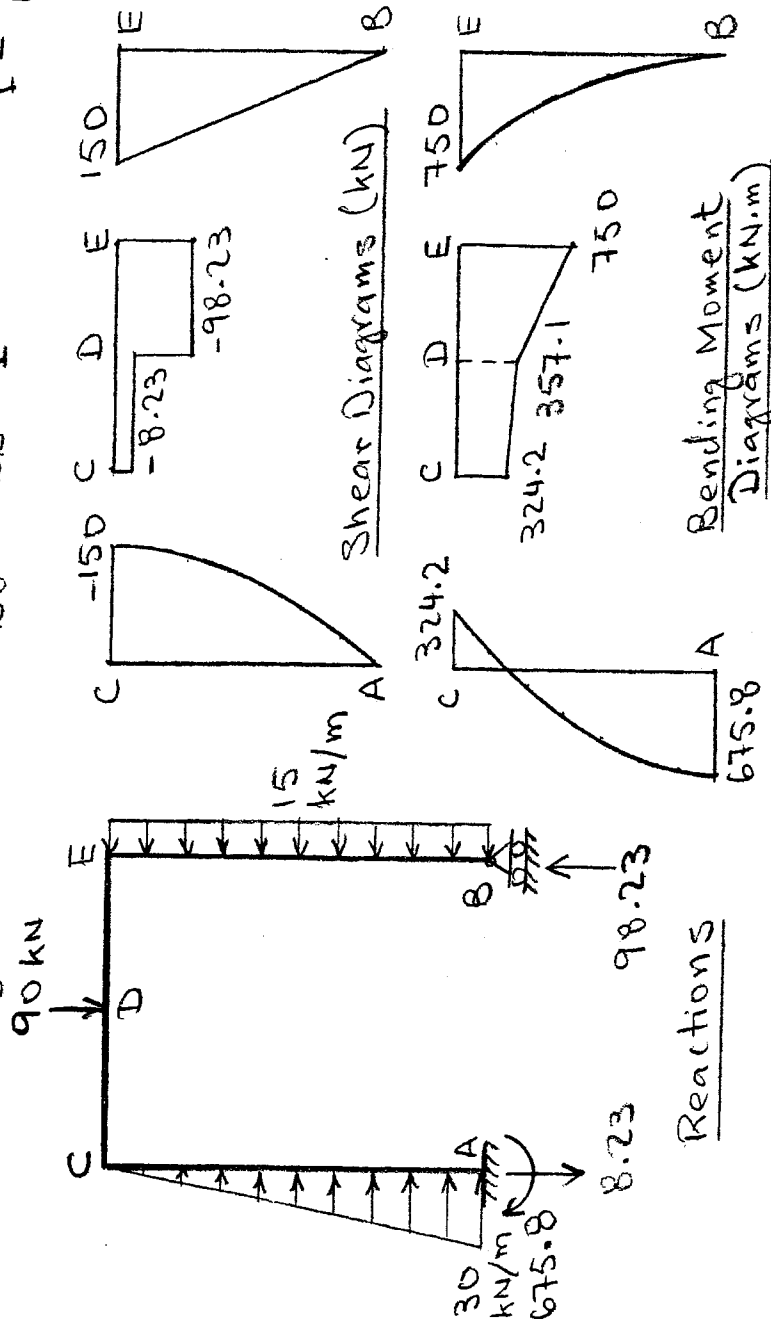
13.23 (contd.)

Using the virtual work method:

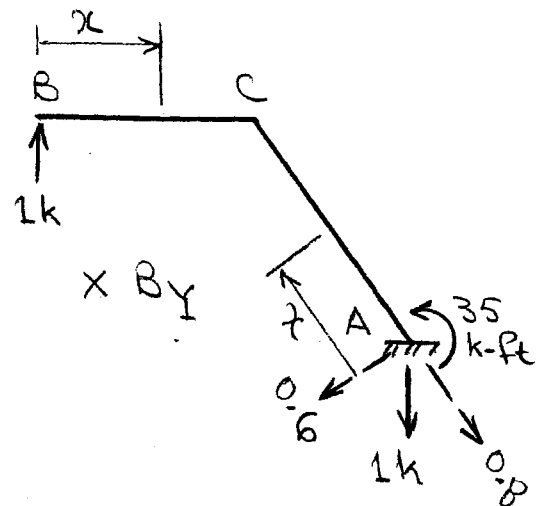
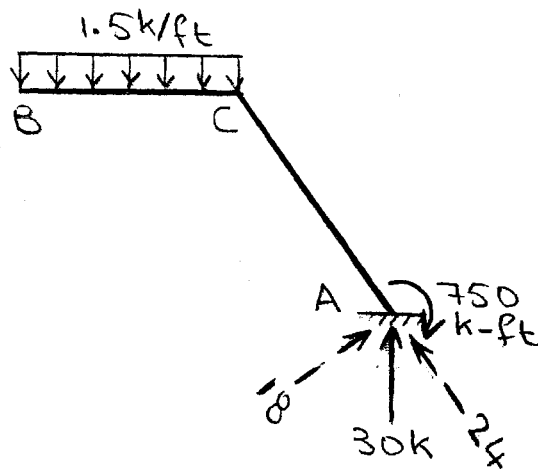
$$\Delta_{B0} = \sum \int \frac{M_0 m_B}{EI} dx = \frac{1}{EI} \left[\frac{1}{2} \int_0^{10} (150x - 1110 - \frac{x^3}{2}) (8) dx + \int_0^4 (-750) x dx + \int_4^8 (-390 - 90x) x dx \right] = -\frac{48200 \text{ kN}\cdot\text{m}^3}{EI}$$

$$f_{BB} = \sum \int \frac{m_B^2}{EI} dx = \frac{1}{EI} \left[\frac{1}{2} \int_0^{10} 64 dx + \int_0^8 x^2 dx \right] = \frac{490.67 \text{ kN}\cdot\text{m}^3/\text{kN}}{EI}$$

Compatibility Equation: $\Delta_{B0} + f_{BB} B_Y = 0 \quad B_Y = \frac{48200}{490.67} = 98.23 \text{ kN} \uparrow$



13.24



Using the virtual work method:

$$\Delta_{B0} = \frac{1}{EI} \left[\int_0^{20} \left(-\frac{1.5x^2}{2} \right) (1x) dx + \int_0^{25} (18x - 750) (-0.6x + 35) dx \right]$$

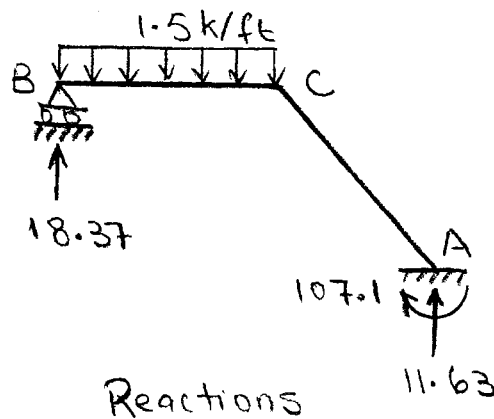
$$= -\frac{405000 \text{ k-ft}^3}{EI}$$

$$f_{BB} = \frac{1}{EI} \left[\int_0^{20} (1x)^2 dx + \int_0^{25} (-0.6x + 35)^2 dx \right]$$

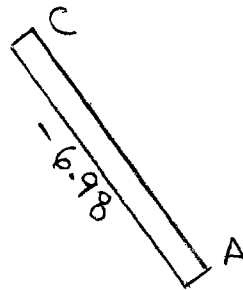
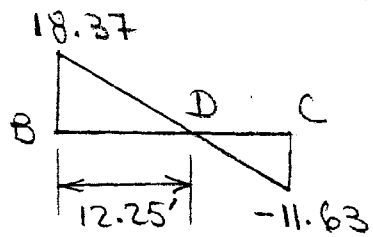
$$= \frac{22041.667 \text{ k-ft}^3/\text{k}}{EI}$$

Compatibility Equation: $\Delta_{B0} + f_{BB} B_Y = 0$

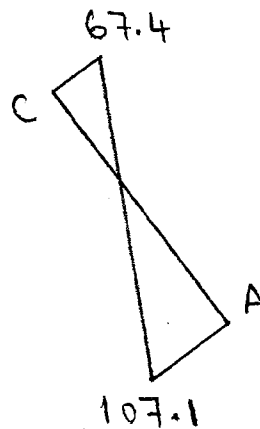
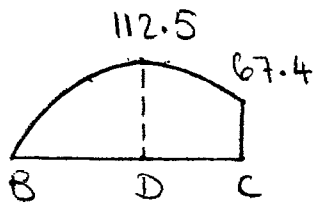
$$B_Y = \frac{405000}{22041.667} = 18.37 \text{ k} \uparrow$$



13.24 (contd.)

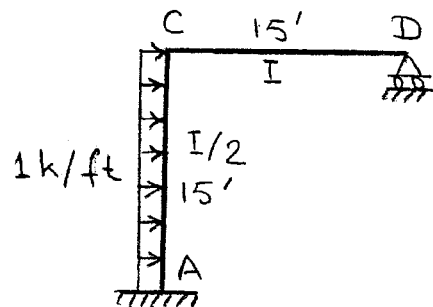


Shear Diagrams (k)

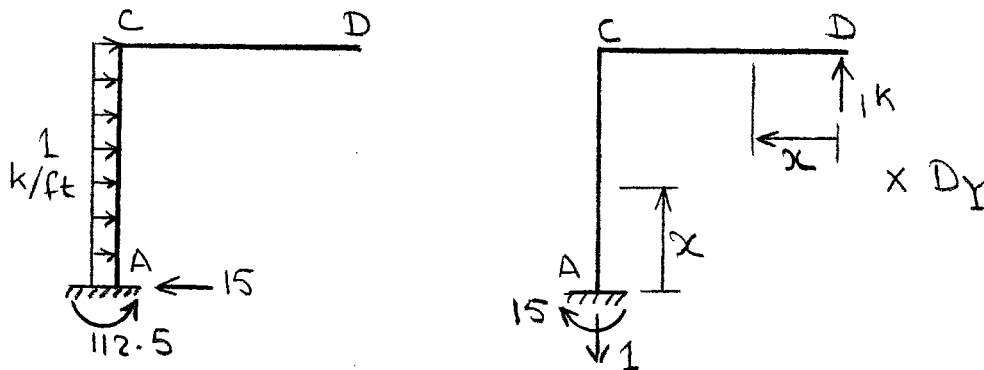


Bending Moment Diagrams (k-ft)

13-25 As the frame is symmetric subjected to antisymmetric loading, only a half of the structure needs to be analyzed.



Substructure for Analysis



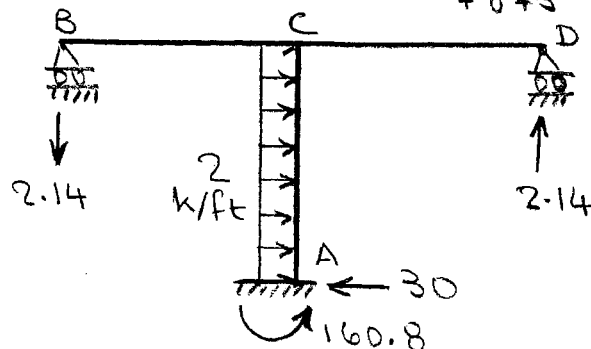
Using the virtual work method:

$$\Delta_{D0} = \frac{1}{EI} \left[2 \int_0^{15} (-112.5 + 15x - \frac{x^2}{2}) (15) dx \right] = -\frac{16875}{EI}$$

$$f_{DD} = \frac{1}{EI} \left[2 \int_0^{15} (15)^2 dx + \int_0^{15} (1x)^2 dx \right] = \frac{7875}{EI}$$

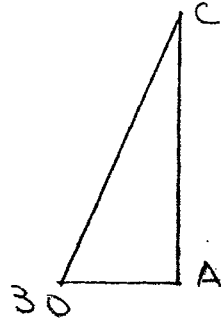
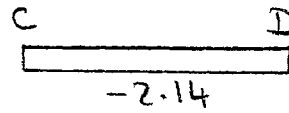
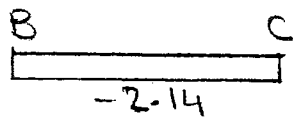
Compatibility Equation:

$$\Delta_{D0} + f_{DD} D_Y = 0 \quad D_Y = \frac{16875}{7875} = 2.14 \text{ k } \uparrow$$

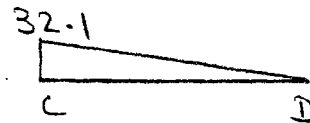
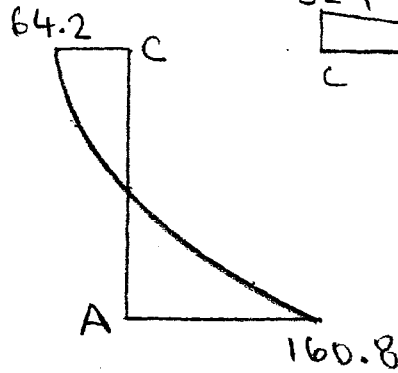


Reactions

13.25 (contd.)

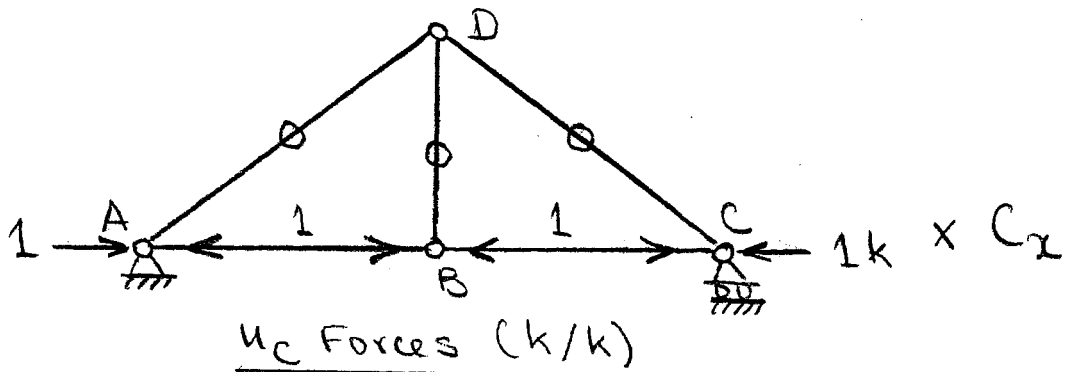
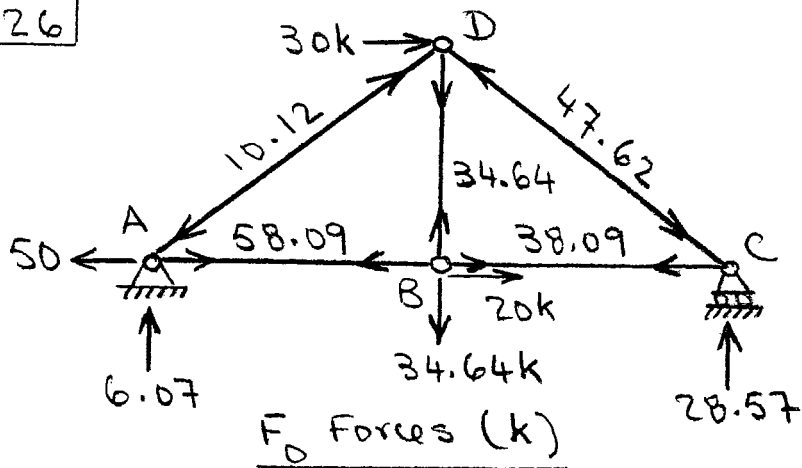


Shear Diagrams (k)



Bending Moment Diagrams (k-ft)

13.26

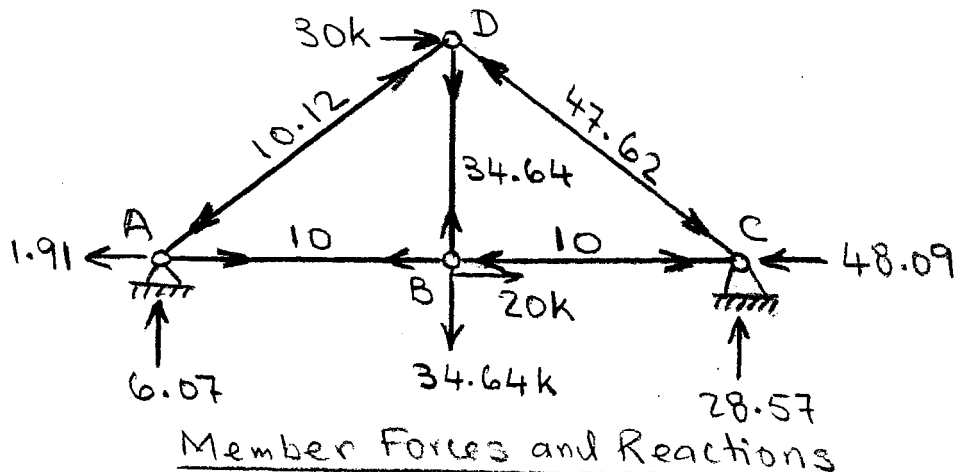


$$\Delta_{CO} = \frac{1}{EA} \sum F_D u_C L = \frac{1}{EA} [58.09(-1)8 + 38.09(-1)8]$$

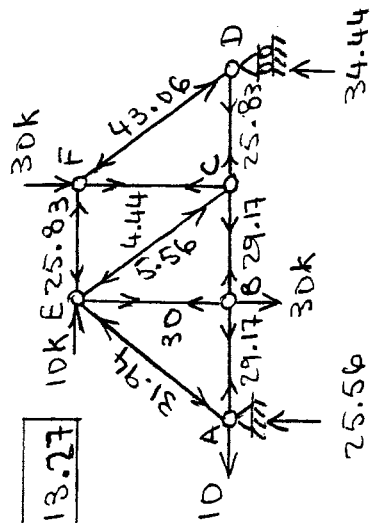
$$= - \frac{769.44 \text{ k-ft}}{EA}$$

$$f_{CC} = \frac{1}{EA} \sum u_C^2 L = \frac{1}{EA} [2(-1)^2 8] = \frac{16 \text{ ft}}{EA}$$

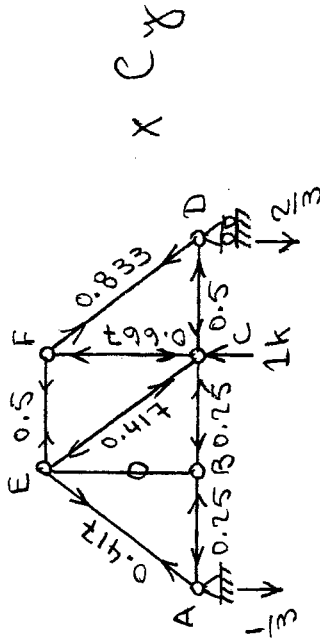
$$C_x = - \frac{\Delta_{CO}}{f_{CC}} = \frac{769.44}{16} = 48.09 \text{ k} \leftarrow$$



13.27

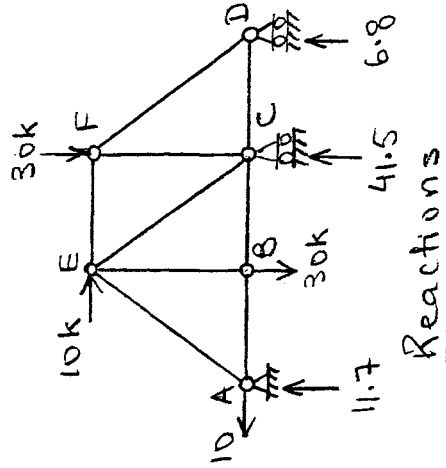


F_0 Forces



u_C Forces

Member	L (in.)	F_0 (k)	u_C (k/k)	$F_0 u_C L$ (k-in.)	$u_C^2 L$ (in.)	$F = F_0 + u_C C_y$ (k)
AB	180	29.17	-0.25	-1312.65	11.25	18.8
BC	180	29.17	-0.25	-1312.65	11.25	18.8
CD	180	25.83	-0.5	-2324.7	45	5.1
EF	180	-25.83	0.5	-2324.7	45	-5.1
AE	300	-31.94	0.417	-3995.69	52.17	-14.6
CE	300	-5.56	-0.417	695.56	52.17	-22.9
DF	300	-43.06	0.833	-10760.69	208.17	-8.5
BE	240	30	0	0	0	30
CF	240	4.44	-0.667	-710.76	106.77	-23.2
		Σ		-22046.28	531.78	

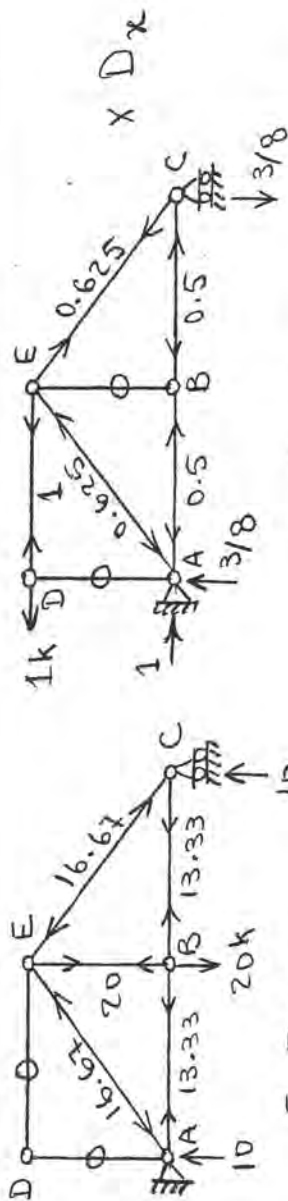


Reactions

$$\Delta_{CO} = \frac{1}{EA} \Sigma F_0 u_C L = \frac{-22046.28 \text{ (k-in.)}}{EA} ; f_{CC} = \frac{1}{EA} \Sigma u_C^2 L = \frac{531.78 \text{ (in.)}}{EA}$$

$$C_y = - \frac{\Delta_{CO}}{f_{CC}} = \frac{22046.28}{531.78} = 41.5 \text{ k} \uparrow$$

13.28

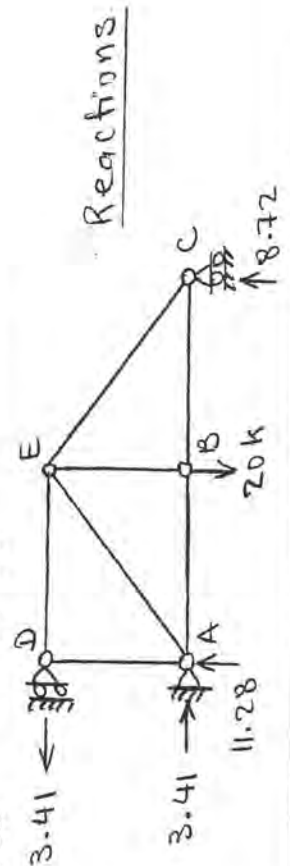


u_D Forces

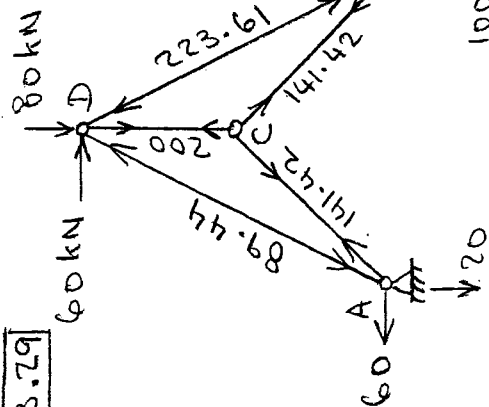
Member	L (in.)	A (in. ²)	F_0 (k)	u_D (k/in.)	$F_0 u_D L/A$ (k/in.)	$u_D^2 L/A$ (1/in.)	$F = F_0 + u_D D x$ (k)
AB	192	8	13.33	-0.5	-159.96	6	11.63
BC	192	8	13.33	-0.5	-159.96	6	11.63
DE	192	8	0	1	0	24	3.41
AE	240	6	-16.67	-0.625	416.75	15.63	-18.8
CE	240	8	-16.67	0.625	-312.56	11.72	-14.54
AD	144	6	0	0	0	0	0
BE	144	6	20	0	0	0	20
			Σ		-215.73	63.35	

$$\Delta_{D0} = \frac{1}{E} \Sigma \frac{F_0 u_D L}{A} = -\frac{215.73}{E} \text{ (k/in.)}; \quad f_{DD} = \frac{1}{E} \Sigma \frac{u_D^2 L}{A} = \frac{63.35}{E} \text{ (1/in.)}$$

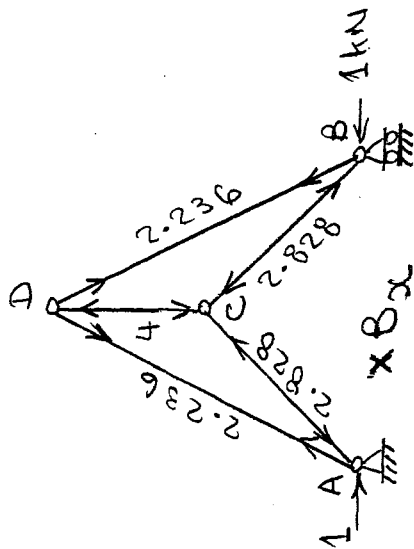
$$D_x = -\frac{\Delta_{D0}}{f_{DD}} = \frac{215.73}{63.35} = 3.41 \text{ k} \leftarrow$$



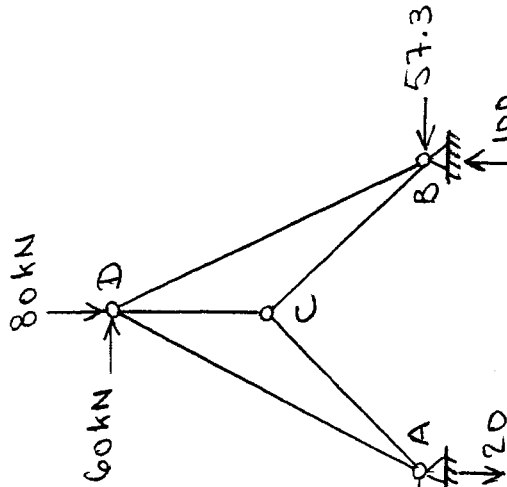
13.29



F_0 forces



u_B forces



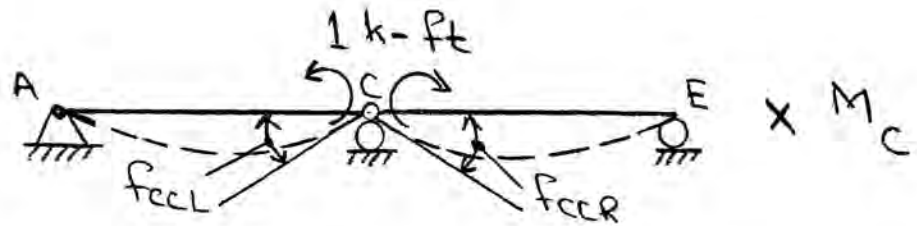
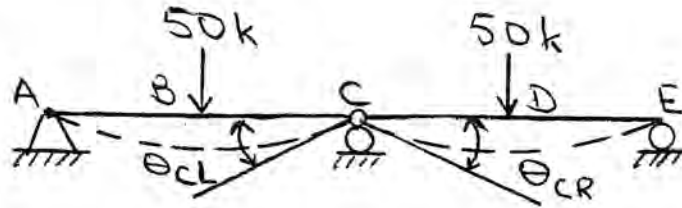
Reactions

Member	L (m)	F_0 (kN)	u_B (kN/m)	$F_0 u_B L$ (kN-m)	$u_B^2 L$ (m)	$F = F_0 + u_B \delta x$ (kN)
AD	11.18	-89.44	2.236	-2235.86	55.9	38.7
BD	11.18	-223.61	2.236	-5589.91	55.9	-95.5
AC	7.07	141.42	-2.828	-2827.55	56.54	-20.6
BC	7.07	141.42	-2.828	-2827.55	56.54	-20.6
CD	5	200	-4	-4000	80	-29.2
		Σ		-17480.87	304.88	

$$\Delta_{B0} = \frac{1}{EA} \Sigma f_0 u_B L = - \frac{17480.87 \text{ (kN-m)}}{EA} \quad ; \quad f_{BB} = \frac{1}{EA} \Sigma u_B^2 L = \frac{304.88 \text{ (m)}}{EA}$$

$$\delta x = - \frac{\Delta_{B0}}{f_{BB}} = \frac{17480.87}{304.88} = 57.3 \text{ kN} \leftarrow$$

13.30



Using beam deflection formulas:

$$\theta_{CL} = \theta_{CR} = \frac{1800 \text{ k-ft}^2}{EI}$$

$$\theta_{\text{corel.}} = \theta_{CL} + \theta_{CR} = \frac{3600 \text{ k-ft}^2}{EI}$$

$$f_{CL} = f_{CR} = \frac{8 \text{ ft}}{EI}$$

$$f_{\text{corel.}} = f_{CL} + f_{CR} = \frac{16 \text{ ft}}{EI}$$

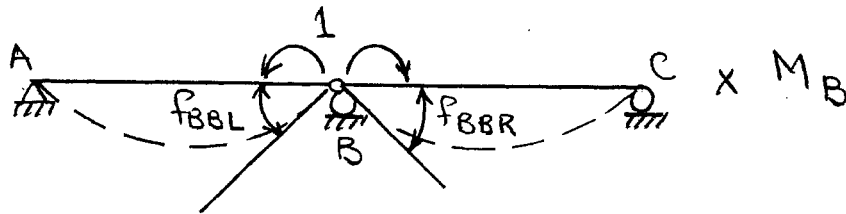
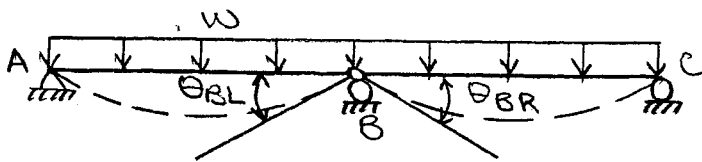
Compatibility Equation:

$$\theta_{\text{corel.}} + f_{\text{corel.}} M_C = 0$$

$$M_C = -\frac{3600}{16} = -225 \text{ k-ft}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.9.

13.31



Using beam deflection formulas:

$$\theta_{BL} = \theta_{BR} = \frac{wL^3}{24EI}$$

$$\theta_{B \text{ rel.}} = \theta_{BL} + \theta_{BR} = \frac{wL^3}{12EI}$$

$$P_{BBL} = P_{BBR} = \frac{L}{3EI}$$

$$P_{BB \text{ rel.}} = P_{BBL} + P_{BBR} = \frac{2L}{3EI}$$

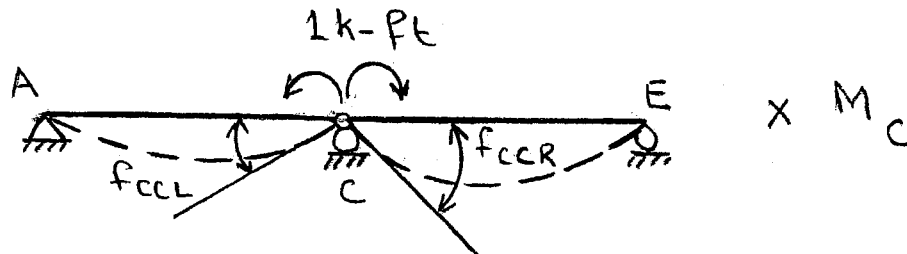
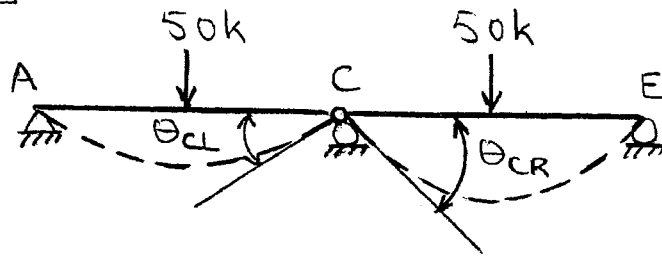
Compatibility Equation:

$$\theta_{B \text{ rel.}} + P_{BB \text{ rel.}} M_B = 0$$

$$M_B = -\frac{wL^3}{12EI} \left(\frac{3EI}{2L} \right) = -\frac{wL^2}{8}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.10.

13.32



Using beam deflection formulas:

$$\theta_{CL} = \frac{400}{EI} \quad ; \quad \theta_{CR} = \frac{800}{EI}$$

$$\theta_{C\text{rel.}} = \theta_{CL} + \theta_{CR} = \frac{1200 \text{ k-ft}^2}{EI}$$

$$f_{CCL} = \frac{2.67}{EI} \quad ; \quad f_{CCR} = \frac{5.33}{EI}$$

$$f_{C\text{rel.}} = f_{CCL} + f_{CCR} = \frac{8 \text{ ft}}{EI}$$

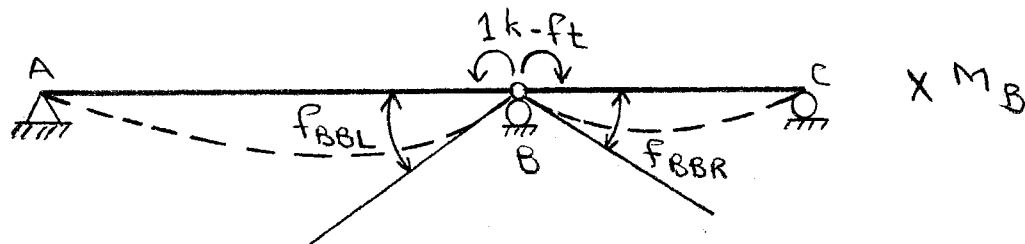
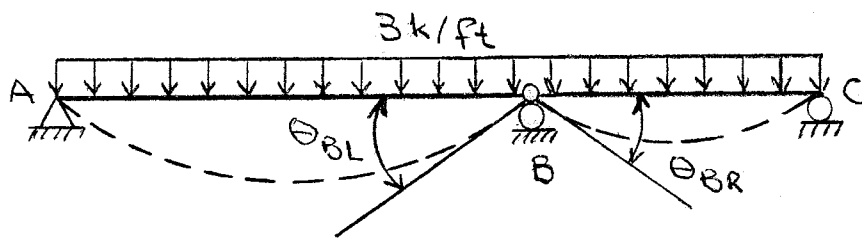
Compatibility Equation:

$$\theta_{C\text{rel.}} + f_{C\text{rel.}} M_C = 0$$

$$M_C = -\frac{1200}{8} = -150 \text{ k-ft}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.11.

13.33



Using beam deflection formulas:

$$\theta_{BL} = \frac{976.56 \text{ k-ft}^2}{EI} ; \quad \theta_{BR} = \frac{421.88 \text{ k-ft}^2}{EI}$$

$$\theta_{B\text{rel.}} = \theta_{BL} + \theta_{BR} = \frac{1398.44 \text{ k-ft}^2}{EI}$$

$$F_{BBL} = \frac{4.17 \text{ ft}}{EI} ; \quad F_{BBR} = \frac{5 \text{ ft}}{EI}$$

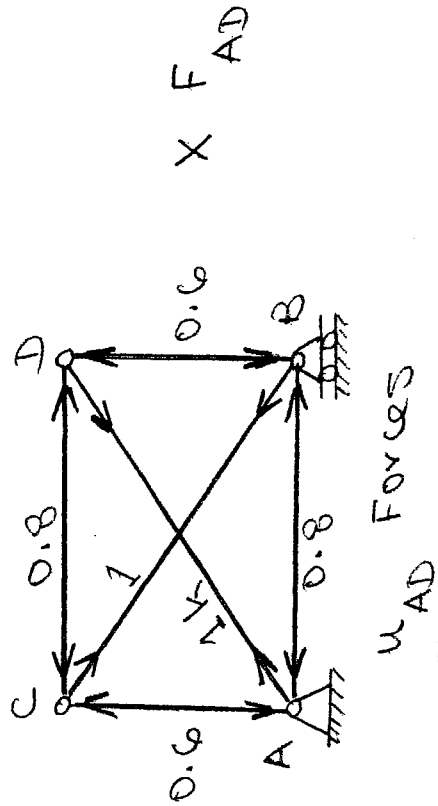
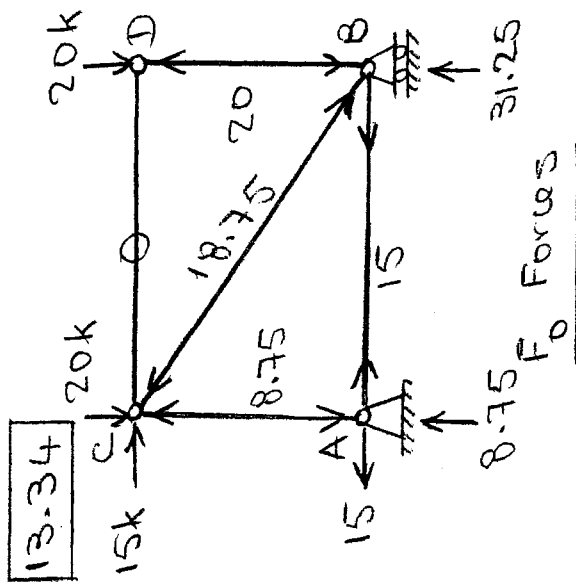
$$F_{BB\text{rel.}} = F_{BBL} + F_{BBR} = \frac{9.17 \text{ ft}}{EI}$$

Compatibility Equation:

$$\theta_{B\text{rel.}} + F_{BB\text{rel.}} M_B = 0$$

$$M_B = - \frac{1398.44}{9.17} = \underline{-152.6 \text{ k-ft}}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.12.

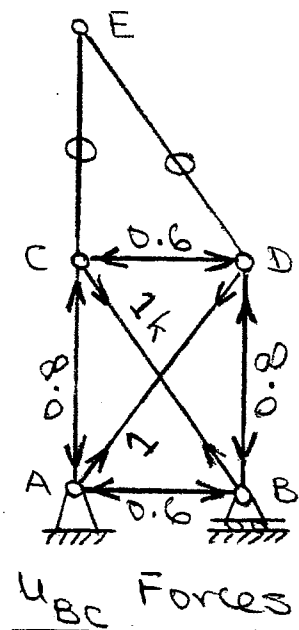
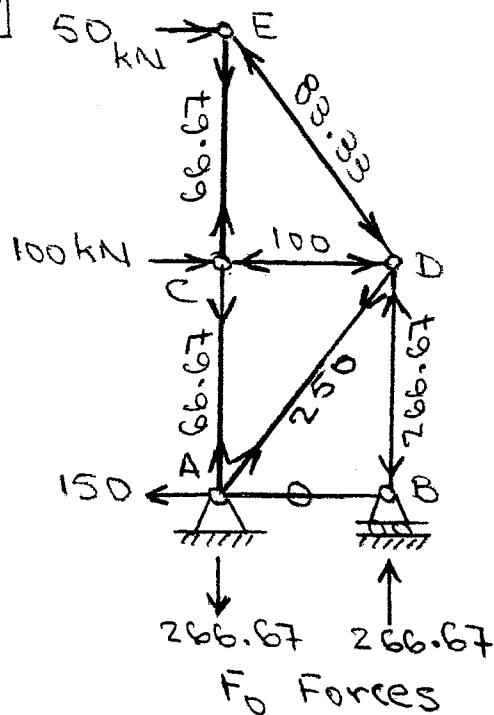


Member	L (in.)	A (in ²)	F_0 (k)	u_{AD} (k/k)	$F_0 u_{AD} L/A$ (k/in.)	$u_{AD}^2 L/A$ (1/in.)	$F = F_0 + u_{AD} F_{AD}$ (k)
AB	192	6	15	-0.8	-384	20.48	11.4
CD	192	6	0	-0.8	0	20.48	-3.8
AC	144	6	-8.75	-0.6	126	8.64	-11.5
BD	144	6	-20	-0.6	288	8.64	-22.7
BC	240	8	-18.75	1	-562.5	30	-14.2
AD	240	8	0	1	0	30	4.5
				Σ	-532.5	118.24	

$$\Delta_{ADD} = \frac{1}{E} \Sigma \frac{F_0 u_{AD} L}{A} = -\frac{532.5 (k \cdot in.)}{E}; \quad f_{AD, AD} = \frac{1}{E} \Sigma \frac{u_{AD}^2 L}{A} = \frac{118.24 (1/in.)}{E}$$

$$F_{AD} = -\frac{\Delta_{ADD}}{f_{AD, AD}} = \frac{532.5}{118.24} = 4.5 k (T)$$

13.35



Member	L (m)	F_0 (kN)	u_{BC} (kN/kN)	$F_0 u_{BC} L$ (kN.m)	$u_{BC}^2 L$ (m)	$F = F_0 + u_{BC} F_{BC}$ (kN)
AB	3	0	-0.6	0	1.08	71.9
CD	3	-100	-0.6	180	1.08	-28.1
AC	4	66.67	-0.8	-213.33	2.56	162.5
BD	4	-266.67	-0.8	853.33	2.56	-170.8
CE	4	66.67	0	0	0	66.7
AD	5	250	1	1250	5	130.2
DE	5	-83.33	0	0	0	-83.3
BC	5	0	1	0	5	-119.8
Σ				2070	17.28	

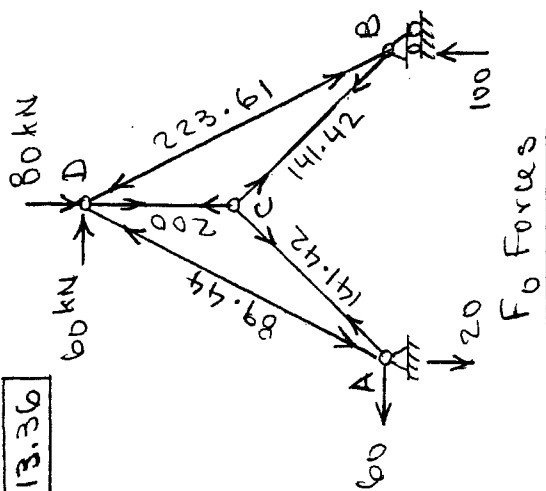
$$\Delta_{BCD} = \frac{1}{EA} \Sigma F_0 u_{BC} L = \frac{2070 \text{ (kN.m)}}{EA}$$

$$f_{BC,BC} = \frac{1}{EA} \Sigma u_{BC}^2 L = \frac{17.28 \text{ (m)}}{EA}$$

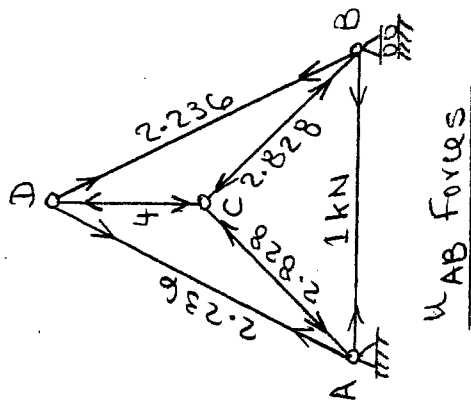
$$F_{BC} = - \frac{\Delta_{BCD}}{f_{BC,BC}} = - \frac{2070}{17.28} = -119.8 \text{ kN}$$

$$= \underline{119.8 \text{ kN (C)}}$$

13.36



F_0 Forces



u_{AB} Forces

Member	L (m)	F_0 (kN)	u_{AB} (kN/kN)	$F_0 u_{AB} L$ (kN-m)	$u_{AB}^2 L$ (m)	$F = F_0 + u_{AB} F_{AB}$
AD	11.18	-89.44	2.236	-2235.86	55.9	34.7
BD	11.18	-223.61	2.236	-5589.91	55.9	-99.5
AC	7.07	141.42	-2.828	-2827.55	56.54	-15.6
BC	7.07	141.42	-2.828	-2827.55	56.54	-15.6
CD	5	200	-4	-4000	80	-22
AB	10	0	1	0	10	55.5
Σ					314.88	

$$\Delta_{ABD} = \frac{1}{EA} \Sigma F_0 u_{AB} L = - \frac{17480.87}{EA} ; \quad F_{AB,AB} = \frac{1}{EA} \Sigma u_{AB}^2 L = \frac{314.88}{EA}$$

$$F_{AB} = - \frac{\Delta_{ABD}}{F_{AB,AB}} = \frac{17480.87}{314.88} = 55.5 \text{ kN (T)}$$

13.37 Using beam deflection formulas:

$$\Delta_{B0} = -\frac{wL^4}{384EI} = -\frac{17(25)(16)^4}{384EI} = -\frac{72533.33 \text{ kN.m}^3}{EI}$$

$$\Delta_{C0} = -\frac{wL^4}{8EI} = -\frac{25(16)^4}{8EI} = -\frac{204800 \text{ kN.m}^3}{EI}$$

$$f_{BB} = \frac{L^3}{24EI} = \frac{(16)^3}{24EI} = \frac{170.67 \text{ kN.m}^3/\text{kN}}{EI}$$

$$f_{BC} = f_{CB} = \frac{5L^3}{48EI} = \frac{5(16)^3}{48EI} = \frac{426.67 \text{ kN.m}^3/\text{kN}}{EI}$$

$$f_{CC} = \frac{L^3}{3EI} = \frac{(16)^3}{3EI} = \frac{1365.33 \text{ kN.m}^3/\text{kN}}{EI}$$

Compatibility Equations:

$$\Delta_{B0} + f_{BB} B_y + f_{BC} C_y = 0$$

$$\Delta_{C0} + f_{CB} B_y + f_{CC} C_y = 0$$

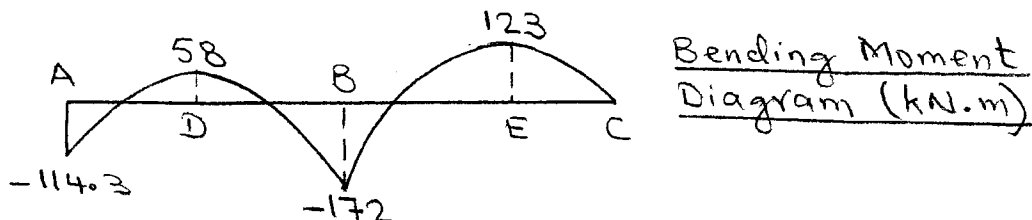
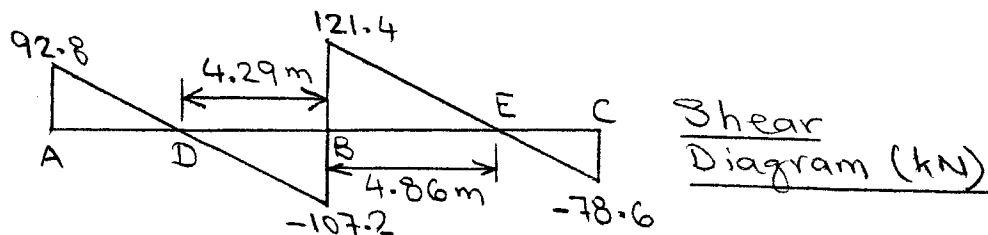
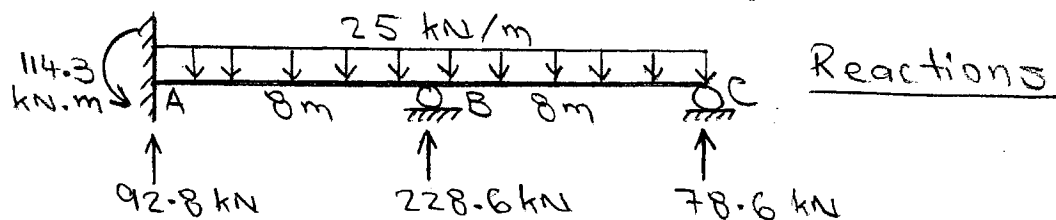
$$-72533.33 + (170.67)B_y + (426.67)C_y = 0$$

$$-204800 + (426.67)B_y + (1365.33)C_y = 0$$

Solving these equations, we obtain

$$B_y = 228.6 \text{ kN} \uparrow$$

$$C_y = 78.6 \text{ kN} \uparrow$$



13.38 Using beam deflection formulas:

$$\Delta_{B0} = -\frac{819166.67 \text{ k-ft}^3}{EI}; \quad \Delta_{D0} = -\frac{280000 \text{ k-ft}^3}{EI}$$

$$f_{BB} = \frac{21333.33 \text{ k-ft}^3/\text{k}}{EI}; \quad f_{BD} = f_{DB} = \frac{6666.67 \text{ k-ft}^3/\text{k}}{EI}$$

$$f_{DD} = \frac{2666.67 \text{ k-ft}^3/\text{k}}{EI}$$

Compatibility Equations:

$$\Delta_{B0} + f_{BB} B_y + f_{BD} D_y = 0$$

$$\Delta_{D0} + f_{DB} B_y + f_{DD} D_y = 0$$

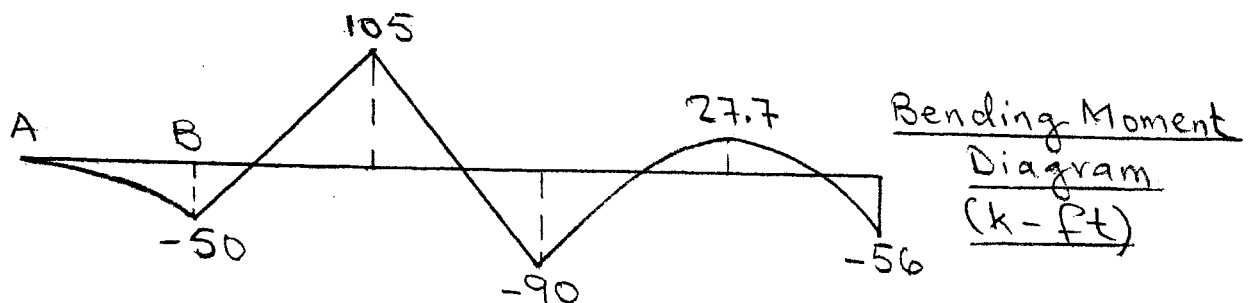
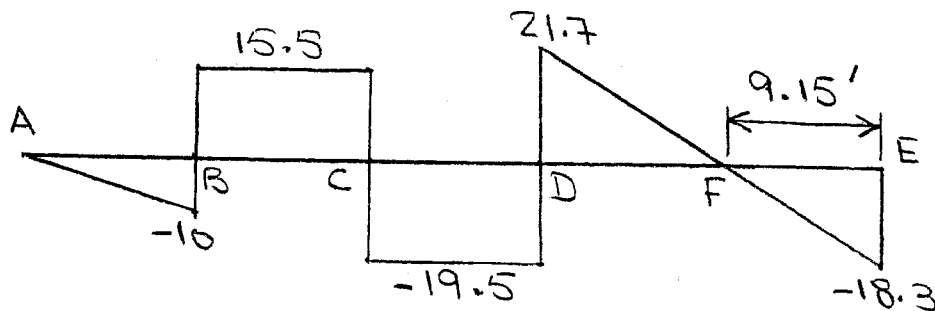
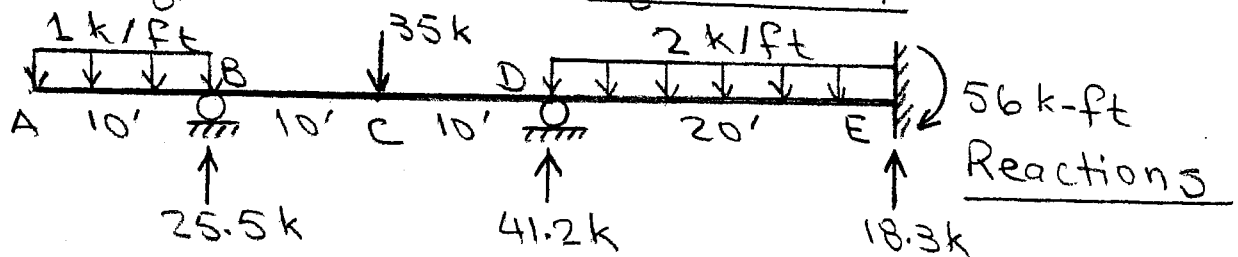
$$-819166.67 + (21333.33) B_y + (6666.67) D_y = 0$$

$$-280000 + (6666.67) B_y + (2666.67) D_y = 0$$

Solving these equations, we obtain

$$B_y = 25.5 \text{ k} \uparrow$$

$$D_y = 41.2 \text{ k} \uparrow$$



13.39 The bending moments at C and E are selected as the redundants. Using beam deflection formulas, we obtain:

$$\theta_{C \text{ rel.}} = \theta_{CL} + \theta_{CR} = \frac{1}{EI} (768 + 336) = \frac{1104 \text{ kN.m}^2}{EI}$$

$$\theta_{E \text{ rel.}} = \theta_{EL} + \theta_{ER} = \frac{1}{EI} (384 + 600) = \frac{984 \text{ kN.m}^2}{EI}$$

$$f_{C \text{ rel.}} = f_{CCL} + f_{CCR} = \frac{1}{EI} \left(\frac{10}{3} + \frac{10}{6} \right) = \frac{5 \text{ kN.m}^2/\text{kN.m}}{EI}$$

$$f_{CE} = f_{EC} = \frac{10}{12EI} = \frac{0.833 \text{ kN.m}^2/\text{kN.m}}{EI}$$

$$f_{EE \text{ rel.}} = f_{EEL} + f_{EER} = \frac{1}{EI} \left(\frac{10}{6} + \frac{8}{3} \right) = \frac{4.33 \text{ kN.m}^2/\text{kN.m}}{EI}$$

Compatibility Equations: $\theta_{C \text{ rel.}} + f_{C \text{ rel.}} M_C + f_{CE} M_E = 0$

$\theta_{E \text{ rel.}} + f_{EC} M_C + f_{EE \text{ rel.}} M_E = 0$

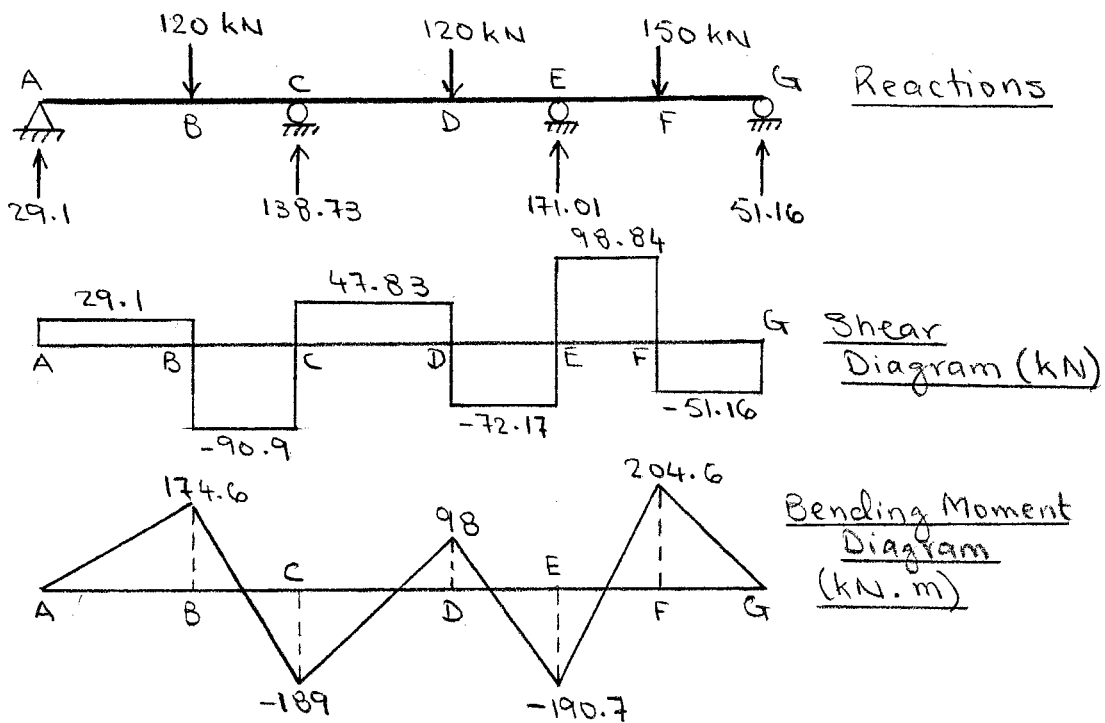
$$1104 + 5M_C + 0.833M_E = 0$$

$$984 + 0.833M_C + 4.33M_E = 0$$

Solving these equations, we obtain

$$M_C = -189 \text{ kN.m}$$

$$M_E = -190.7 \text{ kN.m}$$



13.40 The bending moments at B and C are selected as the redundants. Using beam deflection formulas, we obtain:

$$\theta_{B\text{rel.}} = \theta_{BL} + \theta_{BR} = \frac{1}{EI} (1440 + 720) = \frac{2160 \text{ k-ft}^2}{EI}$$

$$\theta_{C\text{rel.}} = \theta_{CL} + \theta_{CR} = \frac{1}{EI} (720 + 1440) = \frac{2160 \text{ k-ft}^2}{EI}$$

$$f_{BB\text{rel.}} = f_{BBL} + f_{BBR} = \frac{1}{EI} (8 + 4) = \frac{12 \text{ k-ft}^2/\text{k-ft}}{EI}$$

$$f_{CC\text{rel.}} = f_{CCL} + f_{CCR} = \frac{1}{EI} (4 + 8) = \frac{12 \text{ k-ft}^2/\text{k-ft}}{EI}$$

$$f_{BC} = f_{CB} = \frac{2 \text{ k-ft}^2/\text{k-ft}}{EI}$$

Compatibility Equations:

$$\theta_{B\text{rel.}} + f_{BB\text{rel.}} M_B + f_{BC} M_C = 0$$

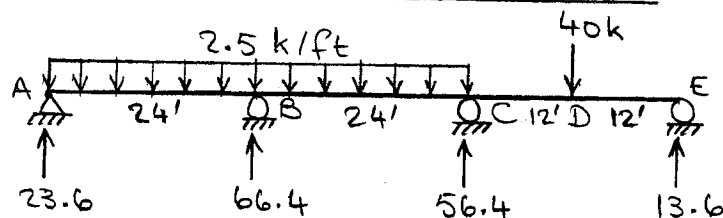
$$\theta_{C\text{rel.}} + f_{CB} M_B + f_{CC\text{rel.}} M_C = 0$$

$$2160 + 12 M_B + 2 M_C = 0$$

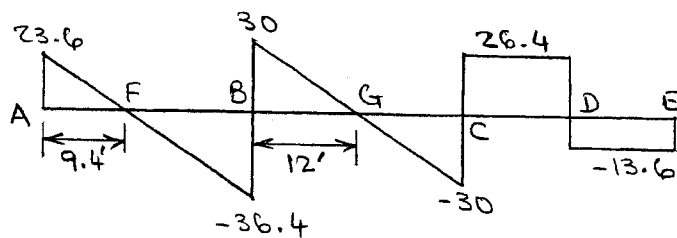
$$2160 + 2 M_B + 12 M_C = 0$$

Solving these equations, we obtain

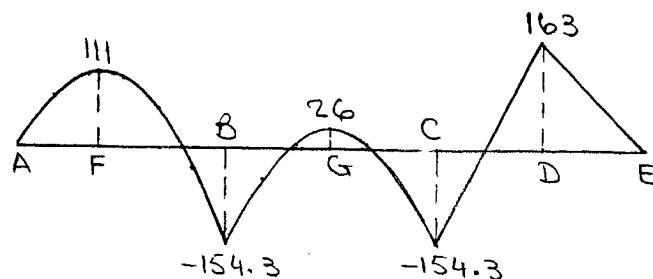
$$M_B = M_C = -154.3 \text{ k-ft}$$



Reactions

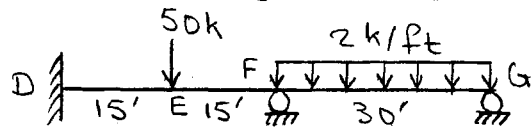


Shear Diagram (k)



Bending Moment Diagram (k-ft)

13.41 As the beam and the loading are symmetric, we will analyze only the right half DG of the beam.



Using beam deflection formulas:

$$\Delta_{FO} = -\frac{1085625 \text{ k-ft}^3}{EI} ; \Delta_{GO} = -\frac{3076875 \text{ k-ft}^3}{EI}$$

$$f_{FF} = \frac{9000 \text{ k-ft}^3/\text{k}}{EI} ; f_{FG} = f_{GF} = \frac{22500 \text{ k-ft}^3/\text{k}}{EI}$$

$$f_{GG} = \frac{72000 \text{ k-ft}^3/\text{k}}{EI}$$

Compatibility Equations:

$$\Delta_{FO} + f_{FF} F_y + f_{FG} G_y = 0$$

$$\Delta_{GO} + f_{GF} F_y + f_{GG} G_y = 0$$

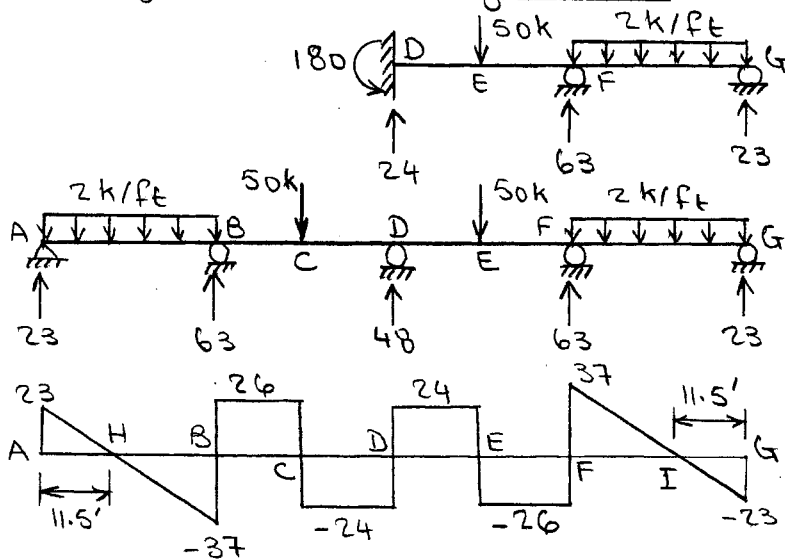
$$-1085625 + 9000 F_y + 22500 G_y = 0$$

$$-3076875 + 22500 F_y + 72000 G_y = 0$$

Solving these equations, we obtain

$$F_y = 63 \text{ k} \uparrow$$

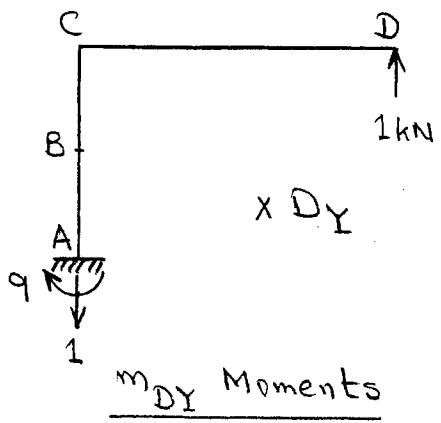
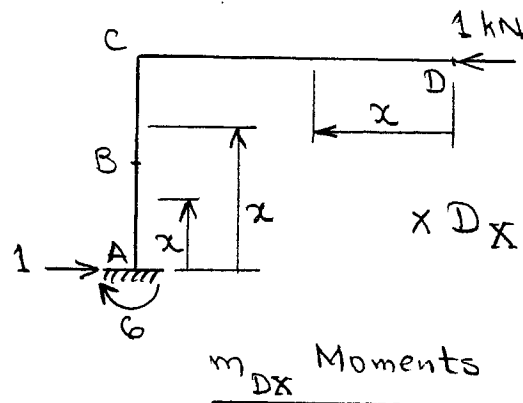
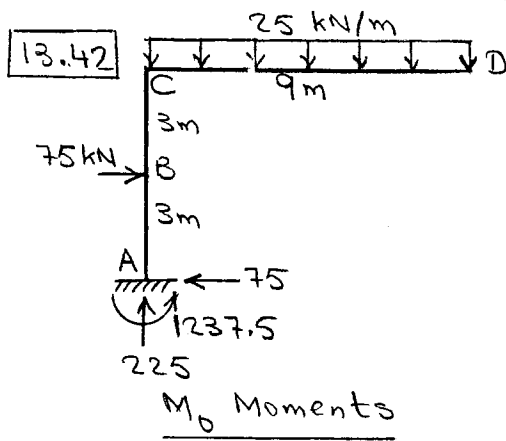
$$G_y = 23 \text{ k} \uparrow$$



Reactions

Shear
Diagram (k)

Bending
Moment Diagram
(k-ft)



Segment	x Coordinate		M_0 (kN.m)	m_{Dx} (kN.m/kN)	m_{Dy} (kN.m/kN)
	Origin	Limits (m)			
AB	A	0-3	$-1237.5 + 75x$	$6 - x$	9
BC	A	3-6	-1012.5	$6 - x$	9
DC	D	0-9	$-12.5x^2$	0	x

$$\Delta_{DXO} = \sum \int \frac{M_0 m_{Dx}}{EI} dx = - \frac{19912.5 \text{ kN-m}^3}{EI}$$

$$\Delta_{DYO} = \sum \int \frac{M_0 m_{Dy}}{EI} dx = - \frac{78215.625 \text{ kN-m}^3}{EI}$$

$$f_{DX,DX} = \sum \int \frac{m_{Dx}^2}{EI} dx = \frac{72 \text{ kN-m}^3/\text{kN}}{EI}$$

$$f_{DY,DY} = \sum \int \frac{m_{Dy}^2}{EI} dx = \frac{729 \text{ kN-m}^3/\text{kN}}{EI}$$

$$f_{DX,DY} = f_{DY,DX} = \sum \int \frac{m_{Dx} m_{Dy}}{EI} dx = \frac{162 \text{ kN-m}^3/\text{kN}}{EI}$$

13.42 (contd.) Compatibility Equations:

$$\Delta_{DX0} + f_{DX,DX} D_X + f_{DX,DY} D_Y = 0$$

$$\Delta_{DY0} + f_{DY,DX} D_X + f_{DY,DY} D_Y = 0$$

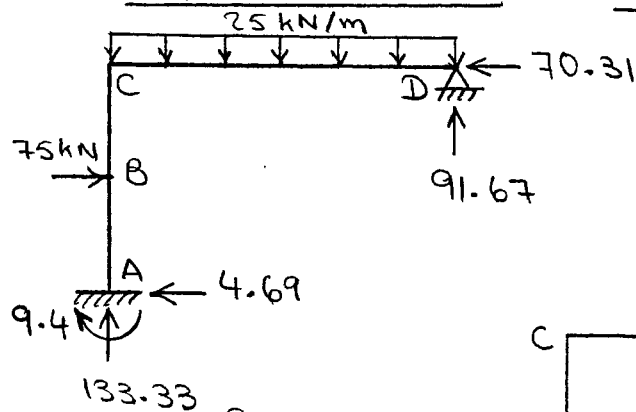
$$-19912.5 + 72 D_X + 162 D_Y = 0$$

$$-78215.625 + 162 D_X + 729 D_Y = 0$$

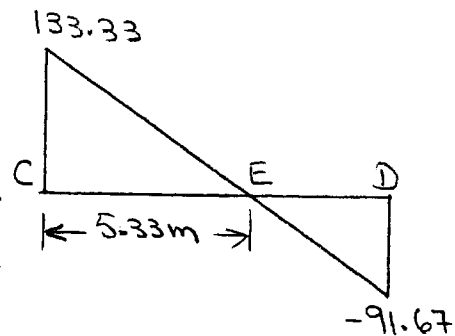
Solving these equations, we obtain

$$D_X = 70.3 \text{ kN} \leftarrow$$

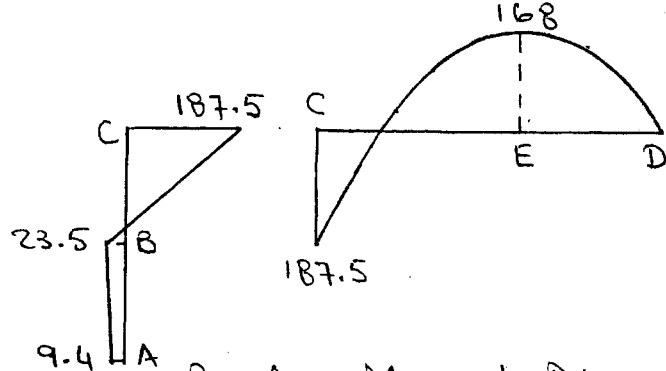
$$D_Y = 91.7 \text{ kN} \uparrow$$



Reactions

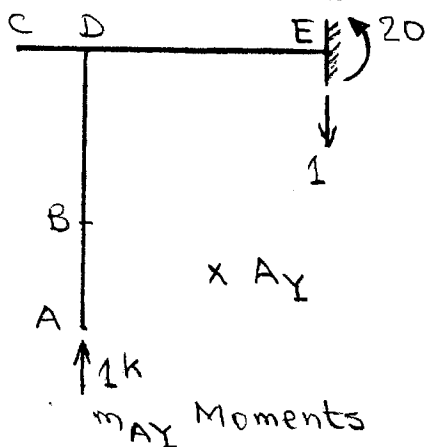
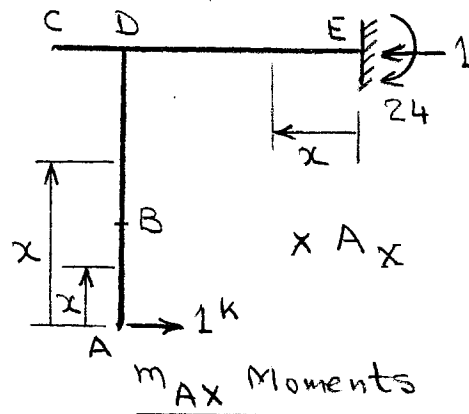
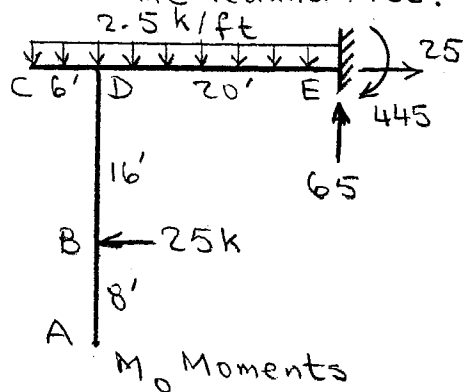


Shear Diagrams (kN)



Bending Moment Diagrams (kN.m)

13.43 The reactions A_x and A_y are selected as the redundants.



Segment	x Coordinate		M_0 (k-ft)	m_{AX} (k-ft/k)	m_{AY} (k-ft/k)
	Origin	Limits (ft)			
AB	A	0-8	0	$-1x$	0
BD	A	8-24	$25(x-8)$	$-1x$	0
ED	E	0-20	$-445 + 65x - 1.25x^2$	-24	$20-x$

$$\Delta_{AXD} = \sum \int \frac{M_0 m_{AX}}{EI} dx = - \frac{78133.33 \text{ k-ft}^3}{EI}$$

$$\Delta_{AYD} = \sum \int \frac{M_0 m_{AY}}{EI} dx = - \frac{19000 \text{ k-ft}^3}{EI}$$

$$f_{AX,AX} = \sum \int \frac{m_{AX}^2}{EI} dx = \frac{16128 \text{ k-ft}^3/k}{EI}$$

$$f_{AY,AY} = \sum \int \frac{m_{AY}^2}{EI} dx = \frac{2666.67 \text{ k-ft}^3/k}{EI}$$

$$f_{AX,AY} = f_{AY,AX} = \sum \int \frac{m_{AX} m_{AY}}{EI} dx = - \frac{4800 \text{ k-ft}^3/k}{EI}$$

13.43 (contd.) Compatibility Equations:

$$\Delta_{AX0} + f_{AX,AX} A_X + f_{AX,AY} A_Y = 0$$

$$\Delta_{AY0} + f_{AY,AX} A_X + f_{AY,AY} A_Y = 0$$

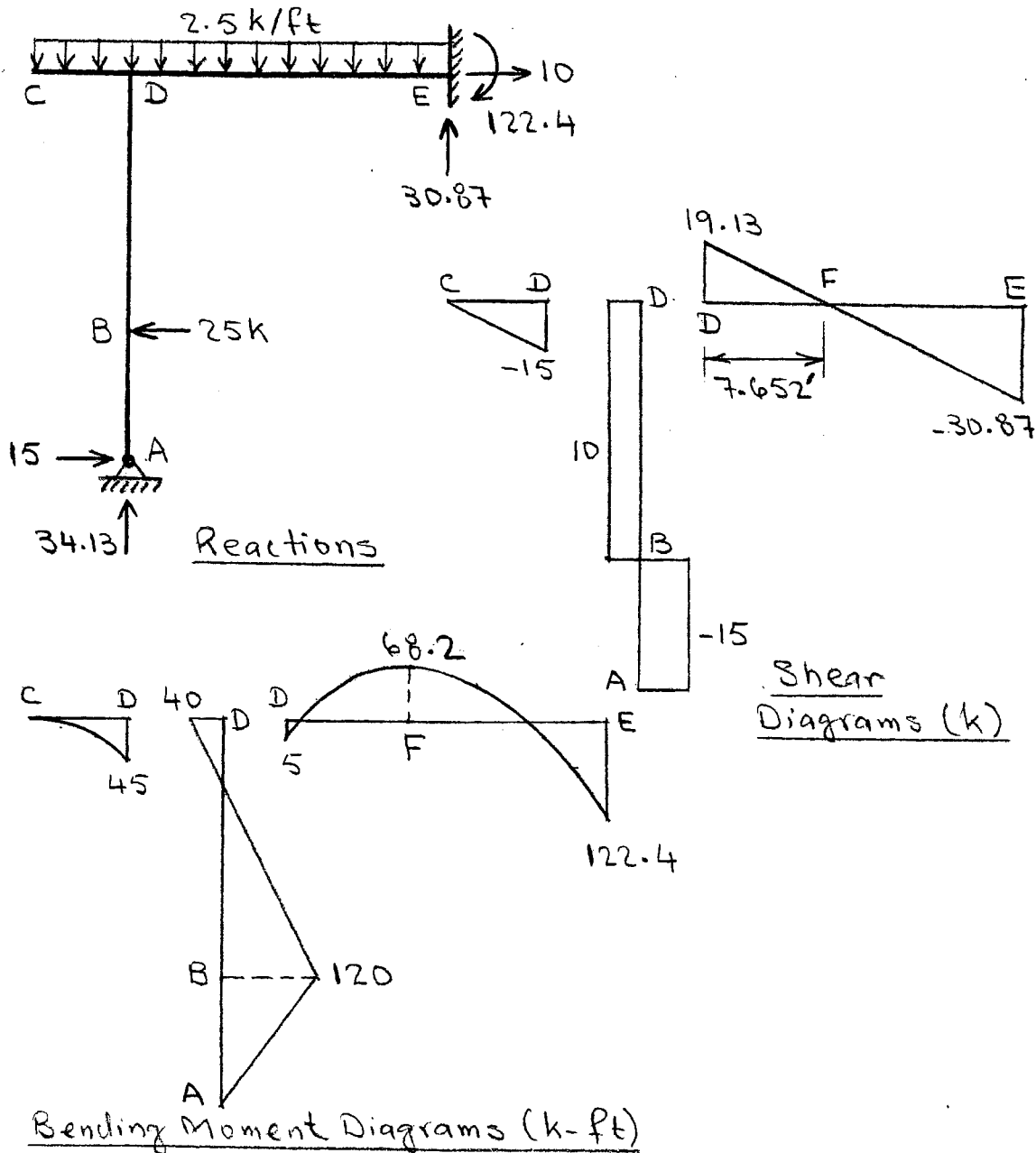
$$-78133.33 + 16128 A_X - 4800 A_Y = 0$$

$$-19000 - 4800 A_X + 2666.67 A_Y = 0$$

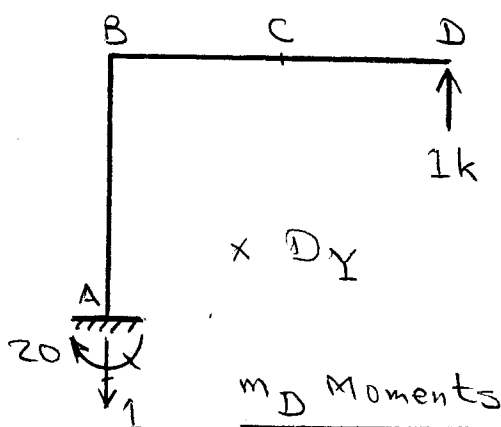
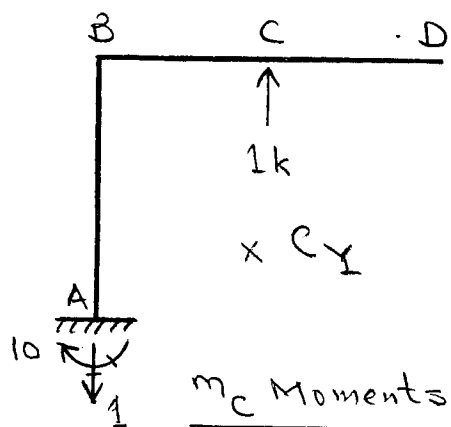
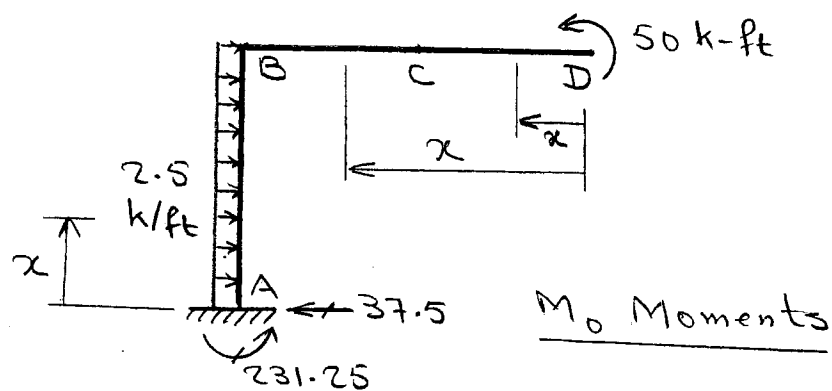
Solving these equations, we obtain:

$$A_X = 15 \text{ k} \rightarrow$$

$$A_Y = 34.13 \text{ k} \uparrow$$



13.44



Member	x coordinate		M_o (k-ft)	m_C	m_D
	Origin	Limits (ft)			
AB	A	0-15	$-231.25 + 37.5x - 1.25x^2$	10	20
CB	D	10-20	50	$1(x-10)$	$1x$
DC	D	0-10	50	0	$1x$

$$\Delta_{CO} = \sum \int \frac{M_o m_C}{EI} dx = - \frac{4062.5 \text{ k-ft}^3}{EI}$$

$$\Delta_{DO} = \sum \int \frac{M_o m_D}{EI} dx = - \frac{3125 \text{ k-ft}^3}{EI}$$

$$f_{CC} = \sum \int \frac{m_C^2}{EI} dx = \frac{1833.33 \text{ k-ft}^3/\text{k}}{EI}$$

$$f_{DD} = \sum \int \frac{m_D^2}{EI} dx = \frac{8666.67 \text{ k-ft}^3/\text{k}}{EI}$$

$$f_{CD} = f_{DC} = \sum \int \frac{m_C m_D}{EI} dx = \frac{3833.33 \text{ k-ft}^3/\text{k}}{EI}$$

13.44 (Contd.) Compatibility Equations:

$$\Delta_{CO} + f_{CC} C_Y + f_{CD} D_Y = 0$$

$$\Delta_{DO} + f_{DC} C_Y + f_{DD} D_Y = 0$$

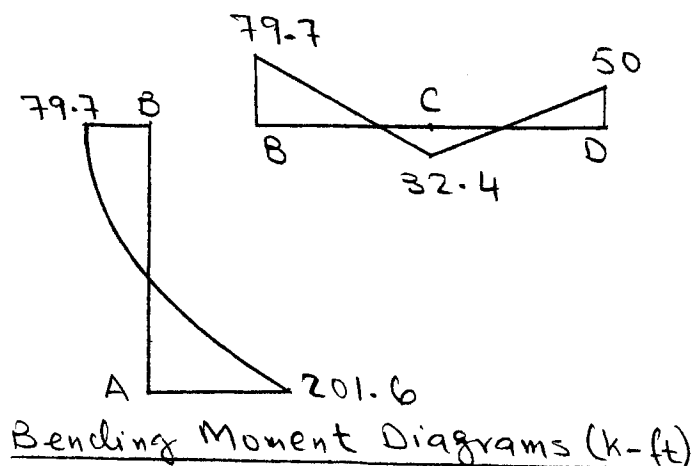
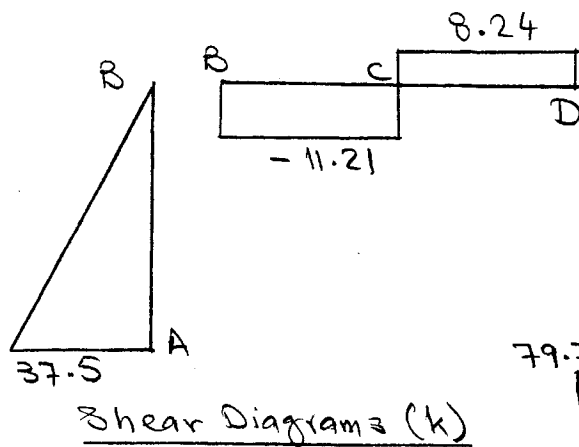
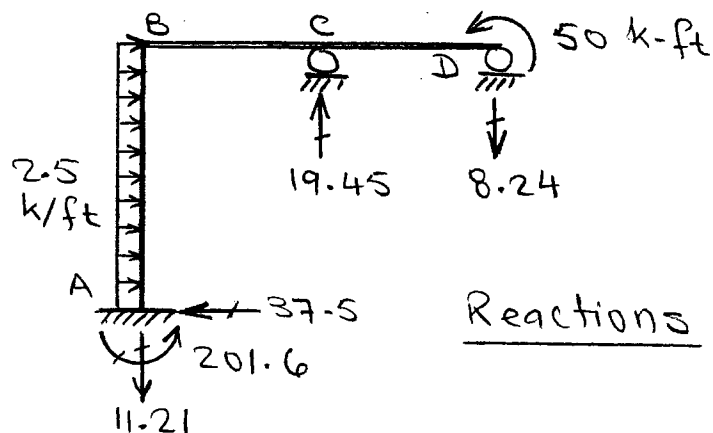
$$-4062.5 + 1833.33 C_Y + 3833.33 D_Y = 0$$

$$-3125 + 3833.33 C_Y + 8666.67 D_Y = 0$$

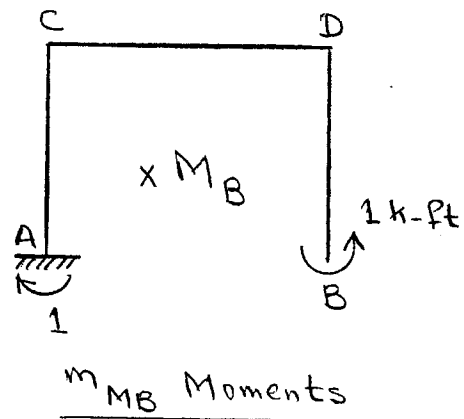
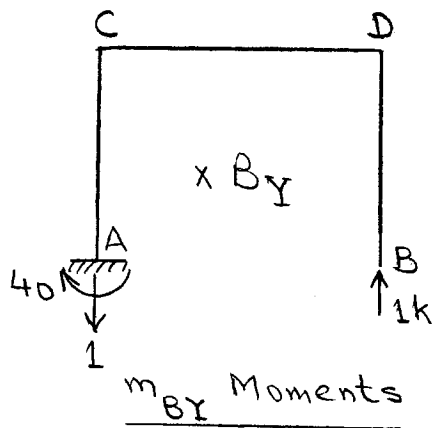
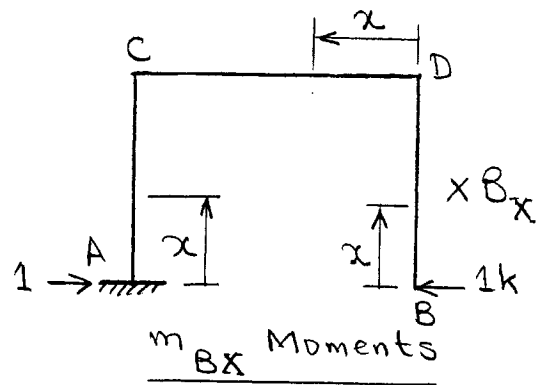
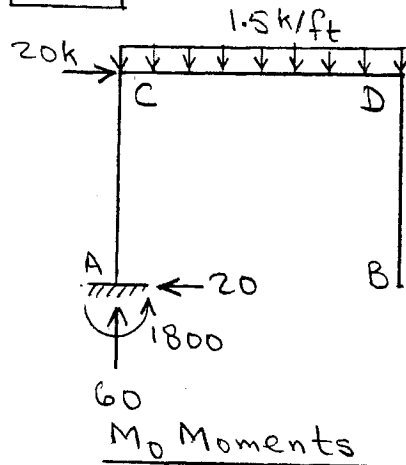
Solving these equations, we obtain:

$$C_Y = 19.45 \text{ k} \uparrow$$

$$D_Y = -8.24 \text{ k} = 8.24 \text{ k} \downarrow$$



13.45



Member	x Coordinate		Moment of Inertia	M_0 (k-ft)	m_{Bx}	m_{By}	m_{MB}
	Origin	Limits (ft)					
AC	A	0-60	I	$-1800 + 20x$	$-x$	40	1
DC	D	0-40	2I	$-0.75x^2$	-30	x	1
BD	B	0-40	I	0	x	0	-1

$$\Delta_{Bx0} = \sum \int \frac{M_0 m_{Bx}}{EI} dx = \frac{870000 \text{ k-ft}^3}{EI}$$

$$\Delta_{By0} = \sum \int \frac{M_0 m_{By}}{EI} dx = -\frac{2040000 \text{ k-ft}^3}{EI}$$

$$\Theta_{B0} = \sum \int \frac{M_0 m_{MB}}{EI} dx = -\frac{53000 \text{ k-ft}^2}{EI}$$

$$f_{Bx, Bx} = \sum \int \frac{m_{Bx}^2}{EI} dx = \frac{36000 \text{ k-ft}^3/\text{k}}{EI}$$

$$13.45 \text{ (Contd.)} \quad f_{BY, BY} = \sum \int \frac{m_{BY}^2}{EI} dx = \frac{58666.67}{EI}$$

$$f_{MB, MB} = \sum \int \frac{m_{MB}^2}{EI} dx = \frac{80}{EI}$$

$$f_{BX, BY} = f_{BY, BX} = \sum \int \frac{m_{BX} m_{BY}}{EI} dx = -\frac{30000}{EI}$$

$$f_{BX, MB} = f_{MB, BX} = \sum \int \frac{m_{BX} m_{MB}}{EI} dx = -\frac{1500}{EI}$$

$$f_{BY, MB} = f_{MB, BY} = \sum \int \frac{m_{BY} m_{MB}}{EI} dx = \frac{1600}{EI}$$

Compatibility Equations:

$$\Delta_{BX0} + f_{BX, BX} B_X + f_{BX, BY} B_Y + f_{BX, MB} M_B = 0$$

$$\Delta_{BY0} + f_{BY, BX} B_X + f_{BY, BY} B_Y + f_{BY, MB} M_B = 0$$

$$\Theta_{B0} + f_{MB, BX} B_X + f_{MB, BY} B_Y + f_{MB, MB} M_B = 0$$

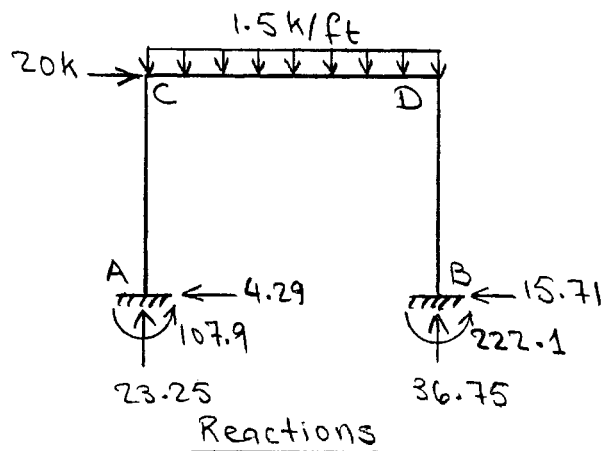
$$870000 + 36000 B_X - 30000 B_Y - 1500 M_B = 0$$

$$-2040000 - 30000 B_X + 58666.67 B_Y + 1600 M_B = 0$$

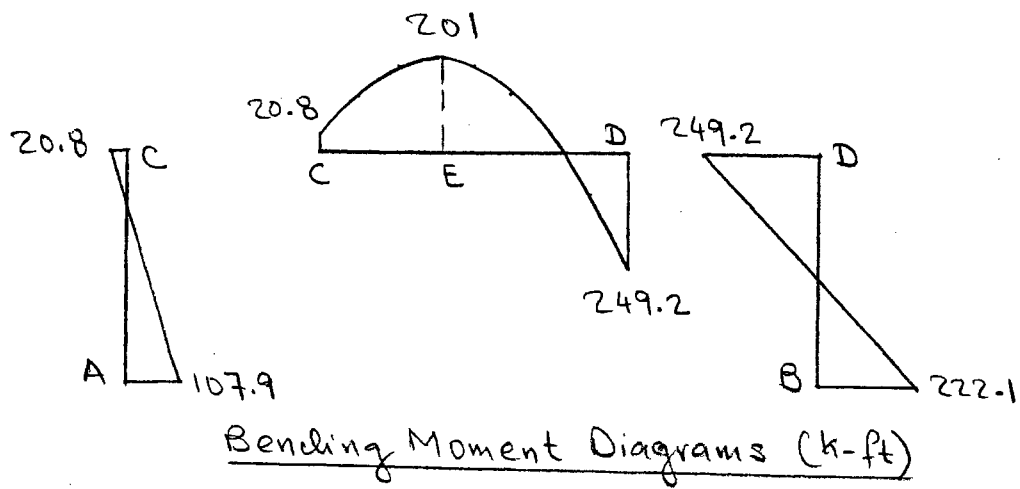
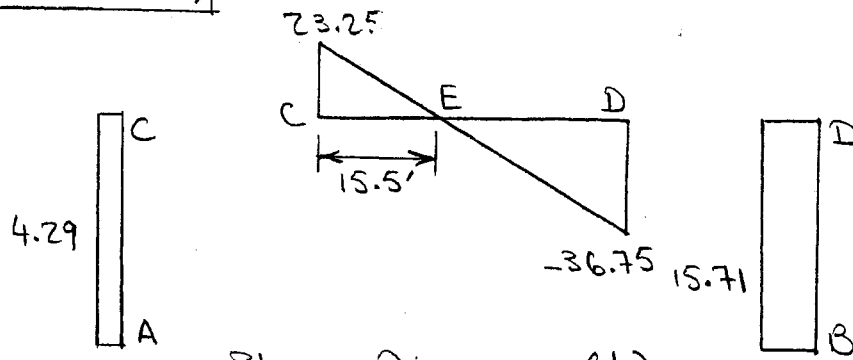
$$-53000 - 1500 B_X + 1600 B_Y + 80 M_B = 0$$

Solving these equations, we obtain:

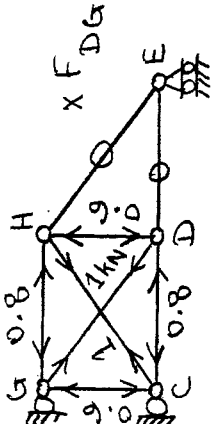
$$\underline{B_X = 15.71 \text{ k} \leftarrow} \quad \underline{B_Y = 36.75 \text{ k} \uparrow} \quad \underline{M_B = 222.1 \text{ k-ft} \curvearrowright}$$



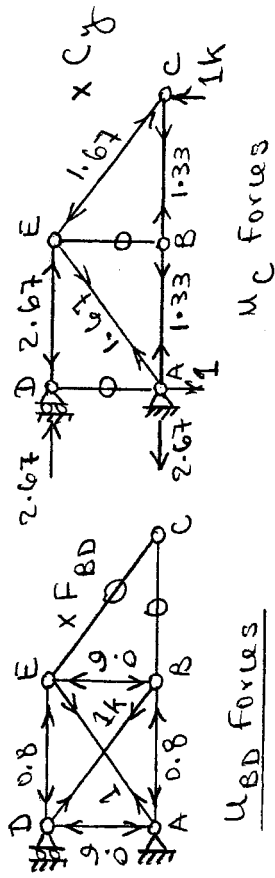
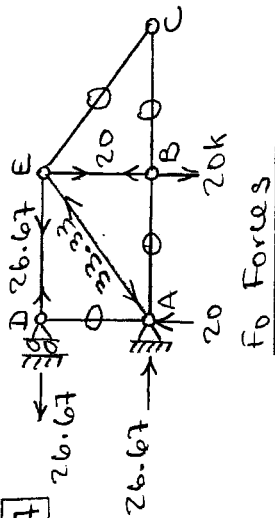
13.45 (contd.)



13.46


$$f_{DG} = - \frac{540}{36.72} = -14.7 \text{ kN} = \underline{\underline{14.7 \text{ kN (C)}}}$$

13.47



Member	L (in.)	A (in ²)	F ₀ (k)	u _{BD} (k/k)	u _C (k/k)	$\frac{F_0 u_{BD} L}{A}$	$\frac{F_0 u_C L}{A}$	$\frac{u_{BD}^2 L}{A}$	$\frac{u_C^2 L}{A}$	$\frac{u_{BD} u_C L}{A}$	$F = F_0 + u_{BD} F_{BD} + u_C C_y$ (k)
AB	192	8	0	-0.8	1.33	0	0	15.36	42.45	-25.54	-1.44
BC	192	8	0	0	1.33	0	0	0	42.45	0	8.29
DE	192	8	26.67	-0.8	-2.67	-512.66	-1709.01	15.36	171.09	51.26	0.31
AD	144	6	0	-0.6	0	0	0	8.64	0	0	-7.3
BE	144	6	20	-0.6	0	-288	0	8.64	0	0	12.7
AE	240	6	-33.33	1	-1.67	-1333.2	-2276.44	40	111.56	66.8	-10.77
CE	240	6	0	0	-1.67	0	0	0	83.67	0	-10.4
BD	240	6	0	1	0	0	0	40	0	0	12.16
Σ						-2133.26	-3935.45	128	451.22	92.52	

$$\Delta_{BD} = \frac{-2133.26}{E}; \quad \Delta_{CD} = -\frac{3935.45}{E}; \quad f_{BD, BD} = \frac{128}{E}; \quad f_{CC} = \frac{451.22}{E}$$

$$f_{BD, C} = f_{C, BD} = \frac{92.52}{E}$$

Compatibility Equations:

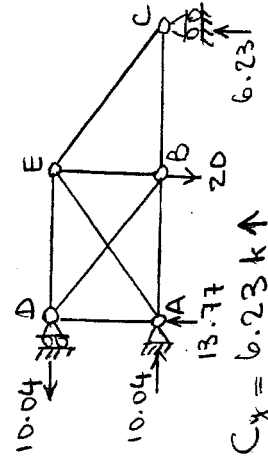
$$\Delta_{BD} + f_{BD, BD} F_{BD} + f_{BD, C} C_y = 0$$

$$\Delta_{CD} + f_{C, BD} F_{BD} + f_{CC} C_y = 0$$

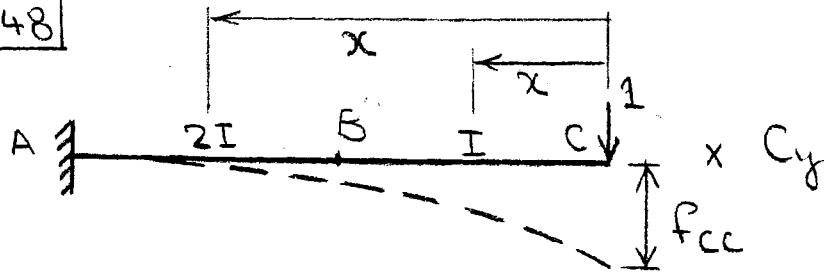
$$-2133.26 + 128 F_{BD} + 92.52 C_y = 0$$

$$-3935.45 + 92.52 F_{BD} + 451.22 C_y = 0$$

Solving these equations, we obtain: $F_{BD} = 12.16 \text{ k(T)}$; $C_y = 6.23 \text{ k} \uparrow$



13.48



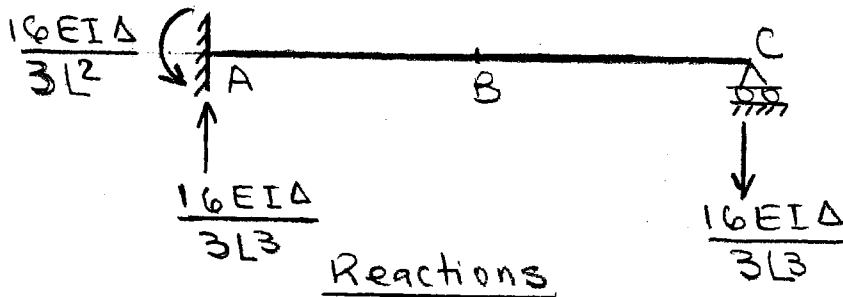
Using the virtual work method:

$$F_{cc} = \frac{1}{EI} \left[\int_0^{L/2} (-1x)^2 dx + \frac{1}{2} \int_{L/2}^L (-1/2x)^2 dx \right]$$

$$= \frac{3L^3}{16EI}$$

Compatibility Equation: $F_{cc} C_y = \Delta$

$$C_y = \frac{16EI\Delta}{3L^3} \downarrow$$



13.49 From the solution of Problem 13.1

$$\Delta_{DD} = -\frac{14760 \text{ kN.m}^3}{EI} = -\frac{14760}{200(3250)} = -0.0227 \text{ m}$$

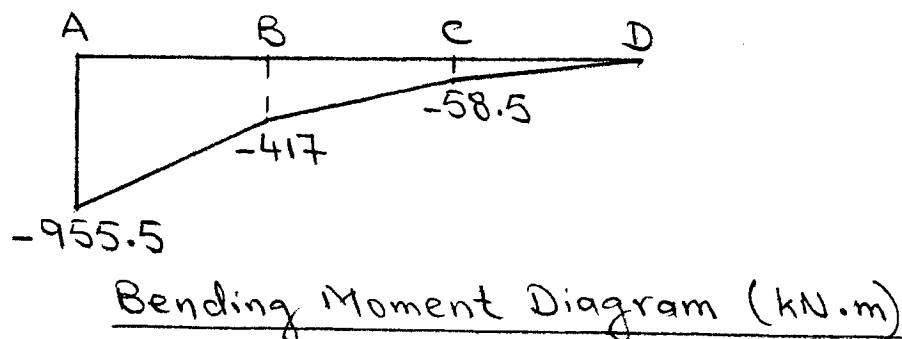
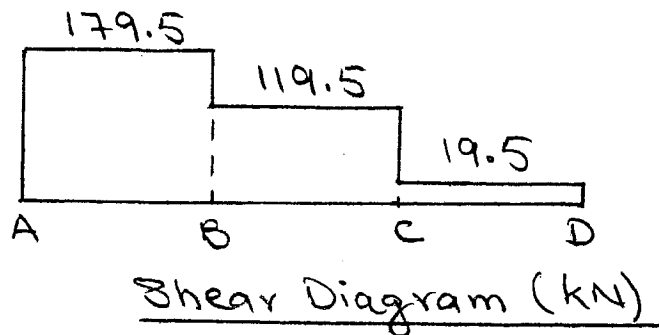
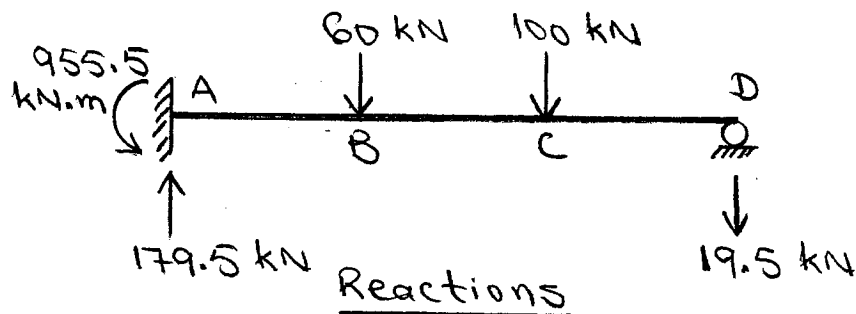
$$f_{DD} = \frac{243 \text{ kN.m}^3/\text{kN}}{EI} = \frac{243}{200(3250)} = 0.000374 \text{ m/kN}$$

Compatibility Equation:

$$\Delta_{DD} + f_{DD} D_y = \Delta_D$$

$$-0.0227 + (0.000374) D_y = -0.03$$

$$D_y = -\frac{0.0073}{0.000374} = -19.5 \text{ kN} = \underline{19.5 \text{ kN} \downarrow}$$



13.50 From the solution of Problem 13.9:

$$\Delta_{C0} = -\frac{158400 \text{ k-ft}^3}{EI} = -\frac{158400 (12)^3}{29000 (1500)} = -6.292 \text{ in.}$$

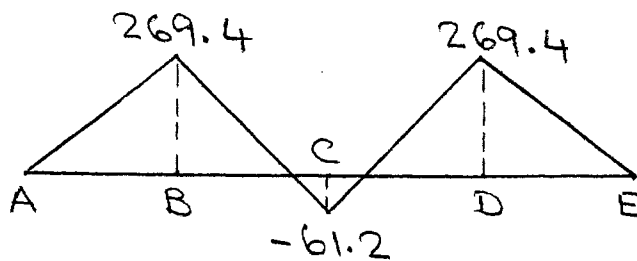
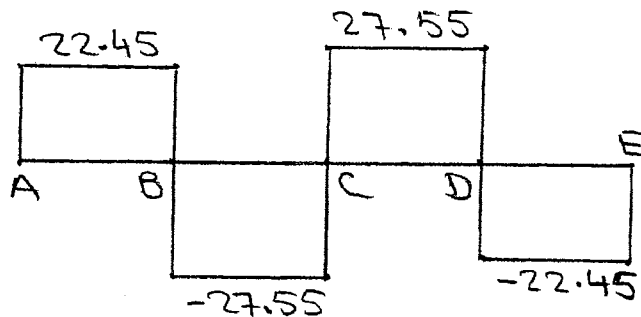
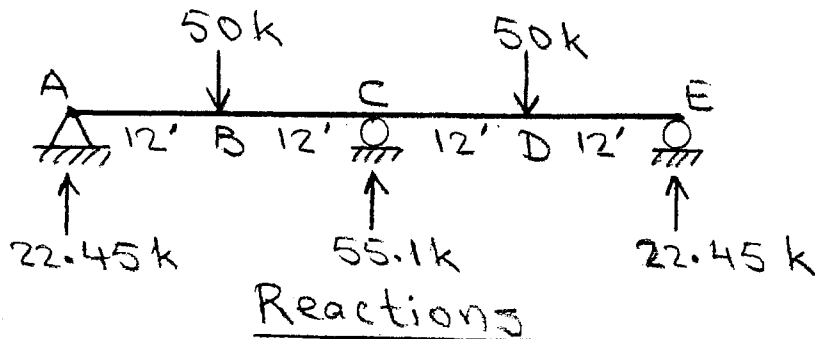
$$f_{CC} = \frac{2304 \text{ k-ft}^3/\text{k}}{EI} = \frac{2304 (12)^3}{29000 (1500)} = 0.0915 \text{ in./k}$$

Compatibility Equation:

$$\Delta_{C0} + f_{CC} C_y = \Delta_C$$

$$-6.292 + (0.0915) C_y = -1.25$$

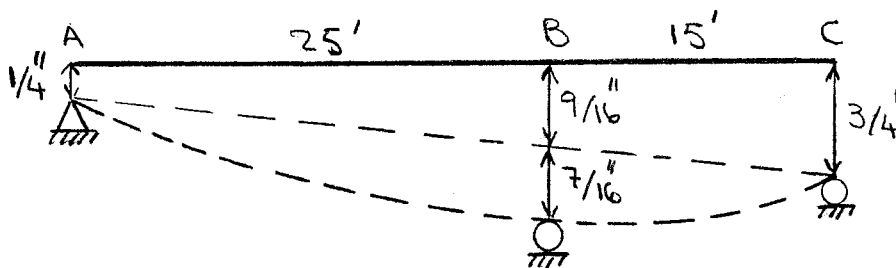
$$C_y = \frac{5.042}{0.0915} = 55.1 \text{ k} \uparrow$$



13.51 From the solution of Problem 13.12:

$$\Delta_{B0} = -\frac{61450.2 \text{ k-ft}^3}{EI} = -\frac{61450.2 (12)^3}{29000 (2500)} = -1.465 \text{ in.}$$

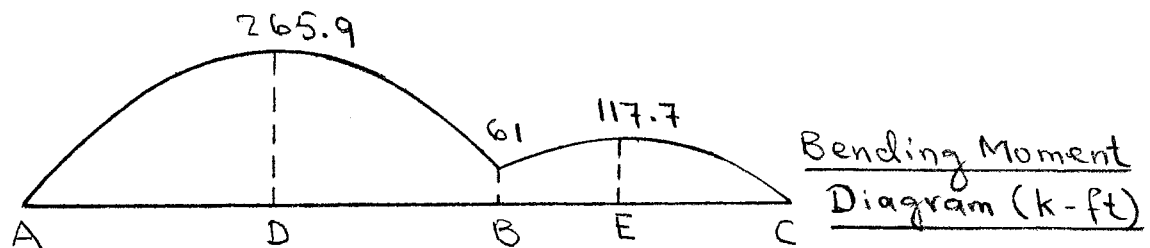
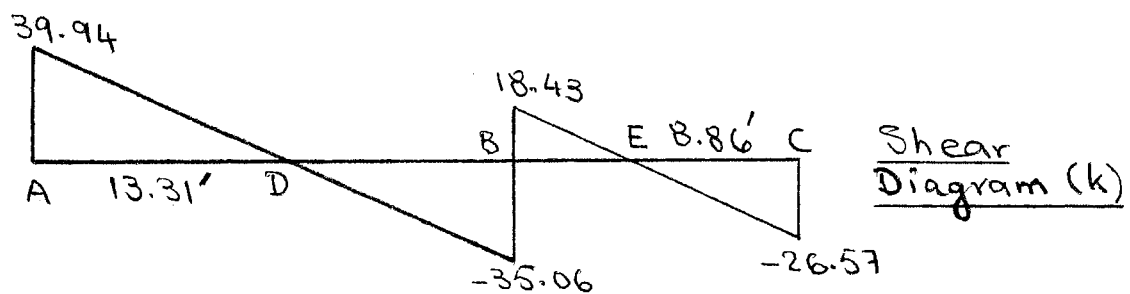
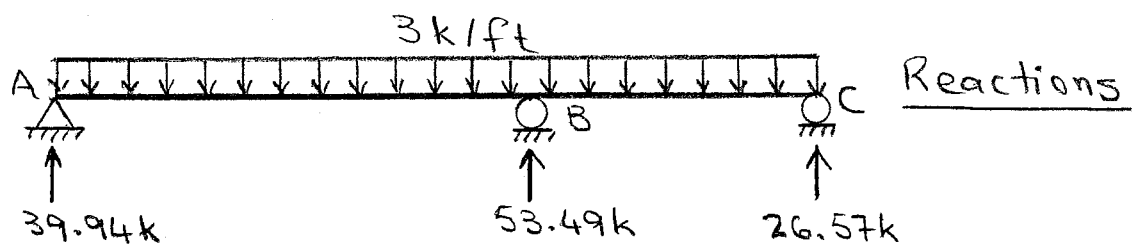
$$f_{BB} = \frac{805.66 \text{ k-ft}^3/\text{k}}{EI} = \frac{805.66 (12)^3}{29000 (2500)} = 0.0192 \text{ in./k}$$



Compatibility Equation:

$$-1.465 + 0.0192 B_y = -\frac{7}{16}$$

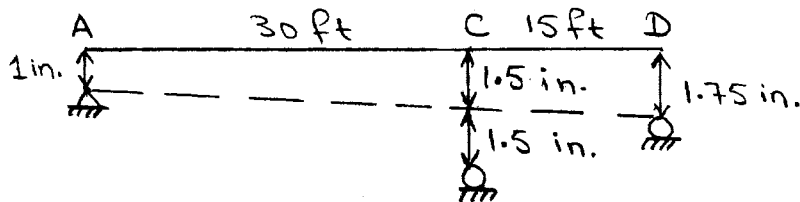
$$B_y = 53.49 \text{ k} \uparrow$$



13.52 From the solution of Problem 13.27 :

$$\Delta_{CO} = - \frac{22046.28 \text{ (k-in)}}{EA} = - \frac{22046.28}{29000 (6)} = - 0.127 \text{ in.}$$

$$f_{CC} = \frac{531.78 \text{ (in.)}}{EA} = \frac{531.78}{29000 (6)} = 0.00306 \text{ in./k}$$

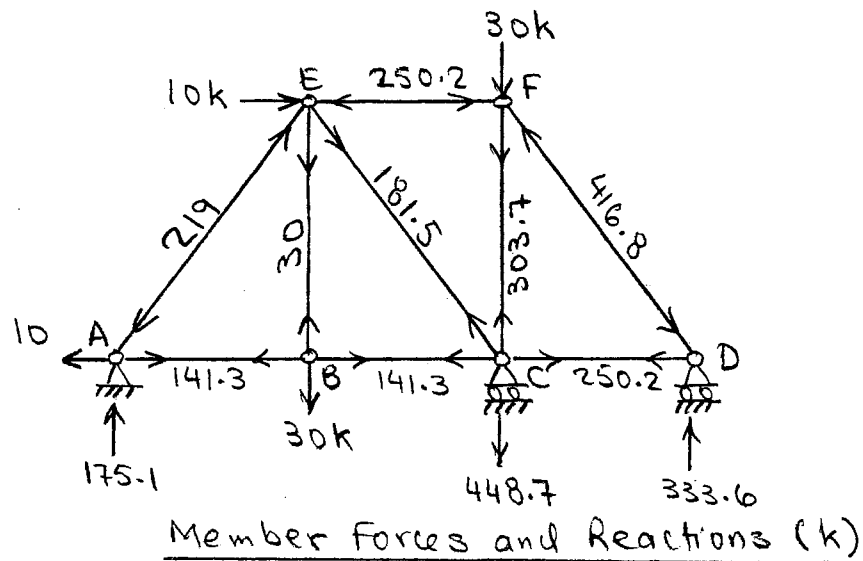


Compatibility Equation:

$$- 0.127 + (0.00306) C_y = - 1.5$$

$$C_y = - 448.7 \text{ k} = \underline{448.7 \text{ k} \downarrow}$$

By using the F_0 and u_c forces computed in the solution of Problem 13.17, we obtain the following member forces.



13.53 From the solution of Problem 13.37:

$$\Delta_{BD} = -\frac{72533.33 \text{ kN}\cdot\text{m}^3}{EI} = -\frac{72533.33}{70(1300)} = -0.797 \text{ m}$$

$$\Delta_{CO} = -\frac{204800 \text{ kN}\cdot\text{m}^3}{EI} = -\frac{204800}{70(1300)} = -2.251 \text{ m}$$

$$f_{BB} = \frac{170.67 \text{ kN}\cdot\text{m}^3/\text{kN}}{EI} = \frac{170.67}{70(1300)} = 0.00188 \text{ m/kN}$$

$$f_{BC} = f_{CB} = \frac{426.67 \text{ kN}\cdot\text{m}^3/\text{kN}}{EI} = \frac{426.67}{70(1300)} = 0.00469 \text{ m/kN}$$

$$f_{CC} = \frac{1365.33 \text{ kN}\cdot\text{m}^3/\text{kN}}{EI} = \frac{1365.33}{70(1300)} = 0.015 \text{ m/kN}$$

Compatibility Equations:

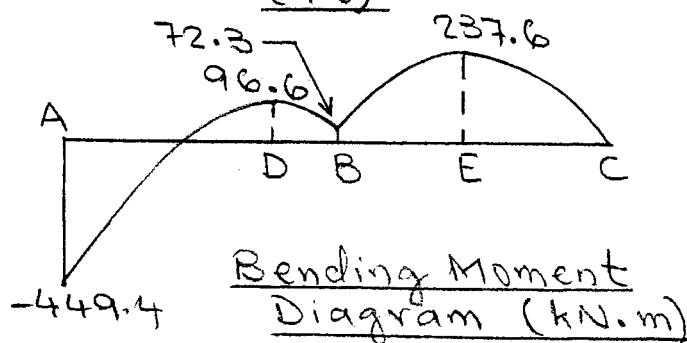
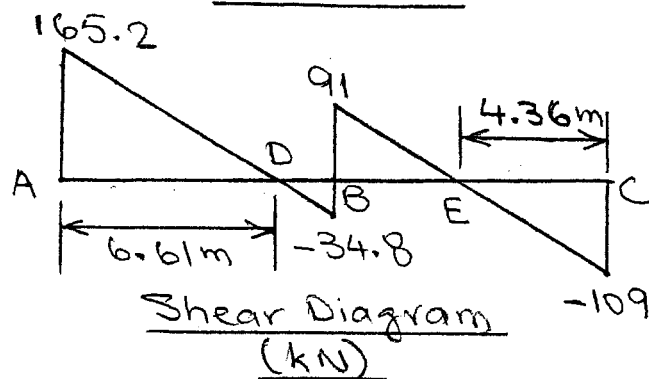
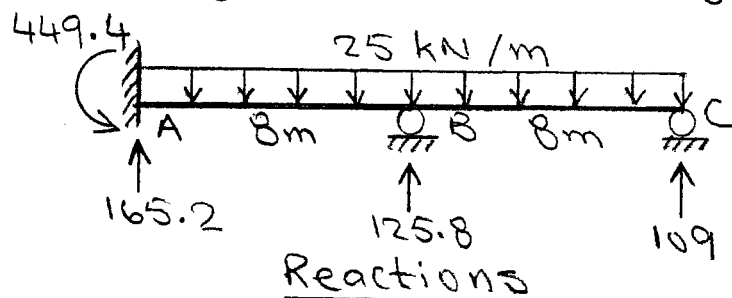
$$-0.797 + (0.00188)B_y + (0.00469)C_y = -0.05$$

$$-2.251 + (0.00469)B_y + (0.015)C_y = -0.025$$

Solving these equations, we obtain

$$B_y = 125.8 \text{ kN} \uparrow$$

$$C_y = 109 \text{ kN} \uparrow$$



13.54 From the solution of Problem 13.39:

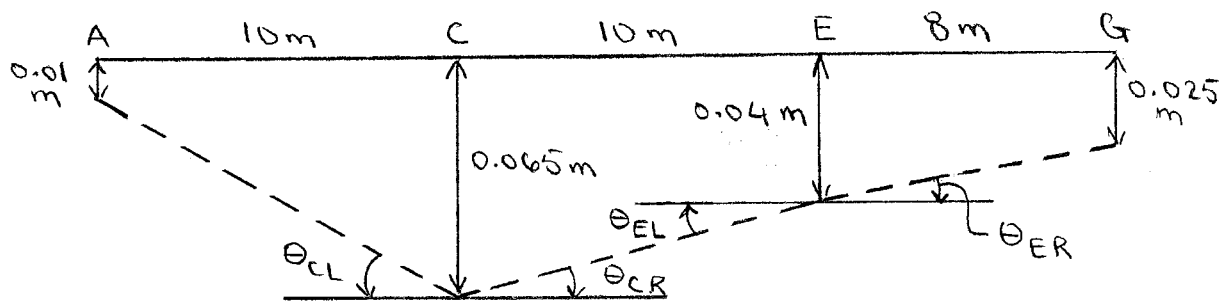
$$\theta_{corel.} = \frac{1104 \text{ kN.m}^2}{EI} = \frac{1104}{200(500)} = 0.01104 \text{ rad.}$$

$$\theta_{Eorel.} = \frac{984 \text{ kN.m}^2}{EI} = 0.00984 \text{ rad.}$$

$$f_{ccrel.} = \frac{5 \text{ kN.m}^2/\text{kN.m}}{EI} = 0.00005 \text{ rad./kN.m}$$

$$f_{EErel.} = \frac{4.33 \text{ kN.m}^2/\text{kN.m}}{EI} = 0.0000433 \text{ rad./kN.m}$$

$$f_{CE} = f_{EC} = \frac{0.833 \text{ kN.m}^2/\text{kN.m}}{EI} = 0.00000833 \text{ rad./kN.m}$$



$$\theta_{CL} = \frac{0.065 - 0.01}{10} = 0.0055 \text{ rad.}$$

$$\theta_{CR} = \frac{0.065 - 0.04}{10} = 0.0025 \text{ rad.}$$

$$\theta_C = \theta_{CL} + \theta_{CR} = 0.008 \text{ rad.}$$

$$\theta_{EL} = \frac{0.04 - 0.065}{10} = -0.0025 \text{ rad.}$$

$$\theta_{ER} = \frac{0.04 - 0.025}{8} = 0.001875 \text{ rad.}$$

$$\theta_E = \theta_{EL} + \theta_{ER} = -0.000625 \text{ rad.}$$

Compatibility Equations:

$$\theta_{corel.} + f_{ccrel.} M_C + f_{CE} M_E = \theta_C$$

$$\theta_{Eorel.} + f_{EC} M_C + f_{EErel.} M_E = \theta_E$$

$$1104 + 5 M_C + 0.833 M_E = 800$$

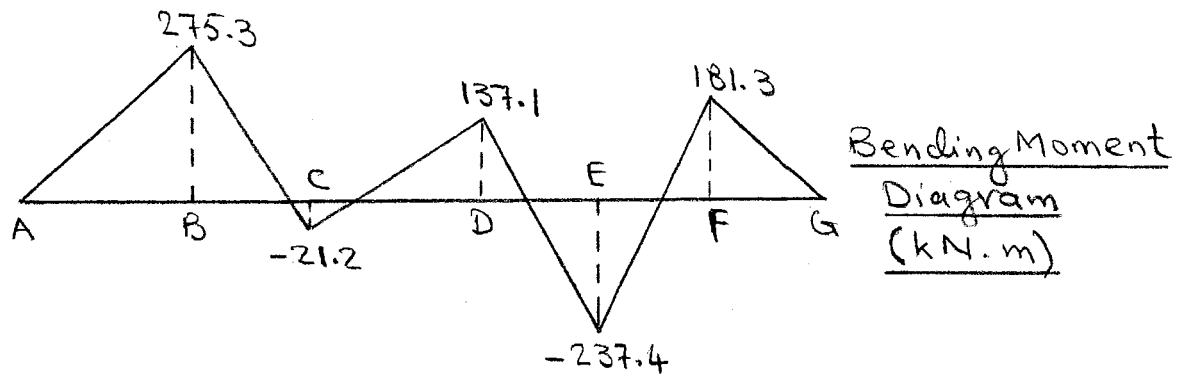
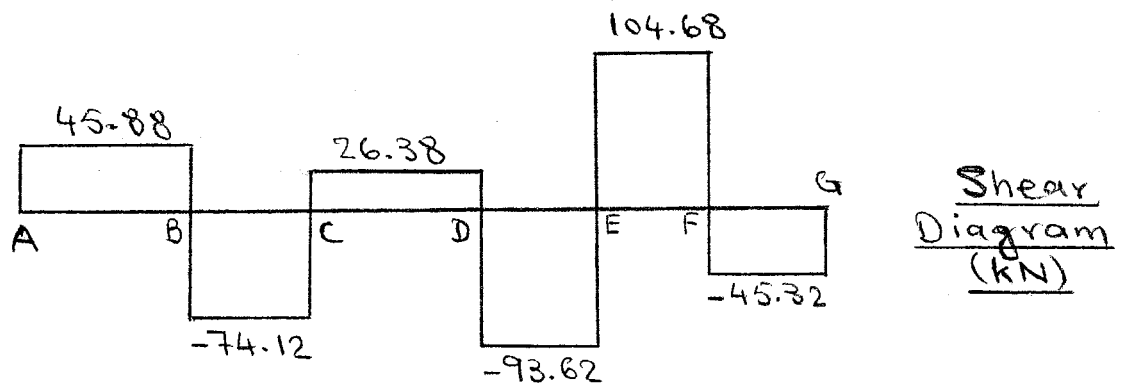
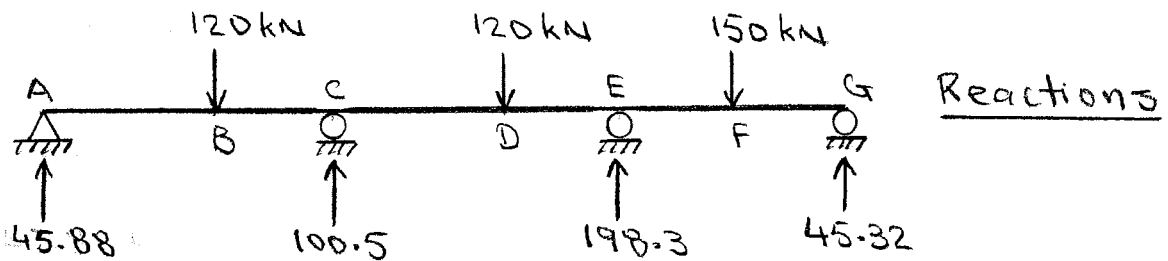
$$984 + 0.833 M_C + 4.33 M_E = -62.5$$

13.54 (contd.)

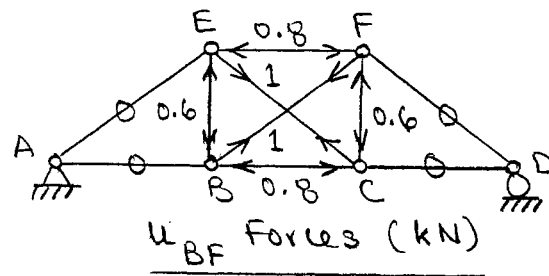
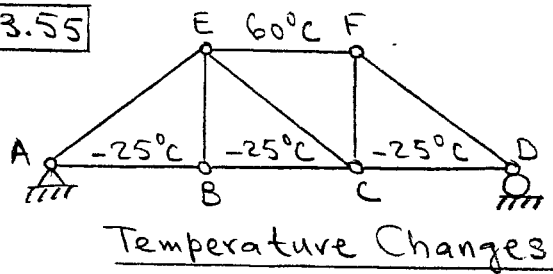
Solving these equations, we obtain

$$M_C = -21.23 \text{ kN.m}$$

$$M_E = -237.42 \text{ kN.m}$$



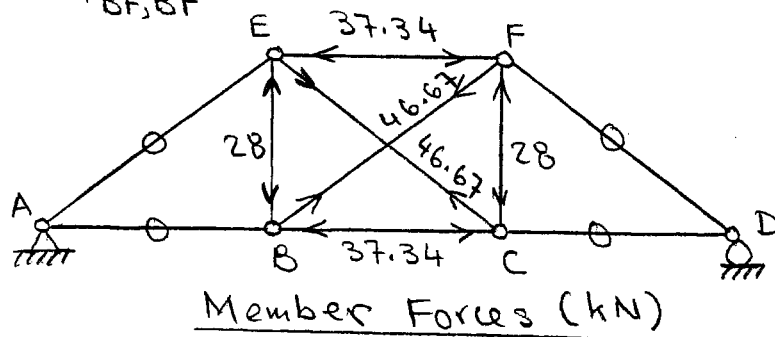
13.55



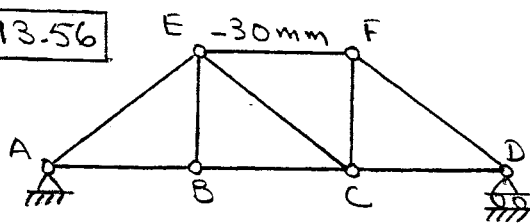
$$\Delta_{BFO} = \alpha \sum (\Delta T) L u_{BF} = 1.2 \times 10^{-5} (8) [-25(-0.8) + 60(-0.8)] = -0.002688 \text{ m} = -2.688 \text{ mm}$$

$$\begin{aligned} f_{BF,BF} &= \frac{1}{EA} \sum u_{BF}^2 L \\ &= \frac{1}{200(10^6)(0.003)} [2(1)^2(10) + 2(-0.8)^2(8) + 2(-0.6)^2(6)] = 57.6 (10^{-6}) \text{ m/kN} \\ &= 0.0576 \text{ mm/kN} \end{aligned}$$

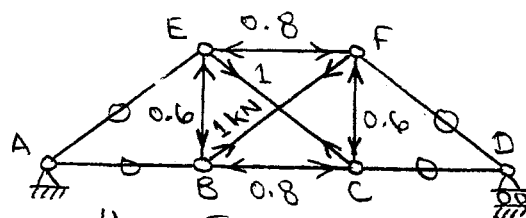
$$F_{BF} = - \frac{\Delta_{BFO}}{f_{BF,BF}} = 46.67 \text{ kN (T)}$$



13.56



Fabrication Errors

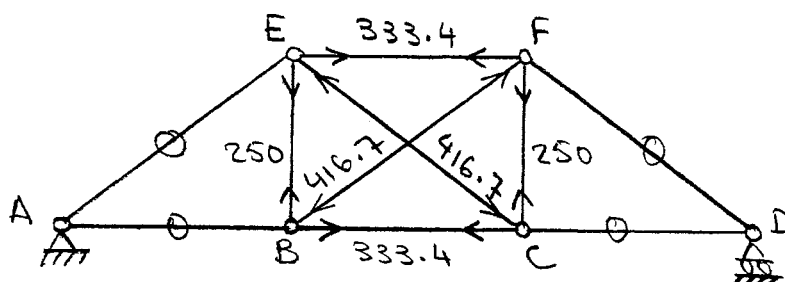


u_{BF} Forces (kN)

$$\Delta_{BFO} = \sum \delta u_{BF} = -30(-0.8) = 24 \text{ mm}$$

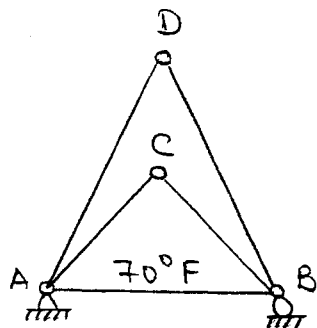
$$\begin{aligned} f_{BF,BF} &= \frac{1}{EA} \sum u_{BF}^2 L \\ &= \frac{1}{200(10^6)(0.003)} [2(1)^2 10 + 2(-0.8)^2 8 + 2(-0.6)^2 6] \\ &= 57.6(10^{-6}) \text{ m/kN} = 0.0576 \text{ mm/kN} \end{aligned}$$

$$\begin{aligned} F_{BF} &= - \frac{\Delta_{BFO}}{f_{BF,BF}} = - \frac{24}{0.0576} = -416.7 \text{ kN} \\ &= \underline{416.7 \text{ kN (C)}} \end{aligned}$$

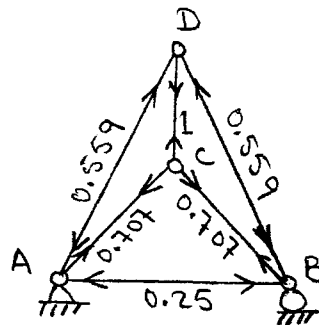


Member Forces (kN)

13.57



Temperature Change

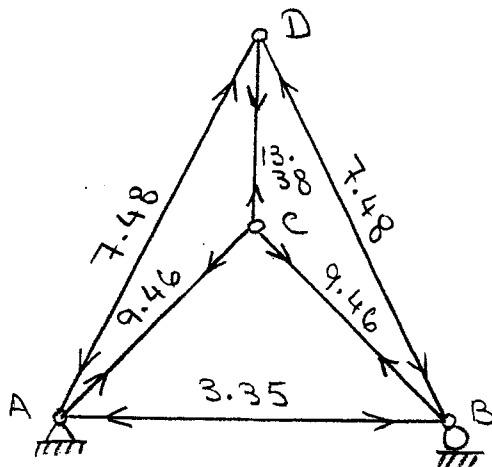


u_{CD} Forces (k)

$$\Delta_{CDO} = \sum \alpha (\Delta T) L u_{CD} = 6.5 \times 10^{-6} (70) (20) (-0.25) \\ = -0.002275 \text{ ft} = -0.0273 \text{ in.}$$

$$P_{CD,CD} = \frac{1}{EA} \sum u_{CD}^2 L \\ = \frac{1}{29000 (8)} [(1)^2 (10) + (-0.25)^2 (20) + 2 (-0.559)^2 \times \\ (22.36) + 2 (0.707)^2 (14.14)] = 0.00017 \text{ ft/k} \\ = 0.00204 \text{ in./k.}$$

$$F_{CD} = - \frac{\Delta_{CDO}}{P_{CD,CD}} = \underline{13.38 \text{ k (T)}}$$



Member Forces (k)

Chapter Fourteen

Three Moment Equation and the Method of Least Work

CHAPTER 14

14.1

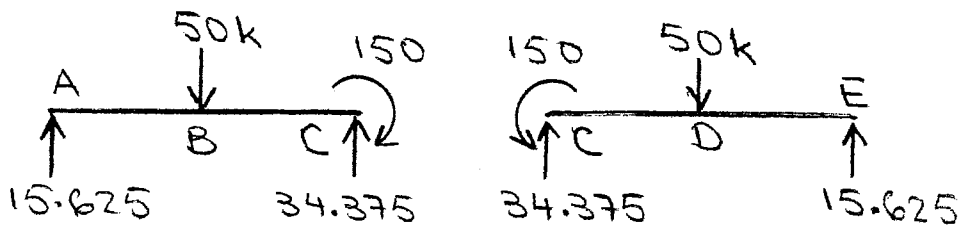
Three-Moment Equation at Joint C:

$$M_A = M_E = 0$$

$$2M_C \left(\frac{16}{2I} + \frac{16}{I} \right) = - \frac{50(16)^2(0.5)(1-0.25)}{2I} - \frac{50(16)^2(0.5)(1-0.25)}{I}$$

$$48 M_C = -7200$$

$$\underline{M_C = -150 \text{ k-ft}}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.11.

14.2

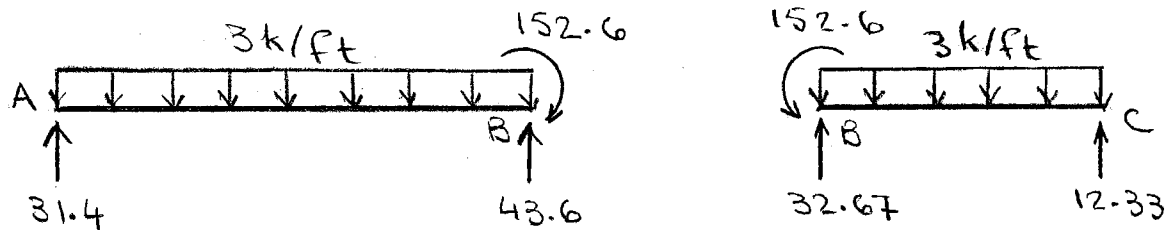
Three - Moment Equation at Joint B:

$$M_A = M_C = 0$$

$$2M_B \left(\frac{25}{2I} + \frac{15}{I} \right) = - \frac{3(25)^3}{4(2I)} - \frac{3(15)^3}{4I}$$

$$55M_B = -8390.625$$

$$M_B = -152.6 \text{ k-ft}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.12.

14.3

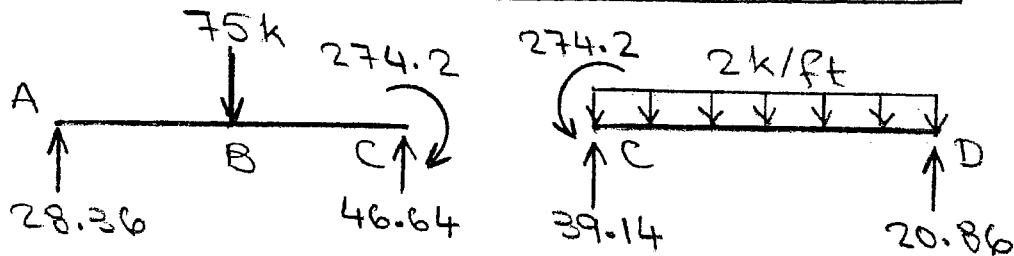
Three-Moment Equation at Joint C:

$$M_A = M_D = 0$$

$$2M_C \left(\frac{30}{3I} + \frac{30}{I} \right) = - \frac{75(30)^2(0.5)(1-0.25)}{3I} - \frac{2(30)^3}{4I}$$

$$80M_C = -21937.5$$

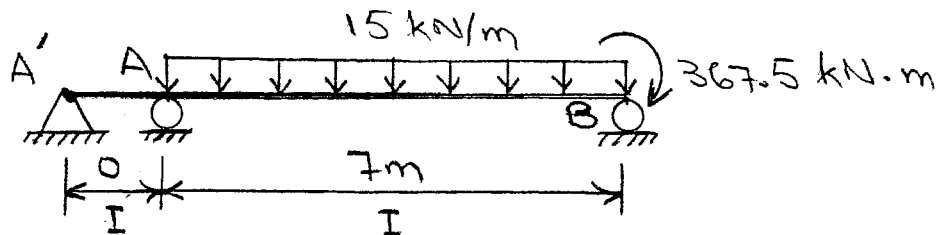
$$M_C = -274.2 \text{ k-ft}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.13.

14.4



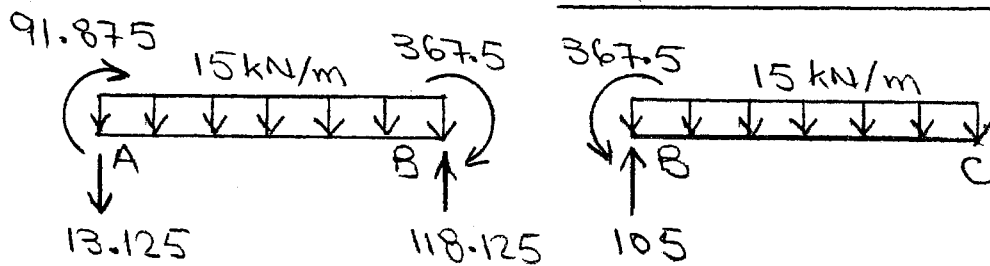
Three - Moment Equation at Joint A:

$$M_{A'} = 0, \quad M_B = -367.5 \text{ kN.m}$$

$$2M_A(0+7) - 367.5(7) = -\frac{1}{4}(15)(7)^3$$

$$14M_A = 1286.25$$

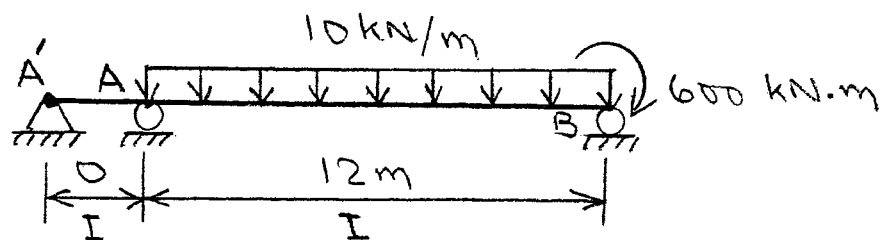
$$M_A = 91.875 \text{ kN.m}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.15.

14.5



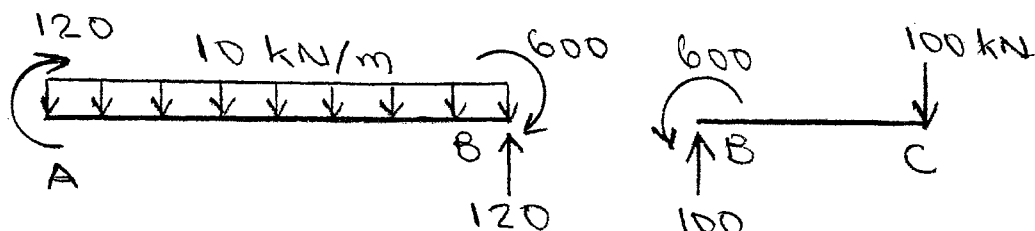
Three-Moment Equation at Joint A:

$$M_{A'} = 0, \quad M_B = -600 \text{ kN.m}$$

$$2M_A(0 + 12) - 600(12) = -\frac{1}{4}(10)(12)^3$$

$$24M_A = 2880$$

$$M_A = 120 \text{ kN.m}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.16.

14.6

Three-Moment Equation at Joint C:

$$M_A = 0$$

$$2M_C \left(\frac{10}{I} + \frac{10}{2I} \right) + M_E \left(\frac{10}{2I} \right) = - \frac{120(10)^2(0.6)(1-0.36)}{I} - \frac{120(10)^2(0.4)(1-0.16)}{2I}$$

$$30M_C + 5M_E = -6624 \quad (1)$$

Three-Moment Equation at Joint E:

$$M_G = 0$$

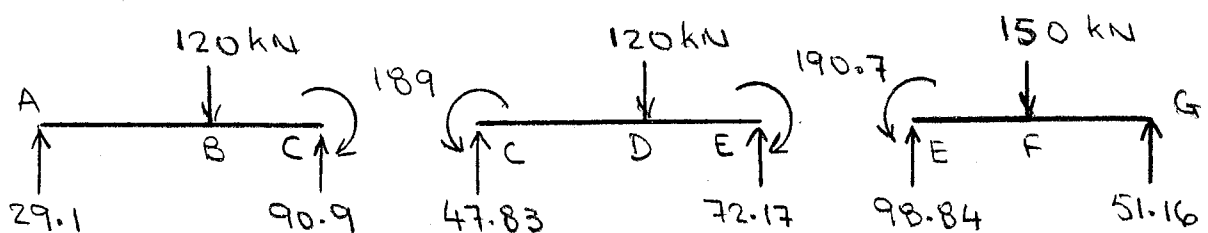
$$M_C \left(\frac{10}{2I} \right) + 2M_E \left(\frac{10}{2I} + \frac{8}{I} \right) = - \frac{120(10)^2(0.6)(1-0.36)}{2I} - \frac{150(8)^2(0.5)(1-0.25)}{I}$$

$$5M_C + 26M_E = -5904 \quad (2)$$

Solving Eqs. (1) and (2) simultaneously, we obtain:

$$M_C = -189 \text{ kN}\cdot\text{m}$$

$$M_E = -190.7 \text{ kN}\cdot\text{m}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.39.

14.7

Three-Moment Equation at Joint B:

$$M_A = 0$$

$$2M_B \left(\frac{24}{I} + \frac{24}{2I} \right) + M_C \left(\frac{24}{2I} \right) = -\frac{2.5(24)^3}{4I} - \frac{2.5(24)^3}{4(2I)}$$

$$72M_B + 12M_C = -12960 \quad (1)$$

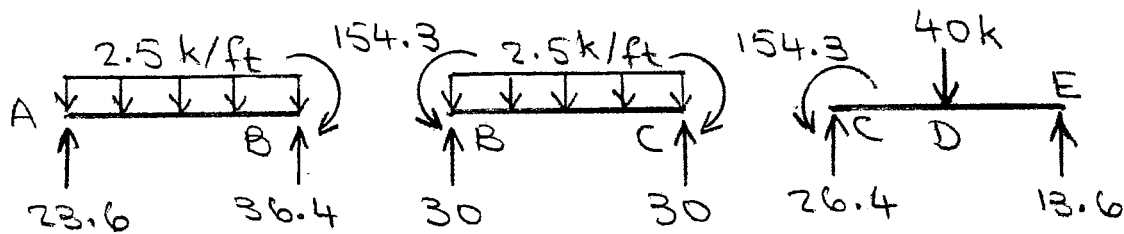
Three-Moment Equation at Joint C: $M_E = 0$

$$M_B \left(\frac{24}{2I} \right) + 2 \left(\frac{24}{2I} + \frac{24}{I} \right) = -\frac{2.5(24)^3}{4(2I)} - \frac{40(24)^2(0.5)(1-0.25)}{I}$$

$$12M_B + 72M_C = -12960 \quad (2)$$

Solving Eqs. (1) and (2) simultaneously, we obtain:

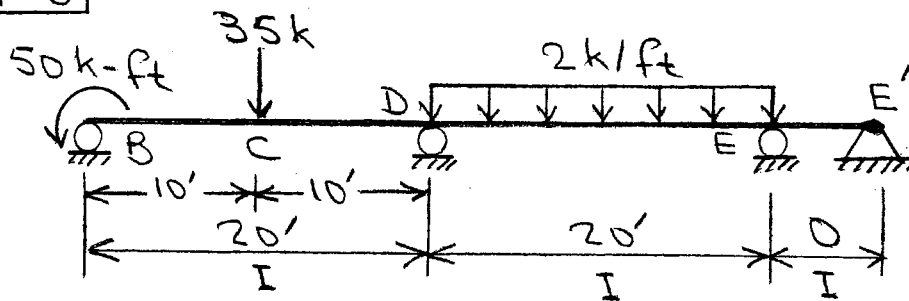
$$\underline{M_B = M_C = -154.3 \text{ k-ft}}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.40.

14.8



Three-Moment Equation at Joint D:

$$M_B = -50 \text{ k-ft}$$

$$-50(20) + 2M_D(20+20) + M_E(20) = -35(20)^2(0.5)(0.75) - \frac{1}{4}(2)(20)^3$$

$$80M_D + 20M_E = -8250 \quad (1)$$

Three-Moment Equation at Joint E:

$$M_{E'} = 0$$

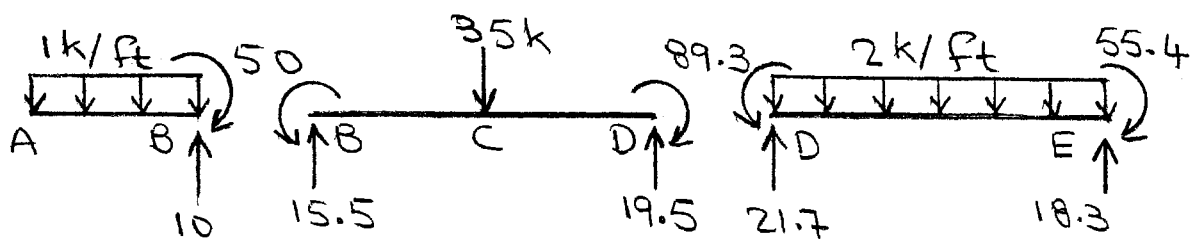
$$M_D(20) + 2M_E(20+0) = -\frac{1}{4}(2)(20)^3$$

$$20M_D + 40M_E = -4000 \quad (2)$$

Solving Eqs. (1) and (2) simultaneously, we obtain:

$$M_D = -89.3 \text{ k-ft}$$

$$M_E = -55.4 \text{ k-ft}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.38.

14.9

Three-Moment Equation at Joint B:

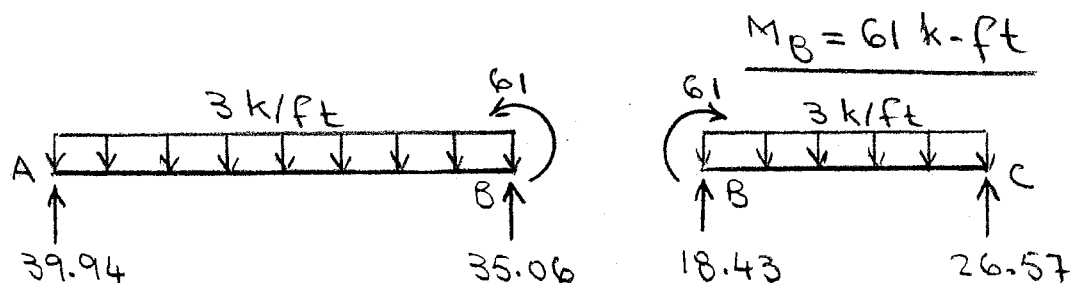
$$M_A = M_C = 0$$

$$2M_B \left(\frac{25}{2I} + \frac{15}{I} \right) = -\frac{3(25)^3}{4(2I)} - \frac{3(15)^3}{4I} - 6E \left[\frac{0.25-1}{25(12)} + \frac{0.75-1}{15(12)} \right]$$

$$55M_B = -8390.625 + 0.0233EI$$

Substituting $EI = \frac{29000(2500)}{(12)^2} = 503472.2 \text{ k-ft}^2$, we obtain:

$$55M_B = 3357$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.51.

14.10

Three-Moment Equation at Joint C: $M_A = 0$

$$2M_C \left(\frac{10}{I} + \frac{10}{2I} \right) + M_E \left(\frac{10}{2I} \right) = \frac{-120(10)^2(0.6)(1-0.36)}{I} \\ - \frac{120(10)^2(0.4)(1-0.16)}{2I} - 6E \left[\frac{0.01-0.065}{10} + \frac{0.04-0.065}{10} \right]$$

Substituting $EI = 200(500) = 100000 \text{ kN.m}^2$, we obtain:

$$30M_C + 5M_E = -1824 \quad (1)$$

Three-Moment Equation at Joint E: $M_G = 0$

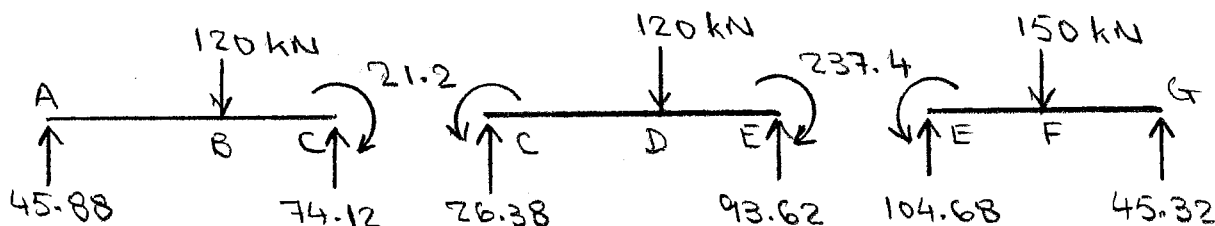
$$M_C \left(\frac{10}{2I} \right) + 2M_E \left(\frac{10}{2I} + \frac{8}{I} \right) = - \frac{120(10)^2(0.6)(1-0.36)}{2I} \\ - \frac{150(8)^2(0.5)(1-0.25)}{I} - 6E \left[\frac{0.065-0.04}{10} + \frac{0.025-0.04}{8} \right]$$

$$5M_C + 26M_E = -6279 \quad (2)$$

Solving Eqs. (1) and (2) simultaneously, we obtain

$$\underline{M_C = -21.2 \text{ kN.m}}$$

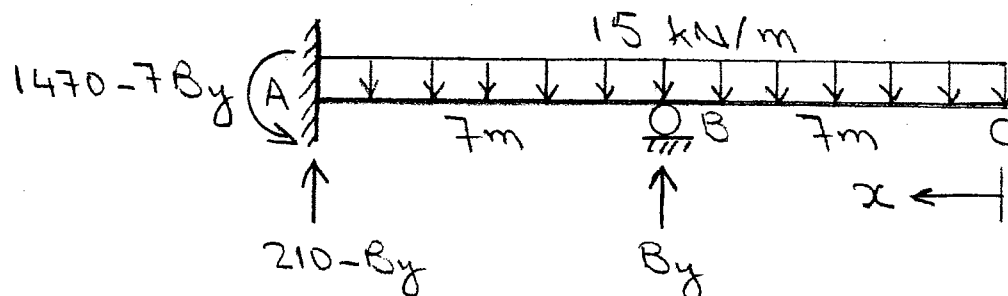
$$\underline{M_E = -237.4 \text{ kN.m}}$$



Span End Moments and Shears

For reactions, and shear and bending moment diagrams, see solution of Problem 13.54.

14.11



Segment	x Coordinate		M	$\frac{\partial M}{\partial B_y}$
	Origin	Limits (m)		
CB	C	0-7	$-7.5x^2$	0
BA	C	7-14	$-7.5x^2 + B_y(x-7)$	$x-7$

$$\frac{\partial U}{\partial B_y} = \int \left(\frac{\partial M}{\partial B_y} \right) \frac{M}{EI} dx = 0$$

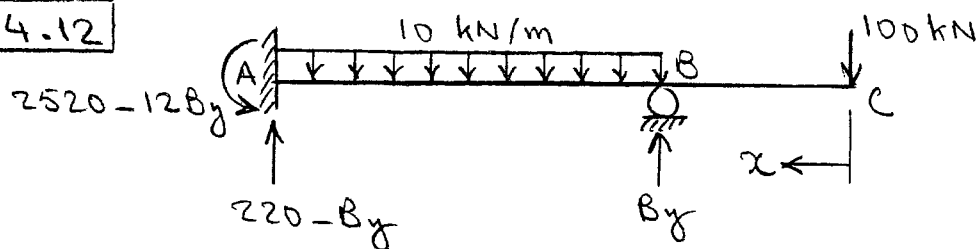
$$\frac{1}{EI} \int_7^{14} (x-7) [-7.5x^2 + B_y(x-7)] dx = 0$$

$$-25510.625 + (114.33) B_y = 0$$

$$\underline{B_y = 223.125 \text{ kN} \uparrow}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.15.

14.12



Segment	x coordinate		M	$\frac{\partial M}{\partial B_y}$
	Origin	Limits (m)		
CB	C	0-6	$-100x$	0
BA	C	6-18	$-100x - 5(x-6)^2 + B_y(x-6)$	$x-6$

$$\frac{\partial U}{\partial B_y} = \int \left(\frac{\partial M}{\partial B_y} \right) \frac{M}{EI} dx = 0$$

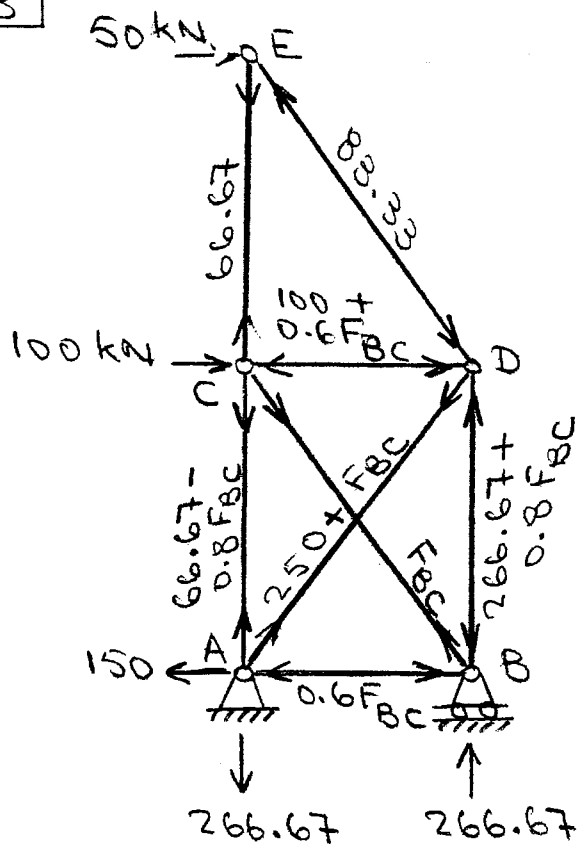
$$\frac{1}{EI} \int_6^{18} (x-6) [-100x - 5(x-6)^2 + B_y(x-6)] dx = 0$$

$$-126720 + 576 B_y = 0$$

$$B_y = \underline{220 \text{ kN} \uparrow}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.16.

14.13



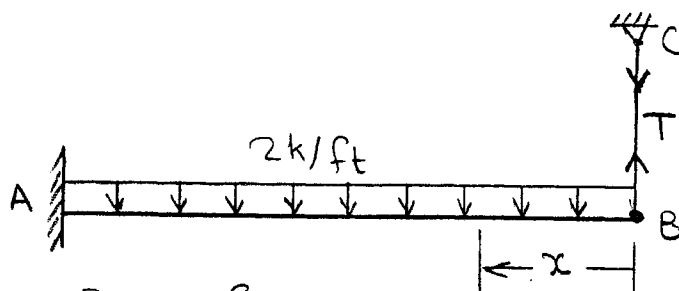
Member	L (m)	F	$\frac{\partial F}{\partial F_{BC}}$	$\left(\frac{\partial F}{\partial F_{BC}}\right) FL$	F (kN)
AB	3	$-0.6 F_{BC}$	-0.6	$1.08 F_{BC}$	71.9
CD	3	$-100 - 0.6 F_{BC}$	-0.6	$180 + 1.08 F_{BC}$	-28.1
AC	4	$66.67 - 0.8 F_{BC}$	-0.8	$-213.33 + 2.56 F_{BC}$	162.5
BD	4	$-266.67 - 0.8 F_{BC}$	-0.8	$853.33 + 2.56 F_{BC}$	-170.8
CE	4	66.67	0	0	66.7
AD	5	$250 + F_{BC}$	1	$1250 + 5 F_{BC}$	130.2
DE	5	-83.33	0	0	-83.3
BC	5	F_{BC}	1	$5 F_{BC}$	-119.8
Σ				$2070 + 17.28 F_{BC}$	

$$\frac{\partial U}{\partial F_{BC}} = \frac{1}{EA} \Sigma \left(\frac{\partial F}{\partial F_{BC}}\right) FL = 0$$

$$\frac{1}{EA} (2070 + 17.28 F_{BC}) = 0$$

$$F_{BC} = -119.8 \text{ kN} = \underline{119.8 \text{ kN (C)}}$$

14-14



$$\frac{\partial U}{\partial T} = \sum \left(\frac{\partial F}{\partial T} \right) \frac{FL}{AE} + \int \left(\frac{\partial M}{\partial T} \right) \frac{M}{EI} dx = 0$$

Cable BC: $F = T$; $\frac{\partial F}{\partial T} = 1$

Beam AB: $M = Tx - x^2$; $\frac{\partial M}{\partial T} = x$

$$\frac{\partial U}{\partial T} = \frac{1}{E} \left[\frac{T(5)(12)^2}{0.5} + \frac{(12)^4}{500} \int_0^{15} x(Tx - x^2) dx \right]$$

$$= 48096T - 524880 = 0$$

$$\underline{T = 10.9 \text{ k (T)}}$$

Chapter Fifteen

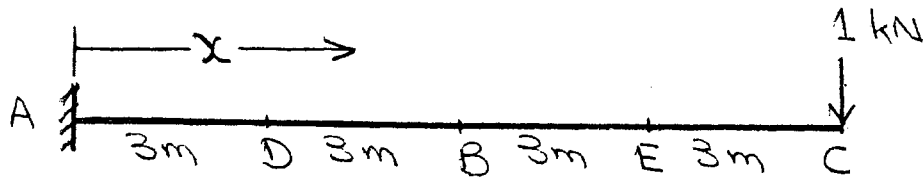
Influence Lines for

Statically Indeterminate

Structures

CHAPTER 15

15.1 Select C_y ($\uparrow +$) as the redundant.



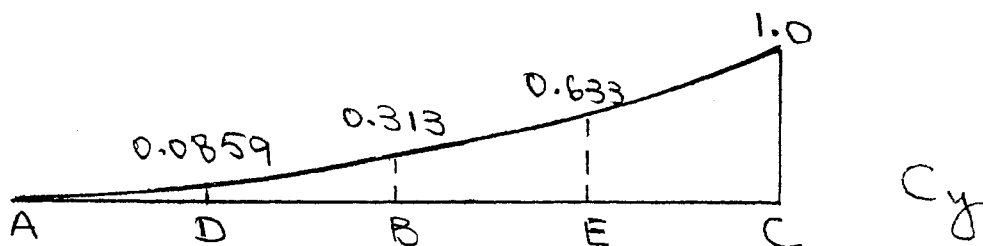
$$C_y = -\frac{f_{cx}}{\bar{f}_{cc}} = -\frac{f_{xc}}{\bar{f}_{cc}}$$

Using beam deflection formulas:

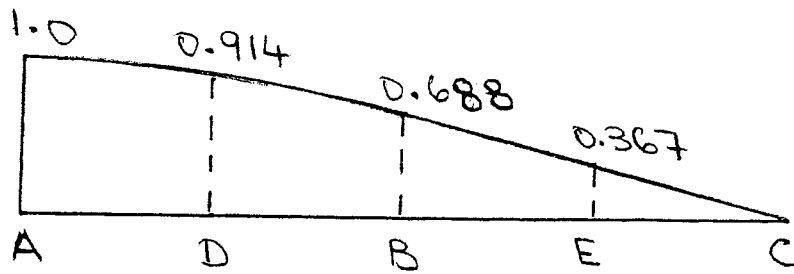
$$f_{xc} = -\frac{1}{6EI} (x^3 - 36x^2)$$

$$\bar{f}_{cc} = -f_{cc} = + \frac{576 \text{ kN}\cdot\text{m}^3/\text{kN}}{EI}$$

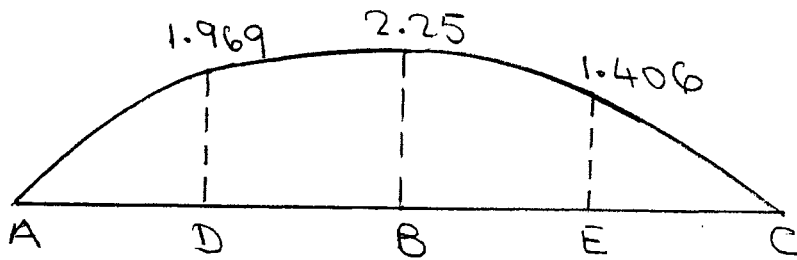
x (m)	$EI f_{xc}$	C_y (kN/kN)	A_y (kN/kN)	M_A (kN·m/kN)	S_B (kN/kN)	M_B (kN·m/kN)
0	0	0	1	0	0	0
3	-49.5	0.0859	0.914	1.969	-0.0859	0.516
6	-180	0.313	0.688	2.25	-0.313(L) 0.687(R)	1.875
9	-364.5	0.633	0.367	1.406	0.367	0.797
12	-576	1	0	0	0	0



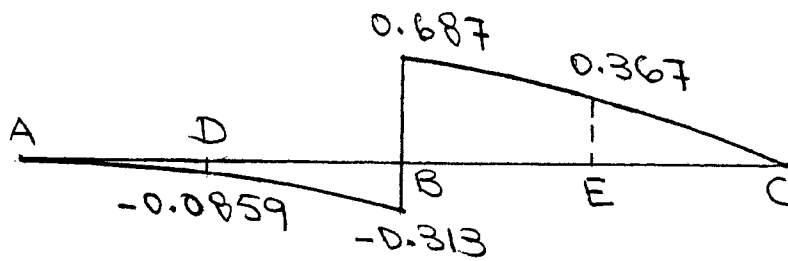
15.1 (Contd.)



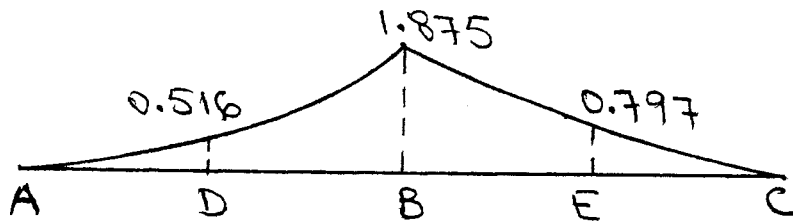
A_y



$M_A (\curvearrow +)$

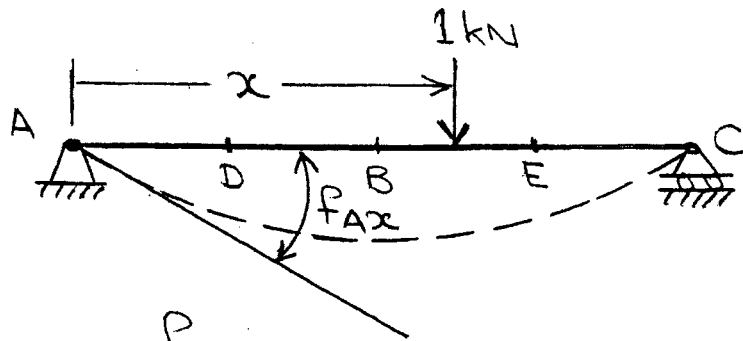


S_B



M_B

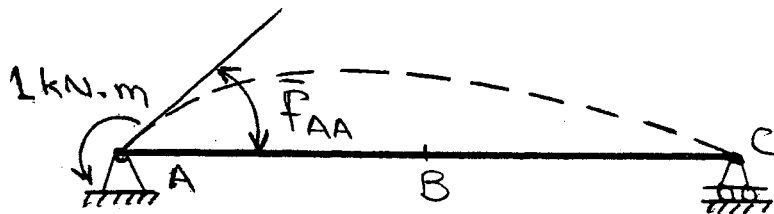
15.2 Select M_A ($\rightarrow +$) as the redundant.



$$M_A = - \frac{f_{Ax}}{f_{AA}}$$

Using beam deflection formulas:

$$f_{Ax} = - \frac{1}{72EI} (x^3 - 36x^2 + 288x)$$

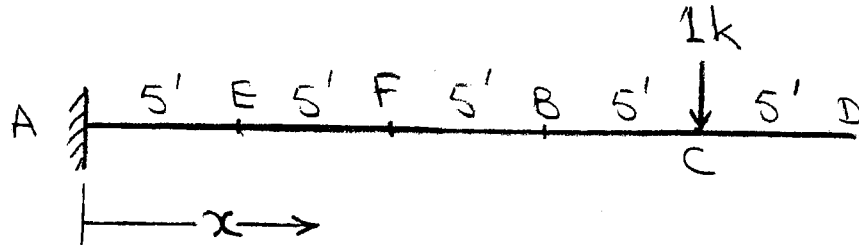


$$f_{AA} = + \frac{4 \text{ kN.m}^2 / \text{kN.m}}{EI}$$

x (m)	$EI f_{Ax}$	M_A
0	0	0
3	-7.875	1.969
6	-9	2.25
9	-5.625	1.406
12	0	0

For influence lines, see solution of Problem 15.1.

15.3 Select C_y ($\uparrow +$) as the redundant.



$$C_y = -\frac{f_{cx}}{\bar{f}_{cc}} = -\frac{f_{xc}}{\bar{f}_{cc}}$$

Using beam deflection formulas:

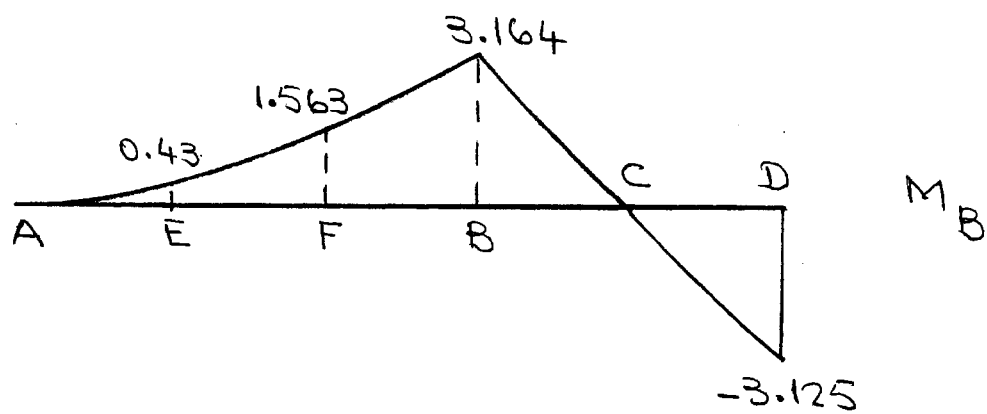
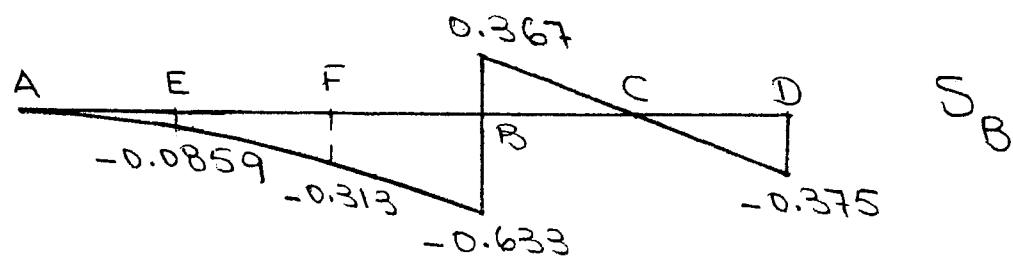
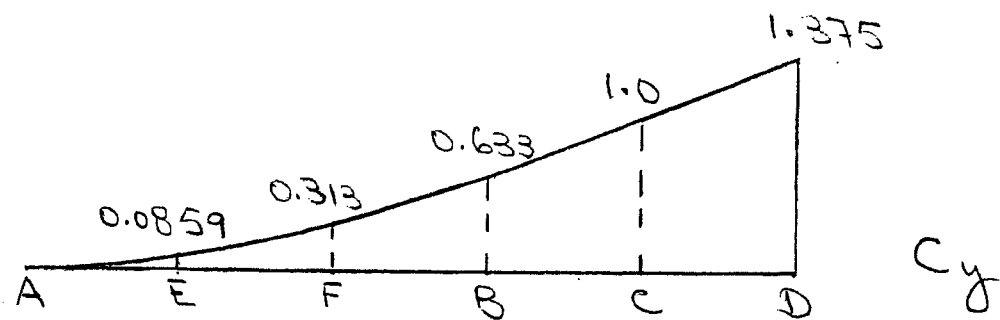
for $0 \leq x \leq 20'$ $f_{xc} = \frac{1}{6EI} (x^3 - 60x^2)$

for $20' \leq x \leq 25'$ $f_{xc} = \frac{200}{3EI} (20 - 3x)$

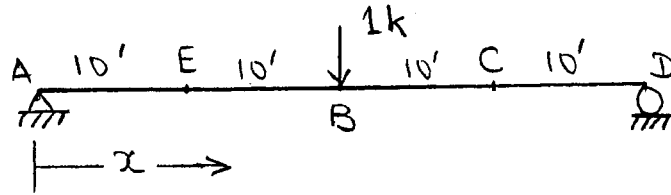
$$\bar{f}_{cc} = -f_{cc} = + \frac{2666.67 \text{ k-ft}^3/\text{k}}{EI}$$

x (ft)	$EI f_{xc}$	C_y (k/k)	S_B (k/k)	M_B (k-ft/k)
0	0	0	0	0
5	-229.17	0.0859	-0.0859	0.43
10	-833.33	0.313	-0.313	1.563
15	-1687.5	0.633	-0.633(L) 0.367(R)	3.164
20	-2666.67	1	0	0
25	-3666.67	1.375	-0.375	-3.125

15.3 (Contd.)



15.4 Select B_y ($\uparrow +$) as the redundant.



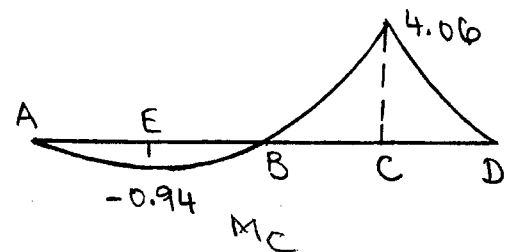
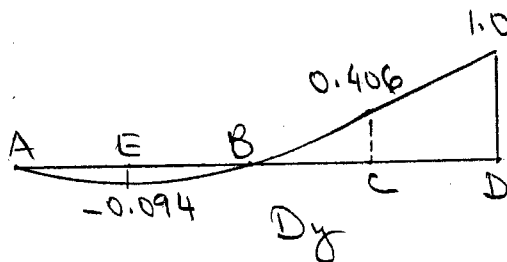
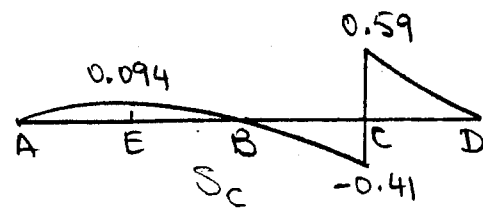
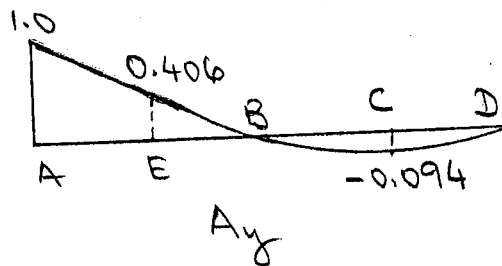
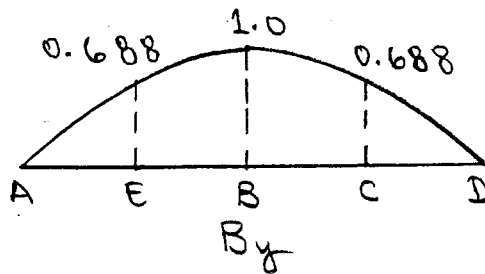
$$B_y = -\frac{f_{Bx}}{\bar{f}_{BB}} = -\frac{f_{xB}}{\bar{f}_{BB}}$$

Using beam-deflection formulas:

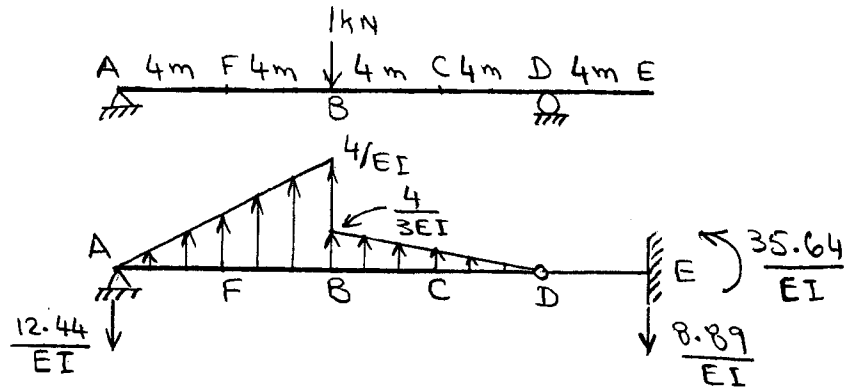
$$f_{EB} = f_{CB} = -\frac{916.7 \text{ k-ft}^3/\text{k}}{EI}$$

$$f_{BB} = -\frac{1333.3 \text{ k-ft}^3/\text{k}}{EI}$$

$$\bar{f}_{BB} = -f_{BB} = +\frac{1333.3 \text{ k-ft}^3/\text{k}}{EI}$$



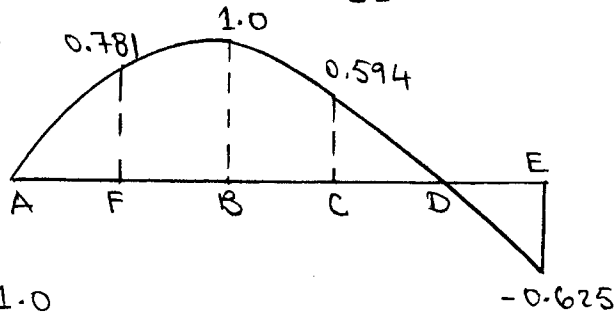
15.5 Select B_y ($\uparrow +$) as the redundant.



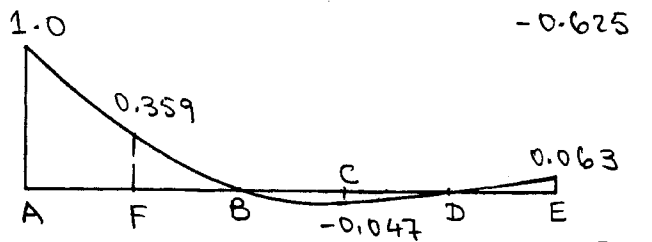
$$f_{BF} = f_{FB} = -\frac{44.43}{EI} ; \quad f_{BB} = -\frac{56.85}{EI}$$

$$f_{BC} = f_{CB} = -\frac{33.72}{EI} ; \quad f_{BE} = f_{EB} = +\frac{35.64}{EI}$$

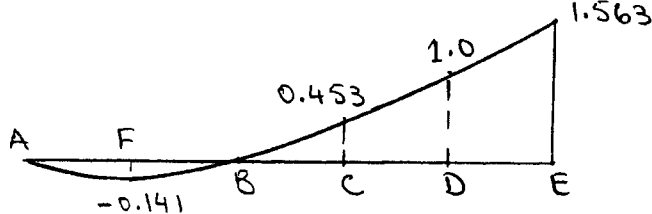
$$\bar{f}_{BB} = -f_{BB} = \frac{56.85}{EI}$$



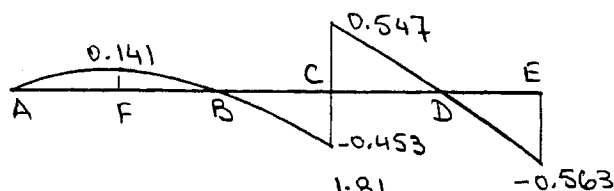
B_y



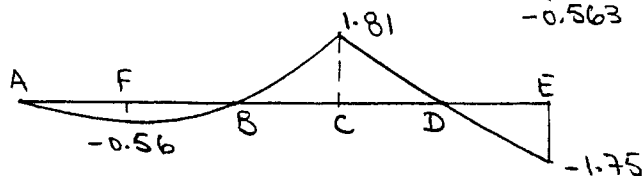
A_y



D_y

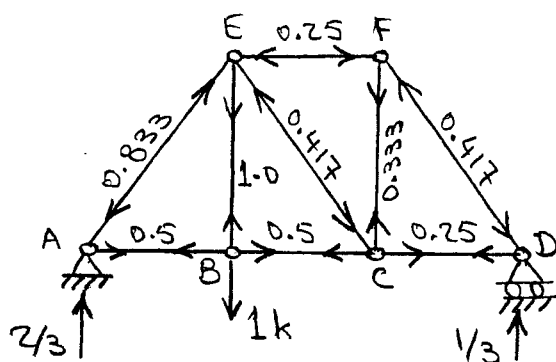


S_C

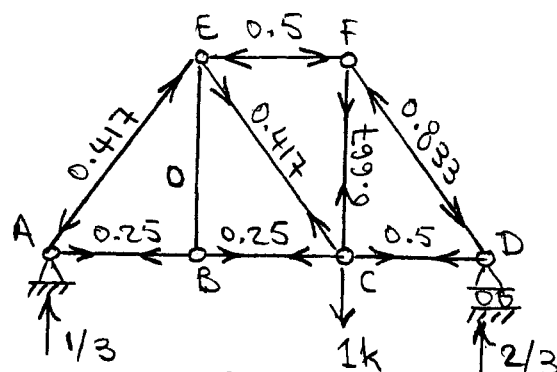


M_C

15.6 Select C_y (\uparrow) as the redundant.



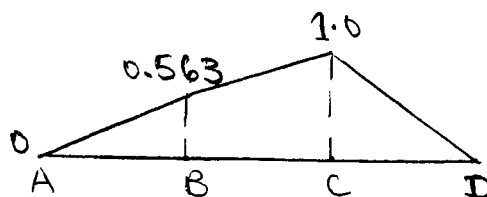
u_B Forces



u_C Forces

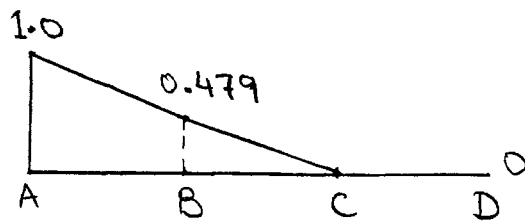
Member	L (ft)	u_B	u_C	$u_B u_C L$	$u_C^2 L$
AB	15	0.5	0.25	1.875	0.938
BC	15	0.5	0.25	1.875	0.938
CD	15	0.25	0.5	1.875	3.75
EF	15	-0.25	-0.5	1.875	3.75
BE	20	1.0	0	0	0
CF	20	0.333	0.667	4.442	8.898
AE	25	-0.833	-0.417	8.684	4.347
CE	25	-0.417	0.417	-4.347	4.347
DF	25	-0.417	-0.833	8.684	17.347
Σ				24.963	44.315

Unit load at B: $C_y = -\frac{24.963}{44.315} = -0.563 \text{ k or } 0.563 \text{ k} \uparrow$

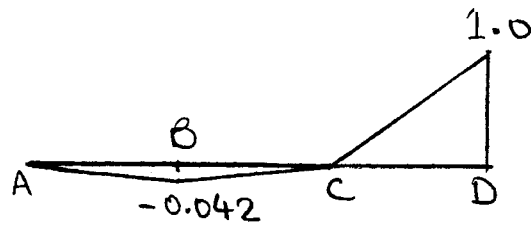


IL for C_y

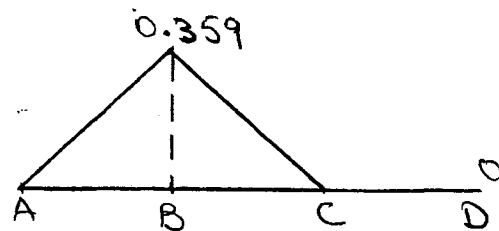
15.6 (cont'd.)



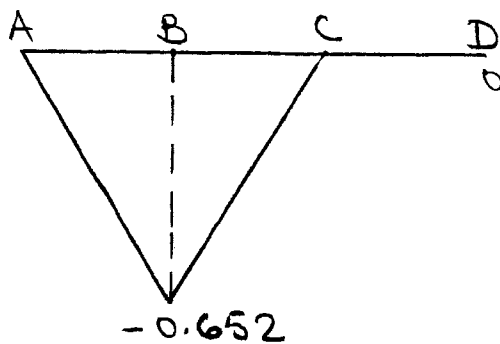
IL for A_y



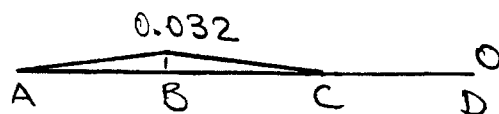
IL for D_y



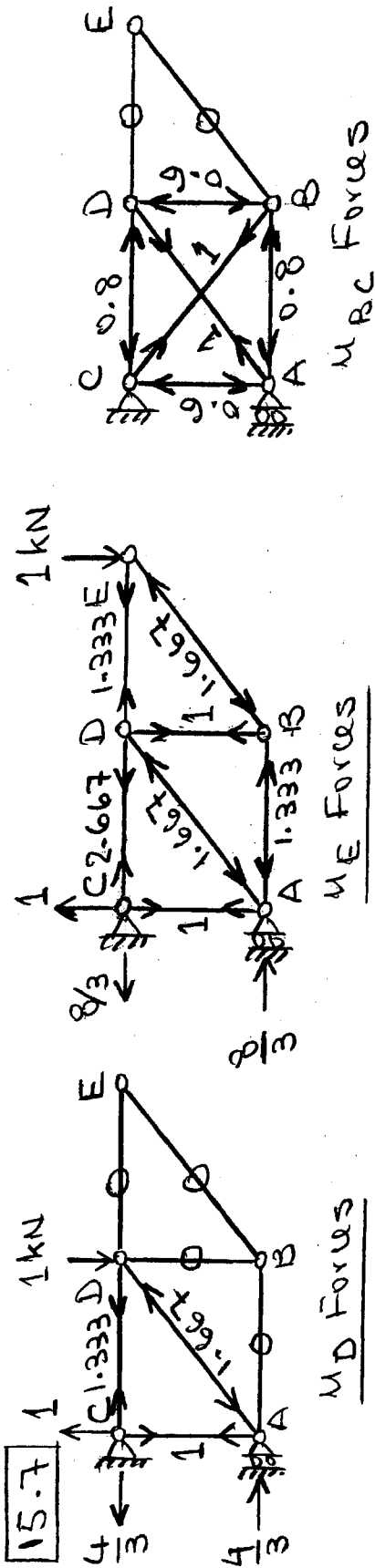
IL for F_{BC}



IL for F_{CE}



IL for F_{EF}



Member	L (m)	u_D	u_E	u_{BC}	$u_{D^H_{BC}L}$	$u_{E^H_{BC}L}$	$u_{B^2_{CL}}$
AB	8	0	-1.333	-0.8	0	8.533	5.12
CD	8	1.333	2.667	-0.8	-8.533	-17.067	5.12
DE	8	0	1.333	0	0	0	0
AC	6	1	1	-0.6	-3.6	-3.6	2.16
BD	6	0	1	-0.6	0	-3.6	2.16
AD	10	-1.667	-1.667	1	-16.667	-16.667	10
BE	10	0	-1.667	0	0	0	0
BC	10	0	0	1	0	0	10
Σ					-28.8	-32.4	34.56

15.7 (contd.)

Unit load at D:

$$f_{BC,D} + f_{BC,BC} F_{BC} = 0$$

$$F_{BC} = \frac{28.8}{34.56} = 0.833 \text{ kN/kN (T)}$$

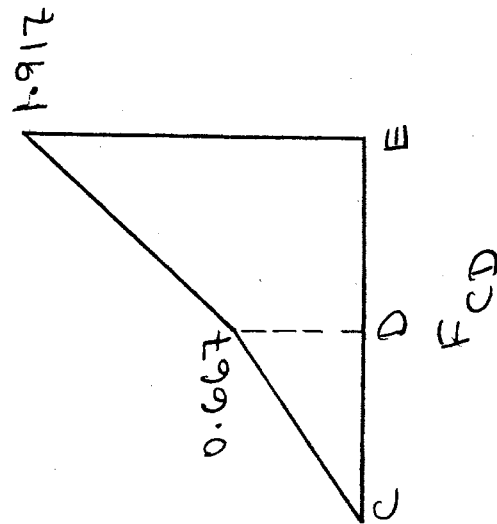
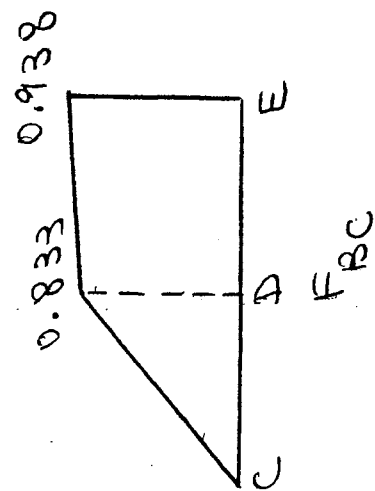
$$F_{CD} = u_D + u_{BC} F_{BC} = 1.333 - 0.8(0.833) = 0.667 \text{ kN/kN (T)}$$

$$f_{BC,E} + f_{BC,BC} F_{BC} = 0$$

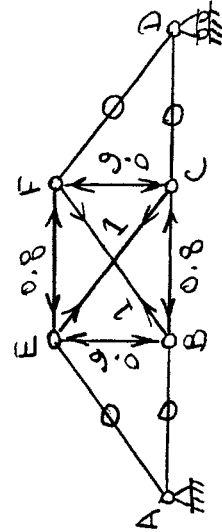
$$F_{BC} = \frac{32.4}{34.56} = 0.938 \text{ kN/kN (T)}$$

$$F_{CD} = u_E + u_{BC} F_{BC} = 2.667 - 0.8(0.938) = 1.917 \text{ kN/kN (T)}$$

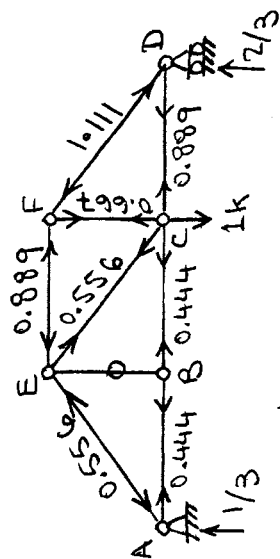
Unit load at E:



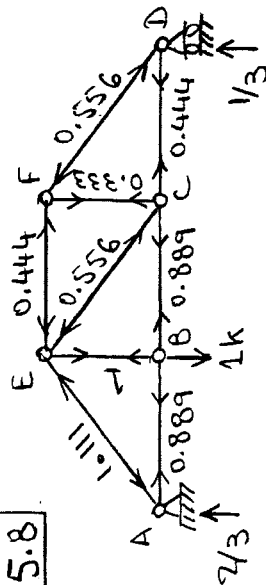
15.8



$\times F_{BF}$
 u_{BF} Forces



u_C Forces



u_B Forces

Member	L (in.)	A (in. ²)	u_B	u_C	u_{BF}	$\frac{u_B u_{BF} L}{A}$	$\frac{u_C u_{BF} L}{A}$	$\frac{u_{BF}^2 L}{A}$
AB	192	8	0.889	0.444	0	0	0	0
BC	192	8	0.889	0.444	-0.8	-17.07	-8.52	15.36
CD	192	8	0.444	0.889	0	0	0	0
EF	192	8	-0.444	-0.889	-0.8	8.52	17.07	15.36
BE	144	6	1	0	-0.6	-14.4	0	8.64
CF	144	6	0.333	0.667	-0.6	-4.8	-9.6	8.64
AE	240	8	-1.111	-0.556	0	0	0	0
DF	240	8	-0.556	-1.111	0	0	0	0
CE	240	6	-0.556	0.556	1	-22.24	22.24	40
BF	240	6	0	0	1	0	0	40
Σ						-49.99	21.19	128

15.8 (contd.)

Unit load at B:

$$F_{BF} = \frac{49.99}{128} = 0.39 \text{ k}$$

$$F_{BC} = 0.889 - 0.8(0.39) = 0.577 \text{ k}$$

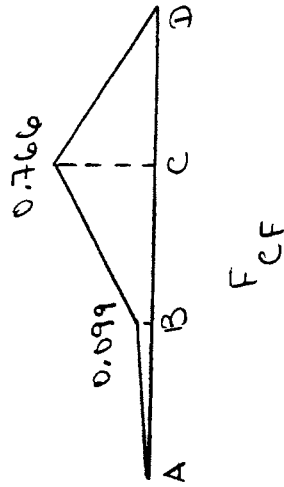
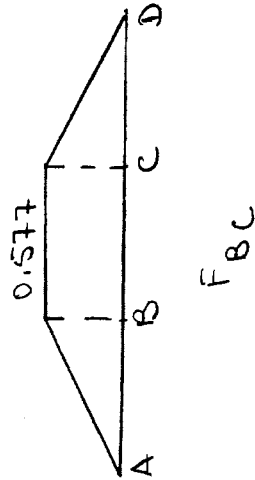
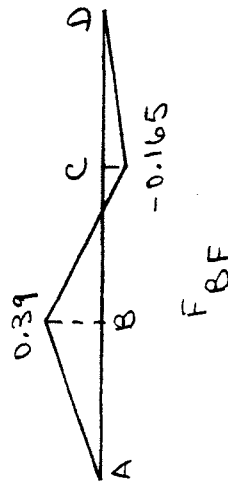
$$F_{CF} = 0.333 - 0.6(0.39) = 0.099 \text{ k}$$

Unit load at C:

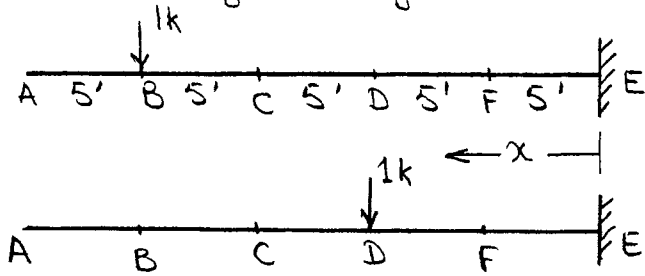
$$F_{BF} = -\frac{21.19}{128} = -0.165 \text{ k}$$

$$F_{BC} = 0.444 - 0.8(-0.165) = 0.577 \text{ k}$$

$$F_{CF} = 0.667 - 0.6(-0.165) = 0.766 \text{ k}$$



15.9 Select B_y and D_y as the redundants.



Compatibility Eqs.: $f_{Bx} + \bar{f}_{BB} B_y + \bar{f}_{BD} D_y = 0$

$f_{Dx} + \bar{f}_{DB} B_y + \bar{f}_{DD} D_y = 0$

Using beam deflection formulas:

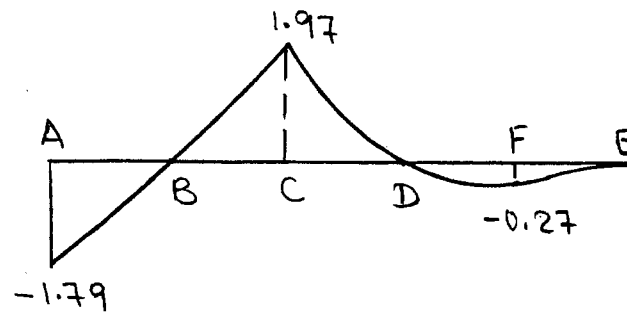
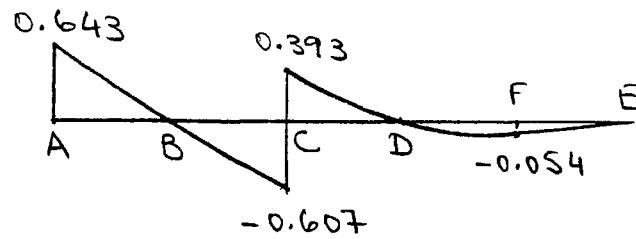
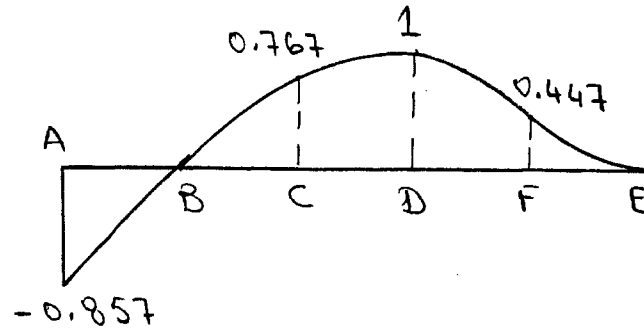
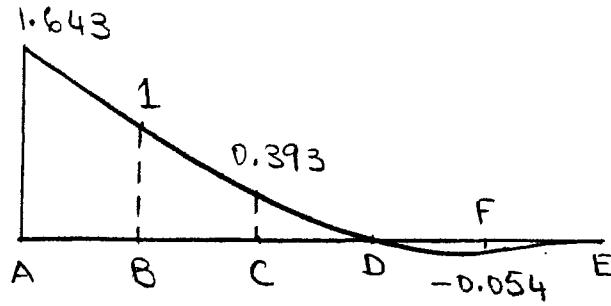
$$f_{Bx} = f_{xB} = \begin{cases} \frac{1}{6EI} (x^3 - 60x^2) & \text{for } 0 \leq x \leq 20' \\ \frac{200}{3EI} (20 - 3x) & \text{for } 20' \leq x \leq 25' \end{cases}$$

$$f_{Dx} = f_{xD} = \begin{cases} \frac{1}{6EI} (x^3 - 30x^2) & \text{for } 0 \leq x \leq 10' \\ \frac{50}{3EI} (10 - 3x) & \text{for } 10' \leq x \leq 25' \end{cases}$$

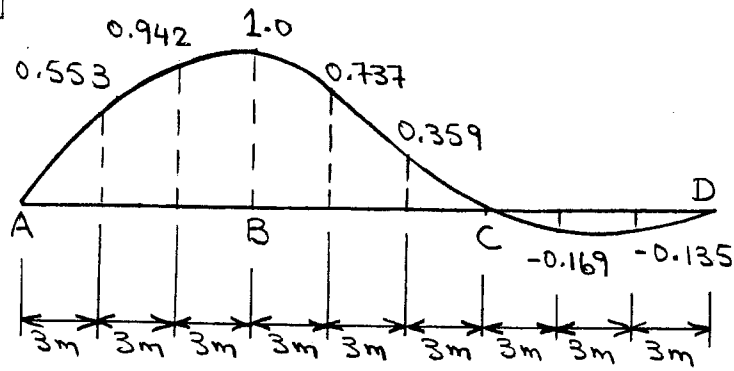
$\bar{f}_{BB} = \frac{2666.7}{EI}$; $\bar{f}_{BD} = \bar{f}_{DB} = \frac{833.3}{EI}$; $\bar{f}_{DD} = \frac{333.3}{EI}$

x (ft.)	$EI f_{Bx}$	$EI f_{Dx}$	B_y	D_y	$\sum C$	M_C
0	0	0	0	0	0	0
5	-229.2	-104.2	-0.054	0.447	-0.054	-0.27
10	-833.3	-333.3	0	1	0	0
15	-1687.5	-583.3	0.393	0.767	-0.607(L) 0.393(R)	1.97
20	-2666.7	-833.3	1	0	0	0
25	-3666.7	-1083.3	1.643	-0.857	0.643	-1.79

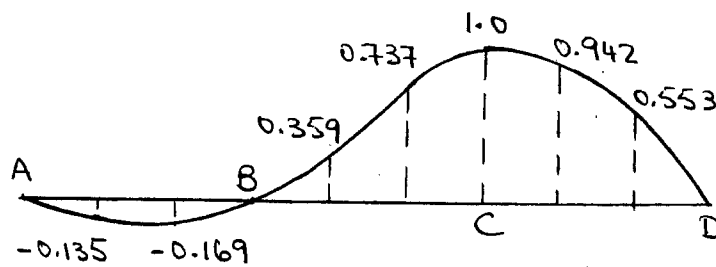
15.9 (Contd.)



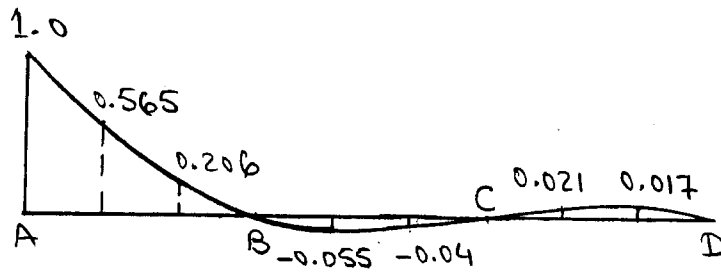
15.10



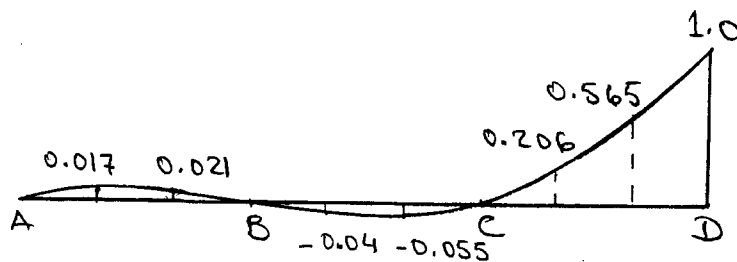
B_y



C_y

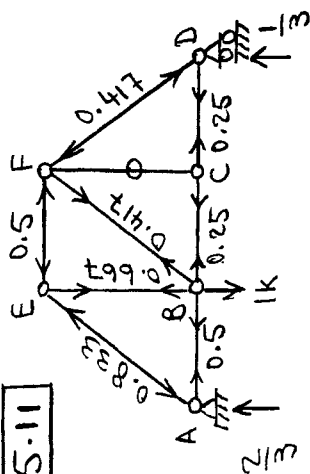


A_y

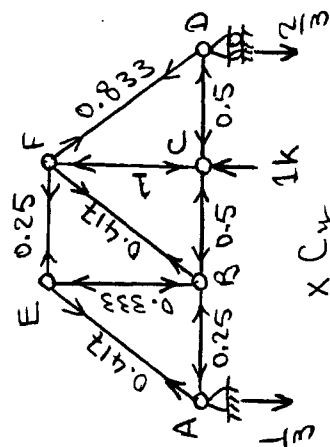


D_y

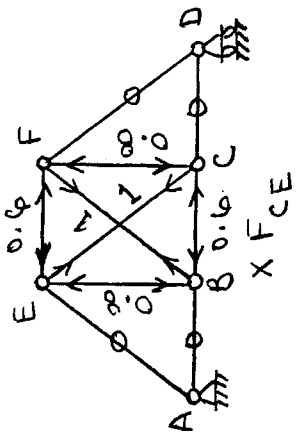
15.11



u_B Forces



u_C Forces



u_{CE} Forces

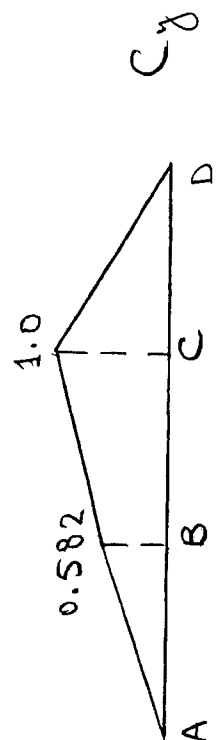
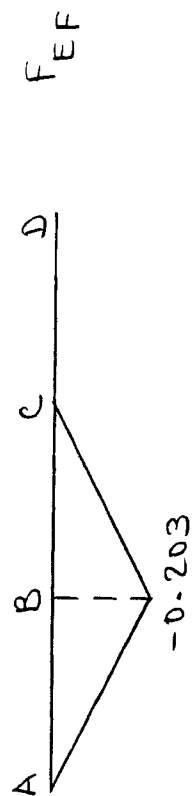
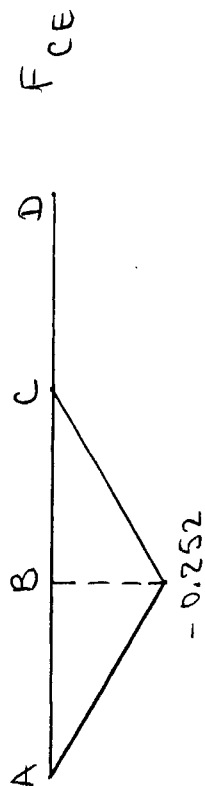
Member	L (ft)	u_B	u_C	u_{CE}	u_{BCE}	$u_C^2 L$	$u_{CE}^2 L$	$u_{CE} u_{CE} L$
AB	15	0.5	-0.25	0	-1.875	0.938	0	0
BC	15	0.25	-0.5	-0.6	-1.875	3.75	5.4	4.5
CD	15	0.25	-0.5	0	-1.875	3.75	0	0
EF	15	-0.5	0.25	-0.6	-1.875	0.938	5.4	-2.25
BE	20	0.667	-0.333	-0.8	-4.442	2.218	12.8	5.328
CF	20	0	-1	-0.8	0	20	12.8	16
AE	25	-0.833	0.417	0	-8.684	4.347	0	0
DF	25	-0.417	0.833	0	-8.684	17.347	0	0
BF	25	0.417	0.417	1	4.347	4.347	25	10.425
CE	25	0	0	1	0	0	25	0
Σ					-24.963	57.635	86.4	34.003

Unit load at B: $-24.963 + 57.635 C_y + 34.003 F_{CE} = 0$

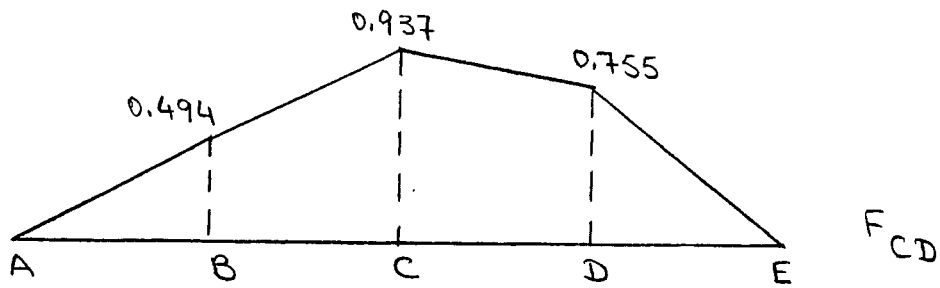
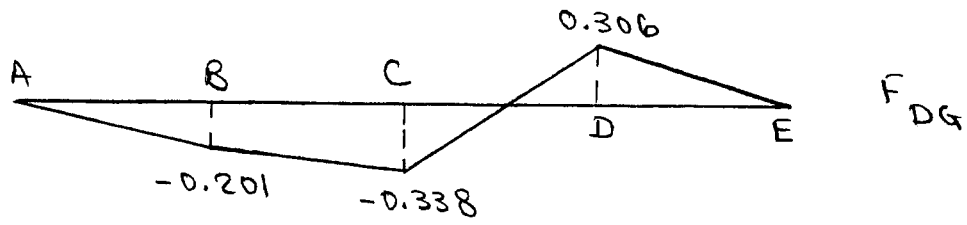
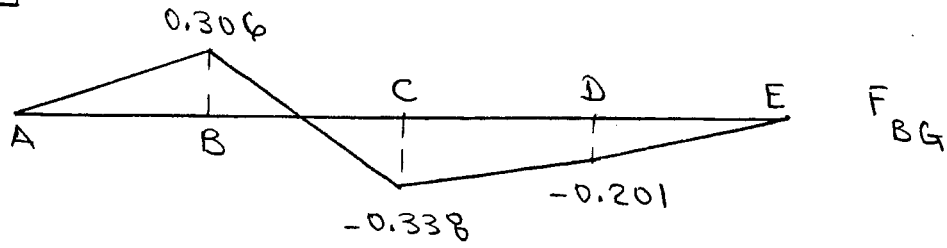
$2.003 + 34.003 C_y + 86.4 F_{CE} = 0$

Solving these equations, we obtain: $C_y = 0.582 \text{ k}$ $F_{CE} = -0.252 \text{ k}$

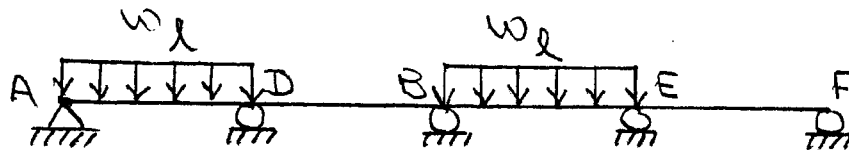
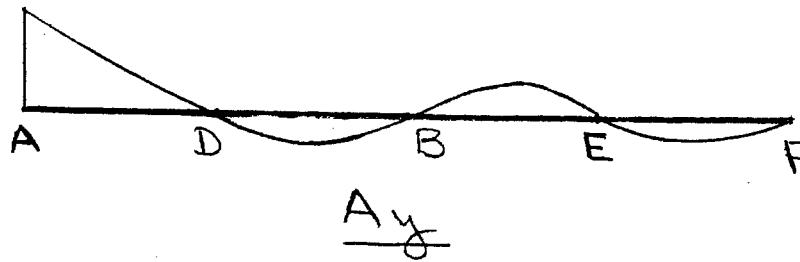
15-11 (Contd.)



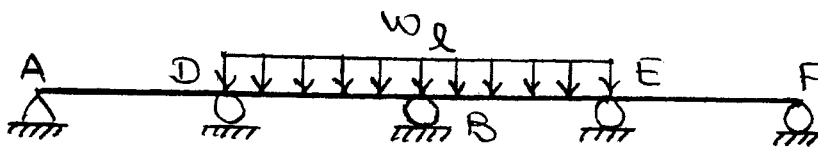
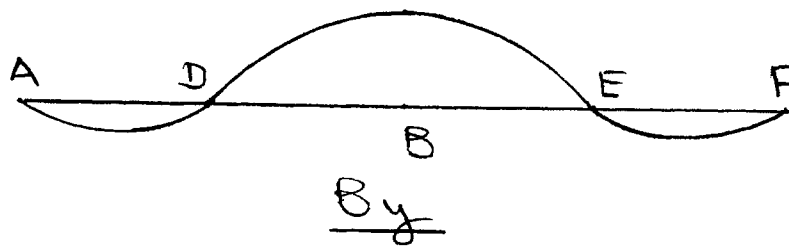
15.12



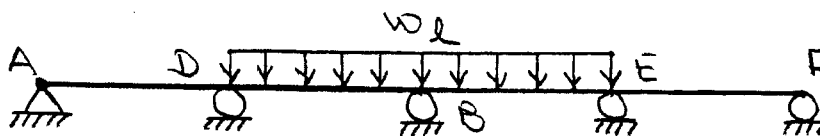
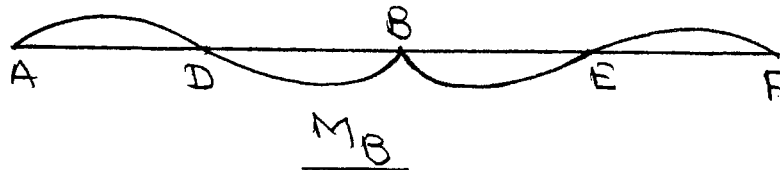
15.13



Live Load Arrangement for Maximum Positive A_y

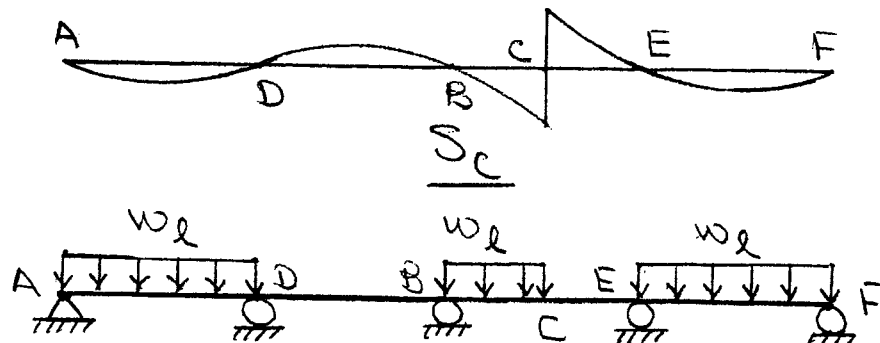


Live Load Arrangement for Maximum Positive B_y

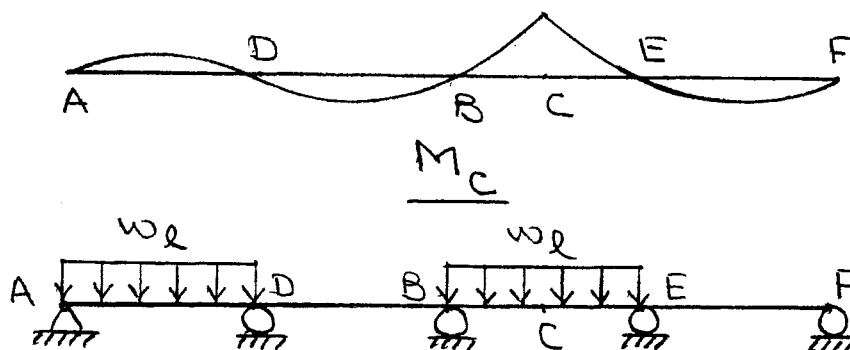


Live Load Arrangement for Maximum Negative M_B

15.13 (Contd.)

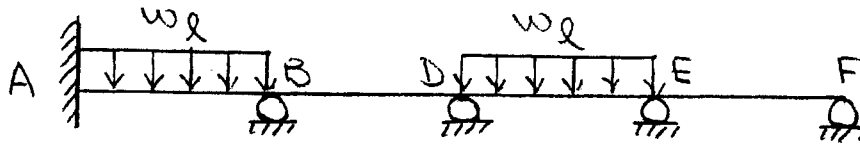
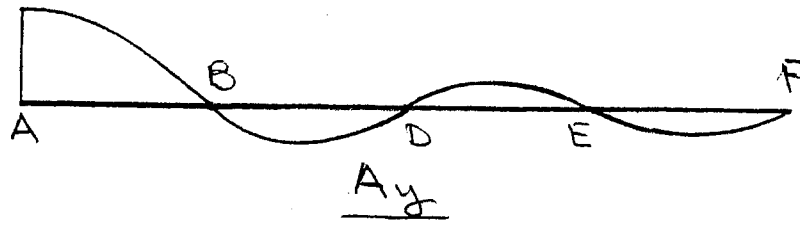


Live Load Arrangement for Maximum Negative S_C

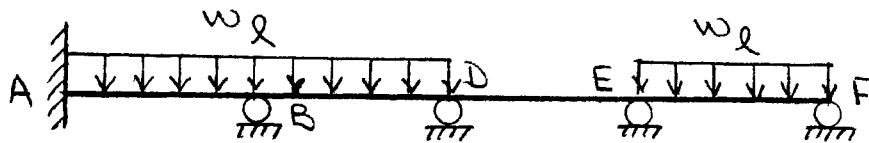
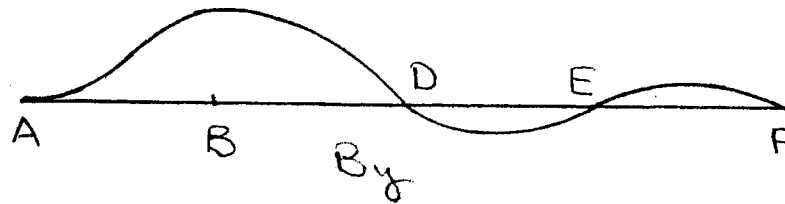


Live Load Arrangement for Maximum Positive M_C

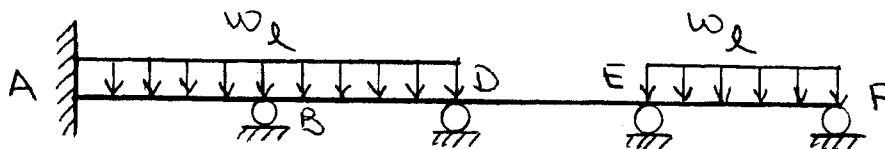
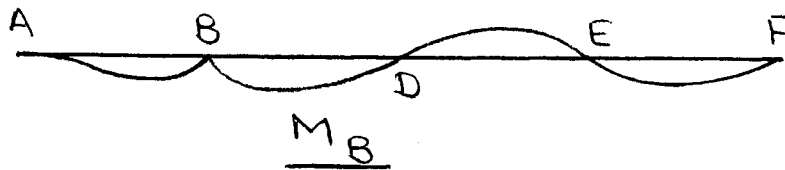
15.14



Live Load Arrangement for Maximum Positive A_y

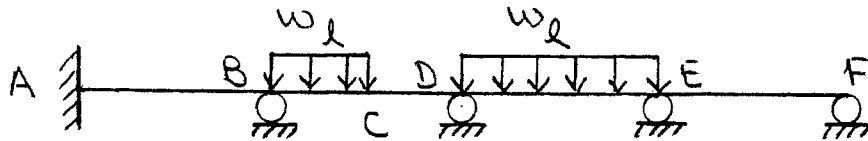
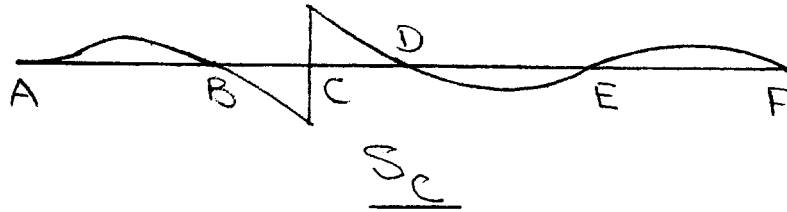


Live Load Arrangement for Maximum Positive B_y

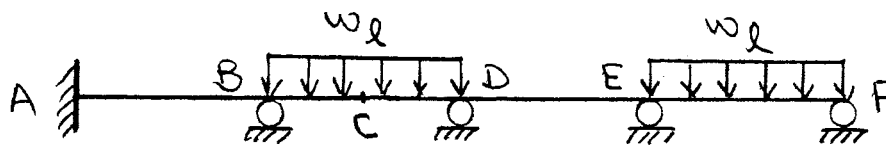
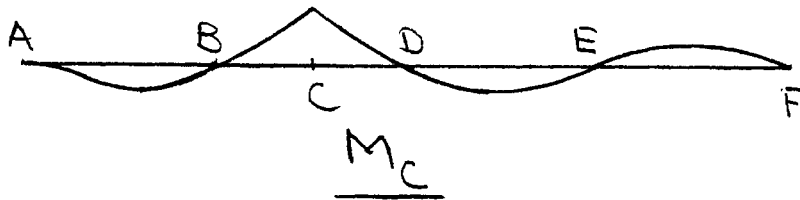


Live Load Arrangement for Maximum Negative M_B

15.14 (contd.)

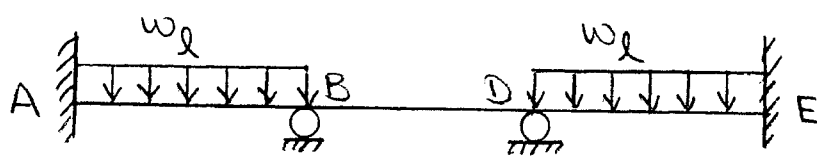
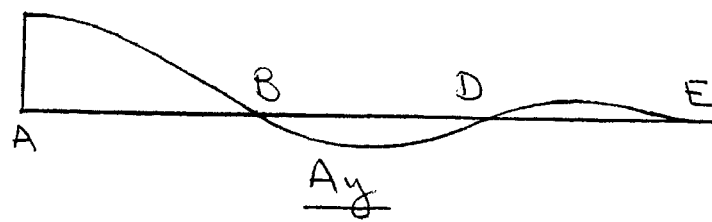


Live Load Arrangement for Maximum Negative S_c

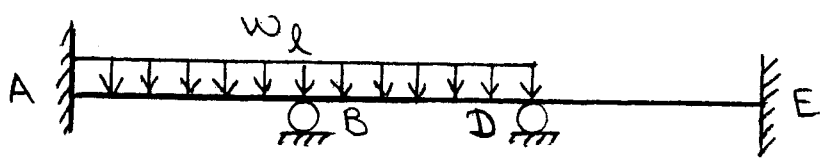
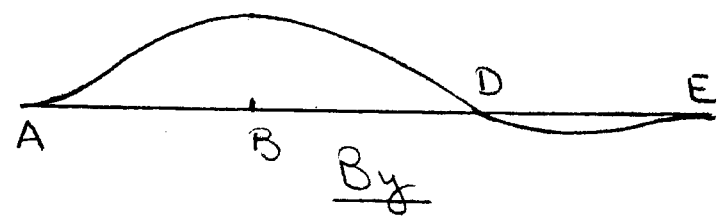


Live Load Arrangement for Maximum Positive M_c

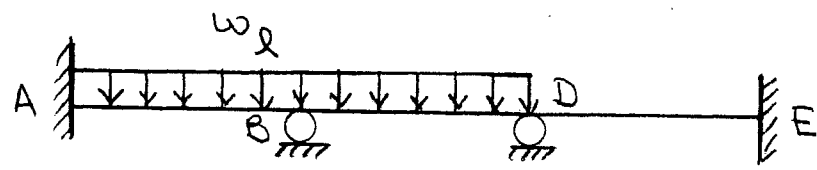
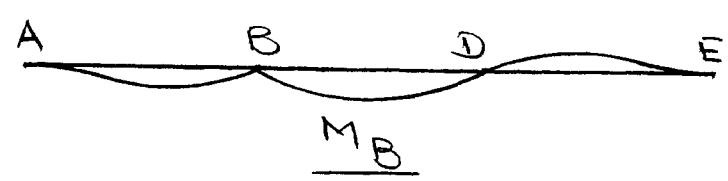
15.15



Live Load Arrangement for Maximum Positive A_y

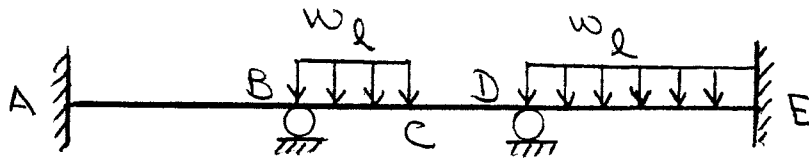
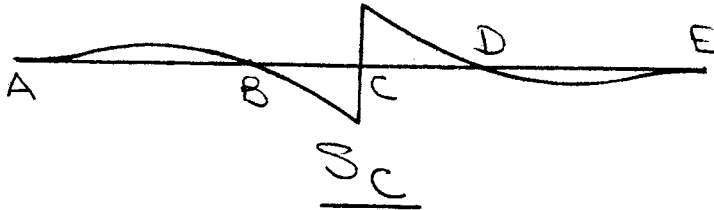


Live Load Arrangement for Maximum Positive B_y

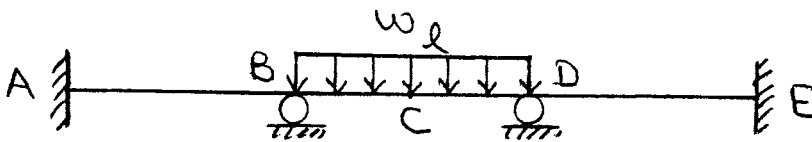
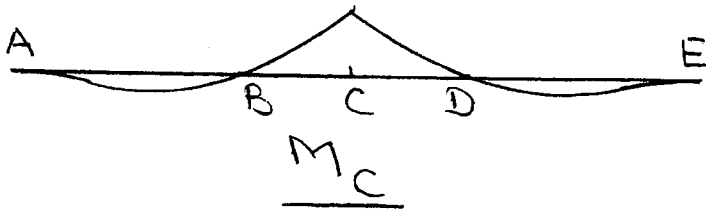


Live Load Arrangement for Maximum Negative M_B

15.15 (Contd.)

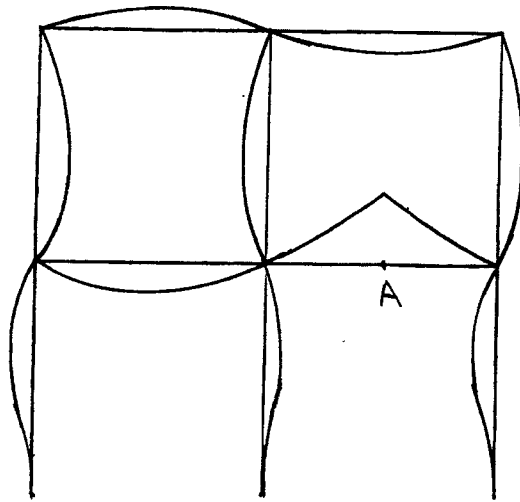


Live Load Arrangement for Maximum Negative S_C

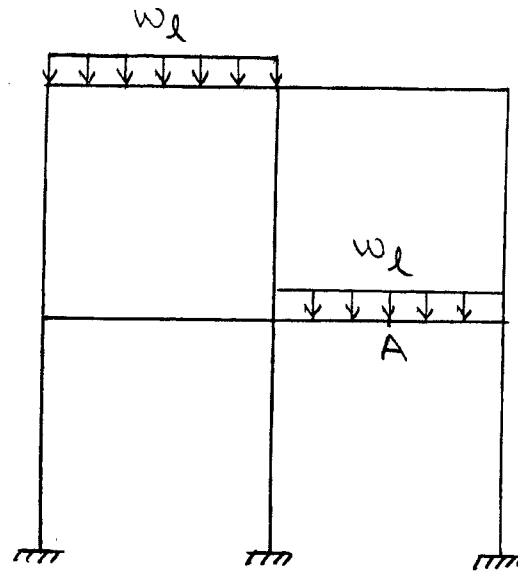


Live Load Arrangement for Maximum Positive M_C

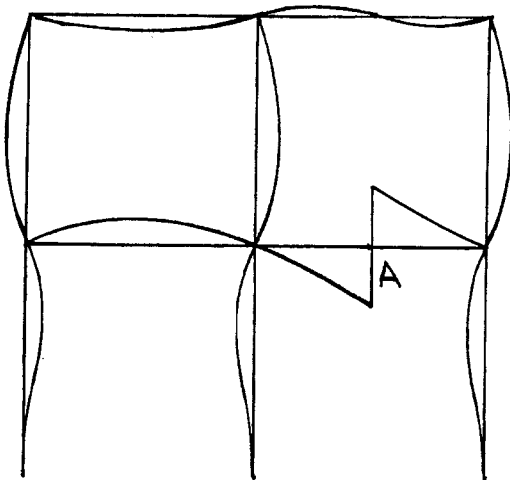
15.16



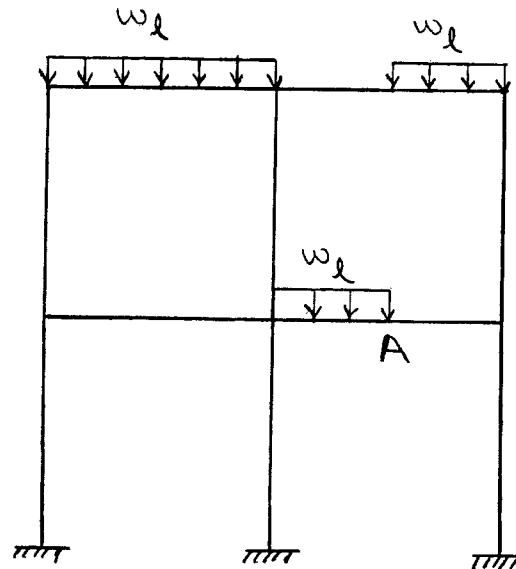
M_A



Live Load Arrangement
for Max. Positive M_A

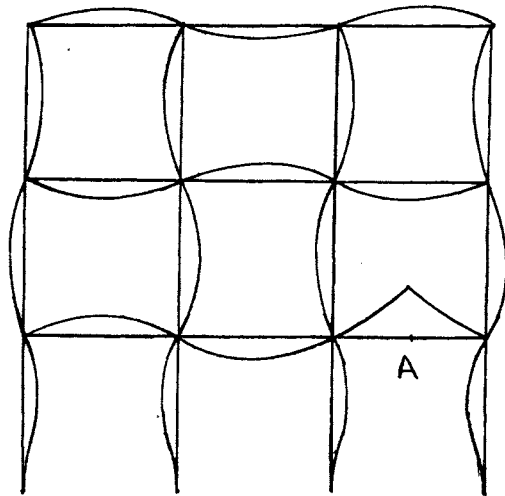


S_A

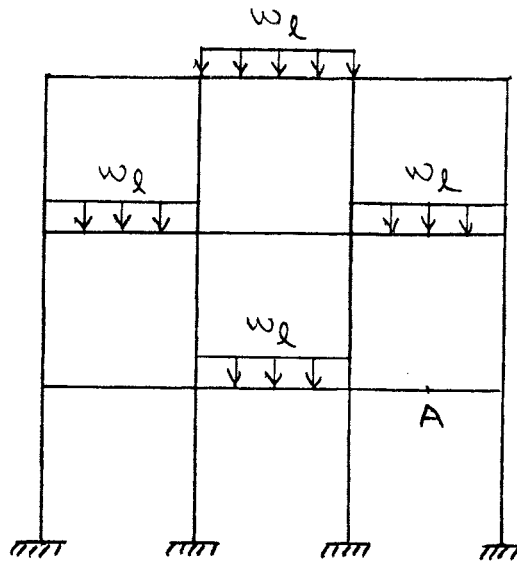


Live Load Arrangement
for Max. Negative S_A

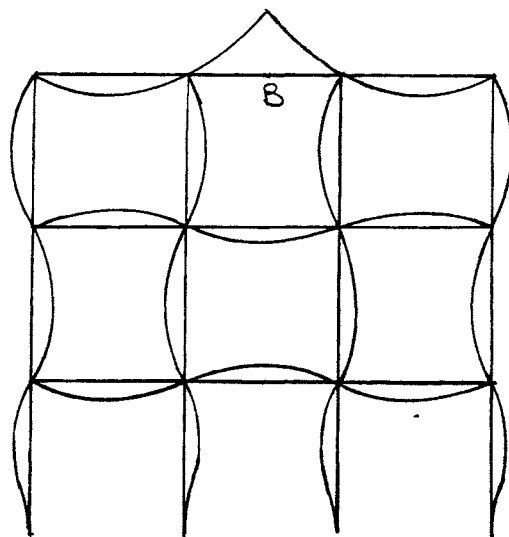
15.17



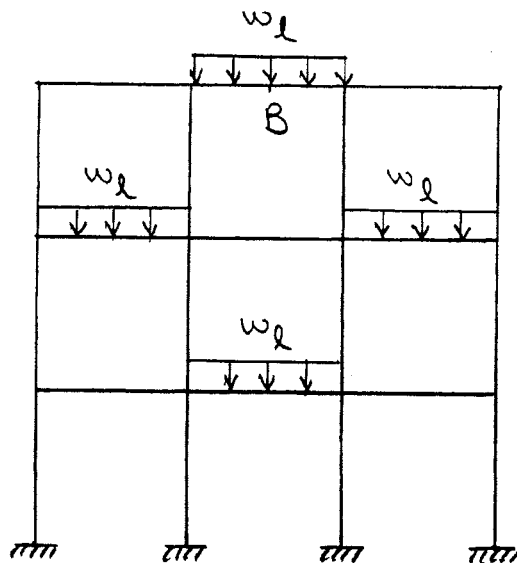
Qualitative I.L. for M_A



Live Load Arrangement
for Max. Negative M_A



Qualitative I.L. for M_B



Live Load Arrangement
for Max. Positive M_B

Chapter Sixteen

Slope-Deflection Method

CHAPTER 16

16-1 Fixed end moments:

$$FEM_{AC} = +40 \text{ k-ft}; \quad FEM_{CA} = -80 \text{ k-ft}$$

$$FEM_{CE} = +37.5 \text{ k-ft}; \quad FEM_{EC} = -37.5 \text{ k-ft}$$

Slope-deflection equations:

$$M_{AC} = 0.0667EI\theta_C + 40; \quad M_{CA} = 0.133EI\theta_C - 80$$

$$M_{CE} = 0.133EI\theta_C + 37.5; \quad M_{EC} = 0.0667EI\theta_C - 37.5$$

Equilibrium equation: $M_{CA} + M_{CE} = 0$

$$(0.133EI\theta_C - 80) + (0.133EI\theta_C + 37.5) = 0$$

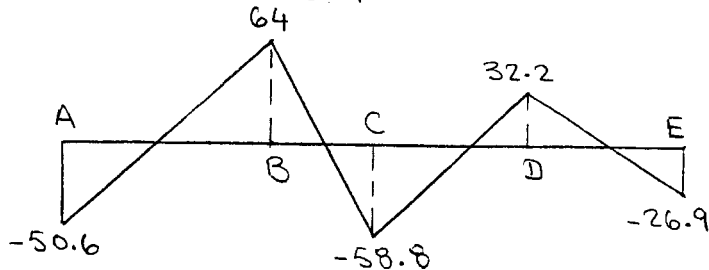
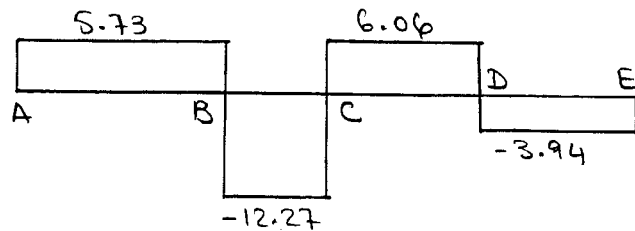
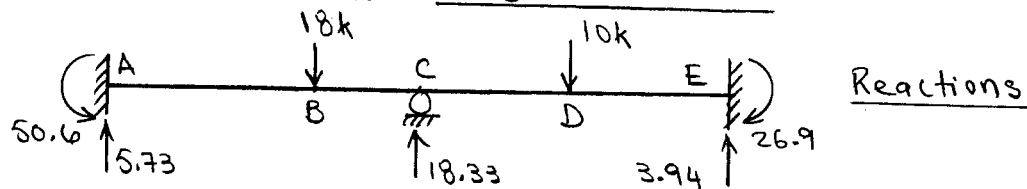
$$0.267EI\theta_C - 42.5 = 0$$

$$EI\theta_C = 159.2 \text{ k-ft}^2$$

Member end moments. Substituting the numerical value of $EI\theta_C$ into the slope-deflection equations, we obtain:

$$M_{AC} = 50.6 \text{ k-ft}; \quad M_{CA} = -58.8 \text{ k-ft}$$

$$M_{CE} = 58.8 \text{ k-ft}; \quad M_{EC} = -26.9 \text{ k-ft}$$



16.2 Fixed-end Moment:

$$FEM_{AB} = \frac{1.5(30)^2}{12} + \frac{20(30)}{8} = 187.5 \text{ k-ft}$$

$$FEM_{BA} = -187.5 \text{ k-ft}; FEM_{BC} = \frac{3(20)^2}{12} = 100 \text{ k-ft}$$

$$FEM_{CB} = -100 \text{ k-ft}.$$

Slope-deflection equations:

$$M_{AB} = 0.0667EI\theta_B + 187.5; M_{BA} = 0.133EI\theta_B - 187.5$$

$$M_{BC} = 0.2EI\theta_B + 100; M_{CB} = 0.1EI\theta_B - 100$$

Equilibrium equation: $M_{BA} + M_{BC} = 0$

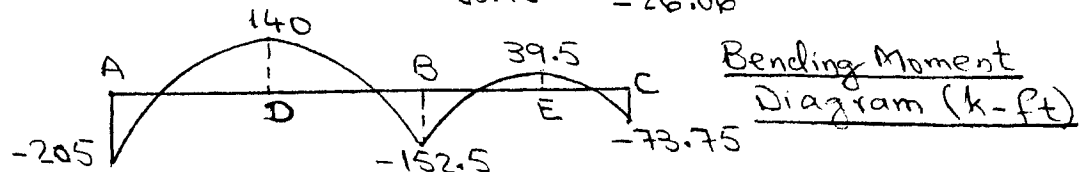
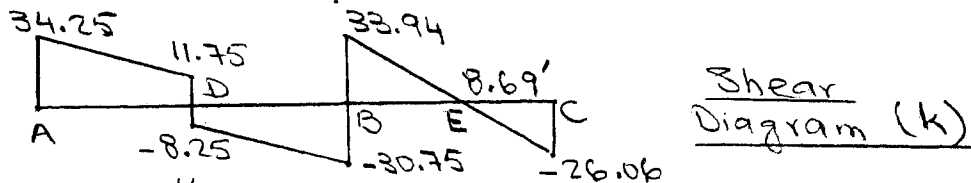
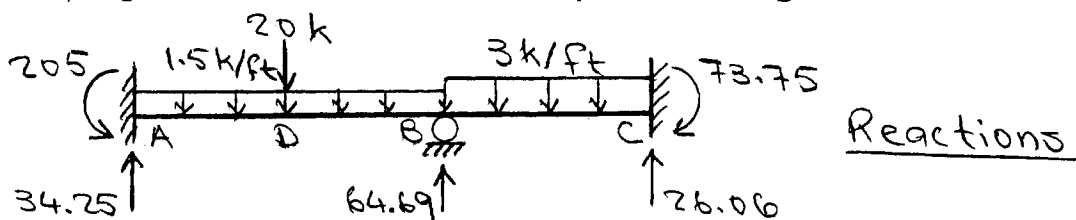
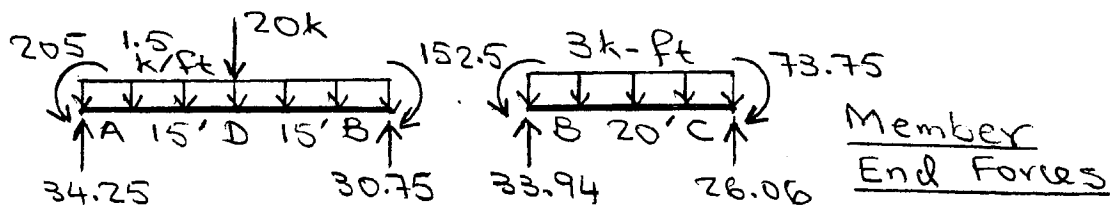
$$(0.133EI\theta_B - 187.5) + (0.2EI\theta_B + 100) = 0$$

$$EI\theta_B = 262.5 \text{ k-ft}^2$$

Member end moments: Substituting the numerical value of $EI\theta_B$ into the slope-deflection equations, we obtain:

$$M_{AB} = 205 \text{ k-ft}; M_{BA} = -152.5 \text{ k-ft}$$

$$M_{BC} = 152.5 \text{ k-ft}; M_{CB} = -73.75 \text{ k-ft}$$



16.3 Fixed end moments:

$$FEM_{BE} = +400 \text{ kN.m}; FEM_{EB} = -400 \text{ kN.m}$$

Slope-deflection equations:

$$M_{AB} = 0.222 EI \theta_B; M_{BA} = 0.444 EI \theta_B$$

$$M_{BE} = 0.444 EI \theta_B + 400; M_{EB} = 0.222 EI \theta_B - 400$$

Equilibrium equation: $M_{BA} + M_{BE} = 0$

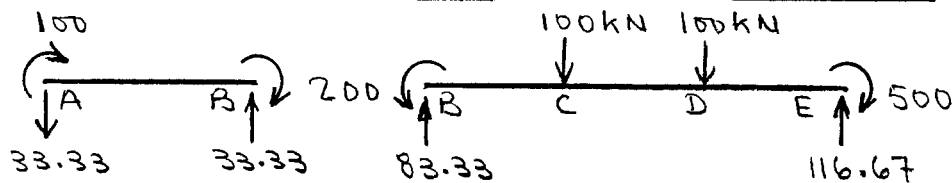
$$0.444 EI \theta_B + (0.444 EI \theta_B + 400) = 0$$

$$EI \theta_B = -450 \text{ kN.m}^2$$

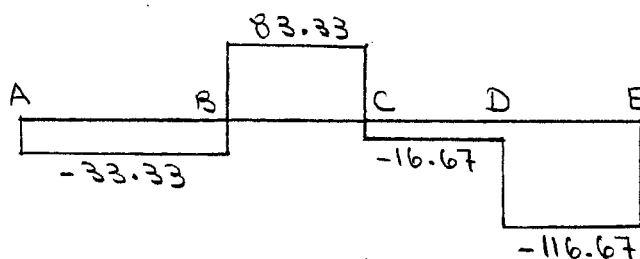
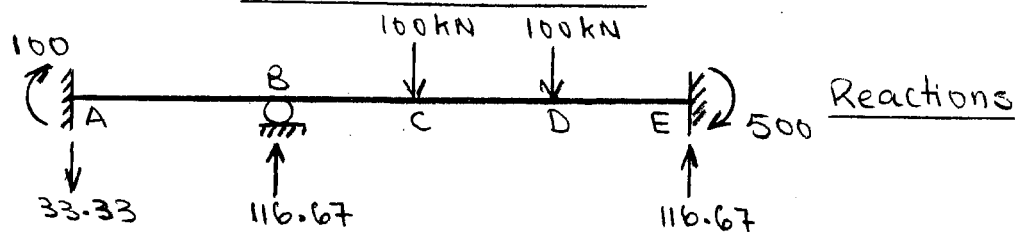
Member end moments: Substituting the numerical value of $EI \theta_B$ into the slope-deflection equations, we obtain

$$M_{AB} = -150 \text{ kN.m}; M_{BA} = -200 \text{ kN.m}$$

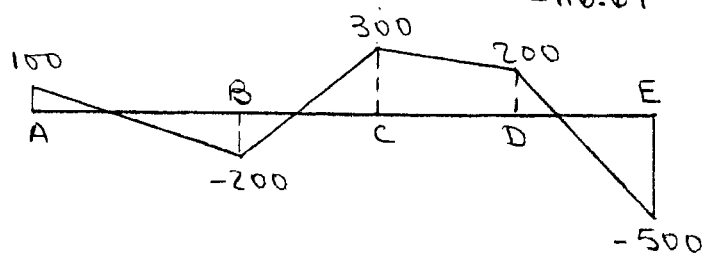
$$M_{BE} = 200 \text{ kN.m}; M_{EB} = -500 \text{ kN.m}$$



Member End Forces



Shear
Diagram (kN)



Bending Moment
Diagram (kN.m)

16.4 Fixed-end moments:

$$FEM_{AB} = \frac{25(8)^2}{12} = 133.33 \text{ kN.m}; FEM_{BA} = -133.33 \text{ kN.m}$$

$$FEM_{BC} = 133.33 \text{ kN.m}; FEM_{CB} = -133.33 \text{ kN.m}$$

Slope-deflection equations: $M_{CB} = 0$

$$M_{AB} = 0.25EI\theta_B + 133.33; M_{BA} = 0.5EI\theta_B - 133.33$$

$$M_{BC} = 0.375EI\theta_B + 200$$

Equilibrium equation: $M_{BA} + M_{BC} = 0$

$$(0.5EI\theta_B - 133.33) + (0.375EI\theta_B + 200) = 0$$

$$EI\theta_B = -76.19 \text{ kN.m}^2$$

Member end moments: Substituting the numerical value of $EI\theta_B$ into the slope-deflection equations, we obtain:

$$\underline{M_{AB} = 114.3 \text{ kN.m}; M_{BA} = -171.4 \text{ kN.m}}$$

$$\underline{M_{BC} = 171.4 \text{ kN.m}}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.37.

16.5

Fixed-end moments:

$$FEM_{AB} = \frac{3(25)^2}{12} = 156.25 \text{ k-ft}; \quad FEM_{BA} = -156.25 \text{ k-ft}$$

$$FEM_{BC} = \frac{3(15)^2}{12} = 56.25 \text{ k-ft}; \quad FEM_{CB} = -56.25 \text{ k-ft}$$

Slope-deflection equations: $M_{AB} = M_{CB} = 0$

$$M_{BA} = \frac{3E(2I)}{25} \theta_B - 156.25 - \frac{156.25}{2} = 0.24 EI \theta_B - 234.38$$

$$M_{BC} = \frac{3EI}{15} \theta_B + 56.25 + \frac{56.25}{2} = 0.2 EI \theta_B + 84.38$$

Equilibrium equation: $M_{BA} + M_{BC} = 0$

$$(0.24 EI \theta_B - 234.38) + (0.2 EI \theta_B + 84.38) = 0$$

$$EI \theta_B = 340.91 \text{ k-ft}^2$$

Member end moments. Substituting the numerical value of $EI \theta_B$ into the slope-deflection equations, we obtain

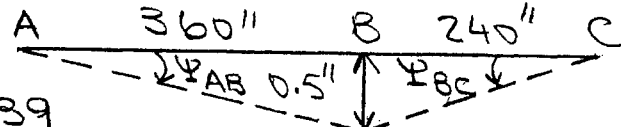
$$M_{BA} = -152.6 \text{ k-ft}; \quad M_{BC} = 152.6 \text{ k-ft}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.12.

16.6 Fixed-end moments;

$$FEM_{AB} = 187.5 \text{ k-ft}; \quad FEM_{BA} = -187.5 \text{ k-ft}$$

$$FEM_{BC} = 100 \text{ k-ft}; \quad FEM_{CB} = -100 \text{ k-ft}$$

Chord rotations: 

$$\Psi_{AB} = -\frac{0.5}{360} = -0.00139$$

$$\Psi_{BC} = +\frac{0.5}{240} = +0.00208$$

Slope-deflection equations:

Using $EI = \frac{29000(1650)}{(12)^2} \text{ k-ft}^2$, we write

$$M_{AB} = 0.0667 EI \theta_B + 92.3 + 187.5 = 0.0667 EI \theta_B + 279.8$$

$$M_{BA} = 0.133 EI \theta_B + 92.3 - 187.5 = 0.133 EI \theta_B - 95.2$$

$$M_{BC} = 0.2 EI \theta_B - 207.7 + 100 = 0.2 EI \theta_B - 107.7$$

$$M_{CB} = 0.1 EI \theta_B - 207.7 - 100 = 0.1 EI \theta_B - 307.7$$

Equilibrium equation: $M_{BA} + M_{BC} = 0$

$$(0.133 EI \theta_B - 95.2) + (0.2 EI \theta_B - 107.7) = 0$$

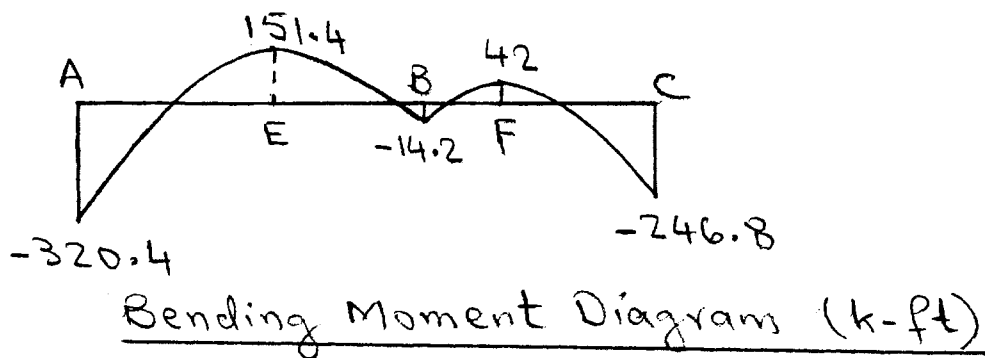
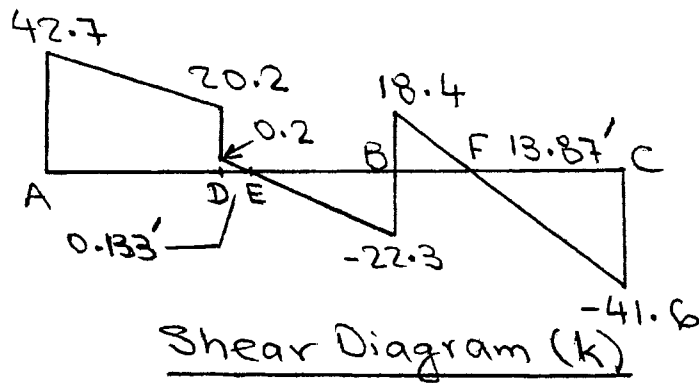
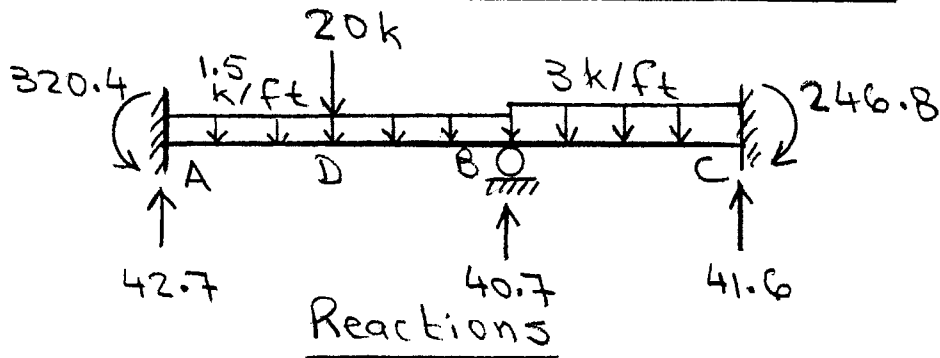
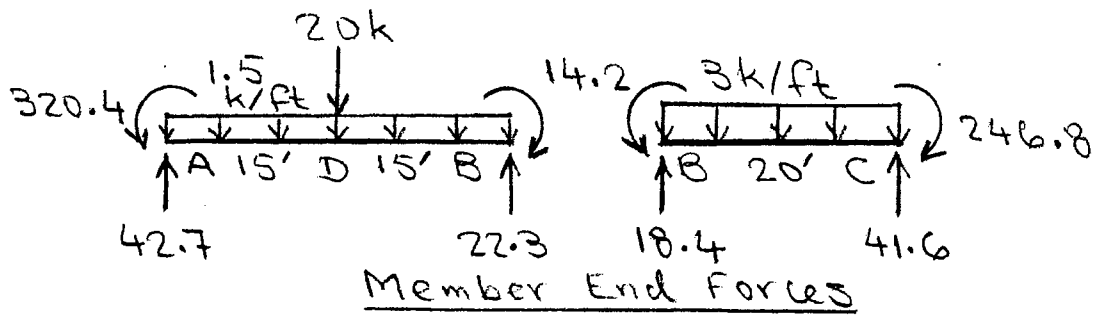
$$EI \theta_B = 609.31 \text{ k-ft}^2$$

Member end moments: Substituting the numerical value of $EI \theta_B$ into the slope-deflection equations, we obtain:

$$\underline{M_{AB} = 320.4 \text{ k-ft};} \quad \underline{M_{BA} = -14.2 \text{ k-ft}}$$

$$\underline{M_{BC} = 14.2 \text{ k-ft};} \quad \underline{M_{CB} = -246.8 \text{ k-ft}}$$

16.6 (contd.)

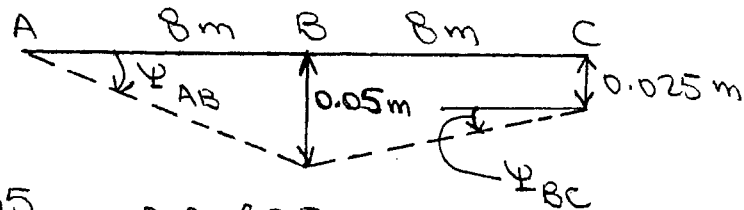


16.7 Fixed-end moments:

$$FEM_{AB} = 133.33 \text{ kN}\cdot\text{m}; \quad FEM_{BA} = -133.33 \text{ kN}\cdot\text{m}$$

$$FEM_{BC} = 133.33 \text{ kN}\cdot\text{m}; \quad FEM_{CB} = -133.33 \text{ kN}\cdot\text{m}$$

Chord rotations:



$$\psi_{AB} = -\frac{0.05}{8} = -0.00625$$

$$\psi_{BC} = +\frac{0.025}{8} = +0.003125$$

Slope-deflection equations: $M_{CB} = 0$

Using $EI = 70(1300) = 91000 \text{ kN}\cdot\text{m}^2$, we write

$$M_{AB} = 0.25EI\theta_B + 426.56 + 133.33 = 0.25EI\theta_B + 559.89$$

$$M_{BA} = 0.5EI\theta_B + 426.56 - 133.33 = 0.5EI\theta_B + 293.23$$

$$M_{BC} = 0.375EI\theta_B - 106.64 + 133.33 + \frac{133.33}{2} \\ = 0.375EI\theta_B + 93.36$$

Equilibrium equation: $M_{BA} + M_{BC} = 0$

$$(0.5EI\theta_B + 293.23) + (0.375EI\theta_B + 93.36) = 0$$

$$EI\theta_B = -441.82 \text{ kN}\cdot\text{m}^2$$

Member end moments: Substituting the numerical value of $EI\theta_B$ into the slope-deflection equations, we obtain:

$$\underline{M_{AB} = 449.4 \text{ kN}\cdot\text{m}}; \quad \underline{M_{BA} = 72.3 \text{ kN}\cdot\text{m}}$$

$$\underline{M_{BC} = -72.3 \text{ kN}\cdot\text{m}}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.53.

16.8 Fixed-end moments:

$$\begin{aligned} FEM_{AB} &= \frac{1.5(25)^2}{12} = 78.125 \text{ k-ft}; FEM_{BA} = -78.125 \text{ k-ft} \\ FEM_{BC} &= \frac{1.5(20)^2}{12} = 50 \text{ k-ft}; FEM_{CB} = -50 \text{ k-ft} \\ FEM_{CD} &= 78.125 \text{ k-ft}; FEM_{DC} = -78.125 \text{ k-ft} \end{aligned}$$

Slope-deflection equations:

$$\begin{aligned} M_{AB} &= 0.08EI\theta_B + 78.125; M_{BA} = 0.16EI\theta_B - 78.125 \\ M_{BC} &= 0.2EI\theta_B + 0.1EI\theta_C + 50; M_{CB} = 0.1EI\theta_B + 0.2EI\theta_C - 50 \\ M_{CD} &= 0.16EI\theta_C + 78.125; M_{DC} = 0.08EI\theta_C - 78.125 \end{aligned}$$

Equilibrium equations:

$$\begin{aligned} M_{BA} + M_{BC} &= 0 \\ M_{CB} + M_{CD} &= 0 \end{aligned}$$

$$0.36EI\theta_B + 0.1EI\theta_C = 28.125$$

$$0.1EI\theta_B + 0.36EI\theta_C = -28.125$$

By solving these equations, we obtain:

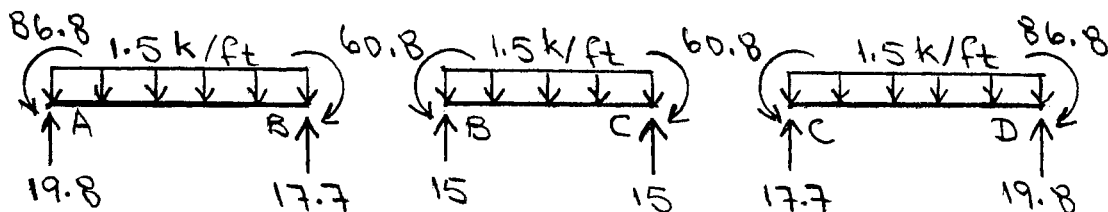
$$EI\theta_B = 108.173 \text{ k-ft}^2; EI\theta_C = -108.173 \text{ k-ft}^2$$

Member end moments: Substituting the numerical values of $EI\theta_B$ and $EI\theta_C$ into the slope-deflection equations, we obtain:

$$M_{AB} = 86.8 \text{ k-ft}; M_{BA} = -60.8 \text{ k-ft}$$

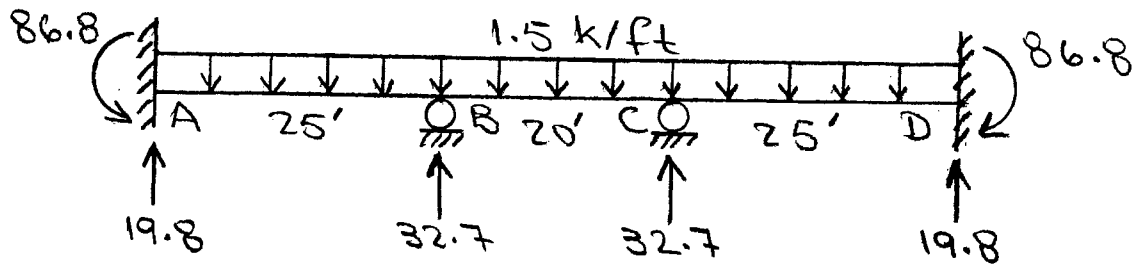
$$M_{BC} = 60.8 \text{ k-ft}; M_{CB} = -60.8 \text{ k-ft}$$

$$M_{CD} = 60.8 \text{ k-ft}; M_{DC} = -86.8 \text{ k-ft}$$

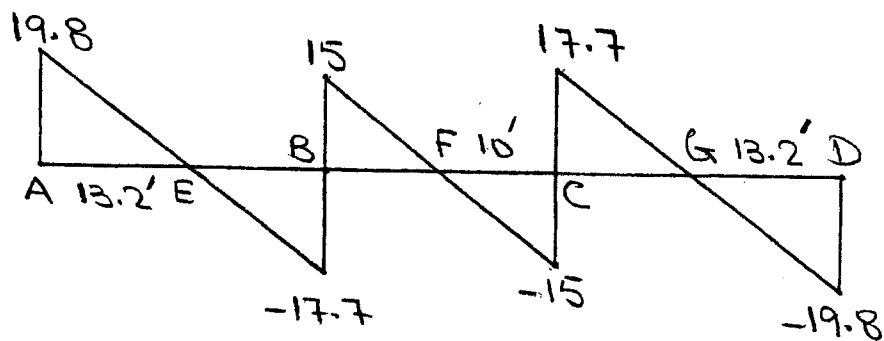


Member End Moments and Shears

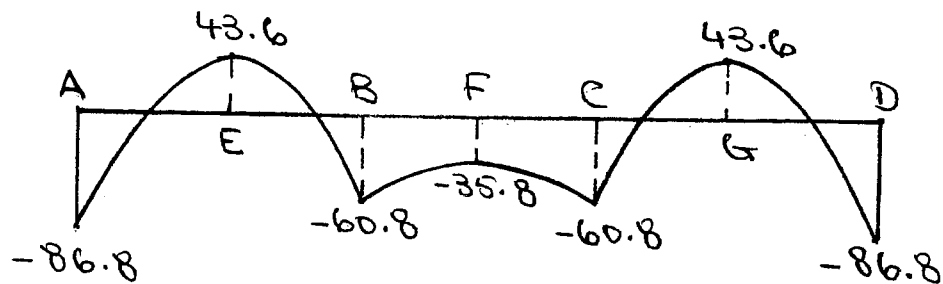
16.8 (Contd.)



Reactions



Shear Diagram (k)



Bending Moment Diagram (k-ft)

16.9 Fixed end moments:

$$FEM_{AB} = \frac{20(8)^2}{12} = 106.7 \text{ kN.m}; FEM_{BA} = -106.7 \text{ kN.m}$$

$$FEM_{BC} = 106.7 \text{ kN.m}; FEM_{CB} = -106.7 \text{ kN.m}$$

$$FEM_{CE} = \frac{60(8)}{8} = 60 \text{ kN.m}; FEM_{EC} = -60 \text{ kN.m}$$

Slope-deflection equations:

$$M_{AB} = 0.25EI\theta_B + 106.7; M_{BA} = 0.5EI\theta_B - 106.7$$

$$M_{BC} = 0.5EI\theta_B + 0.25EI\theta_C + 106.7$$

$$M_{CB} = 0.25EI\theta_B + 0.5EI\theta_C - 106.7$$

$$M_{CE} = 0.5EI\theta_C + 60; M_{EC} = 0.25EI\theta_C - 60$$

Equilibrium equations: $M_{BA} + M_{BC} = 0$

$$M_{CB} + M_{CE} = 0$$

$$EI\theta_B + 0.25EI\theta_C = 0$$

$$0.25EI\theta_B + EI\theta_C = 46.7$$

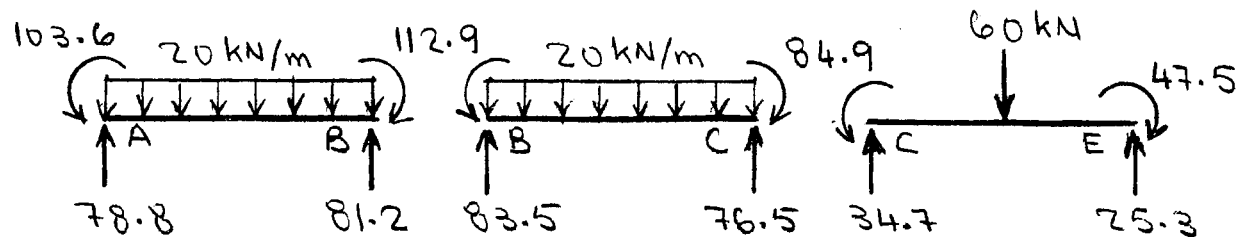
By solving these equations, we obtain

$$EI\theta_B = -12.45 \text{ kN.m}^2; EI\theta_C = 49.81 \text{ kN.m}^2$$

Member end moments: Substituting the numerical values of $EI\theta_B$ and $EI\theta_C$ into the slope-deflection equations, we obtain

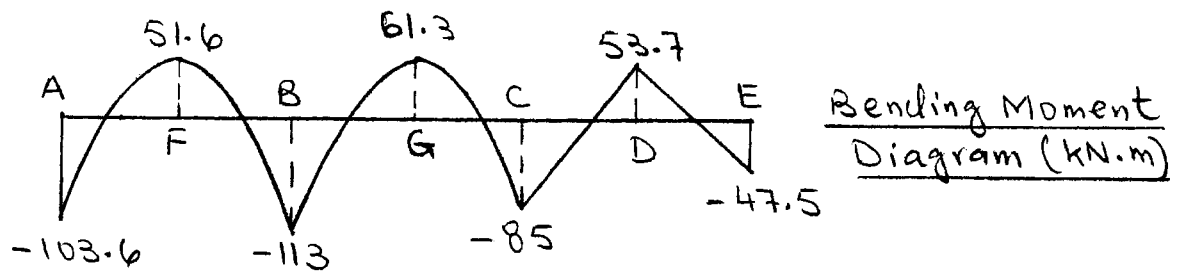
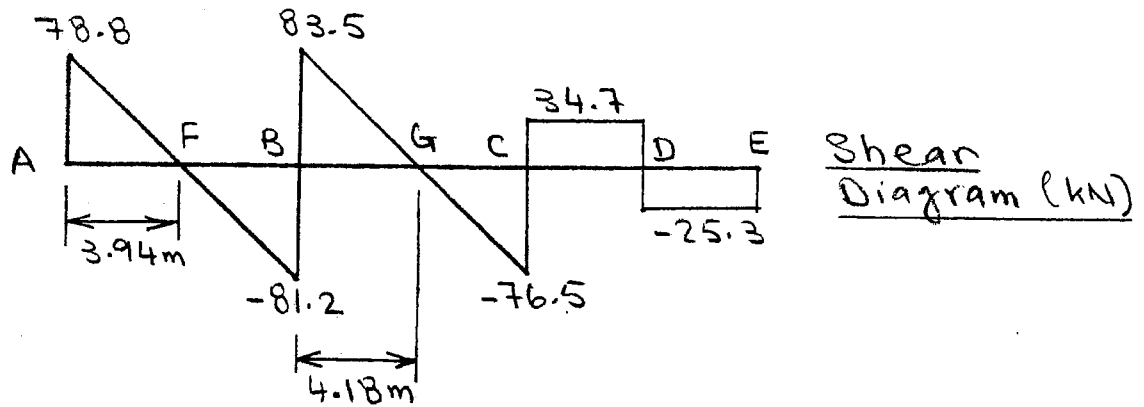
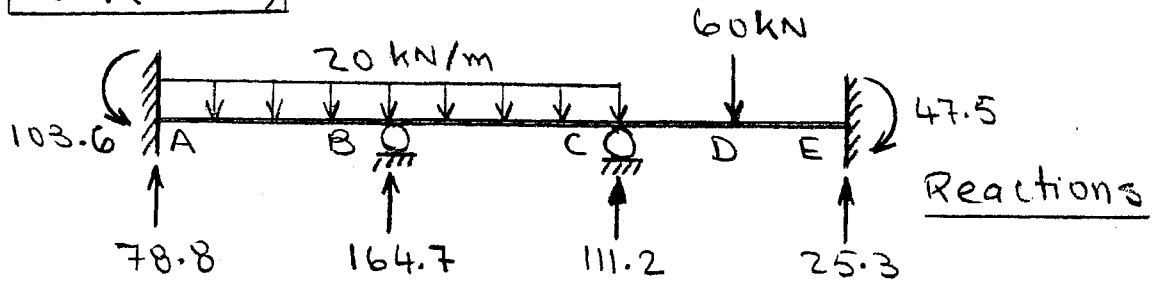
$$M_{AB} = 103.6 \text{ kN.m}; M_{BA} = -112.9 \text{ kN.m}; M_{BC} = 112.9 \text{ kN.m}$$

$$M_{CB} = -84.9 \text{ kN.m}; M_{CE} = 84.9 \text{ kN.m}; M_{EC} = -47.5 \text{ kN.m}$$



Member End Moments and Shears

16.9 (contd.)



16.10 Fixed-end moments:

$$FEM_{AB} = \frac{2(15)^2}{12} = 37.5 \text{ k-ft}; \quad FEM_{BA} = -37.5 \text{ k-ft}$$

$$FEM_{BC} = \frac{3(10)^2}{30} = 10 \text{ k-ft}; \quad FEM_{CB} = -\frac{3(10)^2}{20} = -15 \text{ k-ft}$$

$$FEM_{CD} = 15 \text{ k-ft}; \quad FEM_{DC} = -10 \text{ k-ft}$$

Slope-deflection equations:

$$M_{AB} = 0.133EI\theta_B + 37.5; \quad M_{BA} = 0.267EI\theta_B - 37.5$$

$$M_{BC} = 0.4EI\theta_B + 0.2EI\theta_C + 10$$

$$M_{CB} = 0.2EI\theta_B + 0.4EI\theta_C - 15$$

$$M_{CD} = 0.4EI\theta_C + 15; \quad M_{DC} = 0.2EI\theta_C - 10$$

Equilibrium equations: $M_{BA} + M_{BC} = 0$

$$M_{CB} + M_{CD} = 0$$

$$0.667EI\theta_B + 0.2EI\theta_C = 27.5$$

$$0.2EI\theta_B + 0.8EI\theta_C = 0$$

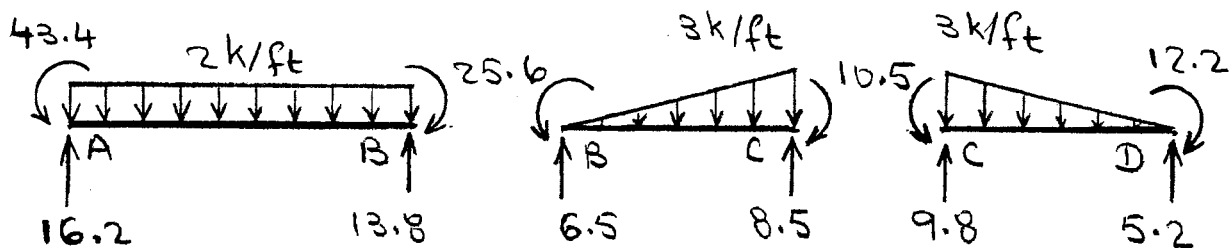
By solving these equations, we obtain

$$EI\theta_B = 44.59 \text{ k-ft}^2; \quad EI\theta_C = -11.15 \text{ k-ft}^2$$

Member end moments: Substituting the numerical values of $EI\theta_B$ and $EI\theta_C$ into the slope-deflection equations, we obtain

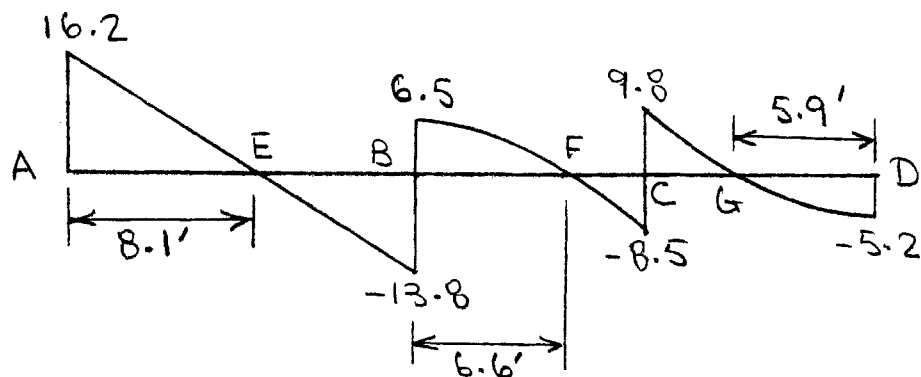
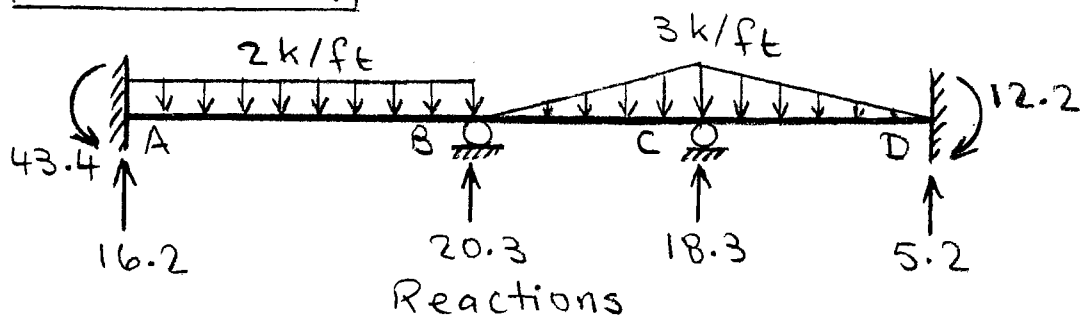
$$M_{AB} = 43.4 \text{ k-ft}; \quad M_{BA} = -25.6 \text{ k-ft}; \quad M_{BC} = 25.6 \text{ k-ft}$$

$$M_{CB} = -10.5 \text{ k-ft}; \quad M_{CD} = 10.5 \text{ k-ft}; \quad M_{DC} = -12.2 \text{ k-ft}$$

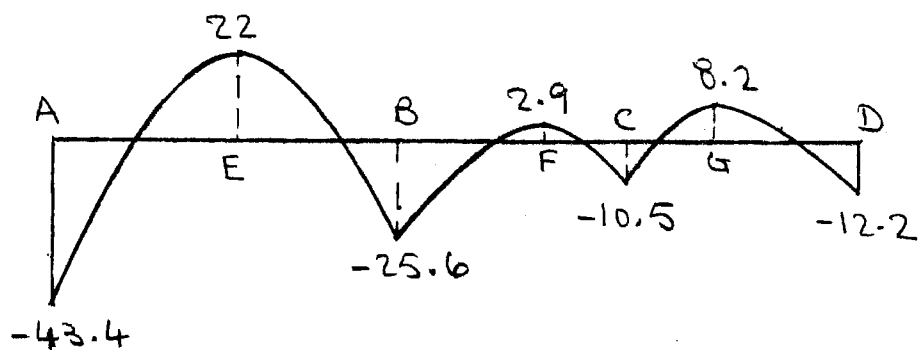


Member End Moments and Shears

16.10 (contd.)

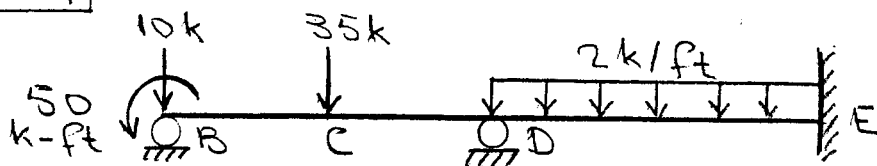


Shear Diagram (k)



Bending Moment Diagram (k-ft)

16.11



Fixed-end moments:

$$FEM_{BD} = \frac{35(20)}{8} = 87.5 \text{ k-ft}; \quad FEM_{DB} = -87.5 \text{ k-ft}$$

$$FEM_{DE} = \frac{2(20)^2}{12} = 66.67 \text{ k-ft}; \quad FEM_{ED} = -66.67 \text{ k-ft}$$

Slope-deflection equations:

$$M_{BD} = 0.2EI\theta_B + 0.1EI\theta_D + 87.5$$

$$M_{DB} = 0.1EI\theta_B + 0.2EI\theta_D - 87.5$$

$$M_{DE} = 0.2EI\theta_D + 66.67; \quad M_{ED} = 0.1EI\theta_D - 66.67$$

Equilibrium equations: $M_{BD} - 50 = 0$

$$M_{DB} + M_{DE} = 0$$

$$0.2EI\theta_B + 0.1EI\theta_D = -37.5$$

$$0.1EI\theta_B + 0.4EI\theta_D = 20.83$$

By solving these equations, we obtain:

$$EI\theta_B = -244.05 \text{ k-ft}^2; \quad EI\theta_D = 113.1 \text{ k-ft}^2$$

Member end moments: Substituting the numerical values of $EI\theta_B$ and $EI\theta_D$ into the slope-deflection equations, we obtain:

$$\underline{M_{BD} = 50 \text{ k-ft}; \quad M_{DB} = -89.3 \text{ k-ft}}$$

$$\underline{M_{DE} = 89.3 \text{ k-ft}; \quad M_{ED} = -55.4 \text{ k-ft}}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.38.

16.12 Fixed-end moments:

$$FEM_{AC} = FEM_{CE} = \frac{120(6)(4)^2}{(10)^2} = 115.2 \text{ kN}\cdot\text{m};$$

$$FEM_{CA} = FEM_{EC} = -\frac{120(4)(6)^2}{(10)^2} = -172.8 \text{ kN}\cdot\text{m}$$

$$FEM_{EG} = \frac{150(8)}{8} = 150 \text{ kN}\cdot\text{m}; \quad FEM_{GE} = -150 \text{ kN}\cdot\text{m}$$

Slope-deflection equations: $M_{AC} = M_{GE} = 0$

$$M_{CA} = \frac{3EI}{10} \theta_C - 172.8 - \frac{115.2}{2} = 0.3EI\theta_C - 230.4$$

$$M_{CE} = \frac{2E(2I)}{10} (2\theta_C + \theta_E) + 115.2 = 0.4EI(2\theta_C + \theta_E) + 115.2$$

$$M_{EC} = 0.4EI(\theta_C + 2\theta_E) - 172.8$$

$$M_{EG} = \frac{3EI}{8} \theta_E + 150 + \frac{150}{2} = 0.375EI\theta_E + 225$$

Equilibrium equations: $M_{CA} + M_{CE} = 0$

$$M_{EC} + M_{EG} = 0$$

$$1.1EI\theta_C + 0.4EI\theta_E = 115.2$$

$$0.4EI\theta_C + 1.175EI\theta_E = -52.2$$

By solving these equations, we obtain

$$EI\theta_C = 137.96 \text{ kN}\cdot\text{m}^2; \quad EI\theta_E = -91.39 \text{ kN}\cdot\text{m}^2$$

Member end moments. Substituting the numerical values of

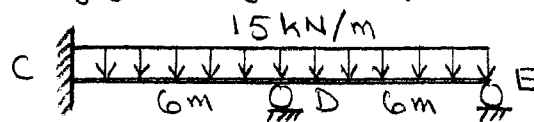
$EI\theta_C$ and $EI\theta_E$ into the slope-deflection equations, we obtain

$$\underline{M_{CA} = -189 \text{ kN}\cdot\text{m}}; \quad \underline{M_{CE} = 189 \text{ kN}\cdot\text{m}}$$

$$\underline{M_{EC} = -190.7 \text{ kN}\cdot\text{m}}; \quad \underline{M_{EG} = 190.7 \text{ kN}\cdot\text{m}}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.39.

16.13 As the beam and the loading are symmetric, we will analyze only the right half CE of the beam.



Fixed-end moments: $FEM_{CD} = 45 \text{ kN.m}$; $FEM_{DC} = -45 \text{ kN.m}$

$FEM_{DE} = 45 \text{ kN.m}$; $FEM_{ED} = -45 \text{ kN.m}$

Slope-deflection equations: $M_{ED} = 0$

$M_{CD} = 0.333 EI \theta_D + 45$; $M_{DC} = 0.667 EI \theta_D - 45$

$M_{DE} = \frac{3EI}{6} (\theta_D) + 45 + \frac{45}{2} = 0.5 EI \theta_D + 67.5$

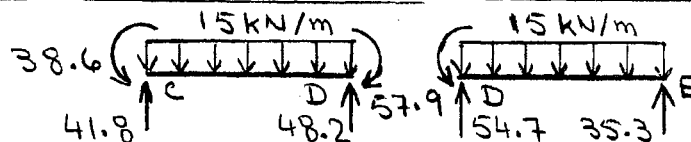
Equilibrium equation: $M_{DC} + M_{DE} = 0$

$1.167 EI \theta_D + 22.5 = 0$; $EI \theta_D = -19.28 \text{ kN.m}^2$

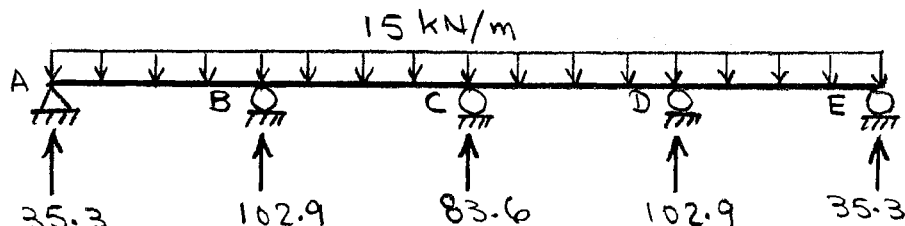
Member end moments: Substituting the numerical value of $EI \theta_D$ into the slope-deflection equations, we obtain

$M_{CD} = 38.6 \text{ kN.m}$; $M_{DC} = -57.9 \text{ kN.m}$; $M_{DE} = 57.9 \text{ kN.m}$

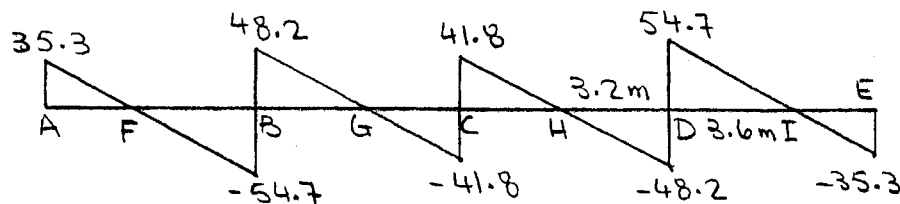
Member End Moments and Shears



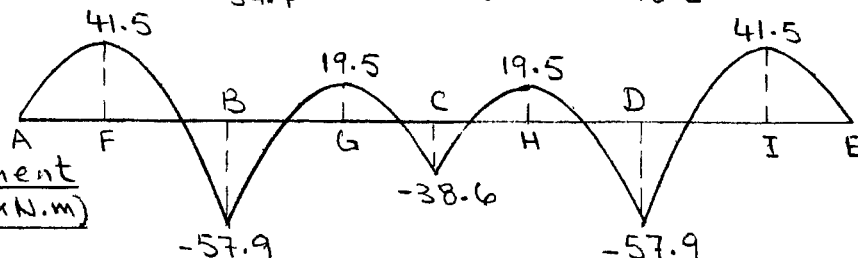
Reactions



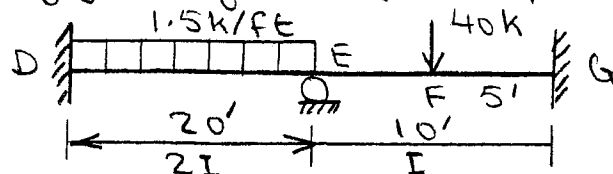
Shear Diagram (kN)



Bending Moment Diagram (kN.m)



16.14 As the beam and loading are symmetric, we will analyze only the right half DG of the beam.



Fixed end moments: $FEM_{DE} = 50 \text{ k-ft}$; $FEM_{ED} = -50 \text{ k-ft}$

$FEM_{EG} = 100 \text{ k-ft}$; $FEM_{GE} = -100 \text{ k-ft}$

Slope deflection equations:

$$M_{DE} = 0.2EI\theta_E + 50; \quad M_{ED} = 0.4EI\theta_E - 50$$

$$M_{EG} = 0.2EI\theta_E + 100; \quad M_{GE} = 0.1EI\theta_E - 100$$

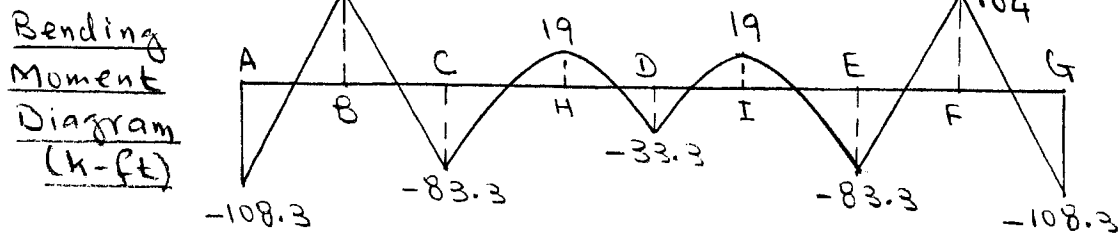
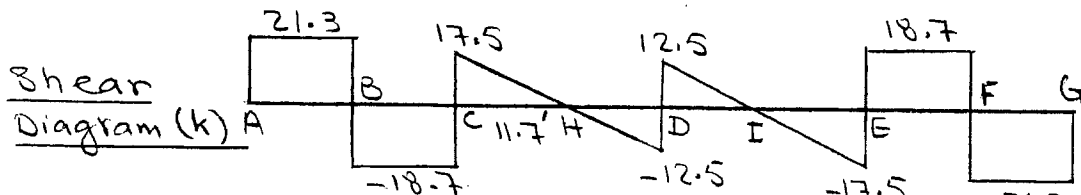
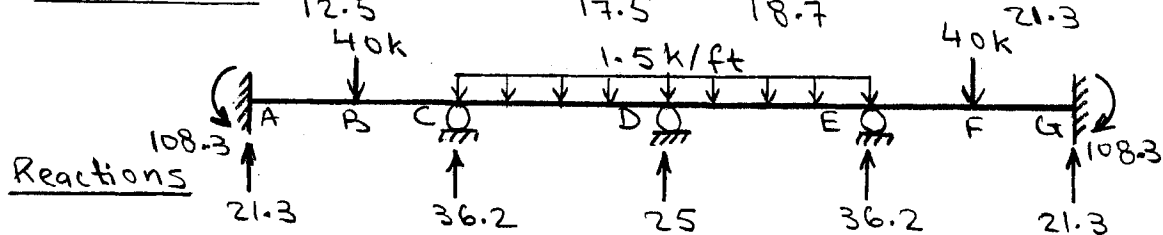
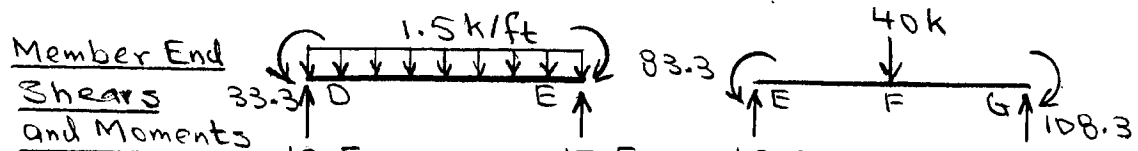
Equilibrium equation: $M_{ED} + M_{EG} = 0$

$$0.6EI\theta_E + 50 = 0 \quad EI\theta_E = -83.33 \text{ k-ft}^2$$

Member end moments: Substituting the numerical value of $EI\theta_E$ into the slope-deflection equations, we obtain

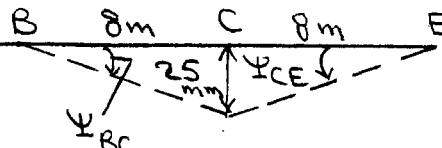
$$M_{DE} = 33.3 \text{ k-ft}; \quad M_{ED} = -83.3 \text{ k-ft}$$

$$M_{EG} = 83.3 \text{ k-ft}; \quad M_{GE} = -108.3 \text{ k-ft}$$



16.15 Fixed end moments:

$$\begin{aligned} FEM_{AB} &= 106.7 \text{ kN.m}; & FEM_{BA} &= -106.7 \text{ kN.m} \\ FEM_{BC} &= 106.7 \text{ kN.m}; & FEM_{CB} &= -106.7 \text{ kN.m} \\ FEM_{CE} &= 60 \text{ kN.m}; & FEM_{EC} &= -60 \text{ kN.m} \end{aligned}$$

Chord rotations: 

$$\psi_{BC} = -\frac{0.025}{8} = -0.003125$$

$$\psi_{CE} = +\frac{0.025}{8} = +0.003125$$

Slope-deflection equations:

$$\begin{aligned} M_{AB} &= 0.25EI\theta_B + 106.7; & M_{BA} &= 0.5EI\theta_B - 106.7 \\ M_{BC} &= 0.5EI\theta_B + 0.25EI\theta_C + 238 \\ M_{CB} &= 0.25EI\theta_B + 0.5EI\theta_C + 24.6 \\ M_{CE} &= 0.5EI\theta_C - 71.3; & M_{EC} &= 0.25EI\theta_C - 191.3 \end{aligned}$$

Equilibrium equations:

$$\begin{aligned} M_{BA} + M_{BC} &= 0 \\ M_{CB} + M_{CE} &= 0 \end{aligned}$$

$$EI\theta_B + 0.25EI\theta_C = -131.3$$

$$0.25EI\theta_B + EI\theta_C = 46.7$$

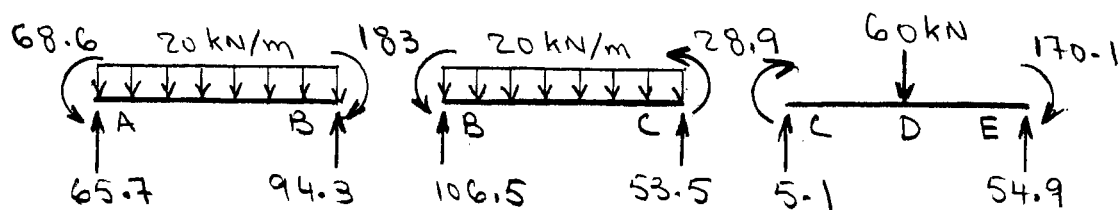
By solving these equations, we obtain

$$EI\theta_B = -152.51 \text{ kN.m}^2; \quad EI\theta_C = 84.83 \text{ kN.m}^2$$

Member end moments: Substituting the numerical values of $EI\theta_B$ and $EI\theta_C$ into the slope-deflection equations, we obtain

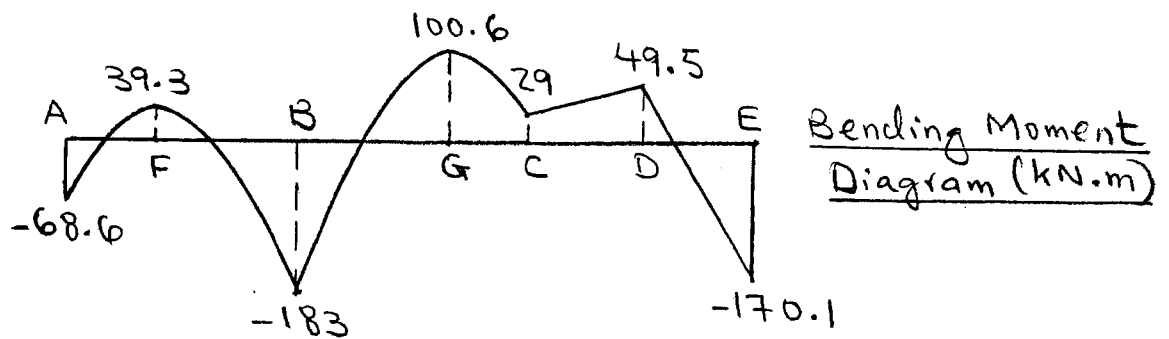
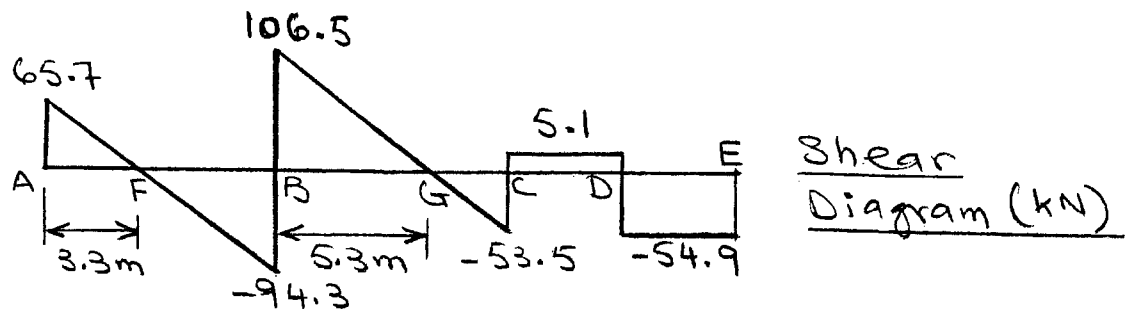
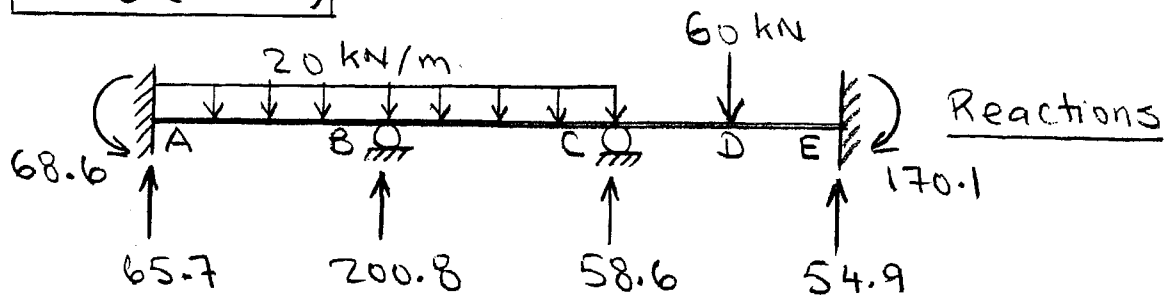
$$M_{AB} = 68.6 \text{ kN.m}; \quad M_{BA} = -183 \text{ kN.m}; \quad M_{BC} = 183 \text{ kN.m}$$

$$M_{CB} = 28.9 \text{ kN.m}; \quad M_{CE} = -28.9 \text{ kN.m}; \quad M_{EC} = -170.1 \text{ kN.m}$$



Member End Moments and Shears

16.15 (contd.)

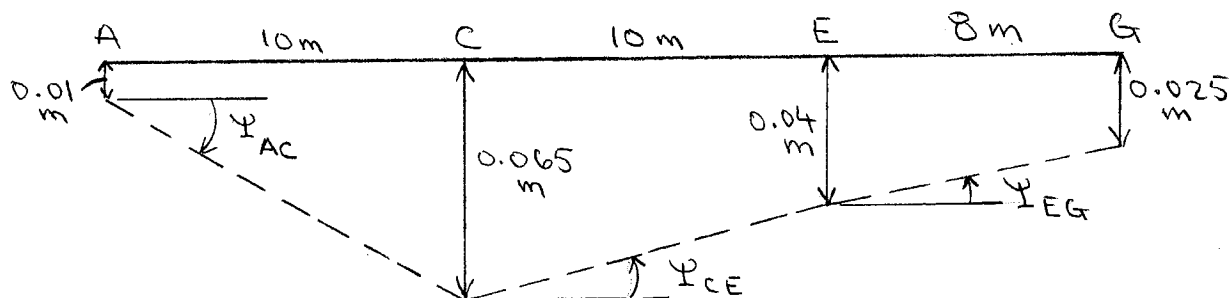


16.16 Fixed-end moments: $FEM_{AC} = FEM_{CE} = 115.2 \text{ kN}\cdot\text{m}$

$$FEM_{CA} = FEM_{EC} = -172.8 \text{ kN}\cdot\text{m}; FEM_{EG} = 150 \text{ kN}\cdot\text{m}$$

$$FEM_{GE} = -150 \text{ kN}\cdot\text{m}$$

Chord rotations:



$$\Psi_{AC} = -\frac{0.055}{10} = -0.0055; \quad \Psi_{CE} = \frac{0.025}{10} = 0.0025$$

$$\Psi_{EG} = \frac{0.015}{8} = 0.001875$$

Slope-deflection equations: $M_{AC} = M_{GE} = 0$

Using $EI = 200(500) = 100000 \text{ kN}\cdot\text{m}^2$, we write:

$$M_{CA} = 0.3EI\theta_C + 165 - 230.4 = 0.3EI\theta_C - 65.4$$

$$M_{CE} = 0.4EI(2\theta_C + \theta_E) - 300 + 115.2 = 0.4EI(2\theta_C + \theta_E) - 184.8$$

$$M_{EC} = 0.4EI(\theta_C + 2\theta_E) - 300 - 172.8 = 0.4EI(\theta_C + 2\theta_E) - 472.8$$

$$M_{EG} = 0.375EI\theta_E - 70.31 + 225 = 0.375EI\theta_E + 154.69$$

Equilibrium equations:

$$M_{CA} + M_{CE} = 0 \Rightarrow 1.1EI\theta_C + 0.4EI\theta_E = 250.2$$

$$M_{EC} + M_{EG} = 0 \Rightarrow 0.4EI\theta_C + 1.75EI\theta_E = 318.11$$

By solving these equations, we obtain

$$EI\theta_C = 147.23 \text{ kN}\cdot\text{m}^2; \quad EI\theta_E = 220.61 \text{ kN}\cdot\text{m}^2$$

Member end moments:

$$M_{CA} = -21.2 \text{ kN}\cdot\text{m}; \quad M_{CE} = 21.2 \text{ kN}\cdot\text{m}$$

$$M_{EC} = -237.4 \text{ kN}\cdot\text{m}; \quad M_{EG} = 237.4 \text{ kN}\cdot\text{m}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 13.54.

16.17 Fixed-end moments:

$$FEM_{AC} = \frac{75(6)}{8} = 56.25 \text{ kN.m}; FEM_{CA} = -56.25 \text{ kN.m}$$

$$FEM_{CD} = \frac{25(9)^2}{12} = 168.75 \text{ kN.m}; FEM_{DC} = -168.75 \text{ kN.m}$$

Slope-deflection equations: $M_{DC} = 0$

$$M_{AC} = 0.333 EI \theta_C + 56.25; FEM_{CA} = 0.667 EI \theta_C - 56.25$$

$$M_{CD} = \frac{3EI}{9} (\theta_C) + 168.75 + \frac{168.75}{2} = 0.333 EI \theta_C + 253.13$$

Equilibrium equation: $M_{CA} + M_{CD} = 0$

$$EI \theta_C = -196.88 \text{ kN.m}^2$$

Member end moments. Substituting the numerical value of $EI \theta_C$ into the slope-deflection equations, we obtain:

$$\underline{M_{AC} = -9.4 \text{ kN.m}; M_{CA} = -187.5 \text{ kN.m}}$$

$$\underline{M_{CD} = 187.5 \text{ kN.m}}$$

For reactions, see solution of Problem 13.42.

16-18 Fixed-end moments:

$$FEM_{AC} = FEM_{CA} = FEM_{CD} = FEM_{DC} = 0$$

$$FEM_{BC} = \frac{2(15)^2}{12} = 37.5 \text{ k-ft}; FEM_{CB} = -37.5 \text{ k-ft}$$

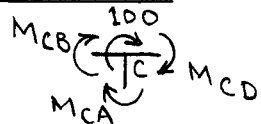
Slope-deflection equations:

$$M_{AC} = 0.133EI\theta_C; M_{CA} = 0.267EI\theta_C$$

$$M_{BC} = 0.133EI\theta_C + 37.5; M_{CB} = 0.267EI\theta_C - 37.5$$

$$M_{CD} = 0.2EI\theta_C; M_{DC} = 0$$

Equilibrium equation:



$$M_{CA} + M_{CB} + M_{CD} + 100 = 0$$

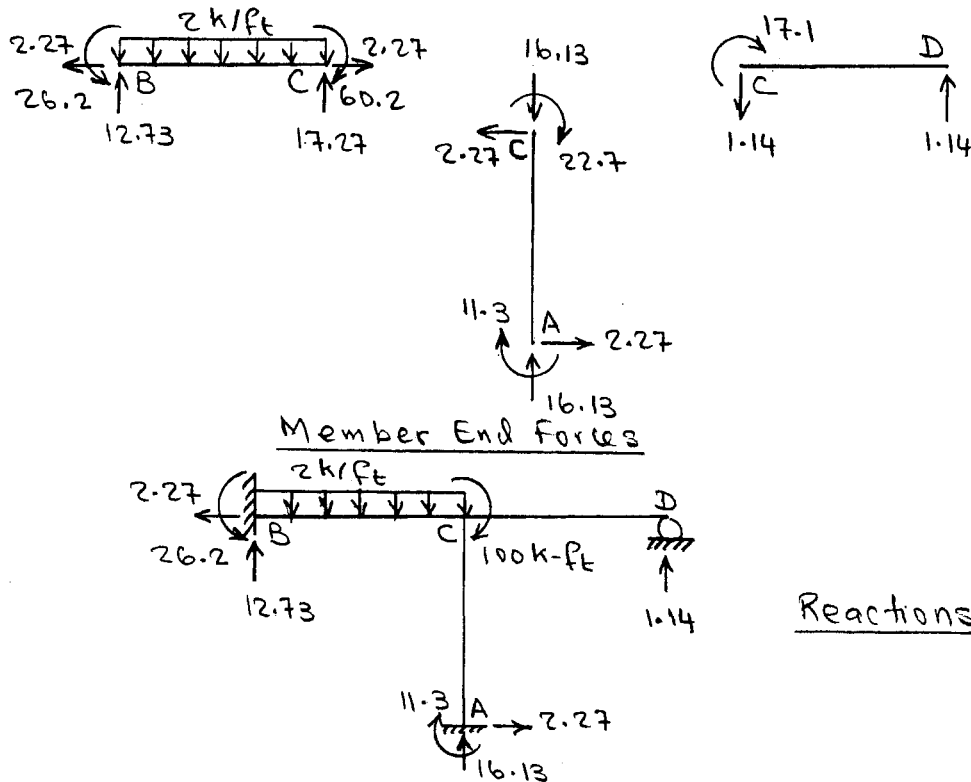
$$(0.267EI\theta_C) + (0.267EI\theta_C - 37.5) + (0.2EI\theta_C) + 100 = 0$$

$$0.734EI\theta_C = -62.5 \quad EI\theta_C = -85.15 \text{ k-ft}^2$$

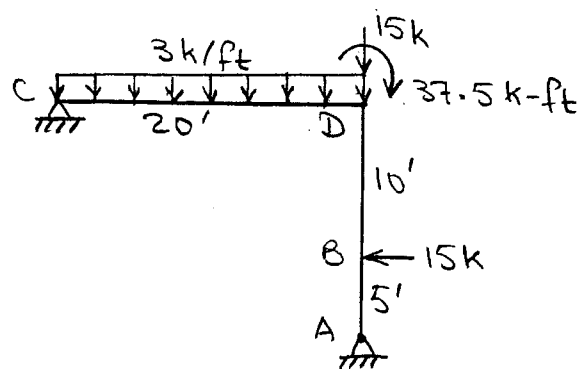
Member end moments: $M_{AC} = -11.3 \text{ k-ft};$

$M_{CA} = -22.7 \text{ k-ft}; M_{BC} = 26.2 \text{ k-ft}; M_{CB} = -60.2 \text{ k-ft};$

$M_{CD} = -17.1 \text{ k-ft}.$



16.19



Fixed-end moments:

$$FEM_{AD} = -\frac{15(5)(10)^2}{(15)^2} = -33.33 \text{ k-ft}$$

$$FEM_{DA} = \frac{15(10)(5)^2}{(15)^2} = 16.67 \text{ k-ft}$$

$$FEM_{CD} = \frac{3(20)^2}{12} = 100 \text{ k-ft}; \quad FEM_{DC} = -100 \text{ k-ft}$$

Slope-deflection equations: $M_{AD} = M_{CD} = 0$

$$M_{DA} = \frac{3EI}{15}(\theta_D) + 16.67 + \frac{33.33}{2} = 0.2EI\theta_D + 33.34$$

$$M_{DC} = \frac{3E(2I)}{20}(\theta_D) - 100 - \frac{100}{2} = 0.3EI\theta_D - 150$$

Equilibrium equation: $M_{DA} + M_{DC} + 37.5 = 0$

$$(0.2EI\theta_D + 33.34) + (0.3EI\theta_D - 150) + 37.5 = 0$$

$$0.5EI\theta_D = 79.16; \quad EI\theta_D = 158.32 \text{ k-ft}^2$$

Member end moments:

$$M_{DA} = 65 \text{ k-ft}; \quad M_{DC} = -102.5 \text{ k-ft}$$

For reactions, see solution of Problem 13.21.

16.20 Fixed-end moments: $FEM_{AC} = FEM_{CA} = 0$

$$FEM_{BD} = FEM_{DB} = 0$$

$$FEM_{CD} = \frac{30(10)^2}{12} = 250 \text{ kN.m}; FEM_{DC} = -250 \text{ kN.m}$$

Slope-deflection equations:

$$M_{AC} = 0.25 EI \theta_C; M_{CA} = 0.5 EI \theta_C$$

$$M_{BD} = 0.25 EI \theta_D; M_{DB} = 0.5 EI \theta_D$$

$$M_{CD} = 0.4 EI \theta_C + 0.2 EI \theta_D + 250$$

$$M_{DC} = 0.2 EI \theta_C + 0.4 EI \theta_D - 250$$

Equilibrium equations: $M_{CA} + M_{CD} = 0$

$$M_{DB} + M_{DC} = 0$$

$$0.9 EI \theta_C + 0.2 EI \theta_D = -250$$

$$0.2 EI \theta_C + 0.9 EI \theta_D = 250$$

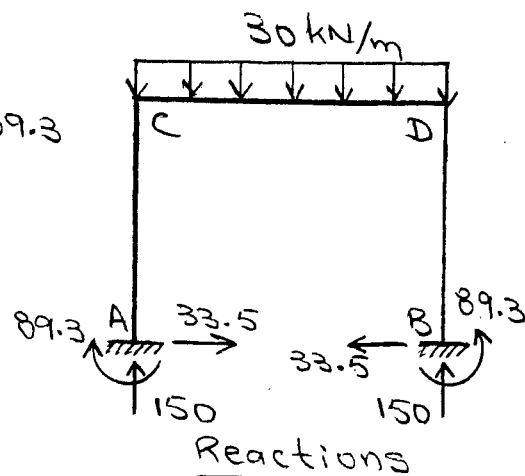
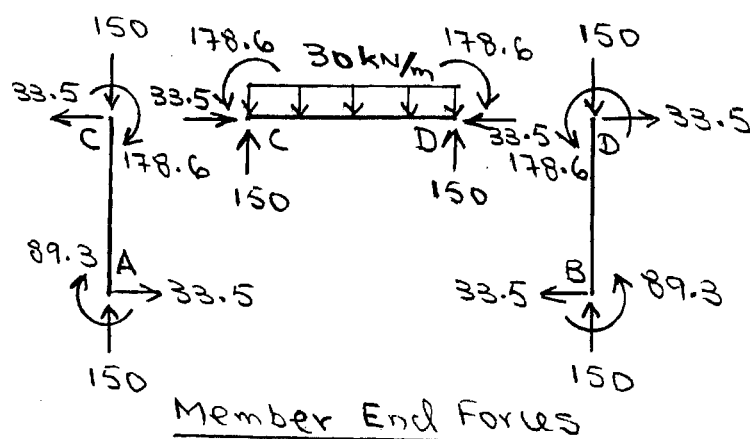
By solving these equations, we obtain:

$$EI \theta_C = -357.14 \text{ kN.m}^2; EI \theta_D = 357.14 \text{ kN.m}^2$$

Member end moments: $M_{AC} = -89.3 \text{ kN.m};$

$M_{CA} = -178.6 \text{ kN.m}; M_{BD} = 89.3 \text{ kN.m}; M_{DB} = 178.6 \text{ kN.m};$

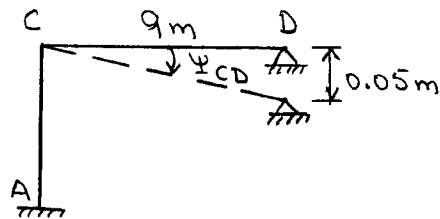
$M_{CD} = 178.6 \text{ kN.m}; M_{DC} = -178.6 \text{ kN.m}.$



16.21 Fixed-end moments:

$$FEM_{AC} = 56.25 \text{ kN.m}; \quad FEM_{CA} = -56.25 \text{ kN.m}$$

$$FEM_{CD} = 168.75 \text{ kN.m}; \quad FEM_{DC} = -168.75 \text{ kN.m}$$



Chord rotation:

$$\psi_{CD} = -\frac{0.05}{9} = -0.00556$$

Slope-deflection equations: $M_{DC} = 0$

$$M_{AC} = 0.333 EI \theta_C + 56.25; \quad M_{CA} = 0.667 EI \theta_C - 56.25$$

Using $EI = 200(400) \text{ kN-m}^2$, we write

$$M_{CD} = 0.333 EI \theta_C + 148.27 + 168.75 + \frac{168.75}{2}$$

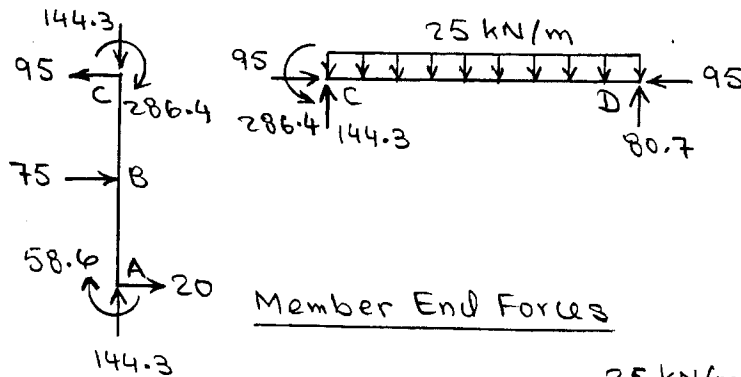
$$= 0.333 EI \theta_C + 401.4$$

Equilibrium equation: $M_{CA} + M_{CD} = 0$

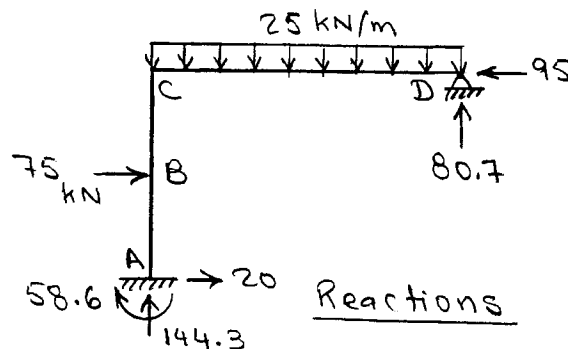
$$EI \theta_C = -345.15 \text{ kN-m}^2$$

Member end moments: $M_{AC} = -58.6 \text{ kN.m}$

$$M_{CA} = -286.4 \text{ kN.m}; \quad M_{CD} = 286.4 \text{ kN.m}$$



Member End Forces



Reactions

16.22 Fixed-end moments:

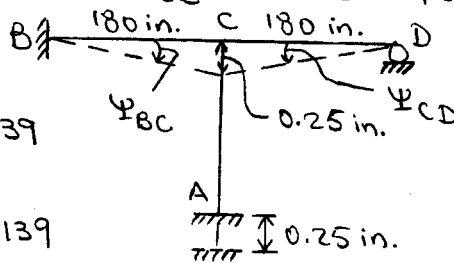
$$FEM_{AC} = FEM_{CA} = FEM_{CD} = FEM_{DC} = 0$$

$$FEM_{BC} = 37.5 \text{ k-ft}; \quad FEM_{CB} = -37.5 \text{ k-ft}.$$

Chord rotations:

$$\Psi_{BC} = -\frac{0.25}{180} = -0.00139$$

$$\Psi_{CD} = +\frac{0.25}{180} = +0.00139$$



Slope-deflection equations. $M_{DC} = 0$

Using $EI = \frac{29000(3500)}{(12)^2} \text{ k-ft}^2$, we write

$$M_{AC} = 0.133 EI \theta_C; \quad M_{CA} = 0.267 EI \theta_C$$

$$M_{BC} = 0.133 EI \theta_C + 391.9 + 37.5 = 0.133 EI \theta_C + 429.4$$

$$M_{CB} = 0.267 EI \theta_C + 391.9 - 37.5 = 0.267 EI \theta_C + 354.4$$

$$M_{CD} = 0.2 EI \theta_C - 196$$

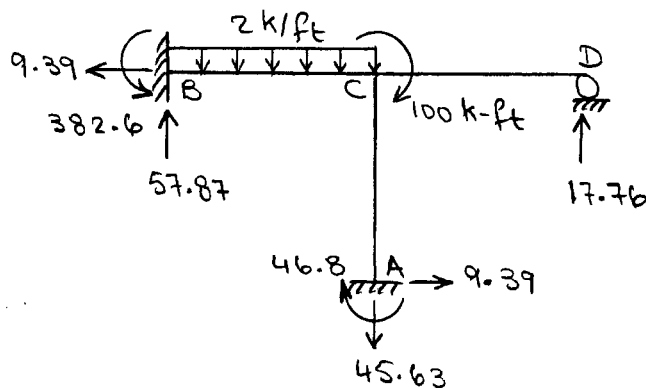
Equilibrium equation: $M_{CA} + M_{CB} + M_{CD} + 100 = 0$

$$0.734 EI \theta_C = -258.4 \quad EI \theta_C = -352 \text{ k-ft}^2$$

Member end moments: $M_{AC} = -46.8 \text{ k-ft};$

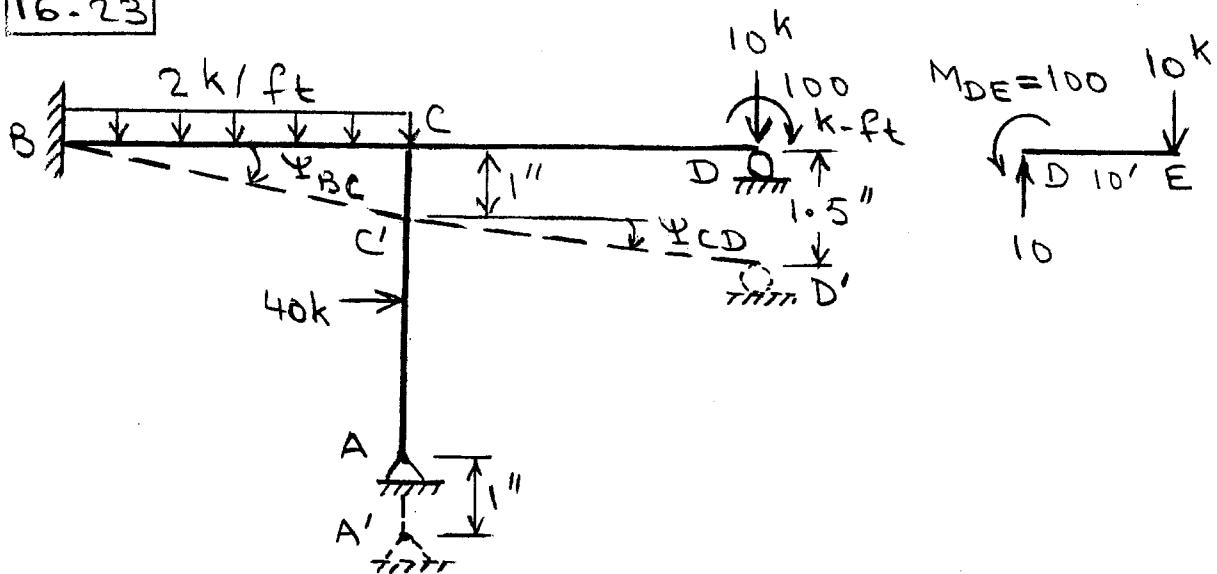
$$M_{CA} = -94 \text{ k-ft}; \quad M_{BC} = 382.6 \text{ k-ft};$$

$$M_{CB} = 260.4 \text{ k-ft}; \quad M_{CD} = -266.4 \text{ k-ft}.$$



Reactions

16.23



Fixed-end moments: $FEM_{CD} = FEM_{DC} = 0$

$FEM_{BC} = 150 \text{ k-ft}$; $FEM_{CB} = -150 \text{ k-ft}$

$FEM_{AC} = 100 \text{ k-ft}$; $FEM_{CA} = -100 \text{ k-ft}$

Chord rotations:

$$\psi_{BC} = -\frac{1}{30(12)} = -0.00728; \quad \psi_{CD} = -\frac{0.5}{30(12)} = -0.00139$$

Slope-deflection equations: $M_{AC} = 0$; $M_{DE} = 100 \text{ k-ft}$

$$M_{CA} = 0.15EI\theta_C - 150; \quad M_{BC} = 0.0667EI\theta_C + 265.7$$

$$M_{CB} = 0.133EI\theta_C - 34.3$$

$$M_{CD} = 0.133EI\theta_C + 0.0667EI\theta_D + 57.9$$

$$M_{DC} = 0.0667EI\theta_C + 0.133EI\theta_D + 57.9$$

Equilibrium equations: $M_{CA} + M_{CB} + M_{CD} = 0$

$$M_{DC} + 100 = 0$$

$$0.417EI\theta_C + 0.0667EI\theta_D = 126.4$$

$$0.0667EI\theta_C + 0.133EI\theta_D = -157.9$$

By solving these equations, we obtain:

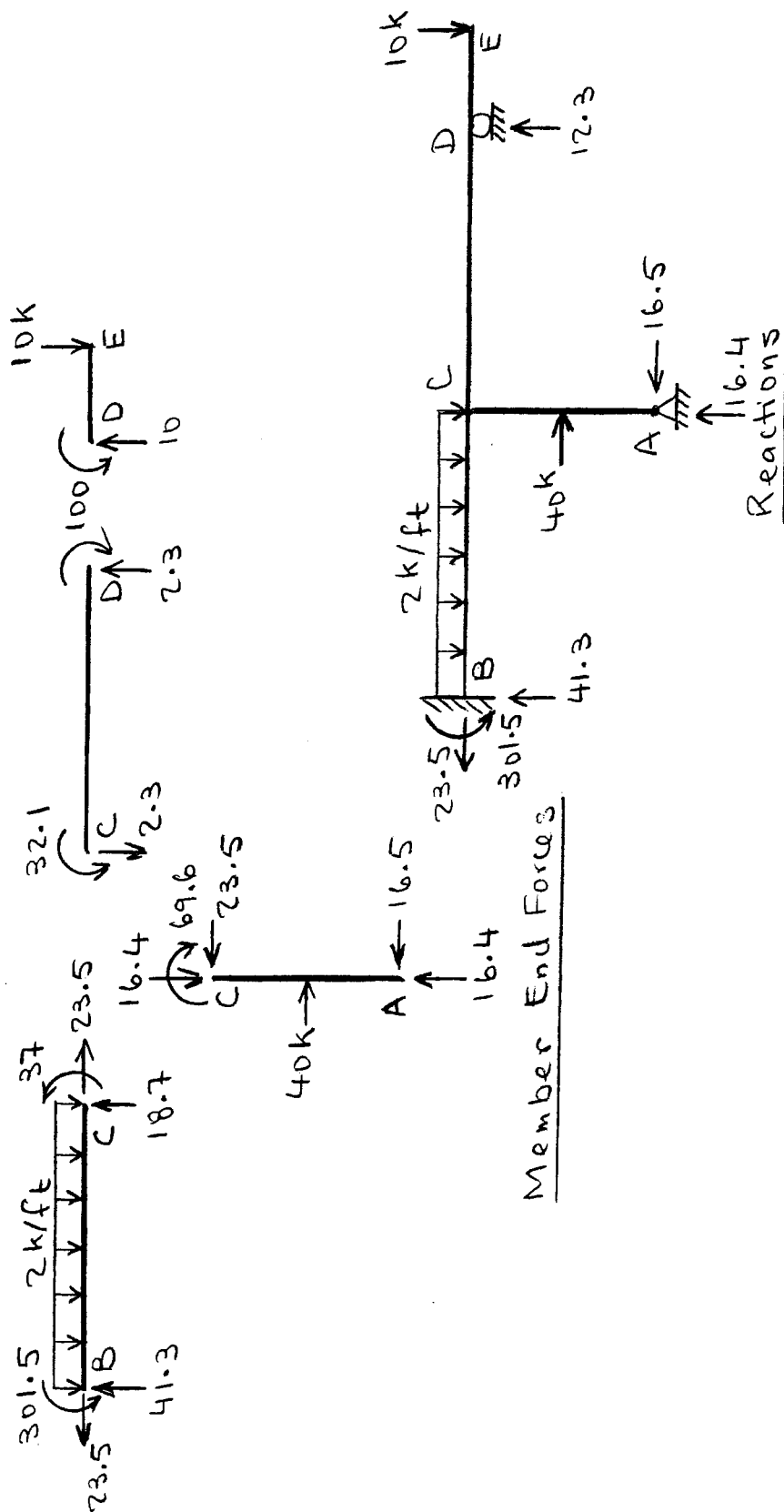
$$EI\theta_C = 536 \text{ k-ft}^2; \quad EI\theta_D = -1456 \text{ k-ft}^2$$

16.23 (contd.)

Member end moments: Substituting the numerical values of $EI\theta_C$ and $EI\theta_D$ into the slope-deflection equations, we obtain

$$M_{CA} = -69.6 \text{ k-ft}; \quad M_{BC} = 301.5 \text{ k-ft}; \quad M_{CB} = 37 \text{ k-ft}; \quad M_{CD} = 32.1 \text{ k-ft}$$

$$M_{DC} = -100 \text{ k-ft.}$$



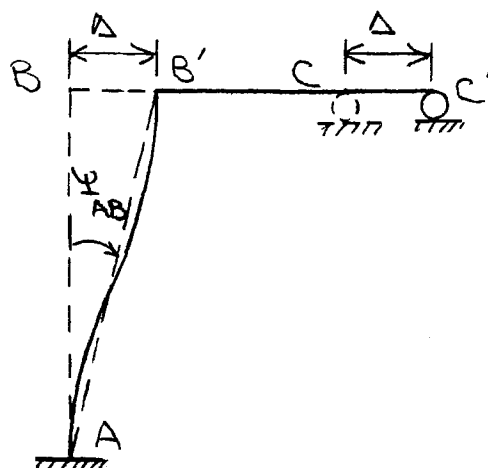
16.24 Fixed-end moments: $FEM_{AB} = FEM_{BA} = 0$

$$FEM_{BC} = \frac{2(15)^2}{12} = 37.5 \text{ k-ft}; \quad FEM_{CB} = -37.5 \text{ k-ft}$$

Chord rotations:

$$\Psi_{AB} = -\frac{\Delta}{20}$$

$$\Psi_{BC} = 0$$



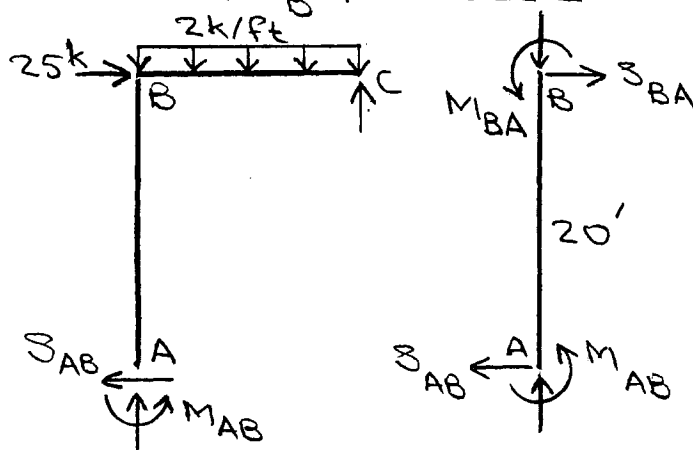
Slope-deflection equations:

$$M_{AB} = 0.1EI\theta_B + 0.015EI\Delta; \quad M_{BA} = 0.2EI\theta_B + 0.015EI\Delta$$

$$M_{BC} = 0.2EI\theta_B + 56.25; \quad \underline{M_{CB} = 0}$$

Equilibrium equations: $M_{BA} + M_{BC} = 0$

$$0.4EI\theta_B + 0.015EI\Delta = -56.25 \quad (1)$$



$$\sum F_x = 0 \quad S_{AB} = 25 \quad \frac{M_{AB} + M_{BA}}{20} = 25$$

$$0.3EI\theta_B + 0.03EI\Delta = 500 \quad (2)$$

By solving equations (1) and (2) simultaneously, we obtain:

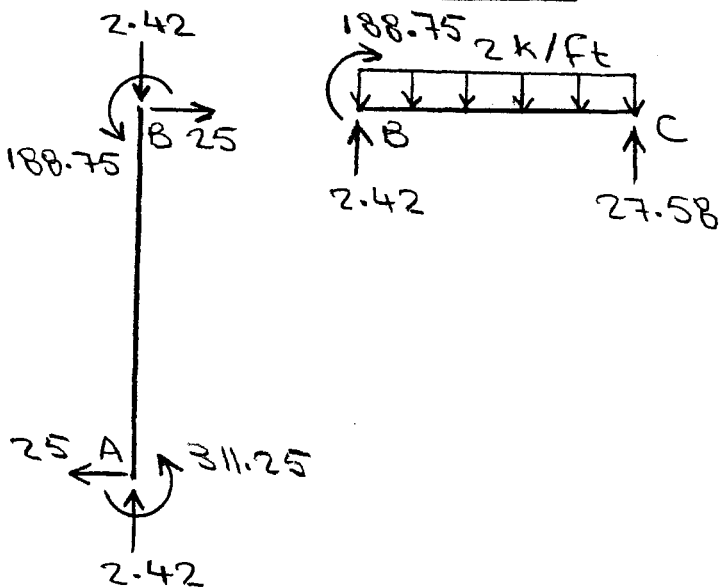
$$EI\theta_B = -1225 \text{ k-ft}^2; \quad EI\Delta = 28916.67 \text{ k-ft}^3$$

16.24 (contd.)

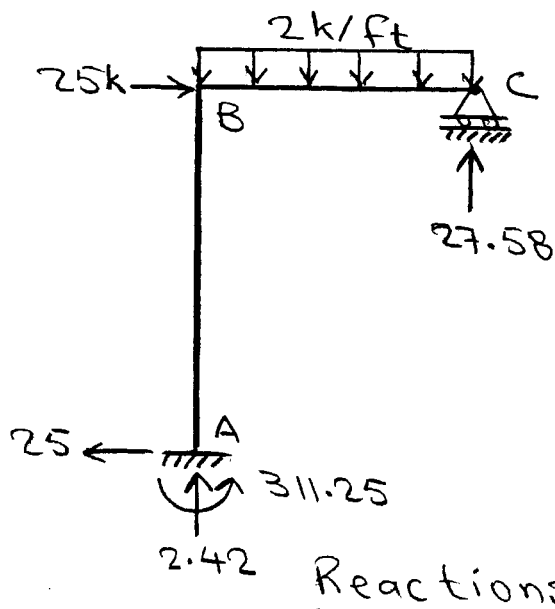
Member end moments: Substituting the numerical values of $EI\theta_B$ and $EI\Delta$ into the slope-deflection equations, we obtain:

$$M_{AB} = 311.25 \text{ k-ft}; \quad M_{BA} = 188.75 \text{ k-ft};$$

$$M_{BC} = -188.75 \text{ k-ft}.$$



Member End Forces



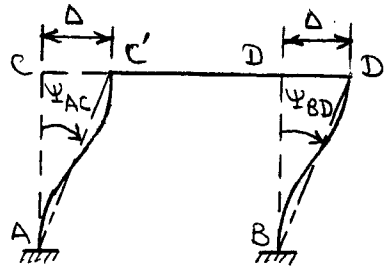
Reactions

16.25 Fixed-end moments. The non-zero fixed-end moments are: $FEM_{CD} = \frac{1.5(40)^2}{12} = 200 \text{ k-ft}$; and $FEM_{DC} = -200 \text{ k-ft}$.

Chord rotations:

$$\Psi_{AC} = \Psi_{BD} = -\frac{\Delta}{30}$$

$$\Psi_{CD} = 0$$



Slope-deflection equations:

$$M_{AC} = 0.0667EI(\theta_C + 0.1\Delta); \quad M_{CA} = 0.0667EI(2\theta_C + 0.1\Delta)$$

$$M_{BD} = 0.0667EI(\theta_D + 0.1\Delta); \quad M_{DB} = 0.0667EI(2\theta_D + 0.1\Delta)$$

$$M_{CD} = 0.1EI(2\theta_C + \theta_D) + 200$$

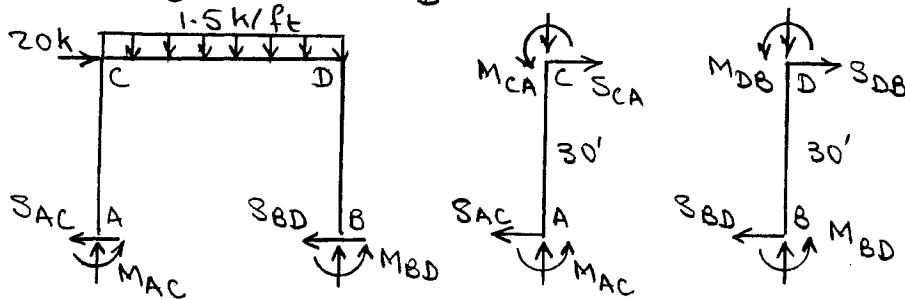
$$M_{DC} = 0.1EI(\theta_C + 2\theta_D) - 200$$

Equilibrium equations: $M_{CA} + M_{CD} = 0$

$$0.333EI\theta_C + 0.1EI\theta_D + 0.00667EI\Delta = -200 \quad (1)$$

$$M_{DC} + M_{DB} = 0$$

$$0.1EI\theta_C + 0.333EI\theta_D + 0.00667EI\Delta = 200 \quad (2)$$



$$\sum F_x = 0 \quad S_{AC} + S_{BD} = 20$$

$$\frac{M_{AC} + M_{CA}}{30} + \frac{M_{BD} + M_{DB}}{30} = 20$$

$$0.00667EI\theta_C + 0.00667EI\theta_D + 0.000889EI\Delta = 20 \quad (3)$$

Solving Eqs. (1) thru (3) simultaneously, we obtain:

$$EI\theta_C = -1308 \text{ k-ft}^2; \quad EI\theta_D = 408 \text{ k-ft}^2; \quad EI\Delta = 29242 \text{ k-ft}^3$$

Member end moments: $M_{AC} = 107.8 \text{ k-ft}$; $M_{CA} = 20.8 \text{ k-ft}$

$M_{BD} = 222.3 \text{ k-ft}$; $M_{DB} = 249.2 \text{ k-ft}$; $M_{CD} = -20.8 \text{ k-ft}$

$M_{DC} = -249.2 \text{ k-ft}$. For reactions, see solution of Problem 13.45.

16.26 Fixed-end moments: $FEM_{AC} = FEM_{CA} = 0$

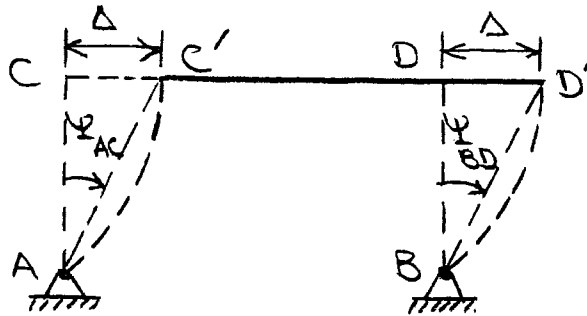
$$FEM_{BD} = FEM_{DB} = 0; FEM_{ED} = \frac{3(30)^2}{12} = 225 \text{ k-ft};$$

$$FEM_{DC} = -225 \text{ k-ft}.$$

Chord rotations:

$$\psi_{AC} = \psi_{BD} = -\frac{\Delta}{15}$$

$$\psi_{CD} = 0$$



Slope-deflection equations: $M_{AC} = M_{BD} = 0$

$$M_{CA} = 0.2EI\theta_C + 0.0133EI\Delta$$

$$M_{DB} = 0.2EI\theta_D + 0.0133EI\Delta$$

$$M_{CD} = 0.133EI\theta_C + 0.0667EI\theta_D + 225$$

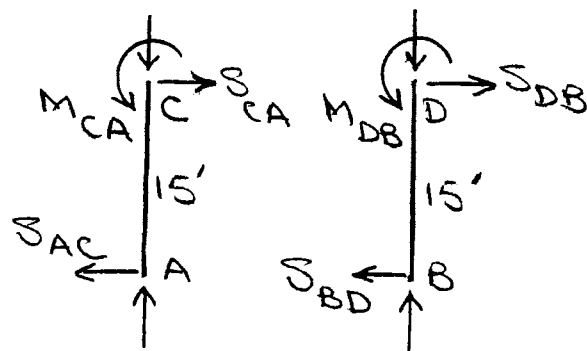
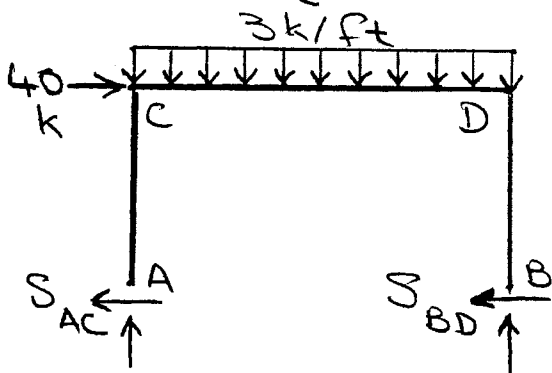
$$M_{DC} = 0.0667EI\theta_C + 0.133EI\theta_D - 225$$

Equilibrium equations: $M_{CA} + M_{ED} = 0$

$$0.333EI\theta_C + 0.0667EI\theta_D + 0.0133EI\Delta = -225 \quad (1)$$

$$M_{DB} + M_{DC} = 0$$

$$0.0667EI\theta_C + 0.333EI\theta_D + 0.0133EI\Delta = 225 \quad (2)$$



$$\sum F_x = 0 \quad S_{AC} + S_{BD} = 40 \quad \frac{M_{CA}}{15} + \frac{M_{DB}}{15} = 40$$

$$0.0133EI\theta_C + 0.0133EI\theta_D + 0.00178\Delta = 40 \quad (3)$$

16.26 (Contd.)

By solving Eqs. (1) thru (3) simultaneously, we obtain:

$$EI\theta_C = -2343.75 \text{ k-ft}^2; \quad EI\theta_D = -656.25 \text{ k-ft}^2;$$

$$EI\Delta = 45000 \text{ k-ft}^3.$$

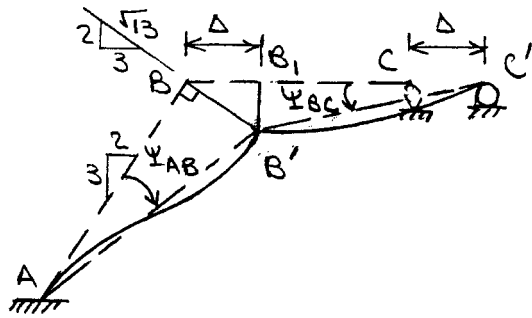
Member end moments:

$$\underline{M_{CA} = 131.25 \text{ k-ft};} \quad \underline{M_{DB} = 468.75 \text{ k-ft};}$$

$$\underline{M_{CD} = -131.25 \text{ k-ft};} \quad \underline{M_{DC} = -468.75 \text{ k-ft}.}$$

For reactions, see solution of Problem 13.22.

15.27 Chord rotations:



$$\psi_{AB} = -\frac{BB'}{L_{AB}} = -\frac{(\sqrt{13}/3)\Delta}{14.42} = -0.0833\Delta$$

$$\psi_{BC} = \frac{B_1B'}{L_{BC}} = \frac{(2/3)\Delta}{12} = 0.0556\Delta$$

Slope-deflection equations: $M_{CB} = 0$

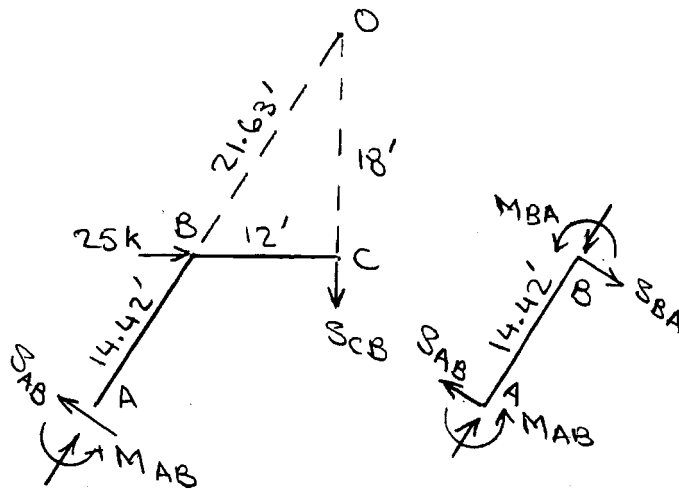
$$M_{AB} = 0.139EI\theta_B + 0.0347EI\Delta$$

$$M_{BA} = 0.277EI\theta_B + 0.0347EI\Delta$$

$$M_{BC} = 0.25EI\theta_B - 0.0139EI\Delta$$

Equilibrium equations: $M_{BA} + M_{BC} = 0$

$$0.527EI\theta_B + 0.0208EI\Delta = 0 \quad (1)$$



$$+\circlearrowleft \sum M_O = 0 \quad M_{AB} - S_{AB}(36.05) + 25(18) = 0$$

$$M_{AB} - \left(\frac{M_{AB} + M_{BA}}{14.42} \right) (36.05) + 450 = 0$$

$$0.901EI\theta_B + 0.139EI\Delta = 450 \quad (2)$$

16.27 (contd.)

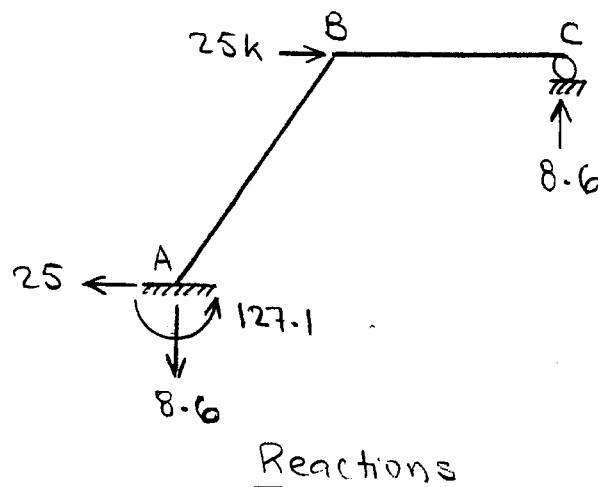
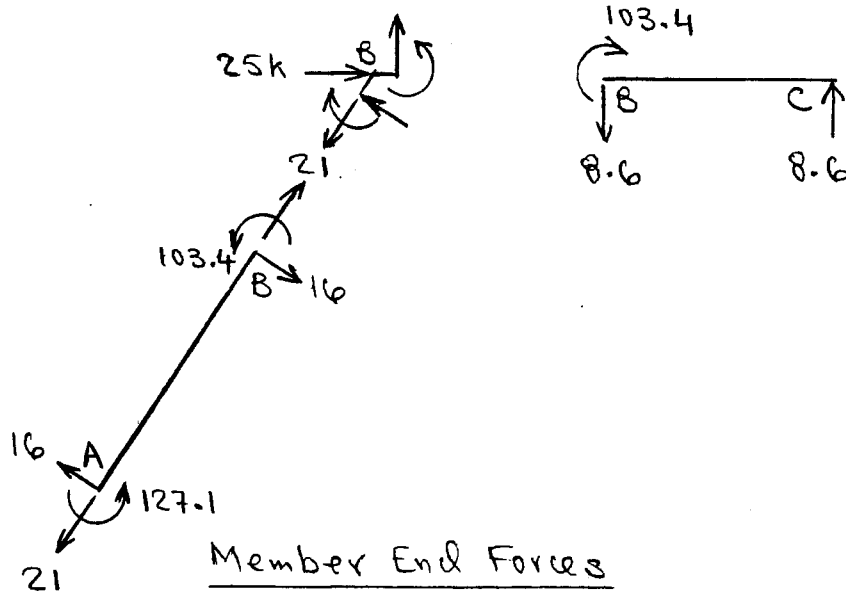
By solving Eqs. (1) and (2), we obtain:

$$EI\theta_B = -171.7 \text{ k-ft}^2; \quad EI\Delta = 4350 \text{ k-ft}^3$$

Member end moments. Substituting the numerical values of $EI\theta_B$ and $EI\Delta$ into the slope-deflection equations, we obtain:

$$M_{AB} = 127.1 \text{ k-ft}; \quad M_{BA} = 103.4 \text{ k-ft};$$

$$M_{BC} = -103.4 \text{ k-ft}.$$



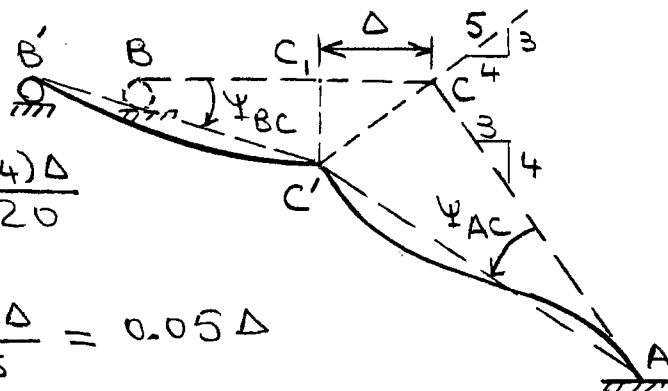
16.28 Fixed end moments:

$$FEM_{BC} = \frac{1.5(20)^2}{12} = 50 \text{ k-ft}; FEM_{CB} = -50 \text{ k-ft}$$

Chord rotations:

$$\psi_{BC} = -\frac{C_1 C'}{L_{BC}} = -\frac{(3/4)\Delta}{20} = -0.0375 \Delta$$

$$\psi_{AC} = \frac{CC'}{L_{AC}} = \frac{(5/4)\Delta}{25} = 0.05 \Delta$$



Slope-deflection equations: $M_{BC} = 0$

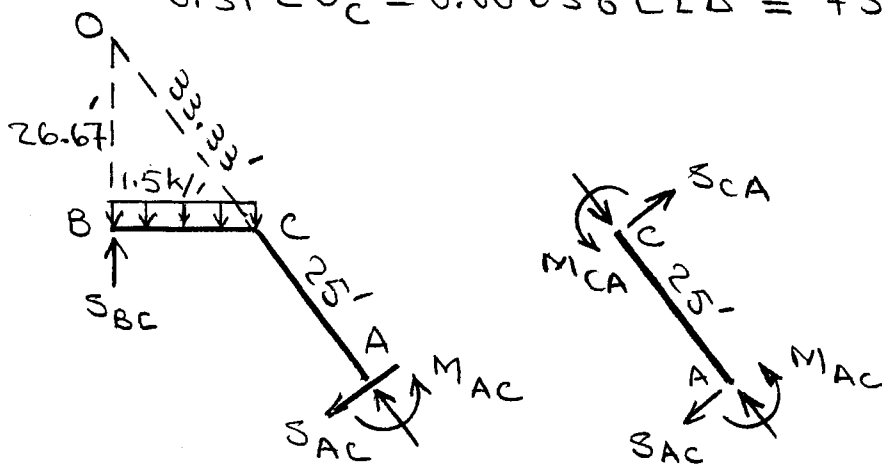
$$M_{CB} = 0.15EI(\theta_C + 0.0375\Delta) - 75$$

$$M_{CA} = 0.08EI(2\theta_C - 0.15\Delta)$$

$$M_{AC} = 0.08EI(\theta_C - 0.15\Delta)$$

Equilibrium equations: $M_{CB} + M_{CA} = 0$

$$0.31EI\theta_C - 0.00638EI\Delta = 75 \quad (1)$$



$$+\circlearrowleft \sum M_O = 0 \quad M_{AC} - S_{AC}(58.33) - 1.5(20)10 = 0$$

$$M_{AC} - \left(\frac{M_{AC} + M_{CA}}{25}\right)(58.33) - 300 = 0$$

$$-0.48EI\theta_C + 0.044EI\Delta = 300 \quad (2)$$

By solving Eqs. (1) and (2), we obtain

$$EI\theta_C = 493 \text{ k-ft}^2; EI\Delta = 12196 \text{ k-ft}^3$$

16.28 (contd.) Member end moments: Substituting the numerical values of $EI\theta_c$ and $EI\Delta$ into the slope-deflection equations, we obtain

$$\underline{M_{CB} = 67.5 \text{ k-ft}}; \quad \underline{M_{CA} = -67.5 \text{ k-ft}}$$

$$\underline{M_{AC} = -106.9 \text{ k-ft}}$$

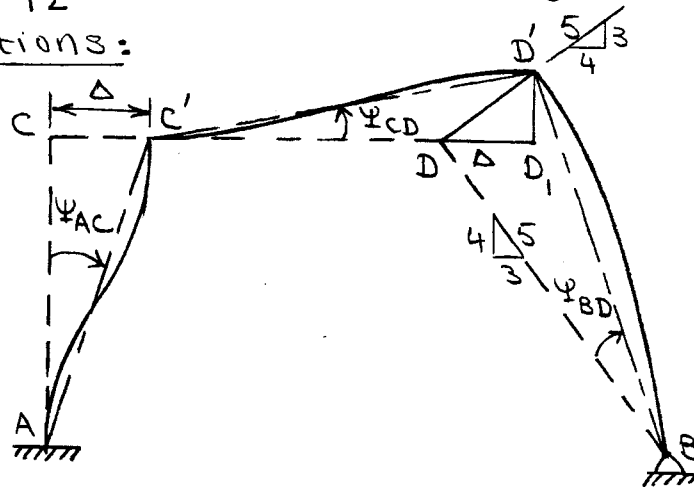
For reactions, see solution of Problem 13.24.

16-29 Fixed-end moments: $FEM_{BD} = FEM_{DB} = 0$

$$FEM_{AC} = \frac{50(4)}{8} = 25 \text{ kN}\cdot\text{m}; FEM_{CA} = -25 \text{ kN}\cdot\text{m}$$

$$FEM_{CD} = \frac{18(5)^2}{12} = 37.5 \text{ kN}\cdot\text{m}; FEM_{DC} = -37.5 \text{ kN}\cdot\text{m}$$

Chord rotations:



$$\Psi_{AC} = -\frac{CC'}{L_{AC}} = -\frac{\Delta}{4} = -0.25\Delta$$

$$\Psi_{BD} = -\frac{DD'}{L_{BD}} = -\frac{(5/4)\Delta}{5} = -0.25\Delta$$

$$\Psi_{CD} = \frac{D_1D'}{L_{CD}} = \frac{(3/4)\Delta}{5} = 0.15\Delta$$

Slope-deflection equations: $M_{BD} = 0$

$$M_{AC} = 0.5EI(\theta_C + 0.75\Delta) + 25$$

$$M_{CA} = 0.5EI(2\theta_C + 0.75\Delta) - 25$$

$$M_{CD} = 0.4EI(2\theta_C + \theta_D - 0.45\Delta) + 37.5$$

$$M_{DC} = 0.4EI(\theta_C + 2\theta_D - 0.45\Delta) - 37.5$$

$$M_{DB} = 0.6EI(\theta_D + 0.25\Delta)$$

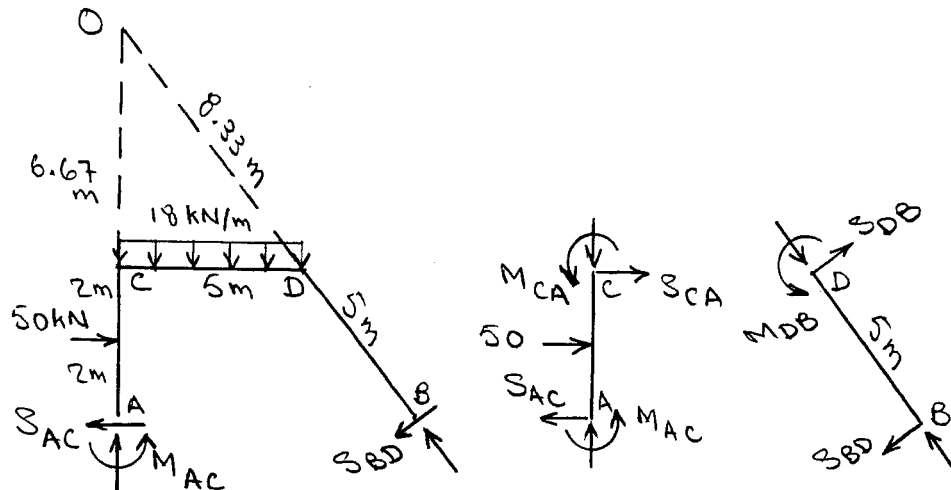
Equilibrium equations: $M_{CA} + M_{CD} = 0$

$$1.8EI\theta_C + 0.4EI\theta_D + 0.195EI\Delta = -12.5 \quad (1)$$

$$M_{DC} + M_{DB} = 0$$

$$0.4EI\theta_C + 1.4EI\theta_D - 0.03EI\Delta = 37.5 \quad (2)$$

16.29 (contd.)



$$+\circlearrowleft \sum M_O = 0$$

$$M_{AC} - S_{AC}(10.67) - S_{BD}(13.33) + 50(8.67) - 18(5)(2.5) = 0$$

$$M_{AC} - \left(25 + \frac{M_{AC} + M_{CA}}{4}\right)10.67 - \left(\frac{M_{DB}}{5}\right)13.33 + 208.5 = 0$$

$$3.5EI\theta_C + 1.6EI\theta_D + 2.025EI\Delta = -33.25 \quad (3)$$

By solving Eqs. (1) thru (3) simultaneously, we obtain:

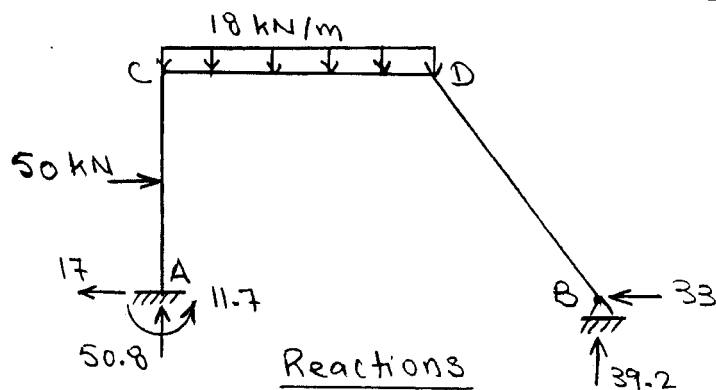
$$EI\theta_C = -11.33 \text{ kN}\cdot\text{m}^2; \quad EI\theta_D = 29.59 \text{ kN}\cdot\text{m}^2$$

$$EI\Delta = -20.22 \text{ kN}\cdot\text{m}^3$$

Member end moments. Substituting the numerical values of $EI\theta_C$, $EI\theta_D$ and $EI\Delta$ into the slope-deflection equations, we obtain:

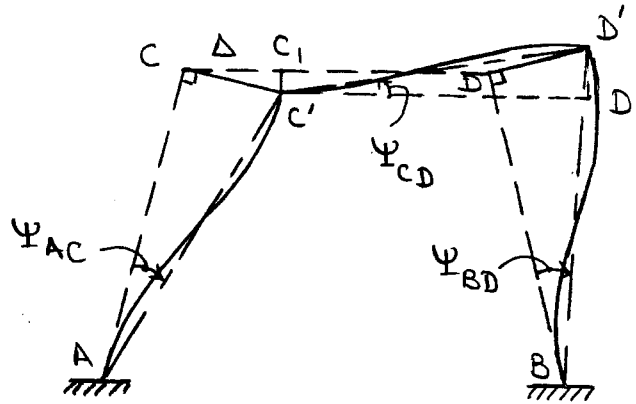
$$\underline{M_{AC} = 11.7 \text{ kN}\cdot\text{m}; \quad M_{CA} = -43.9 \text{ kN}\cdot\text{m};}$$

$$\underline{M_{CD} = 43.9 \text{ kN}\cdot\text{m}; \quad M_{DC} = -14.7 \text{ kN}\cdot\text{m}; \quad M_{DB} = 14.7 \text{ kN}\cdot\text{m}}$$



16.30 Fixed-end moments: The non-zero fixed-end moments are: $FEM_{CD} = \frac{3(16)^2}{12} = 64 \text{ k-ft}$ and $FEM_{DC} = -64 \text{ k-ft}$.

Chord rotations:



$$\Psi_{AC} = -\frac{CC'}{L_{AC}} = -\frac{(\sqrt{7}/4)\Delta}{16.49} = -0.0625\Delta$$

$$\Psi_{BD} = -\frac{DD'}{L_{BD}} = -\frac{(\sqrt{7}/4)\Delta}{16.49} = -0.0625\Delta$$

$$\Psi_{CD} = \frac{D_1D'_1}{L_{CD}} = \frac{2(1/4)\Delta}{16} = 0.03125\Delta$$

Slope-deflection equations:

$$M_{AC} = 0.121EI(\theta_C + 0.188\Delta); M_{CA} = 0.121EI(2\theta_C + 0.188\Delta)$$

$$M_{BD} = 0.121EI(\theta_D + 0.188\Delta); M_{DB} = 0.121EI(2\theta_D + 0.188\Delta)$$

$$M_{CD} = 0.125EI(2\theta_C + \theta_D - 0.0938\Delta) + 64$$

$$M_{DC} = 0.125EI(\theta_C + 2\theta_D - 0.0938\Delta) - 64$$

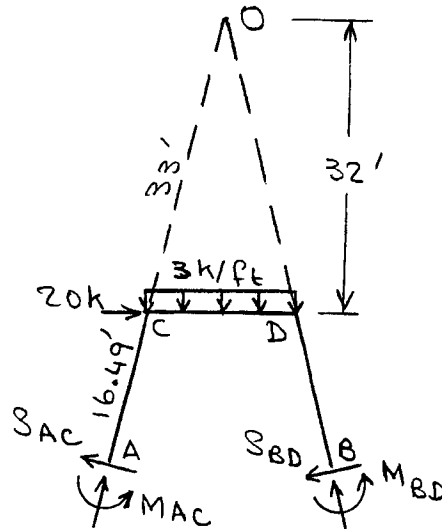
Equilibrium equations: $M_{CA} + M_{CD} = 0$

$$0.492EI\theta_C + 0.125EI\theta_D + 0.011EI\Delta = -64 \quad (1)$$

$$M_{DC} + M_{DB} = 0$$

$$0.125EI\theta_C + 0.492EI\theta_D + 0.011EI\Delta = 64 \quad (2)$$

Free body diagrams of members AC and CD. Member AC shows forces M_{CA} , S_{CA} at C and S_{AC} , M_{AC} at A. Member CD shows forces S_{CD} , S_{DC} at C and D, and M_{CD} at D.



$$M_{AC} + M_{BD} - (8_{AC} + 8_{BD})49.49 + 20(32) = 0$$

$$M_{AC} + M_{BD} - \frac{49.49}{16.49}(M_{AC} + M_{CA} + M_{BD} + M_{DB}) + 640 = 0$$

$$0.968EI\theta_c + 0.968EI\theta_D + 0.227EI\Delta = 640 \quad (3)$$

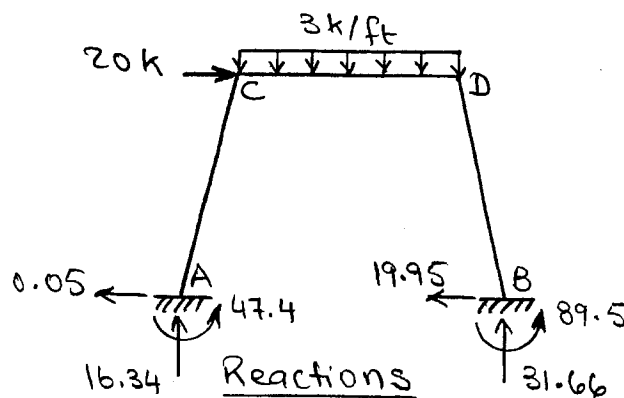
$$EI\theta_C = -233.7 \text{ k-ft}^2, EI\theta_D = 115.1 \text{ k-ft}^2;$$

$$EI \Delta = 3325 \text{ k-ft}^3$$

Member end moments. Substituting the numerical values of $EI\theta_C$, $EI\theta_D$ and $EI\Delta$ into the slope-deflection equations, we obtain: $M_{AC} = 47.4 \text{ k-ft}$;

$$\underline{M_{CA} = 19 \text{ k-ft}; M_{BD} = 89.5 \text{ k-ft}; M_{DB} = 103.5 \text{ k-ft};}$$

$$\underline{M_{CD} = -19 \text{ k-ft}}; \underline{M_{DC} = -103.5 \text{ k-ft}}$$

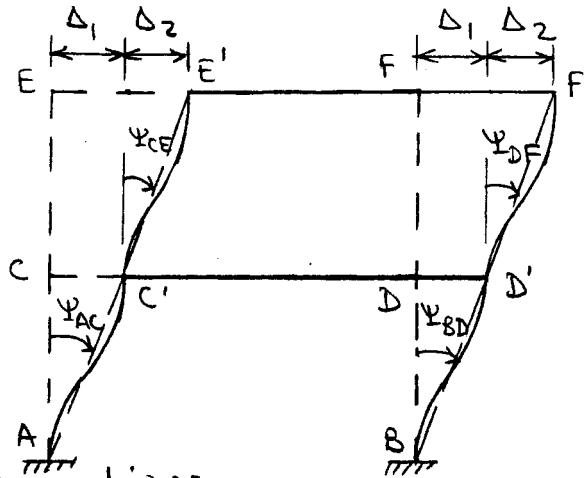


16.31 Chord rotations:

$$\psi_{AC} = \psi_{BD} = -\frac{\Delta_1}{15}$$

$$\psi_{CE} = \psi_{DF} = -\frac{\Delta_2}{15}$$

$$\psi_{CD} = \psi_{EF} = 0$$



Slope-deflection equations:

$$M_{AC} = 0.133EI(\theta_C + 0.2\Delta_1); M_{CA} = 0.133EI(2\theta_C + 0.2\Delta_1)$$

$$M_{BD} = 0.133EI(\theta_D + 0.2\Delta_1); M_{DB} = 0.133EI(2\theta_D + 0.2\Delta_1)$$

$$M_{CE} = 0.133EI(2\theta_C + \theta_E + 0.2\Delta_2)$$

$$M_{EC} = 0.133EI(\theta_C + 2\theta_E + 0.2\Delta_2)$$

$$M_{DF} = 0.133EI(2\theta_D + \theta_F + 0.2\Delta_2)$$

$$M_{FD} = 0.133EI(\theta_D + 2\theta_F + 0.2\Delta_2)$$

$$M_{CD} = 0.133EI(2\theta_C + \theta_D); M_{DC} = 0.133EI(\theta_C + 2\theta_D)$$

$$M_{EF} = 0.133EI(2\theta_E + \theta_F); M_{FE} = 0.133EI(\theta_E + 2\theta_F)$$

Equilibrium equations: $M_{CA} + M_{CD} + M_{CE} = 0$

$$EI(6\theta_C + \theta_D + \theta_E + 0.2\Delta_1 + 0.2\Delta_2) = 0 \quad (1)$$

$$M_{DB} + M_{DC} + M_{DF} = 0$$

$$EI(\theta_C + 6\theta_D + \theta_F + 0.2\Delta_1 + 0.2\Delta_2) = 0 \quad (2)$$

$$M_{EC} + M_{EF} = 0$$

$$EI(\theta_C + 4\theta_E + \theta_F + 0.2\Delta_2) = 0 \quad (3)$$

$$M_{FD} + M_{FE} = 0$$

$$EI(\theta_D + \theta_E + 4\theta_F + 0.2\Delta_2) = 0 \quad (4)$$

$$3\psi_{CE} + 8\psi_{DF} = 9$$

$$\frac{1}{15}[(M_{CE} + M_{EC}) + (M_{DF} + M_{FD})] = 9$$

16.31 (Contd.)

$$EI(3\theta_C + 3\theta_D + 3\theta_E + 3\theta_F + 0.8\Delta_2) = 1015 \quad (5)$$

$$\delta_{AC} + \delta_{BD} = 27$$

$$\frac{1}{15} [(M_{AC} + M_{CA}) + (M_{BD} + M_{DB})] = 27$$

$$EI(3\theta_C + 3\theta_D + 0.8\Delta_1) = 3045 \quad (6)$$

By solving Eqs. (1) thru (6) simultaneously, we

obtain: $EI\theta_C = EI\theta_D = -267.6 \text{ k-ft}^2$;

$EI\theta_E = EI\theta_F = -110.7 \text{ k-ft}^2$; $EI\Delta_1 = 5813 \text{ k-ft}^3$;

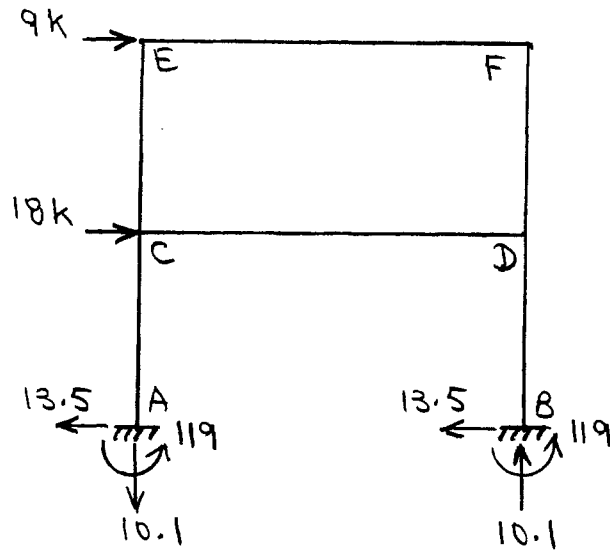
$EI\Delta_2 = 4106 \text{ k-ft}^3$.

Member end moments: $M_{AC} = M_{BD} = 119 \text{ k-ft}$;

$M_{CA} = M_{DB} = 83.5 \text{ k-ft}$; $M_{CE} = M_{DF} = 23.3 \text{ k-ft}$;

$M_{EC} = M_{FD} = 44.2 \text{ k-ft}$; $M_{CD} = M_{DC} = -106.8 \text{ k-ft}$;

$M_{EF} = M_{FE} = -44.2 \text{ k-ft}$.



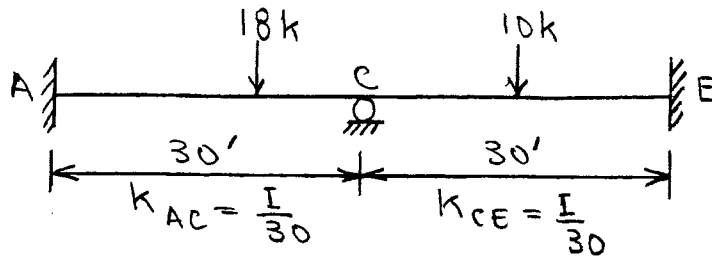
Reactions

Chapter Seventeen

Moment-Distribution Method

CHAPTER 17

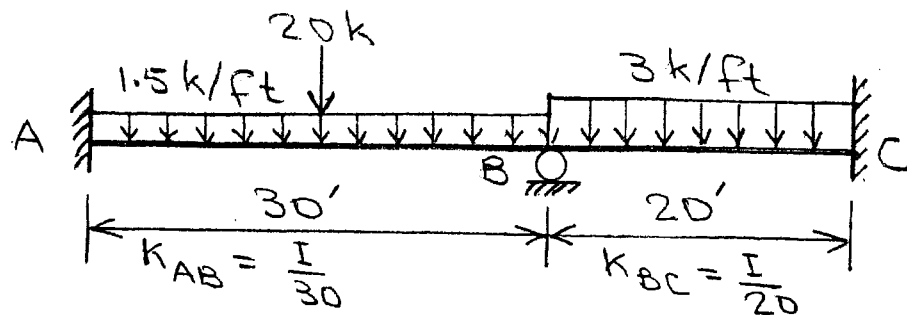
17-1



		0.5	0.5	
DF				
FEM	40	-80	37.5	-37.5
		21.3	21.3	
	10.6			10.6
Final Moments	50.6	-58.7	58.8	-26.9

For reactions, and shear and bending moment diagrams, see solution of Problem 16-1.

17.2

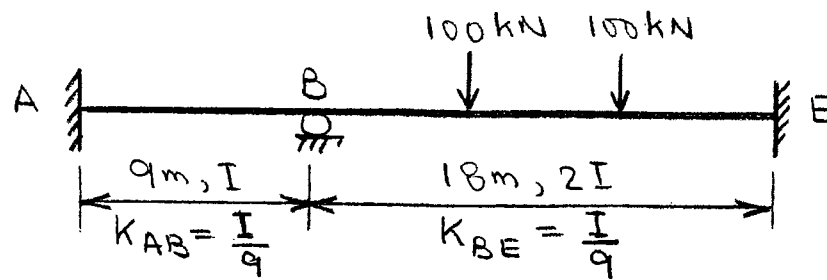


DF
FEM

	0.4	0.6	
187.5	-187.5	100	-100
	35	52.5	
17.5			26.25
Final Moments	205	-152.5	152.5 -73.75

For reactions, and shear and bending moment diagrams, see solution of Problem 16.2.

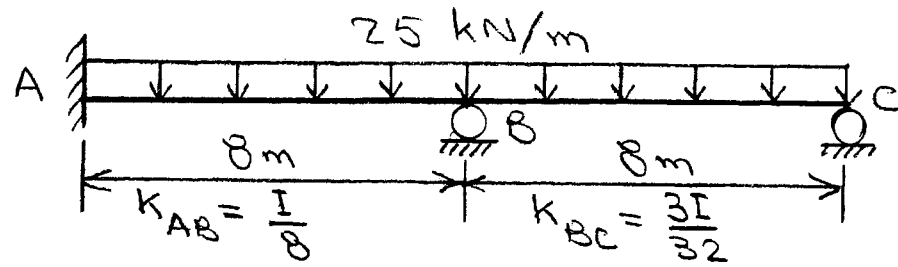
17.3



		0.5	0.5	
DF				
FEM	0	0	400	-400
		-200	-200	
	-100			-100
Final Moments	-100	-200	200	-500

For reactions, and shear and bending moment diagrams, see solution of Problem 16.3.

17.4

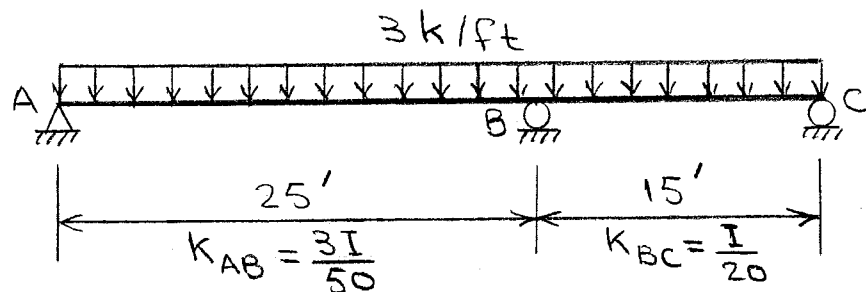


DF
FEM

	0.571	0.429	
133.3	-133.3	133.3	-133.3
		66.7	133.3
	-38.1	-28.6	
-19			
Final Moments	114.3	-171.4	171.4
			0

For reactions, and shear and bending moment diagrams, see solution of Problem 13.37.

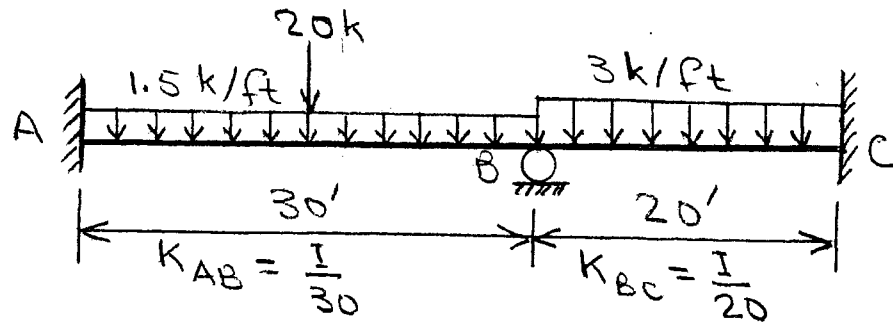
17.5



		0.545	0.455	
DF				
FEM	156.3	-156.3	56.3	-56.3
	-156.3	54.5	45.5	56.3
		-78.1	28.1	
		27.3	22.7	
Final Moments	0	-152.6	152.6	0

For reactions, and shear and bending moment diagrams, see solution of Problem 13.12.

17.6



$$FEM_{AB} = \frac{1.5(30)^2}{12} + \frac{20(30)}{8} + \frac{6(29000)(1650)(0.5)}{(30)^2(12)3}$$

$$= 112.5 + 75 + 92.3 = 279.8 \text{ k-ft}$$

$$FEM_{BA} = -112.5 - 75 + 92.3 = -95.2 \text{ k-ft}$$

$$FEM_{BC} = \frac{3(20)^2}{12} - \frac{6(29000)(1650)(0.5)}{(20)^2(12)3}$$

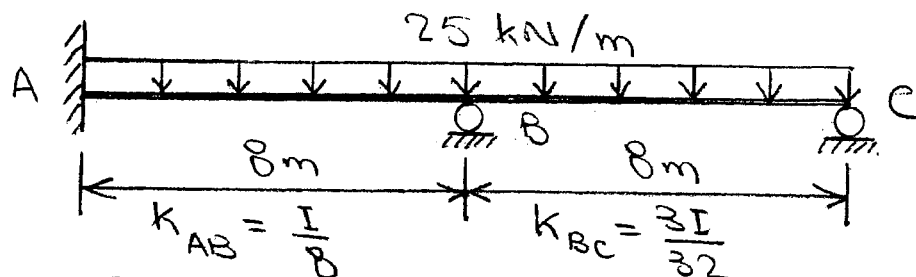
$$= 100 - 207.7 = -107.7 \text{ k-ft}$$

$$FEM_{CB} = -100 - 207.7 = -307.7 \text{ k-ft}$$

		0.4	0.6
DF			
FEM	279.8	-95.2	-107.7
		81.2	121.7
	40.6		60.9
Final Moments	320.4	-14	14
			-246.8

For reactions, and shear and bending moment diagrams, see solution of Problem 16.6.

17.7



$$FEM_{AB} = \frac{25(8)^2}{12} + \frac{6(70)(1300)(0.05)}{(8)^2} = 133.3 + 426.6 = 559.9 \text{ kN.m}$$

$$FEM_{BA} = -133.3 + 426.6 = 293.3 \text{ kN.m}$$

$$FEM_{BC} = 133.3 - \frac{6(70)(1300)(0.025)}{(8)^2} = 133.3 - 213.3 = -80 \text{ kN.m}$$

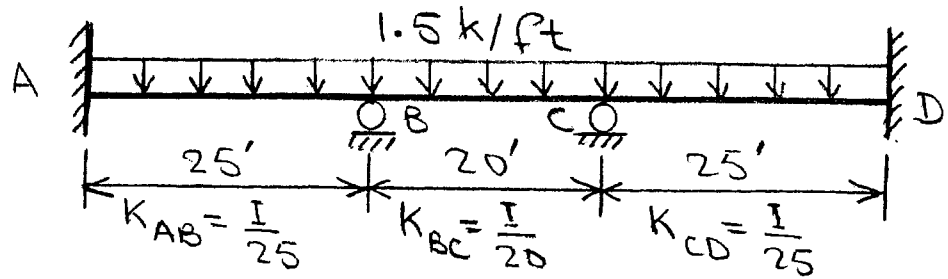
$$FEM_{CB} = -133.3 - 213.3 = -346.6 \text{ kN.m}$$

DF
FEM

	0.571	0.429	
559.9	293.3	-80	-346.6
	-121.8	-91.5	346.6
-60.9		173.3	
	-99	-74.3	
-49.5			
Final Moments	449.5	72.5	-72.5
			0

For reactions, and shear and bending moment diagrams, see solution of Problem 13.53.

17.8



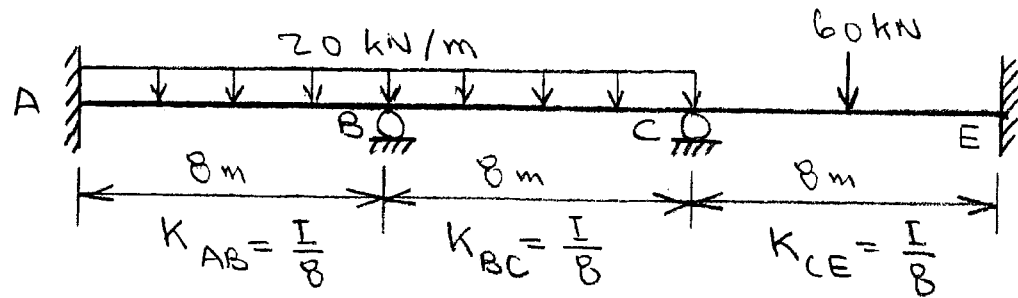
DF
FEM

4/9		5/9		5/9		4/9	
78.1	-78.1	50	-50	78.1	-78.1		
	12.5	15.6	-15.6	-12.5			
6.2		-7.8	7.8		-6.2		
	3.5	4.3	-4.3	-3.5			
1.7		-2.2	2.2		-1.7		
	1	1.2	-1.2	-1			
0.5		-0.6	0.6		-0.5		
	0.3	0.3	-0.3	-0.3			
0.1		-0.2	0.2		-0.1		
	0.1	0.1	-0.1	-0.1			
86.6	-60.7	60.7	-60.7	60.7	-86.6		

Final Moments

For reactions, and shear and bending moment diagrams, see solution of Problem 16.8.

17.9

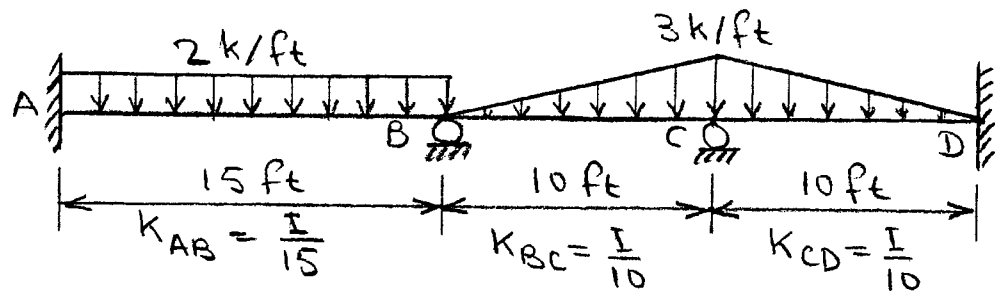


DF
FEM

	0.5 0.5		0.5 0.5		
	106.7	-106.7	106.7	-106.7	60 -60
			23.4	23.4	
		11.7			11.7
	-5.9	-5.9			
	-3		-3		
			1.5	1.5	
		0.8			0.8
	-0.4	-0.4			
	-0.2		-0.2		
			0.1	0.1	
Final Moments	103.5	-113	112.9	-84.9	85 -47.5

For reactions, and shear and bending moment diagrams, see solution of Problem 16.9.

17.10

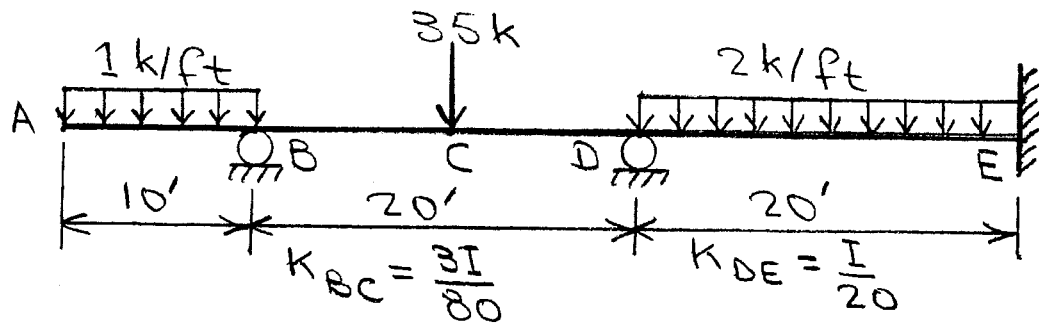


DF
FEM

	0.4	0.6	0.5	0.5	
37.5	-37.5	10	-15	15	-10
5.5	11	16.5	8.3	-4.2	-2.1
0.4	0.8	1.3	0.7	-0.3	-0.2
43.4	-25.6	25.6	-10.5	10.5	-12.3

For reactions, and shear and bending moment diagrams, see solution of Problem 16.10.

17.11

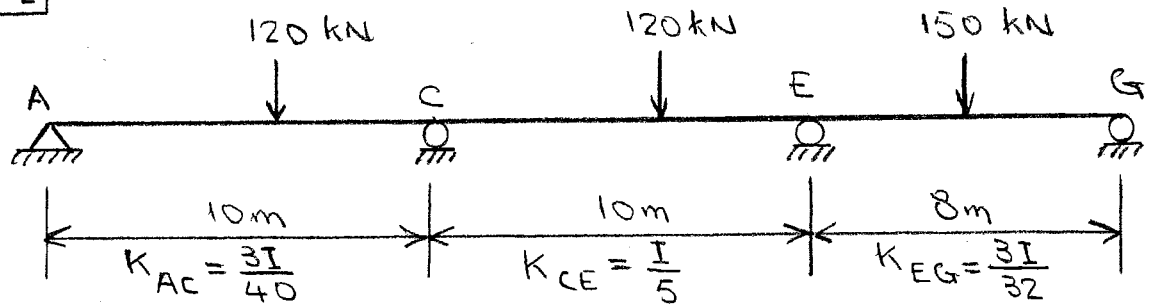


DF
FEM

		1		0.429	0.571	
	-50	87.5		-87.5	66.7	-66.7
		-37.5		8.9	11.9	
			-18.8			5.9
			8.1	10.7		
						5.4
Final Moments	-50	50		-89.3	89.3	-55.4

For reactions, and shear and bending moment diagrams, see solution of Problem 13.38.

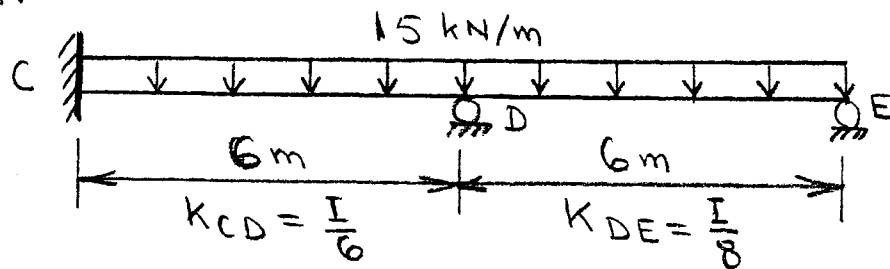
17.12



DF		0.273	0.727	0.681	0.319	
FEM	115.2	-172.8	115.2	-172.8	150	-150
	-115.2	15.7	41.9	15.5	7.3	150
		-57.6	7.8	20.9	75	
		13.6	36.2	-65.3	-30.6	
		8.9	-32.7	18.1		
		23.8	-12.3	-5.8		
		-6.2	11.9			
		1.7	4.5	-8.1	-3.8	
		-4.1	2.3			
		1.1	3	-1.6	-0.7	
		-0.8	1.5			
		0.2	0.6	-1	-0.5	
		-0.5	0.3			
		0.1	0.4	-0.2	-0.1	
Final Moments	0	-189.1	189.1	-190.8	190.8	0

For reactions, and shear and bending moment diagrams, see solution of Problem 13.39.

17.13 As the beam and loading are symmetric, we will analyze only the right half CE of the beam.



DF	0.571	0.429	1
FEM	45	-45	45
		22.5	-45
	-12.8	-9.7	
	-6.4		
Final Moments	38.6	-57.8	57.8
			0

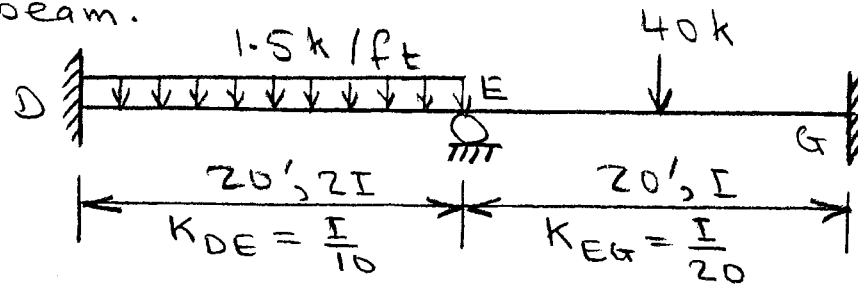
Because of symmetry, the member end moments for the left half of the beam are:

$$M_{AB} = 0; \quad M_{BA} = -57.8 \text{ kN.m}$$

$$M_{BC} = 57.8 \text{ kN.m}; \quad M_{CB} = -38.6 \text{ kN.m}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 16.13.

17.14 As the beam and loading are symmetric, we will analyze only the right half DG of the beam.



DF
FEM

	$\frac{2}{3}$	$\frac{1}{3}$	
50	-50	100	-100
	-33.3	-16.7	
-16.7			-8.3
Final Moments	33.3	-83.3	83.3
			-108.3

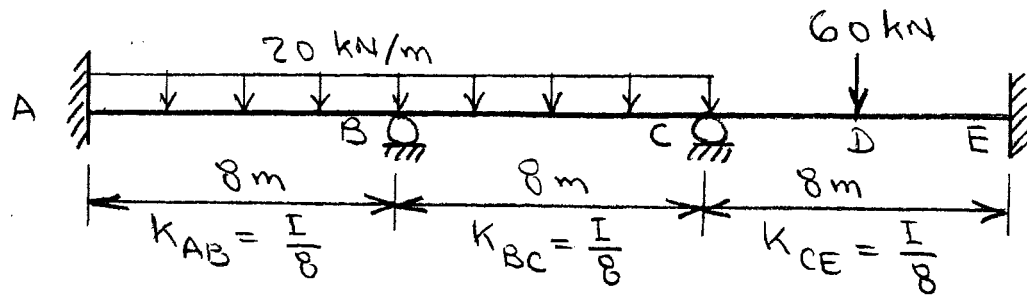
Because of symmetry, the member end moments for the left half of the beam are:

$$\underline{M_{AC} = 108.3 \text{ k-ft}}; \quad \underline{M_{CA} = -83.3 \text{ k-ft}}$$

$$\underline{M_{CD} = 83.3 \text{ k-ft}}; \quad \underline{M_{DC} = -33.3 \text{ k-ft}}$$

For reactions, and shear and bending moment diagrams, see solution of Problem 16.14.

17.15



$$FEM_{AB} = \frac{20(8)^2}{12} = 106.7 \text{ kN.m}; \quad FEM_{BA} = -106.7 \text{ kN.m}$$

$$FEM_{BC} = 106.7 + \frac{6(70)(800)(0.025)}{(8)^2} = 106.7 + 131.3 = 238 \text{ kN.m}$$

$$FEM_{CB} = -106.7 + 131.3 = 24.6 \text{ kN.m}$$

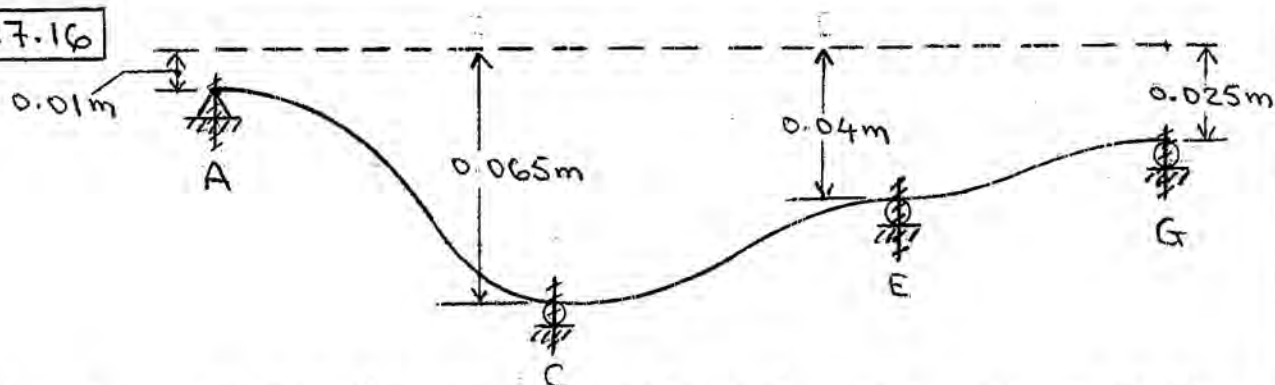
$$FEM_{CE} = \frac{60(8)}{8} - 131.3 = 60 - 131.3 = -71.3 \text{ kN.m}$$

$$FEM_{EC} = -60 - 131.3 = -191.3 \text{ kN.m}$$

DF	0.5		0.5		0.5	
FEM	106.7	-106.7	238	24.6	-71.3	-191.3
		-65.7	-65.7	23.4	23.4	
	-32.8		11.7	-32.8		11.7
		-5.8	-5.8	16.4	16.4	
	-2.9		8.2	-2.9		8.2
		-4.1	-4.1	1.5	1.5	
	-2.1		0.7	-2.1		0.7
		-0.4	-0.4	1	1	
	-0.2		0.5	-0.2		0.5
		-0.2	-0.2	0.1	0.1	
ments	68.7	-182.9	182.9	29	-28.9	-170.2

For reactions, and shear and bending moment diagrams, see solution of Problem 16.15.

17.16



$$FEM_{AC} = \frac{120(6)(4)^2}{(10)^2} + \frac{6(200)(500)(0.055)}{(10)^2} = 115.2 + 330 = 445.2 \text{ kN.m}$$

$$FEM_{CA} = -\frac{120(6)^2(4)}{(10)^2} + 330 = -172.8 + 330 = 157.2 \text{ kN.m}$$

$$FEM_{CE} = 115.2 - \frac{6(200)(1000)(0.025)}{(10)^2} = 115.2 - 300 = -184.8 \text{ kN.m}$$

$$FEM_{EC} = -172.8 - 300 = -472.8 \text{ kN.m}$$

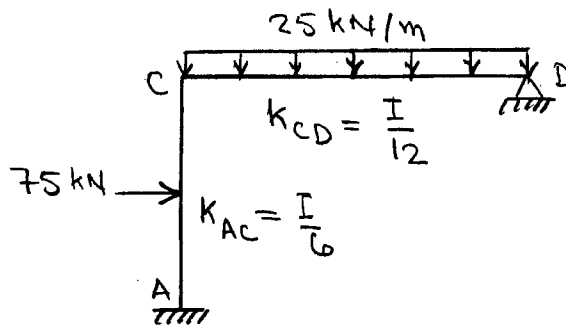
$$FEM_{EG} = \frac{150(8)}{8} - \frac{6(200)(500)(0.015)}{(8)^2} = 150 - 140.6 = 9.4 \text{ kN.m}$$

$$FEM_{GE} = -150 - 140.6 = -290.6 \text{ kN.m}$$

DF	0.273	0.727	0.681	0.319
FEM	445.2	157.2	-184.8	-472.8
	-445.2	7.5	20.1	315.6
		-222.6	157.8	10
		17.7	47.1	-105.8
			-52.9	23.6
		14.4	38.5	-16.1
			-8	19.2
		2.2	5.8	-13.1
			-6.5	2.9
		1.8	4.7	-2
			-1	2.4
		0.3	0.7	-1.6
			-0.8	0.4
		0.2	0.6	-0.3
				-0.1
Final Moments	0	-21.3	21.3	-237.6
				237.6
				0

For reactions, and shear and bending moment diagrams, see solution of Problem 13.54.

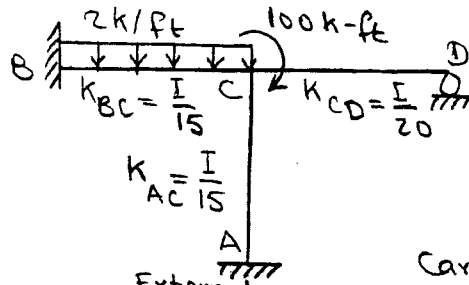
17.17



	AC	CA	CD	DC
DF		2/3	1/3	
FEM	56.3	-56.3	168.8	-168.8
		-75	-37.5	168.8
	-37.5		84.4	
		-56.3	-28.1	
	-28.2			
Final Moments	-9.4	-187.6	187.6	0

for reactions, see solution of Problem 13.42

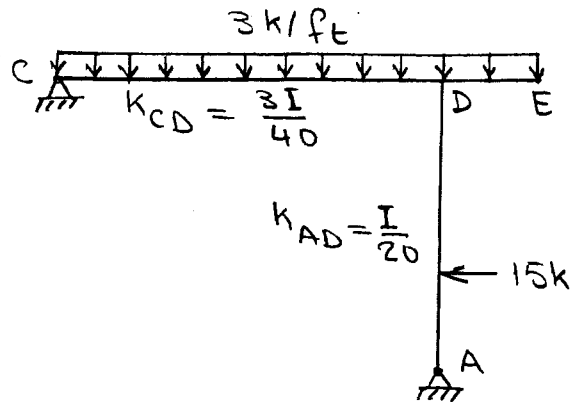
17-18



	BC	CB	External Moment at Joint C	CA	CD	DC	AC
DF		0.364	—	0.364	0.272		
FEM	37.5	-37.5	100	0	0	0	0
		-22.7	—	-22.7	-17.1		
	-11.3						-11.3
Final Moments	26.2	-60.2	100	-22.7	-17.1	0	-11.3

For reactions, see solution of Problem 16.18.

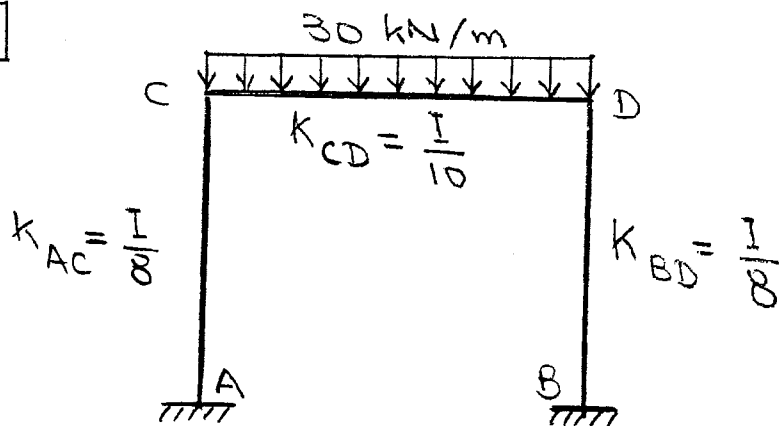
17.19



	CD	DC	DE	DA	AD
DF		0.6	-	0.4	
FEM	100	-100	37.5	16.7	-33.3
	-100	27.5		18.3	33.3
		-50		16.7	
		20		13.3	
Final Moments	0	-102.5	37.5	65	0

For reactions, see solution of Problem 13.21.

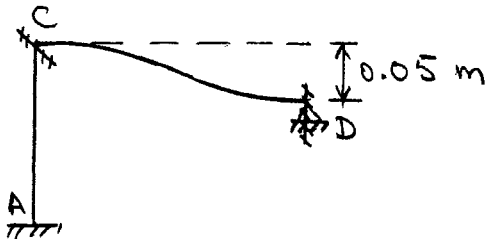
17.20



	AC	CA	CD	DC	DB	BD
DF						
FEM						
	0	0	250	-250	0	0
	-138.9	-111.1	111.1	138.9		
	-69.4	55.6	-55.6	69.4		
	-30.9	-24.7	24.7	30.9		
	-15.4	12.3	-12.3	15.4		
	-6.8	-5.5	5.5	6.8		
	-3.4	2.7	-2.7	3.4		
	-1.5	-1.2	1.2	1.5		
	-0.8	0.6	-0.6	0.8		
	-0.3	-0.3	0.3	0.3		
	-0.2	0.1	-0.1	0.2		
Final Moments	-89.2	-178.4	178.5	-178.5	178.4	89.2

For reactions, see solution of Problem 16.20.

17.21



$$FEM_{AC} = \frac{75(6)}{8} = 56.3 \text{ k-ft}; \quad FEM_{CA} = -56.3 \text{ k-ft}$$

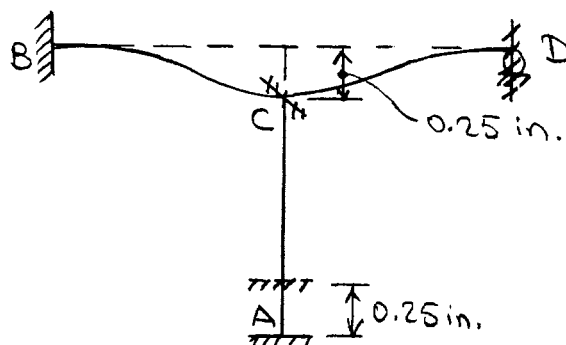
$$FEM_{CD} = \frac{25(9)^2}{12} + \frac{6(200)400(0.05)}{(9)^2} = 168.8 + 296.3 = 465.1 \text{ k-ft}$$

$$FEM_{DC} = -168.1 + 296.3 = 128.2 \text{ k-ft}$$

	AC	CA	CD	DC
DF		2/3	1/3	
FEM	56.3	-56.3	465.1	128.2
		-272.5	-136.3	-128.2
	-136.3		-64.1	
		42.7	21.4	
	21.4			
Final Moments	-58.6	-286.1	286.1	0

For reactions, see solution of Problem 16-21

17.22



$$FEM_{AC} = FEM_{CA} = 0$$

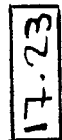
$$FEM_{BC} = \frac{2(15)^2}{12} + \frac{6(29000)(3500)(0.25)}{(15)^2(12)^3} = 37.5 + 391.6 = 429.1 \text{ k-ft}$$

$$FEM_{CB} = -37.5 + 391.6 = 354.1 \text{ k-ft}$$

$$FEM_{CD} = FEM_{DC} = -391.6 \text{ k-ft}$$

		External Moment at Joint C		Carry Over			
		BC	CB	CA	CD	DC	AC
DF			0.364	-	0.364	0.272	
FEM		429.1	354.1	100	0	-391.6	-391.6
			-22.7		-22.7	-17.1	391.6
		-11.3				195.8	
			-71.3		-71.3	-53.2	
		-35.7					
Final Moments		382.1	260.1	100	-94	-266.1	0
							-47

For reactions, see solution of Problem 16.22.



$$K_{BC} = \frac{I}{m^0} \therefore K_{CD} = \frac{I}{m^0} = \frac{I}{\frac{1}{4} \cdot \frac{1}{4} \cdot 4} = \frac{I}{\frac{1}{4}}$$

$$FEM_{BC} = \frac{2(30)^2}{12} + \frac{6(10000)(3000)(1)}{(30)^2(12)^3} = 150 + 115.7 = 265.7 \text{ k-ft}$$

$$FEM_{DC} = 57.9 \text{ k-ft}; \quad FEM_{DE} = 10(10) = 100 \text{ k-ft.}$$

Final Moments

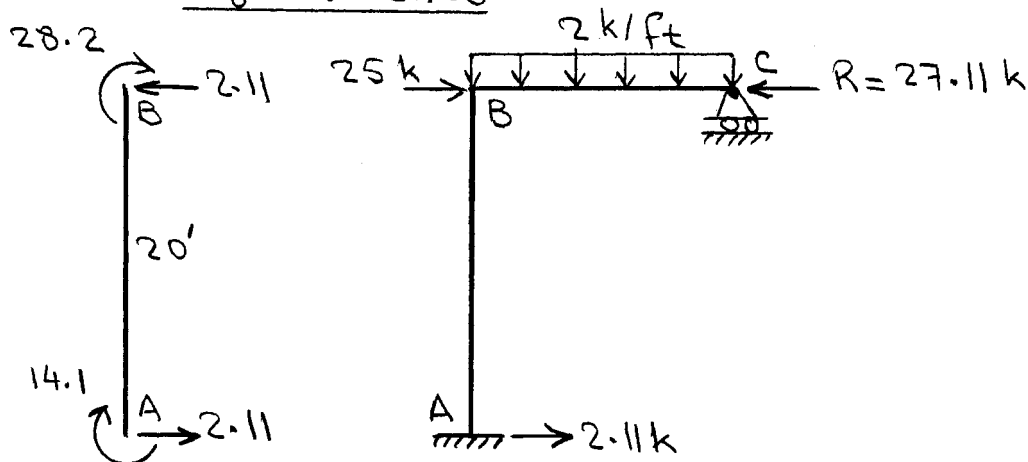
For reactions, see solution of problem 16-23.

17.24 $K_{AB} = \frac{I}{20}$, $K_{BC} = \frac{3}{4} \left(\frac{I}{15} \right) = \frac{I}{20}$

Part I - Sidesway prevented:

AB	BA	BC	CB
	0.5	0.5	
	-18.8	-18.8	
-9.4		18.8	
-4.7		-9.4	
-14.1	-28.2	28.1	0

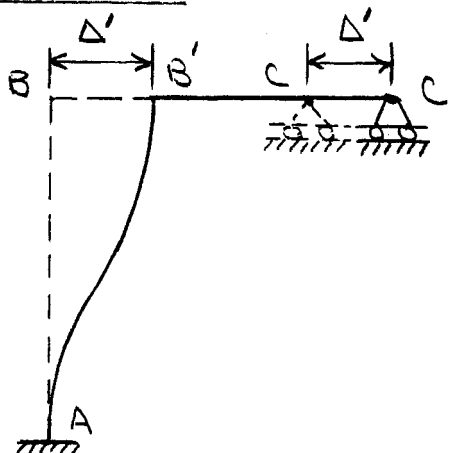
M_D Moments



Part II - Sidesway permitted:

$$FEM_{AB} = FEM_{BA} = \frac{6EI\Delta'}{(20)^2}$$

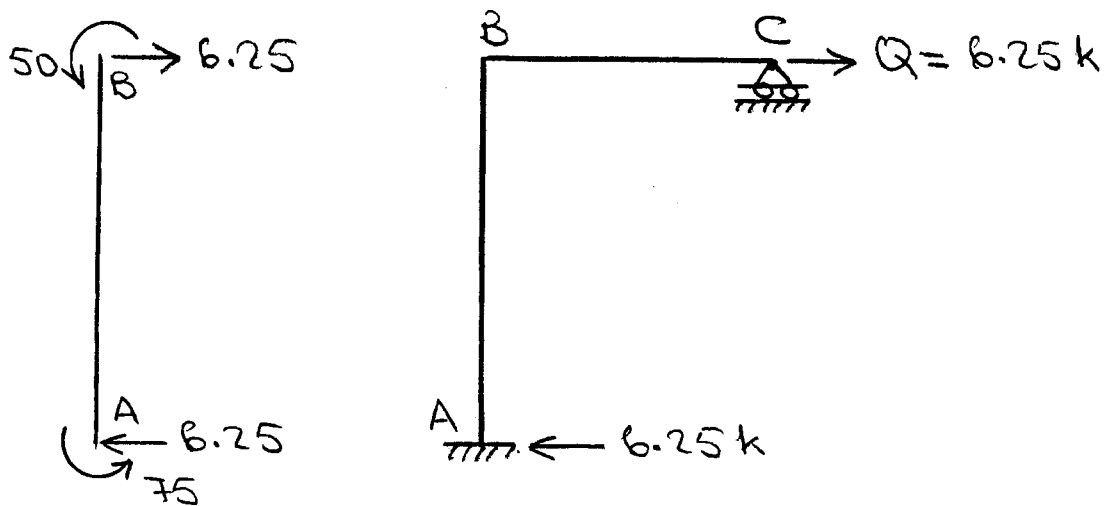
$$\text{Let } FEM_{AB} = FEM_{BA} = 100 \text{ k-ft.}$$



17.24 (contd.)

AB	BA	BC	CB
	0.5	0.5	
100	100	-50	
-25	-50	-50	
75	50	-50	0

M_Q Moments



Actual member end moments:

$$M_{AB} = -14.1 + \left(\frac{27.11}{6.25} \right) 75 = \underline{311.2 \text{ k-ft}}$$

$$M_{BA} = -28.2 + \left(\frac{27.11}{6.25} \right) 50 = \underline{188.7 \text{ k-ft}}$$

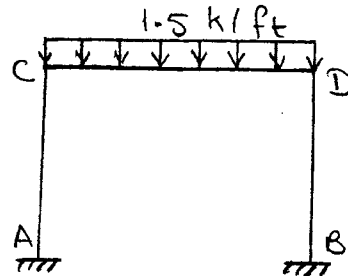
$$M_{BC} = 28.1 - \left(\frac{27.11}{6.25} \right) 50 = \underline{-188.8 \text{ k-ft}}$$

$$\underline{M_{CB} = 0}$$

For reactions, see solution of Problem 16.24.

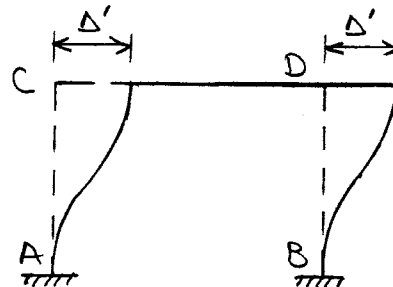
17.25 $K_{AC} = K_{BD} = \frac{I}{30}$; $K_{CD} = \frac{2I}{40} = \frac{I}{20}$

Part I - No Sidesway:



	AC	CA	CD	DC	DB	BD
DF		0.4	0.6	0.6	0.4	
FEM	0	0	200	-200	0	0
		-80	-120	120	80	
	-40		60	-60		40
		-24	-36	36	24	
	-12		18	-18		12
		-7.2	-10.8	10.8	7.2	
	-3.6		5.4	-5.4		3.6
		-2.2	-3.2	3.2	2.2	
	-1.1		1.6	-1.6		1.1
		-0.6	-1.0	1.0	0.6	
	-0.3		0.5	-0.5		0.3
		-0.2	-0.3	0.3	0.2	
	-0.1		0.2	-0.2		0.1
		-0.1	-0.1	0.1	0.1	
M _o Moments	-57.1	-114.3	114.3	-114.3	114.3	57.1

Part II - Sidesway:



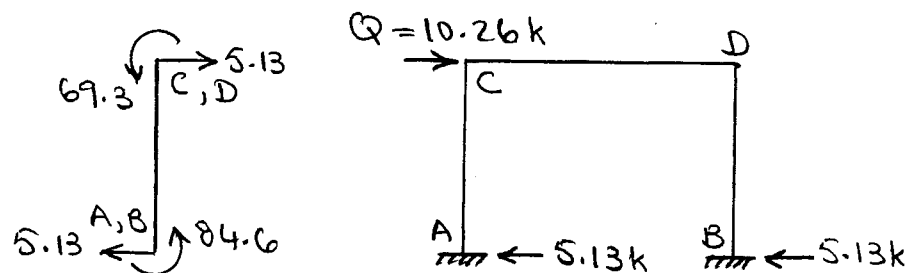
$FEM_{CD} = FEM_{DC} = 0$

$FEM_{AC} = FEM_{CA} = FEM_{BD} = FEM_{DB} = \frac{6EI\Delta'}{(30)^2}$

let $\frac{6EI\Delta'}{(30)^2} = 100 \text{ k-ft}$

17.25 (Contd.)

	AC	CA	CD	DC	DB	BD
DF		0.4	0.6	0.6	0.4	
FEM	100	100	0	0	100	100
	-20	-40	-60	-60	-40	-20
	12	18	18	12		
	6	9	9	6		
	-1.8	-3.6	-5.4	-5.4	-3.6	-1.8
	1.1	1.6	1.6	1.1		
	0.6	0.8	0.8	0.6		
	-0.2	-0.3	-0.5	-0.5	-0.3	-0.2
	0.1	0.2	0.2	0.1		
M_Q Moments	84.6	69.3	-69.3	-69.3	69.3	84.6



Actual member end moments: $M = M_0 + \left(\frac{20}{Q}\right) M_Q$

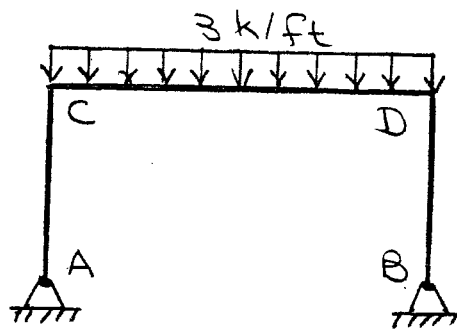
$$M_{AC} = 107.8 \text{ k-ft}; \quad M_{CA} = 20.8 \text{ k-ft};$$

$$M_{CD} = -20.8 \text{ k-ft}; \quad M_{DC} = -249.4 \text{ k-ft};$$

$$M_{DB} = 249.4 \text{ k-ft}; \quad M_{BD} = 222 \text{ k-ft}$$

For reactions, see solution of Problem 13.45

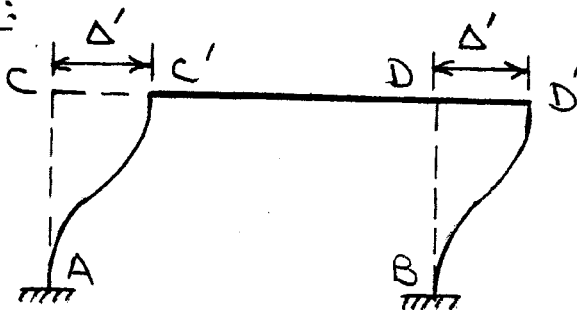
17.26 $K_{AC} = K_{BD} = \frac{3}{4} \left(\frac{I}{15} \right) = \frac{I}{20}$; $K_{CD} = \frac{I}{30}$
Part I - No Sidesway:



	AC	CA	CD	DC	DB	BD
DF		0.6	0.4	0.4	0.6	
FEM	0	0	225	-225	0	0
		-135	-90	90	135	
		-27	45	-45	27	
		-5.4	9	-9	5.4	
		-1.1	1.8	-1.8	1.1	
		-0.2	0.4	-0.4	0.2	
M_0 Moments	0	-168.7	168.7	-168.7	168.7	0

Part II - Sidesway:

$FEM_{CD} = FEM_{DC} = 0$

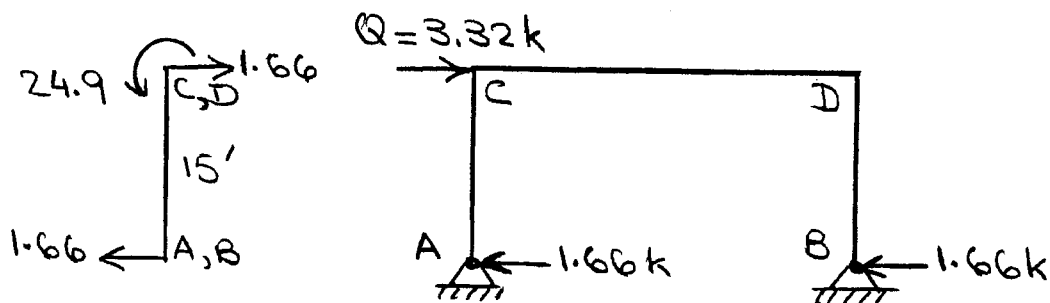


$FEM_{AC} = FEM_{CA} = FEM_{BD} = FEM_{DB} = \frac{6EI\Delta'}{(15)^2}$

Let $\frac{6EI\Delta'}{(15)^2} = 100 \text{ k-ft.}$

17.26 (Contd.)

	AC	CA	CD	DC	DB	BD
DF		0.6	0.4	0.4	0.6	
FEM	100 -100	100 -60	0 -40	0 -40	100 -60	100 -100
		-50 42	-20 28	-20 28	-50 42	
		-8.4	14 -5.6	14 -5.6	-8.4	
		1.7	-2.8 1.1	-2.8 1.1	1.7	
		-0.4	0.6 -0.2	0.6 -0.2	-0.4	
M_Q Moments	0	24.9	-24.9	-24.9	24.9	0



Actual member end moments: $M = M_0 + \left(\frac{40}{Q}\right) M_Q$

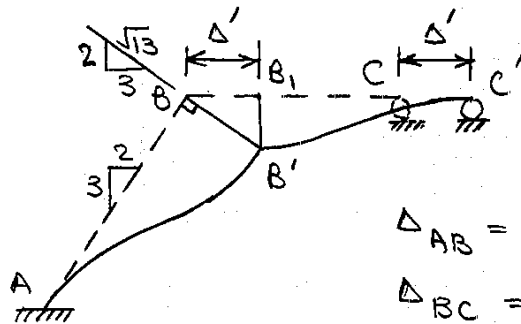
$$\underline{M_{AC} = M_{BD} = 0; \quad M_{CA} = 131.3 \text{ k-ft}}$$

$$\underline{M_{CD} = -131.3 \text{ k-ft}; \quad M_{DC} = -468.7 \text{ k-ft}}$$

$$\underline{M_{DB} = 468.7 \text{ k-ft.}}$$

For reactions, see solution of Problem 13.22.

17.27 $K_{AB} = \frac{I}{14.42}$; $K_{BC} = \frac{3}{4} \left(\frac{I}{12} \right) = \frac{I}{16}$



$$\Delta_{AB} = B'B' = \frac{\sqrt{13}}{3} \Delta' = 1.202 \Delta'$$

$$\Delta_{BC} = B_1B' = \frac{2}{3} \Delta' = 0.667 \Delta'$$

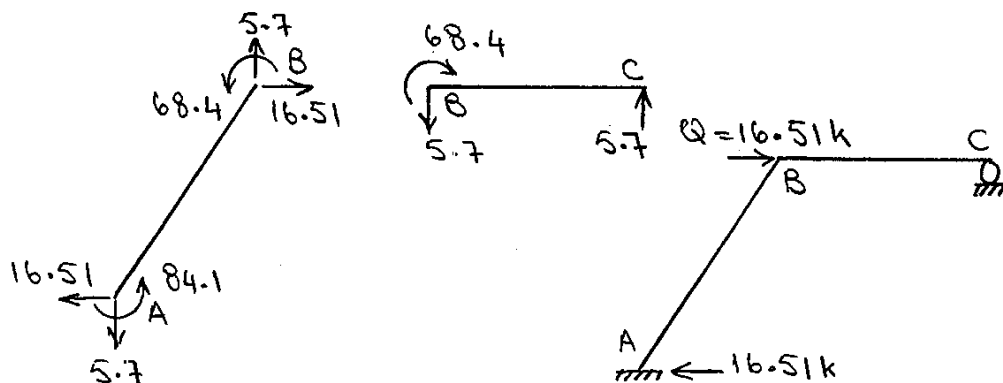
$$FEM_{AB} = FEM_{BA} = \frac{6EI(1.202\Delta')}{(14.42)^2}$$

$$FEM_{BC} = FEM_{CB} = -\frac{6EI(0.667\Delta')}{(12)^2}$$

Let $FEM_{AB} = FEM_{BA} = \frac{6EI(1.202\Delta')}{(14.42)^2} = 100 \text{ k-ft}$
 $EI\Delta' = 2883.2$

$$FEM_{BC} = FEM_{CB} = -80.1 \text{ k-ft}$$

	AB	BA	BC	CB
DF		0.526	0.474	
FEM	100	100	-80.1	-80.1
		-10.5	-9.4	80.1
	-5.3		40.1	
		-21.1	-19	
	-10.6			
M_Q Moments	84.1	68.4	-68.4	0



Actual member end moments : $M = \frac{25}{16.51} M_Q$

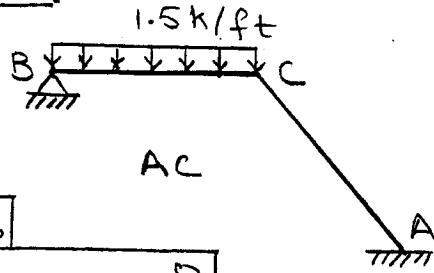
$$M_{AB} = 127.3 \text{ k-ft} ; M_{BA} = 103.6 \text{ k-ft} ;$$

$$M_{BC} = -103.6 \text{ k-ft} ; M_{CB} = 0$$

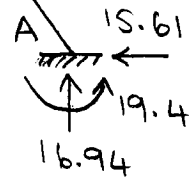
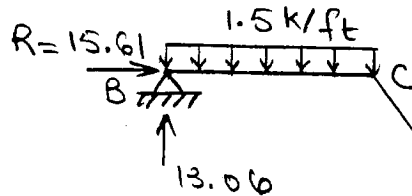
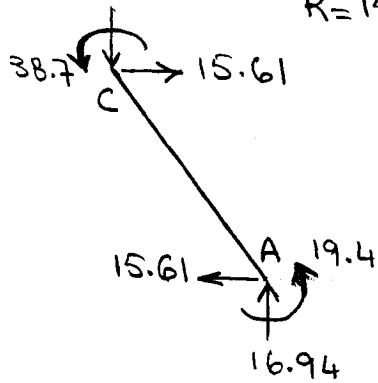
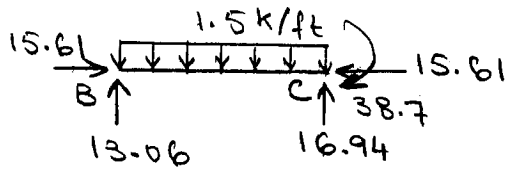
For reactions, see solution of Problem 16.27.

$$17.28 \quad K_{CB} = \frac{3}{4} \left(\frac{I}{25} \right) = \frac{3I}{80}; \quad K_{CA} = \frac{I}{25}$$

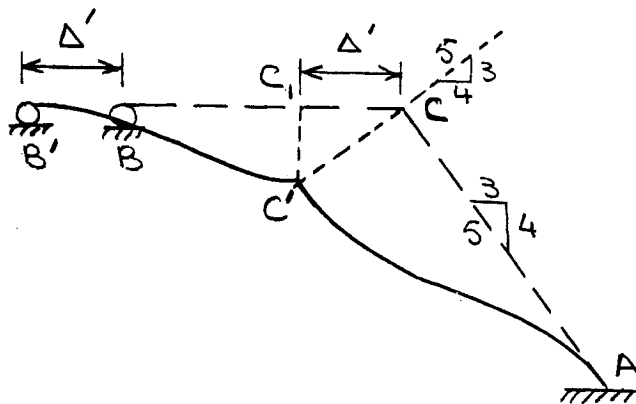
Part I - Sidesway prevented:



	BC	CB	CA	AC
DF		0.484	0.516	
FEM	50	-50	0	0
	-50	24.2	25.8	
		-25		12.9
		12.1	12.9	
				6.5
M_O Moments	0	-38.7	38.7	19.4



Part II - Sideways permitted:



$$\Delta_{BC} = \frac{3}{4} \Delta'$$
$$\Delta_{AC} = \frac{5}{4} \Delta'$$

17.28 (contd.)

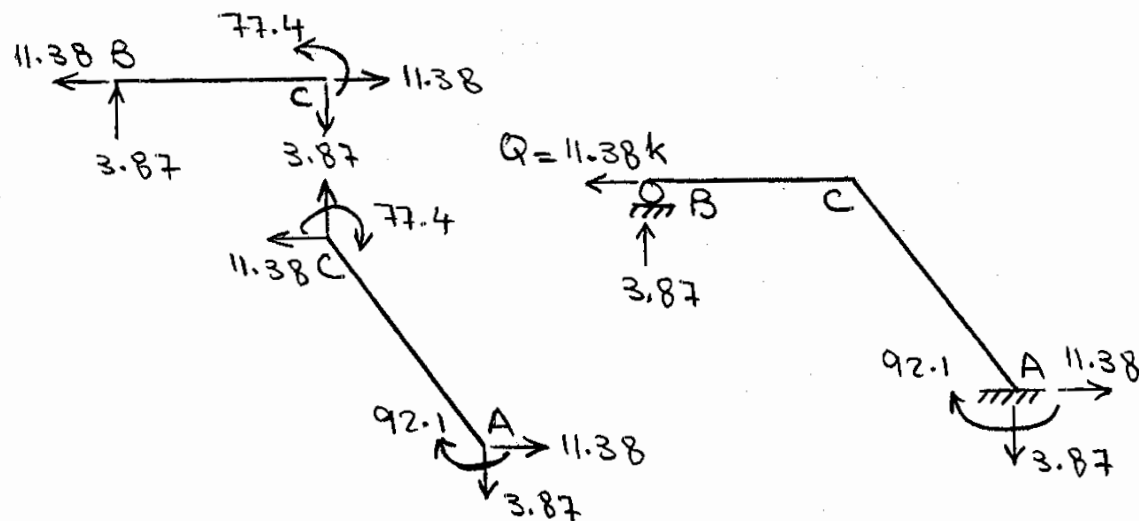
$$FEM_{BC} = FEM_{CB} = \frac{6EI(0.75\Delta')}{(20)^2}$$

$$\text{Let } FEM_{BC} = FEM_{CB} = 100 \text{ k-ft}$$

$$EI\Delta' = 8,888.89$$

$$FEM_{CA} = FEM_{AC} = -\frac{6EI(1.25\Delta')}{(25)^2} = -106.7 \text{ k-ft}$$

	BC	CB	CA	AC
DF		0.484	0.516	
FEM	100 -100	100 3.2	-106.7 3.5	-106.7
		-50 24.2	1.7 25.8	
			12.9	
MQ Moments	0	77.4	-77.4	-92.1

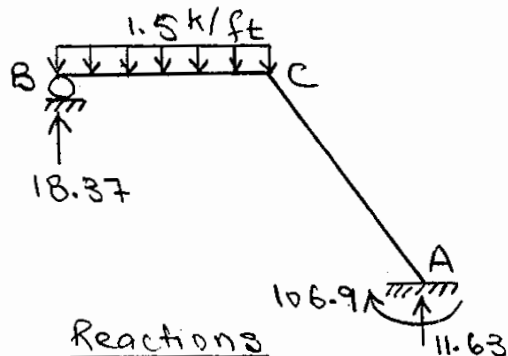


$$\text{Actual Moments: } M = M_0 + \left(\frac{R}{Q}\right) M_Q$$

$$M_{BC} = 0; \quad M_{CB} = 67.5 \text{ k-ft}$$

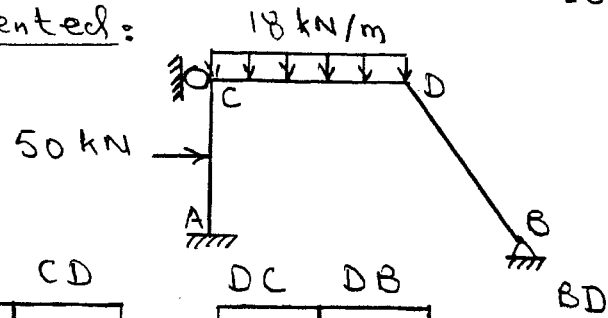
$$M_{CA} = -67.5 \text{ k-ft}$$

$$M_{AC} = -106.9 \text{ k-ft}$$

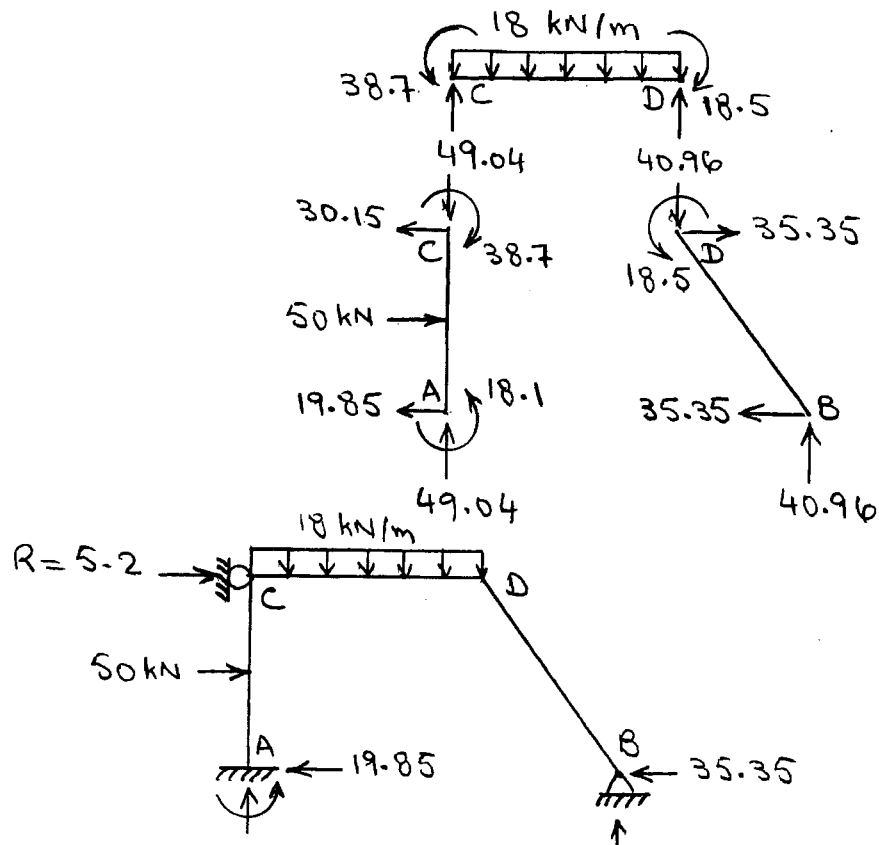


17.29 $K_{AC} = \frac{I}{4}$; $K_{CD} = \frac{I}{5}$; $K_{BD} = \frac{3}{4} \left(\frac{I}{5} \right) = \frac{3I}{20}$

Part I - Sidesway prevented:

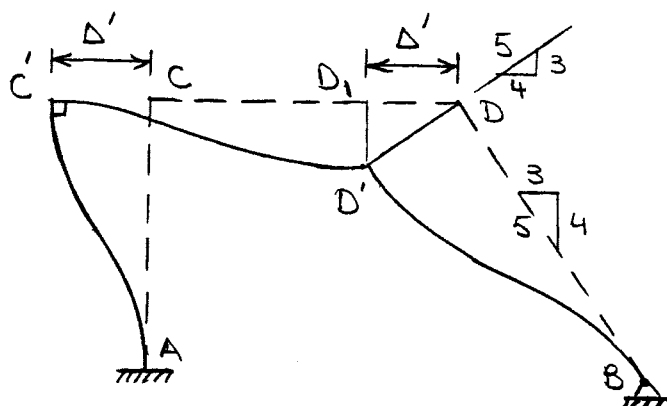


	AC	CA	CD	DC	DB	BD
DF		0.556	0.444	0.571	0.429	
FEM	25	-25	37.5	-37.5	0	0
		-7	-5.5	21.4	16.1	
	-3.5		10.7	-2.8		
		-5.9	-4.8	1.6	1.2	
	-3		0.8	-2.4		
		-0.4	-0.4	1.4	1.0	
	-0.2		0.7	-0.2		
		-0.4	-0.3	0.1	0.1	
	-0.2			-0.2		
				0.1	0.1	
M_0 Moments	18.1	-38.7	38.7	-18.5	18.5	0



17.29 (contd.)

Part II - Sidesway permitted:



$$\Delta_{AC} = \Delta'$$

$$\Delta_{CD} = \frac{3}{4} \Delta' = 0.75 \Delta'$$

$$\Delta_{BD} = \frac{5}{4} \Delta' = 1.25 \Delta'$$

$$FEM_{AC} = FEM_{CA} = -\frac{6EI\Delta'}{(4)^2}$$

$$\text{Let } FEM_{AC} = FEM_{CA} = -100 \text{ kN.m}$$

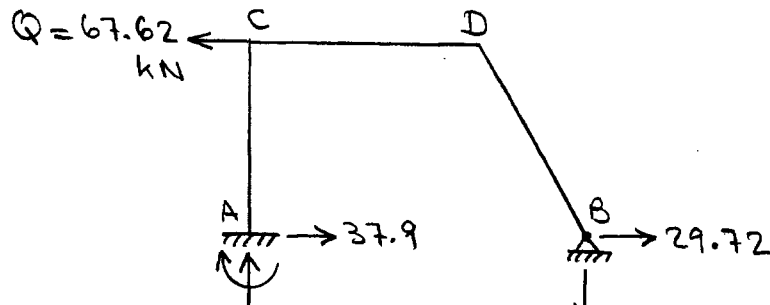
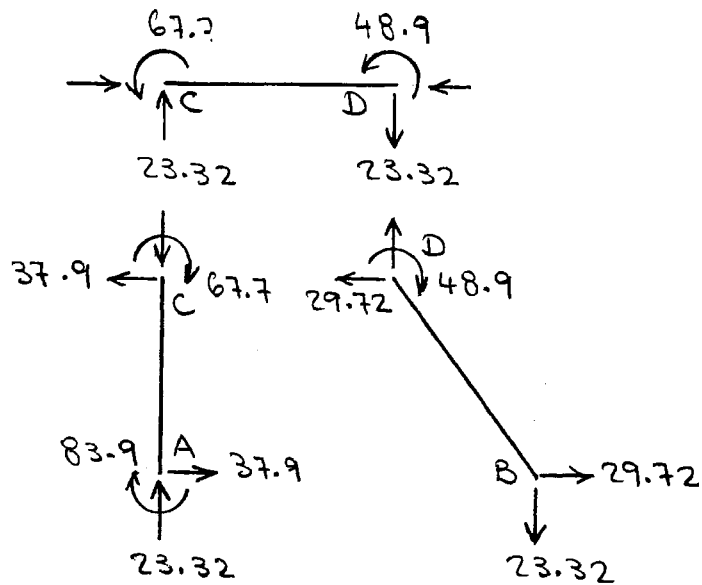
$$EI\Delta' = 266.7$$

$$FEM_{CD} = FEM_{DC} = \frac{6EI(0.75\Delta')}{(5)^2} = 48 \text{ kN.m}$$

$$FEM_{BD} = FEM_{DB} = -\frac{6EI(1.25\Delta')}{(5)^2} = -80 \text{ kN.m}$$

	AC	CA	CD	DC	DB	BD
DF		0.556	0.444	0.571	0.429	
FEM	-100	-100	48	48	-80	-80
		28.9	23.1	18.3	13.7	80
	14.5		9.2	11.6	40	
		-5.1	-4.1	-29.5	-22.1	
	-2.6		-14.8	-2.1		
		8.2	6.6	1.2	0.9	
	4.1		0.6	3.3		
		-0.3	-0.3	-1.9	-1.4	
	-0.2		-1.0	-0.2		
		0.6	0.4	0.1	0.1	
	0.3			0.2		
				-0.1	-0.1	
M _Q Moments	-83.9	-67.7	67.7	48.9	-48.9	0

17.29 (contd.)

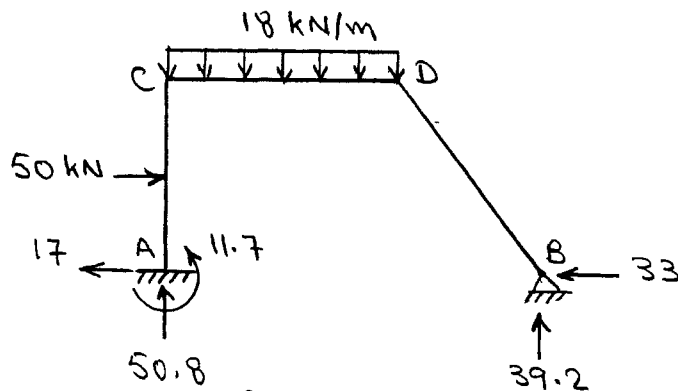


Actual member end moments: $M = M_0 + \left(\frac{R}{Q}\right) M_Q$

$$M_{AC} = 11.7 \text{ kN}\cdot\text{m} ; M_{CA} = -43.9 \text{ kN}\cdot\text{m}$$

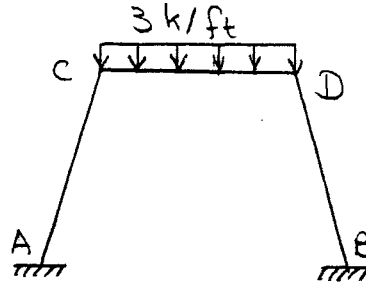
$$M_{CD} = 43.9 \text{ kN}\cdot\text{m} ; M_{DC} = -14.7 \text{ kN}\cdot\text{m}$$

$$M_{DB} = 14.7 \text{ kN}\cdot\text{m} ; M_{BD} = 0$$



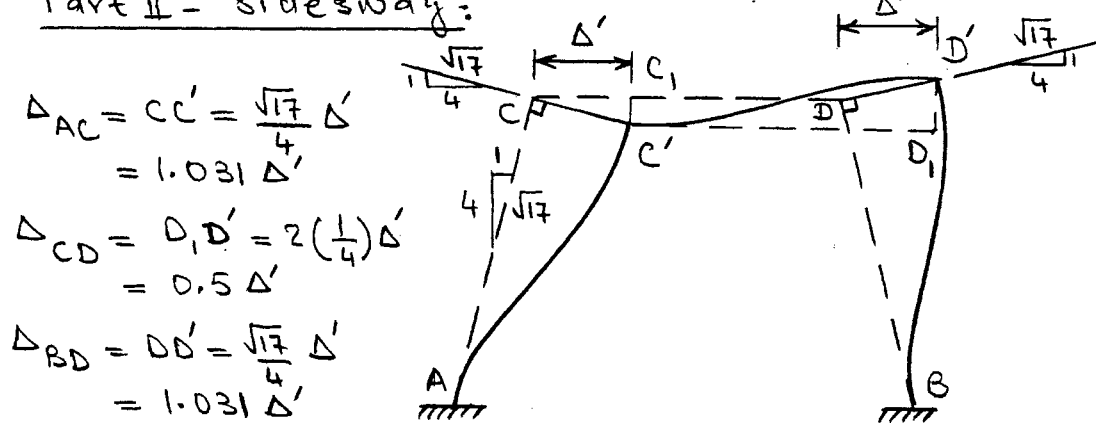
Reactions

Part I - No Sideways: 16.



	CA	CD	DC	DB	BD
DF FEM	0.492	0.508	0.508	0.492	
	0	64	-64	0	0
	-31.5	-32.5	32.5	31.5	
	-15.8	16.3	-16.3		15.8
	-8	-8.3	8.3	8	
	-4	4.2	-4.2		4
	-2.1	-2.1	2.1	2.1	
	-1.1	1.1	-1.1		1.1
	-0.5	-0.6	0.6	0.5	
	-0.3	0.3	-0.3		0.3
	-0.1	-0.2	0.2	0.1	
Moments	-21.2	42.2	-42.2	42.2	21.2

Part II - Sideways:



$$\Delta_{AC} = CC' = \frac{\sqrt{17}}{4} \Delta' = 1.031 \Delta'$$

$$\Delta_{CD} = D_1 D'_1 = 2 \left(\frac{1}{4} \right) \Delta'_1$$

$$= 0.5 \Delta'_1$$

$$\Delta_{BD} = DD' = \frac{\sqrt{17}}{4} \Delta' = 1.031 \Delta'$$

$$FEM_{AC} = FEM_{CA} = \frac{6EI(1.031\Delta')}{(16.49)^2}$$

Let $FEM_{AC} = FEM_{CA} = 100 \text{ k-ft}$

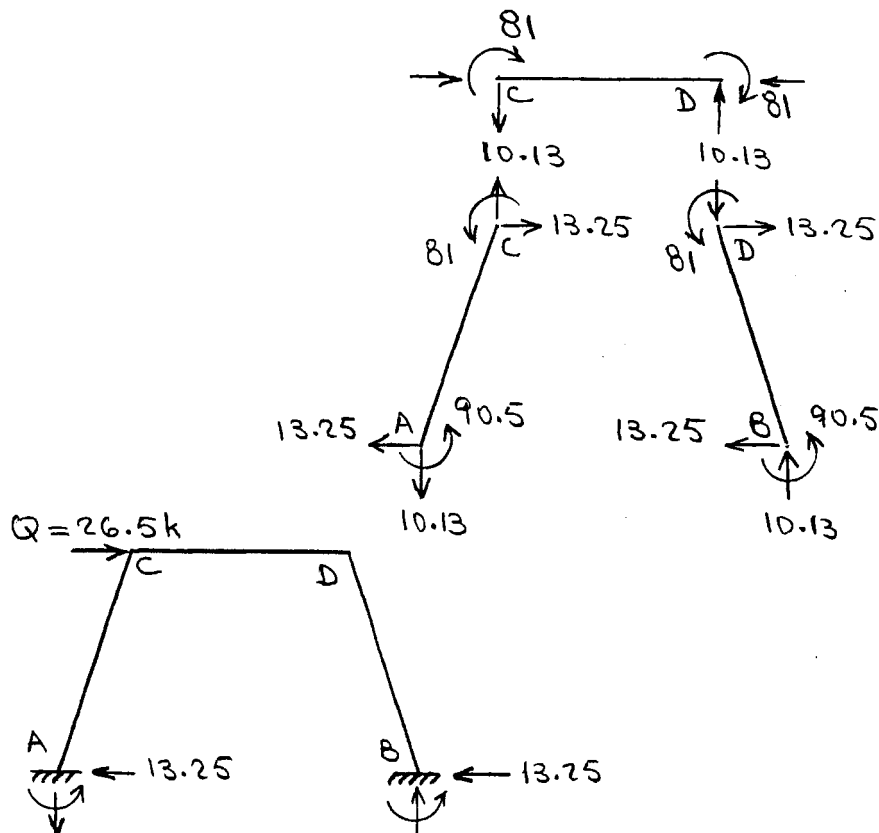
$$EI \Delta' = 4395.7$$

$$FEM_{BD} = FEM_{DB} = \frac{6EI(1.031\Delta')}{(16.49)^2} = 100 \text{ k-ft}$$

$$FEM_{CD} = FEM_{DC} = - \frac{(16.44)^2 \cdot 6EI(0.5\Delta')}{(16)^2} = -51.5 \text{ k-ft.}$$

17.30 (contd.)

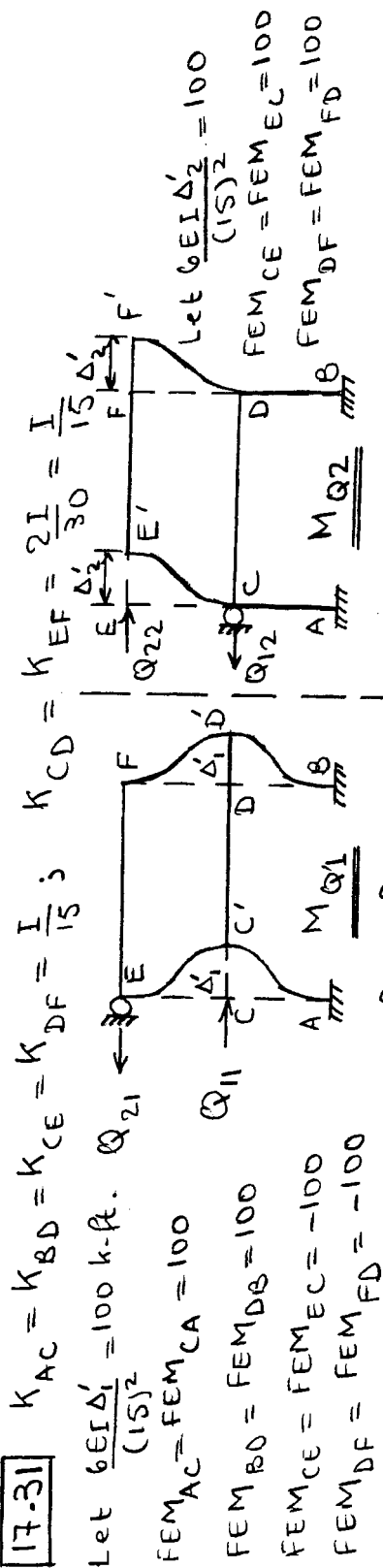
	AC	CA	CD	DC	DB	BD
DF		0.492	0.508	0.508	0.492	
FEM	100	100	-51.5	-51.5	100	100
		-23.9	-24.6	-24.6	-23.9	
	-12	6.1	6.2	6.2	6.1	-12
	3.1	-1.5	-1.6	-1.6	-1.5	3.1
	-0.8	0.4	0.4	0.4	0.4	-0.8
	0.2	-0.1	-0.1	-0.1	-0.1	0.2
M_Q Moments	90.5	81	-81	-81	81	90.5



Actual member end moments: $M = M_0 + \left(\frac{20}{26.5}\right) M_Q$
 $M_{AC} = 47.1 \text{ k-ft}; M_{CA} = 19 \text{ k-ft}; M_{CD} = -19 \text{ k-ft};$
 $M_{DC} = -103.3 \text{ k-ft}; M_{DB} = 103.3 \text{ k-ft}; M_{BD} = 89.5 \text{ k-ft}.$

For reactions, see solution of Problem 16.30.

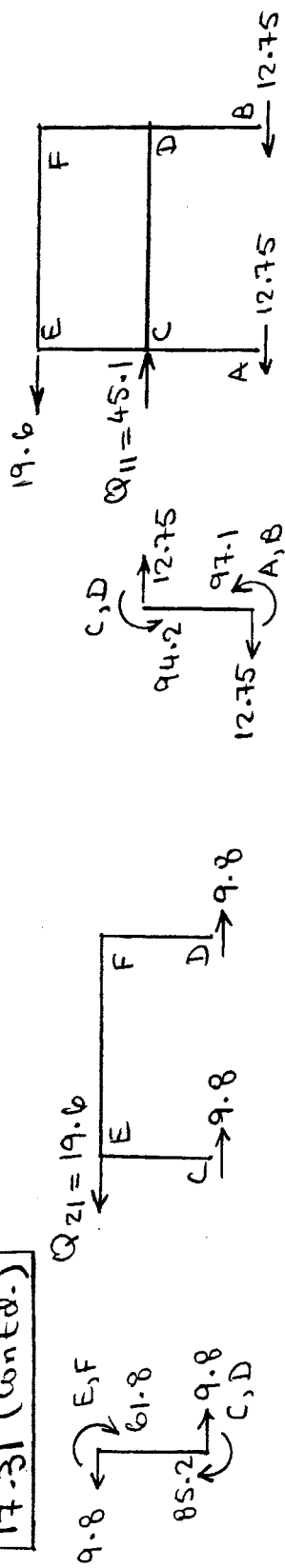
[17.31]



Carry Over

	AC	CA	CD	CE	EC	EF	FE	FD	DF	DC	DB	BD
DF FEM	100	100	0	0	0	0	0	0	0	0	0	100
			25	25	25	25	25	25	25	25	25	
		-8.3	-8.4	-8.3	-12.5	-12.5	-12.5	-12.5	-12.5	-8.4	-8.3	
		-4.2	-4.2	-4.2	-6.3	-6.3	-6.3	-6.3	-6.3	-4.2	-4.2	
		3.5	3.5	3.5	5.3	5.3	5.3	5.3	5.3	3.5	3.5	
		1.8	1.8	1.8	2.7	2.7	2.7	2.7	2.7	1.8	1.8	
		-1.5	-1.5	-1.5	-2.3	-2.3	-2.3	-2.3	-2.3	-1.5	-1.5	
		-0.8	-0.8	-0.8	-1.2	-1.2	-1.2	-1.2	-1.2	-0.8	-0.8	
		0.7	0.7	0.7	1	1	1	1	1	0.7	0.7	
		0.4	0.4	0.4	0.5	0.5	0.5	0.5	0.5	0.4	0.4	
		-0.3	-0.3	-0.3	-0.5	-0.5	-0.5	-0.5	-0.5	-0.3	-0.3	
		-0.2	-0.2	-0.2	-0.3	-0.3	-0.3	-0.3	-0.3	-0.2	-0.2	
		0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3	0.2	0.2	
		0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.1	0.1	
		-0.1	-0.1	-0.1	-0.2	-0.2	-0.2	-0.2	-0.2	-0.1	-0.1	
MQL Moments	97.1	94.2	-8.8	-85.2	-61.8	61.7	61.7	-61.8	-85.2	-8.8	94.2	97.1

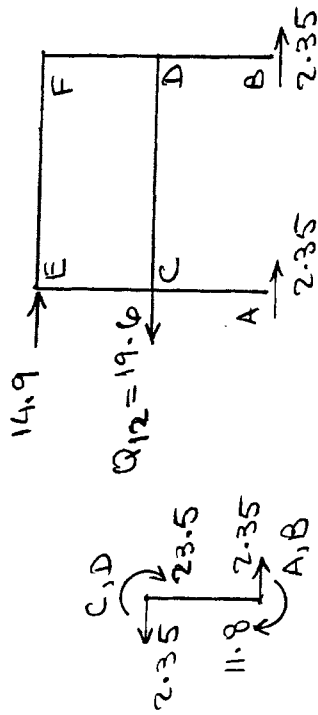
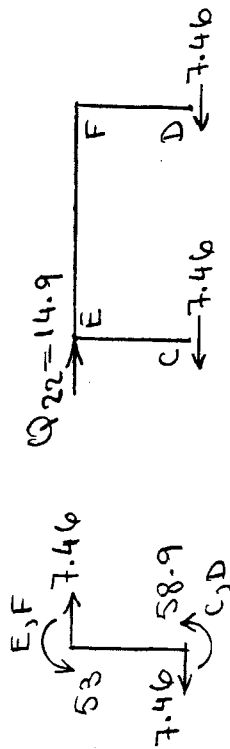
17.31 (contd.)



Carry Over

	AC	CA	CD	CE	EC	EF	FE	FD	DF	DC	DB	BD
	1/3	0	1/3	1/3	1/2	1/2	1/2	1/2	1/3	1/3	1/3	0
DF	0	0	0	100	100	0	0	100	100	0	0	0
FEM	-16.7	-33.3	-33.4	-33.3	-50	-50	-50	-50	-33.3	-33.4	-33.3	0
	13.9	13.9	13.9	20.9	20.9	20.9	20.9	20.9	13.9	13.9	13.9	-16.7
	7	7	7	10.5	10.5	10.5	10.5	7	10.5	7	7	7
	-2.9	-5.8	-2.9	-8.8	-8.8	-8.8	-8.8	-8.8	-5.8	-5.8	-5.8	-2.9
	2.4	2.4	2.4	3.7	3.7	3.7	3.7	3.7	2.4	2.4	2.4	-2.9
	1.2	1.2	1.2	1.9	1.9	1.9	1.9	1.2	1.9	1.2	1.2	1.2
	-0.5	-1	-0.5	-1.6	-1.6	-1.6	-1.6	-1.6	-1	-1	-1	-0.5
	0.2	0.4	0.4	0.7	0.7	0.7	0.7	0.7	0.4	0.4	0.4	0.2
	-0.2	-0.2	-0.2	-0.3	-0.3	-0.3	-0.3	-0.3	-0.2	-0.2	-0.2	-0.2
	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1
	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
M _{Q2} Moments	-11.8	-23.5	-35.4	58.9	53	-52.8	-52.8	53	58.9	-35.4	-23.5	-11.8

17.31 (contd.)



By superimposing the horizontal forces at joints C and E, we write

$$45.1c_1 - 19.6c_2 = 18$$

$$-19.6c_1 + 14.9c_2 = 9$$

By solving these equations, we obtain $c_1 = 1.545$ and $c_2 = 2.636$.

Member end moments: $M = c_1 M_{Q1} + c_2 M_{Q2}$

$$\underline{M_{AC} = M_{BD} = 119 \text{ k-ft}; \quad M_{CA} = M_{DB} = 83.5 \text{ k-ft}}$$

$$\underline{M_{CD} = M_{DC} = -106.8 \text{ k-ft}; \quad M_{CE} = M_{DF} = 23.5 \text{ k-ft}}$$

$$\underline{M_{EC} = M_{FD} = 44 \text{ k-ft}; \quad M_{EF} = M_{FE} = -44 \text{ k-ft.}}$$

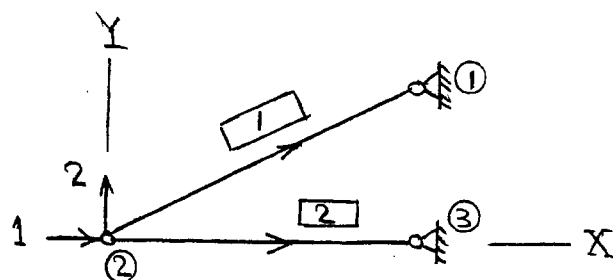
For reactions, see solution of Problem 16.31.

Chapter Eighteen

Introduction to Matrix Structural Analysis

CHAPTER 18

18.1



Member 1: $L = 11.18 \text{ ft}$; $\cos \theta = 0.894$; $\sin \theta = 0.447$

$$[K_1] = EA \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0.0715 & 0.0357 & -0.0715 & -0.0357 \\ & 0.0179 & -0.0357 & -0.0179 \\ \text{Sym.} & & 0.0715 & 0.0357 \\ & & & 0.0179 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix}$$

Member 2: $L = 10 \text{ ft}$; $\cos \theta = 1$; $\sin \theta = 0$

$$[K_2] = EA \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0.1 & 0 & -0.1 & 0 \\ & 0 & 0 & 0 \\ \text{Sym.} & & 0.1 & 0 \\ & & & 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix}$$

Structure stiffness matrix:

$$[S] = EA \begin{bmatrix} 1 & 2 \\ 0.1715 & 0.0357 \\ 0.0357 & 0.0179 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

Joint load vector: $\{P\} = \begin{bmatrix} 0 \\ -24 \end{bmatrix} \text{ k}$

Joint displacements: By solving the equations

$$\{P\} = [S]\{d\}, \text{ we obtain: } \{d\} = \frac{1}{EA} \begin{bmatrix} 477.26 \\ -2292.3 \end{bmatrix}$$

18.1 (contd.) Member forces:

$$\{v_1\} = \frac{1}{EA} \begin{bmatrix} 477.26 \\ -2292.3 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix}$$

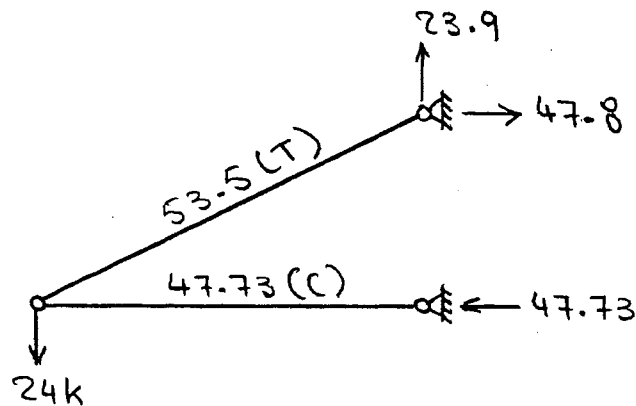
$$\{u_1\} = [T_1] \{v_1\} = \frac{1}{EA} \begin{bmatrix} -598 \\ 0 \end{bmatrix}$$

$$\{Q_1\} = [k_1] \{u_1\} = \begin{bmatrix} -53.5 \\ 53.5 \end{bmatrix} k ; \{F_1\} = [T_1]^T \{Q_1\} = \begin{bmatrix} -47.8 \\ -23.9 \\ 47.8 \\ 23.9 \end{bmatrix} k$$

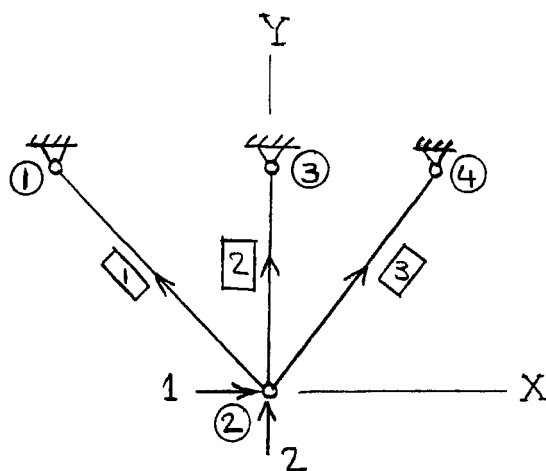
$$\{v_2\} = \frac{1}{EA} \begin{bmatrix} 477.26 \\ -2292.3 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix}$$

$$\{u_2\} = [T_2] \{v_2\} = \frac{1}{EA} \begin{bmatrix} 477.26 \\ 0 \end{bmatrix}$$

$$\{Q_2\} = [k_2] \{u_2\} = \begin{bmatrix} 47.73 \\ -47.73 \end{bmatrix} k ; \{F_2\} = [T_2]^T \{Q_2\} = \begin{bmatrix} 47.73 \\ 0 \\ -47.73 \\ 0 \end{bmatrix} k$$



18.2.



Member 1: $L = 271.53 \text{ in.}; \cos \theta = -0.707; \sin \theta = 0.707$

$$[K_1] = 147.3 \begin{bmatrix} 1 & 2 & 0 & 0 \\ 1 & -1 & -1 & 1 \\ & 1 & 1 & -1 \\ \text{sym.} & & 1 & -1 \\ & & & 1 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix}$$

Member 2: $L = 192 \text{ in.}; \cos \theta = 0; \sin \theta = 1.$

$$[K_2] = 312.5 \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ & 1 & 0 & -1 \\ & & 0 & 0 \\ & & & 1 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix}$$

Member 3: $L = 240 \text{ in.}; \cos \theta = 0.6; \sin \theta = 0.8$

$$[K_3] = 333.3 \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0.36 & 0.48 & -0.36 & -0.48 \\ & 0.64 & -0.48 & -0.64 \\ & & 0.36 & 0.48 \\ & & & 0.64 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix}$$

Structure stiffness matrix:

$$[S] = \begin{bmatrix} 1 & 2 \\ 267.3 & 12.7 \\ 12.7 & 673.1 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

Joint load vector: $\{P\} = \begin{bmatrix} 20 \\ -50 \end{bmatrix} k$

18.2 (contd.) Joint displacements: By solving the equations $\{P\} = [S]\{d\}$, we obtain:

$$\{d\} = \begin{bmatrix} 0.07842 \\ -0.07576 \end{bmatrix} \text{ in.}$$

Member forces:

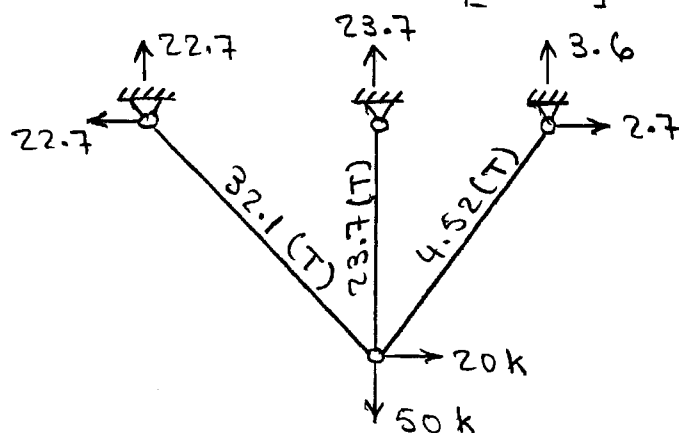
$$\{v_1\} = \{v_2\} = \{v_3\} = \begin{bmatrix} 0.07842 \\ -0.07576 \\ 0 \\ 0 \end{bmatrix} \text{ in.}$$

$$\{u_1\} = [T_1]\{v_1\} = \begin{bmatrix} -0.109 \\ 0 \end{bmatrix} \text{ in.; } \{Q_1\} = [k_1]\{u_1\} = \begin{bmatrix} -32.1 \\ 32.1 \end{bmatrix} \text{ k}$$

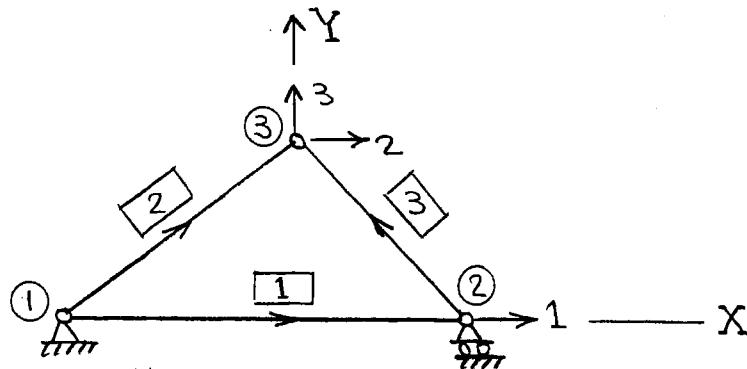
$$\{F_1\} = [T]^T\{Q_1\} = \begin{bmatrix} 22.7 \\ -22.7 \\ -22.7 \\ 22.7 \end{bmatrix} \text{ k} \quad \{u_2\} = \begin{bmatrix} -0.07576 \\ 0 \end{bmatrix} \text{ in.}$$

$$\{Q_2\} = \begin{bmatrix} -23.7 \\ 23.7 \end{bmatrix} \text{ k; } \{F_2\} = \begin{bmatrix} 0 \\ -23.7 \\ 0 \\ 23.7 \end{bmatrix} \text{ k}$$

$$\{u_3\} = \begin{bmatrix} -0.01356 \\ 0 \end{bmatrix} \text{ in.; } \{Q_3\} = \begin{bmatrix} -4.52 \\ 4.52 \end{bmatrix} \text{ k; } \{F_3\} = \begin{bmatrix} -2.71 \\ -3.61 \\ 2.71 \\ 3.61 \end{bmatrix} \text{ k}$$



18.3



Member 1: $L = 14 \text{ m}$; $\cos \theta = 1$; $\sin \theta = 0$

$$[K_1] = \frac{EA}{14} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 1 \\ 0 \end{matrix}$$

Member 2: $L = 10 \text{ m}$; $\cos \theta = 0.8$; $\sin \theta = 0.6$

$$[K_2] = \frac{EA}{10} \begin{bmatrix} 0.64 & 0.48 & -0.64 & -0.48 \\ 0.48 & 0.36 & -0.48 & -0.36 \\ -0.64 & -0.48 & 0.64 & 0.48 \\ -0.48 & -0.36 & 0.48 & 0.36 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 2 \\ 3 \end{matrix}$$

Member 3: $L = 8.485 \text{ m}$; $\cos \theta = -0.707$; $\sin \theta = 0.707$

$$[K_3] = \frac{EA}{16.97} \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{matrix} 1 \\ 0 \\ 2 \\ 3 \end{matrix}$$

Structure stiffness matrix:

$$[S] = EA \begin{bmatrix} 0.1304 & -0.05893 & 0.05893 \\ -0.05893 & 0.1229 & -0.01093 \\ 0.05893 & -0.01093 & 0.09493 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

Joint load vector:

$$\{P\} = \begin{bmatrix} 0 \\ 80 \\ -120 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \text{ kN}$$

18.3 (contd.) Joint displacements: By solving the equations $\{P\} = [S]\{d\}$, we obtain:

$$\{d\} = \frac{1}{EA} \begin{bmatrix} 1439 \\ 1161 \\ -2024 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

Member forces:

$$\{v_1\} = \frac{1}{EA} \begin{bmatrix} 0 \\ 0 \\ 1439 \\ 0 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 1 \\ 0 \end{matrix} \quad \{u_1\} = [T_1]\{v_1\} = \frac{1}{EA} \begin{bmatrix} 0 \\ 1439 \end{bmatrix}$$

$$\{Q_1\} = [k_1]\{u_1\} = \begin{bmatrix} -102.8 \\ 102.8 \end{bmatrix} \text{ kN}$$

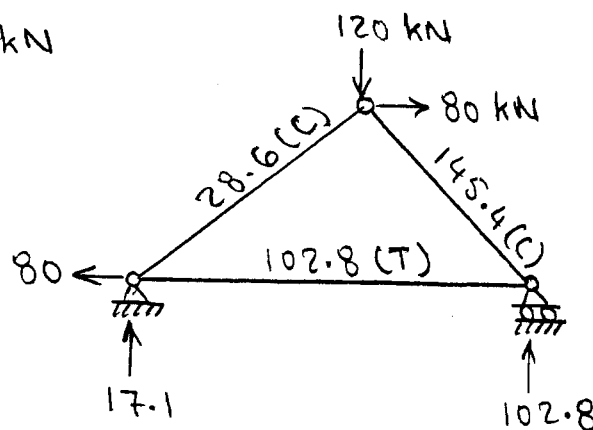
$$\{F_1\} = \begin{bmatrix} -102.8 \\ 0 \\ 102.8 \\ 0 \end{bmatrix} \text{ kN}; \quad \{v_2\} = \frac{1}{EA} \begin{bmatrix} 0 \\ 0 \\ 1161 \\ -2024 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 2 \\ 3 \end{matrix}$$

$$\{u_2\} = \frac{1}{EA} \begin{bmatrix} 0 \\ -285.6 \end{bmatrix}; \quad \{Q_2\} = \begin{bmatrix} 28.56 \\ -28.56 \end{bmatrix} \text{ kN}$$

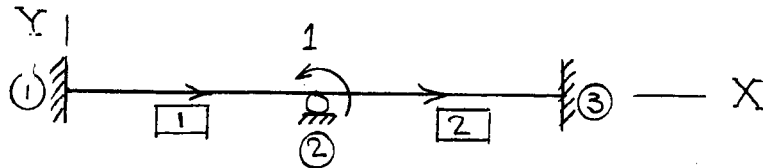
$$\{F_2\} = \begin{bmatrix} 22.85 \\ 17.14 \\ -22.85 \\ -17.14 \end{bmatrix} \text{ kN}; \quad \{v_3\} = \frac{1}{EA} \begin{bmatrix} 1439 \\ 0 \\ 1161 \\ -2024 \end{bmatrix} \begin{matrix} 1 \\ 0 \\ 2 \\ 3 \end{matrix}$$

$$\{u_3\} = \frac{1}{EA} \begin{bmatrix} -1017.4 \\ -2251.8 \end{bmatrix}; \quad \{Q_3\} = \begin{bmatrix} 145.4 \\ -145.4 \end{bmatrix} \text{ kN}$$

$$\{F_3\} = \begin{bmatrix} -102.8 \\ 102.8 \\ 102.8 \\ -102.8 \end{bmatrix} \text{ kN}$$



18.4



$$\begin{array}{l} \text{Member 2} \rightarrow \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \\ \text{Member 1} \rightarrow \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$[K_1] = [K_2] = \frac{EI}{L^3} \begin{bmatrix} 12 & 180 & -12 & 180 \\ & 3600 & -180 & 1800 \\ \text{sym.} & & 12 & -180 \\ & & & 3600 \end{bmatrix} \begin{array}{l} 0 \\ 0 \\ 0 \\ 1 \end{array}$$

$$\{F_{F1}\} = \{Q_{F1}\} = \begin{bmatrix} 4.67 \\ 40 \\ 13.33 \\ -80 \end{bmatrix} \begin{array}{l} 0 \\ 0 \\ 0 \\ 1 \end{array} \quad \{F_{F2}\} = \{Q_{F2}\} = \begin{bmatrix} 5 \\ 37.5 \\ 5 \\ -37.5 \end{bmatrix} \begin{array}{l} 0 \\ 1 \\ 0 \\ 0 \end{array}$$

$$[S] = \frac{EI}{L^3} [7200] 1; \quad \{P_F\} = [-42.5] 1$$

Joint displacements: $[S]\{d\} = -\{P_F\}$

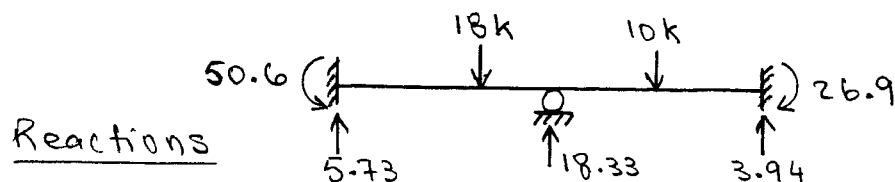
$$\{d\} = \frac{L^3}{EI} [0.005903]$$

Member forces:

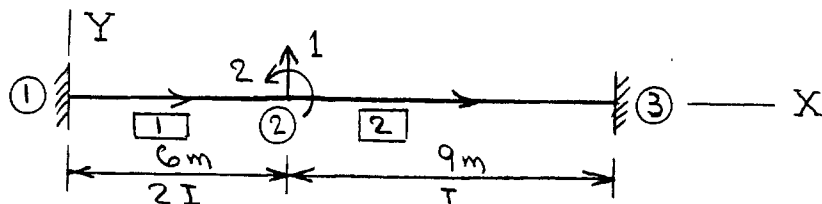
$$\{u_1\} = \{v_1\} = \frac{L^3}{EI} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.005903 \end{bmatrix} \begin{array}{l} 0 \\ 0 \\ 0 \\ 1 \end{array}; \quad \{u_2\} = \{v_2\} = \frac{L^3}{EI} \begin{bmatrix} 0 \\ 0.005903 \\ 0 \\ 0 \end{bmatrix} \begin{array}{l} 0 \\ 1 \\ 0 \\ 0 \end{array}$$

$$\{F_1\} = \{Q_1\} = [k_1]\{u_1\} + \{Q_{F1}\} = \begin{bmatrix} 5.73 \text{ k} \\ 50.6 \text{ k-ft} \\ 12.27 \text{ k} \\ -58.8 \text{ k-ft} \end{bmatrix}$$

$$\{F_2\} = \{Q_2\} = \begin{bmatrix} 6.06 \text{ k} \\ 58.8 \text{ k-ft} \\ 3.94 \text{ k} \\ -26.9 \text{ k-ft} \end{bmatrix}$$



18.5



Member 1:

$$[K_1] = EI \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0.111 & 0.333 & -0.111 & 0.333 \\ 1.333 & -0.333 & 0.111 & -0.333 \\ \text{Sym.} & & & 1.333 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 1 \\ 2 \end{matrix}$$

Member 2:

$$[K_2] = EI \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0.0165 & 0.0741 & -0.0165 & 0.0741 \\ 0.444 & -0.0741 & 0.222 & 0 \\ \text{Sym.} & & 0.0165 & -0.0741 \\ & & & 0.444 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix}$$

Structure stiffness matrix:

$$[S] = EI \begin{bmatrix} 1 & 2 \\ 0.1275 & -0.2589 \\ -0.2589 & 1.777 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

Joint load vector:

$$\{P\} = \begin{bmatrix} -150 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

Joint displacements: By solving the equations

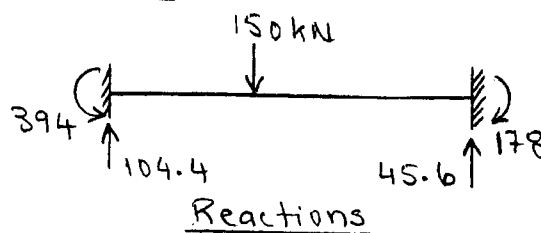
$$\{P\} = [S]\{d\}, \text{ we obtain: } \{d\} = \frac{1}{EI} \begin{bmatrix} -1671 \\ -243.4 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

Member forces:

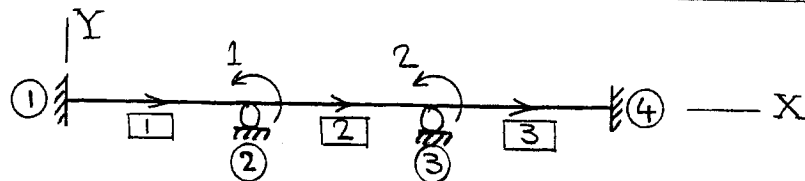
$$\{u_1\} = \{v_1\} = \frac{1}{EI} \begin{bmatrix} 0 \\ 0 \\ -1671 \\ -243.4 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 1 \\ 2 \end{matrix}; \quad \{u_2\} = \{v_2\} = \frac{1}{EI} \begin{bmatrix} -1671 \\ -243.4 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix}$$

$$\{F_1\} = \{Q_1\} = [k_1]\{u_1\} = \begin{bmatrix} 104.4 \text{ kN} \\ 394 \text{ kN.m} \\ -104.4 \text{ kN} \\ 232 \text{ kN.m} \end{bmatrix}$$

$$\{F_2\} = \{Q_2\} = \begin{bmatrix} -45.6 \text{ kN} \\ -232 \text{ kN.m} \\ 45.6 \text{ kN} \\ -178 \text{ kN.m} \end{bmatrix}$$



18.6



$$\begin{array}{l}
 \text{Member 3} \rightarrow \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 \text{Member 2} \rightarrow \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 \text{Member 1} \rightarrow \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{array}$$

$$[k_1] = [k_2] = [k_3] = \frac{EI}{L^3} \begin{bmatrix} 12 & 108 & -12 & 108 \\ 108 & 1296 & -108 & 648 \\ -12 & -108 & 12 & -108 \\ 108 & 648 & -108 & 1296 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \end{matrix}$$

$$\{F_{P1}\} = \{Q_{P1}\} = \begin{bmatrix} 27 \\ 81 \\ 27 \\ -81 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \end{matrix}; \quad \{F_{P3}\} = \{Q_{P3}\} = \begin{bmatrix} 13.5 \\ 40.5 \\ 13.5 \\ -40.5 \end{bmatrix} \begin{matrix} 0 \\ 2 \\ 0 \\ 0 \end{matrix}$$

Structure stiffness matrix:

$$[S] = \frac{EI}{L^3} \begin{bmatrix} 2592 & 648 \\ 648 & 2592 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}; \quad \{P_f\} = \begin{bmatrix} -81 \\ 40.5 \end{bmatrix}$$

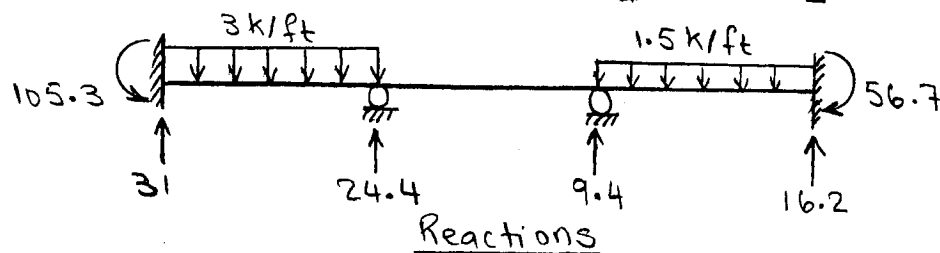
Joint displacements: By solving the equations

$$-\{P_f\} = [S]\{d\}, \text{ we obtain: } \{d\} = \frac{L^3}{EI} \begin{bmatrix} 0.0375 \\ -0.025 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

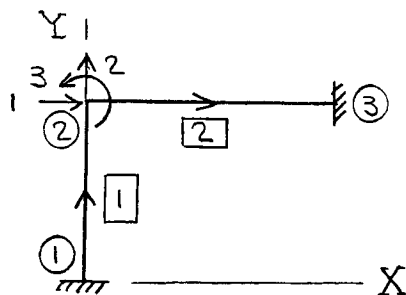
Member forces:

$$\{u_1\} = \frac{L^3}{EI} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.0375 \end{bmatrix}; \quad \{u_2\} = \frac{L^3}{EI} \begin{bmatrix} 0 \\ 0.0375 \\ 0 \\ -0.025 \end{bmatrix}; \quad \{u_3\} = \frac{L^3}{EI} \begin{bmatrix} 0 \\ -0.025 \\ 0 \\ 0 \end{bmatrix}$$

$$\{Q_1\} = \begin{bmatrix} 31 \text{ k} \\ 105.3 \text{ k-ft} \\ 23 \text{ k} \\ -32.4 \text{ k-ft} \end{bmatrix}; \quad \{Q_2\} = \begin{bmatrix} 1.4 \text{ k} \\ 32.4 \text{ k-ft} \\ -1.4 \text{ k} \\ -8.1 \text{ k-ft} \end{bmatrix}; \quad \{Q_3\} = \begin{bmatrix} 10.8 \text{ k} \\ 8.1 \text{ k-ft} \\ 16.2 \text{ k} \\ -56.7 \text{ k-ft} \end{bmatrix}$$



18.7



Member 1: $L = 15 \text{ ft}$; $\cos \theta = 0$; $\sin \theta = 1$

$$[k_1] = EI \times 10^3 \begin{bmatrix} 115.2 & 0 & 0 & -115.2 & 0 & 0 \\ 0 & 3.556 & 26.67 & 0 & -3.556 & 26.67 \\ 0 & 26.67 & 133.3 & 0 & -26.67 & 133.3 \\ \text{Sym.} & & & 115.2 & 0 & 0 \\ & & & 0 & 3.556 & -26.67 \\ & & & & & 26.67 \end{bmatrix}$$

$$\{Q_{f1}\} = \begin{bmatrix} 0 \\ 15 \\ 37.5 \\ 0 \\ 15 \\ -37.5 \end{bmatrix}; [T_1] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}; \{F_{f1}\} = \begin{bmatrix} -15 \\ 0 \\ 37.5 \\ -15 \\ 0 \\ -37.5 \end{bmatrix}$$

$$[K_1] = [T_1]^T [k_1] [T_1] = EI \times 10^3 \begin{bmatrix} 0 & 0 & 0 & 1 & 2 & 3 \\ 3.556 & 0 & -26.67 & -3.556 & 0 & -26.67 \\ 0 & 115.2 & 0 & 0 & -115.2 & 0 \\ 0 & 26.67 & 26.67 & 0 & 133.3 & 0 \\ \text{Sym.} & & & 3.556 & 0 & 26.67 \\ & & & 0 & 115.2 & 0 \\ & & & & & 26.67 \end{bmatrix}$$

Member 2: $L = 20 \text{ ft}$; $\cos \theta = 1$; $\sin \theta = 0$

$$[k_2] = [K_2] = EI \times 10^3 \begin{bmatrix} 1 & 2 & 3 & 0 & 0 & 0 \\ 86.4 & 0 & 0 & -86.4 & 0 & 0 \\ 0 & 1.5 & 15 & 0 & -1.5 & 15 \\ 0 & 15 & 200 & 0 & -15 & 100 \\ \text{Sym.} & & & 86.4 & 0 & 0 \\ & & & 0 & 1.5 & -15 \\ & & & & & 200 \end{bmatrix}$$

$$\{F_{f2}\} = \{Q_{f2}\} = \begin{bmatrix} 0 \\ 10 \\ 50 \\ 0 \\ 10 \\ -50 \end{bmatrix}$$

18.7 (Contd.) Structure matrices:

$$[S] = EI \times 10^{-3} \begin{bmatrix} 89.96 & 0 & 26.67 \\ 0 & 116.7 & 15 \\ 26.67 & 15 & 466.7 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}; \{P_P\} = \begin{bmatrix} -15 \\ 10 \\ 12.5 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

Joint displacements: By solving the equations

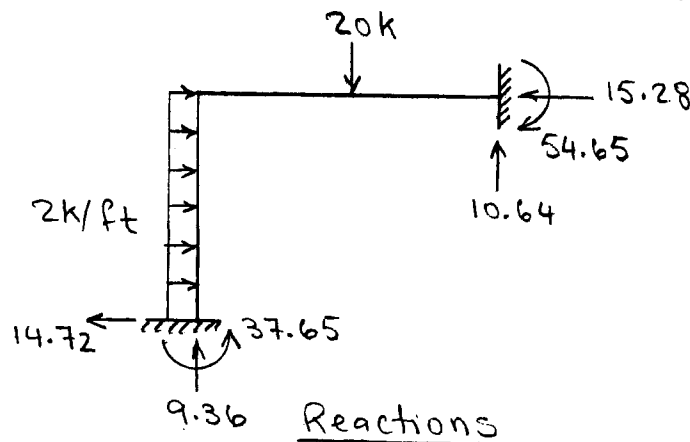
$$-\{P_P\} = [S]\{d\}, \text{ we obtain: } \{d\} = \frac{1}{EI \times 10^{-3}} \begin{bmatrix} 0.1769 \\ -0.08129 \\ -0.03427 \end{bmatrix}$$

Member forces:

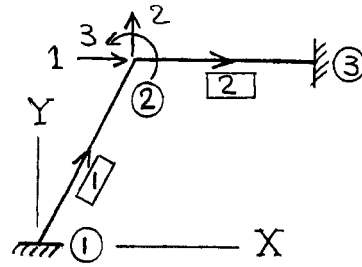
$$\{u_1\} = \frac{1}{EI \times 10^{-3}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.1769 \\ -0.08129 \\ -0.03427 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{matrix}; \{u_2\} = \frac{1}{EI \times 10^{-3}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.08129 \\ -0.1769 \\ -0.03427 \end{bmatrix}$$

$$\{Q_1\} = \begin{bmatrix} 9.36 \text{ k} \\ 14.72 \text{ k} \\ 37.65 \text{ k-ft} \\ -9.36 \text{ k} \\ 15.29 \text{ k} \\ -41.92 \text{ k-ft} \end{bmatrix}; \{F_1\} = \begin{bmatrix} -14.72 \text{ k} \\ 9.36 \text{ k} \\ 37.65 \text{ k-ft} \\ -15.29 \text{ k} \\ -9.36 \text{ k} \\ -41.92 \text{ k-ft} \end{bmatrix}$$

$$\{u_2\} = \{u_2\} = \frac{1}{EI \times 10^{-3}} \begin{bmatrix} 0.1769 \\ -0.08129 \\ -0.03427 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{matrix}; \{F_2\} = \{Q_2\} = \begin{bmatrix} 15.28 \text{ k} \\ 9.36 \text{ k} \\ 41.92 \text{ k-ft} \\ -15.28 \text{ k} \\ 10.64 \text{ k} \\ -54.65 \text{ k-ft} \end{bmatrix}$$



18.8



Member 1: $L = 11.18 \text{ m}$; $\cos \theta = 0.447$; $\sin \theta = 0.894$

$$[k_1] = \begin{bmatrix} 71556 & 0 & 0 & -71556 & 0 & 0 \\ 0 & 687 & 3840 & 0 & -687 & 3840 \\ 0 & 3840 & 28623 & 0 & -3840 & 14311 \\ \text{Sym.} & & & 71556 & 0 & 0 \\ 0 & 0 & 0 & 0 & 687 & -3840 \\ 0 & 0 & 0 & 0 & -3840 & 28623 \end{bmatrix}$$

$$[T_1] = \begin{bmatrix} 0.447 & 0.894 & 0 & 0 & 0 & 0 \\ -0.894 & 0.447 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.447 & 0.894 & 0 \\ 0 & 0 & 0 & -0.894 & 0.447 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[K_1] = [T_1]^T [k_1] [T_1] = \begin{bmatrix} 0 & 0 & 0 & 1 & 2 & 3 \\ 14847 & 28320 & -3433 & -14847 & -28320 & -3433 \\ 0 & 57327 & 1716 & -28320 & -57327 & 1716 \\ 28623 & 3433 & -1716 & 14311 & 0 & 0 \\ \text{Sym.} & & & 14847 & 28320 & 3433 \\ 0 & 0 & 0 & 0 & 57327 & -1716 \\ 0 & 0 & 0 & 0 & -1716 & 28623 \end{bmatrix}$$

Member 2: $L = 10 \text{ m}$; $\cos \theta = 1$; $\sin \theta = 0$

$$[k_2] = [K_2] = \begin{bmatrix} 1 & 2 & 3 & 0 & 0 & 0 \\ 80000 & 0 & 0 & -80000 & 0 & 0 \\ 0 & 960 & 4800 & 0 & -960 & 4800 \\ 0 & 4800 & 32000 & 0 & -4800 & 16000 \\ \text{Sym.} & & & 80000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 960 & -4800 \\ 0 & 0 & 0 & 0 & -4800 & 32000 \end{bmatrix}$$

Structure matrices:

$$[S] = \begin{bmatrix} 1 & 2 & 3 \\ 94847 & 28320 & 3433 \\ 28320 & 58287 & 3084 \\ 3433 & 3084 & 60623 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}; \quad \{P\} = \begin{bmatrix} 0 \\ 0 \\ -150 \end{bmatrix}$$

18.8 (contd.)

Joint displacements: By solving the equations $\{P\} = [S]\{d\}$, we obtain:

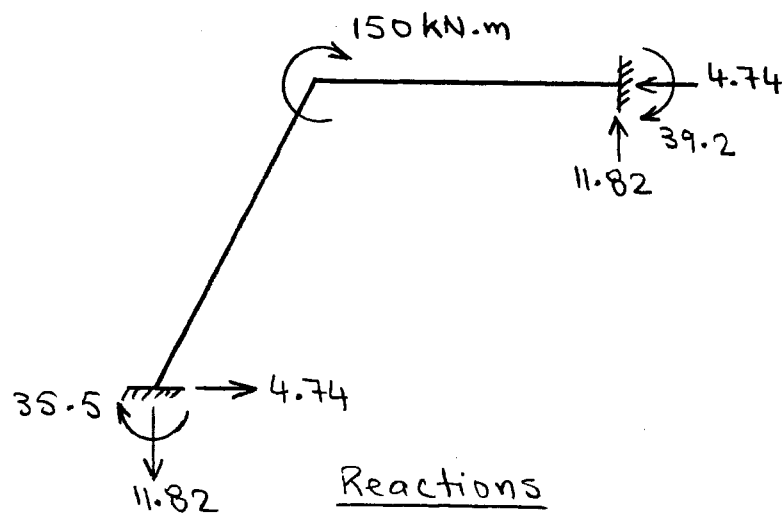
$$\{d\} = 10^{-4} \begin{bmatrix} 0.5924 \text{ m} \\ 1.0259 \text{ m} \\ -24.829 \text{ rad} \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix}$$

Member forces:

$$\{u_1\} = 10^{-4} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.5924 \\ 1.0259 \\ -24.829 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{matrix} ; \{u_3\} = 10^{-4} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1.182 \\ -0.07103 \\ -24.829 \end{bmatrix}$$

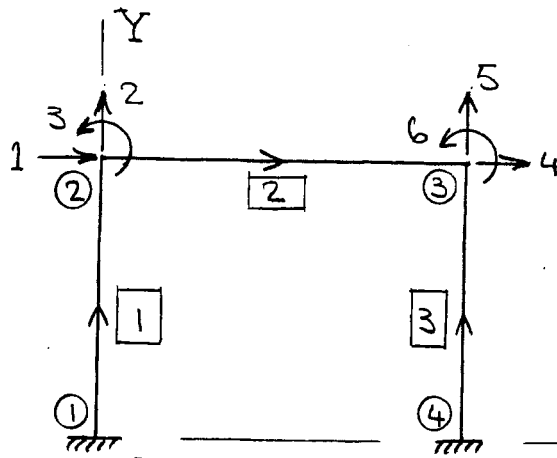
$$\{Q_1\} = \begin{bmatrix} -8.46 \text{ kN} \\ -9.53 \text{ kN} \\ -35.5 \text{ kN}\cdot\text{m} \\ 8.46 \text{ kN} \\ 9.53 \text{ kN} \\ -71 \text{ kN}\cdot\text{m} \end{bmatrix} ; \{F_1\} = \begin{bmatrix} 4.74 \text{ kN} \\ -11.82 \text{ kN} \\ -35.5 \text{ kN}\cdot\text{m} \\ -4.74 \text{ kN} \\ 11.82 \text{ kN} \\ -71 \text{ kN}\cdot\text{m} \end{bmatrix}$$

$$\{u_2\} = \{v_2\} = 10^{-4} \begin{bmatrix} 0.5924 \\ 1.0259 \\ -24.829 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{matrix} ; \{F_2\} = \{Q_2\} = \begin{bmatrix} 4.74 \text{ kN} \\ -11.82 \text{ kN} \\ -79 \text{ kN}\cdot\text{m} \\ -4.74 \text{ kN} \\ 11.82 \text{ kN} \\ -39.2 \text{ kN}\cdot\text{m} \end{bmatrix}$$



Reactions

18.9



Members 1 and 3: $L = 30 \text{ ft}$; $\cos \theta = 0$; $\sin \theta = 1$

$$[k_1] = [k_3] = EI \times 10^{-3} \begin{bmatrix} 80 & 0 & 0 & -80 & 0 & 0 \\ 0 & 0.444 & 6.67 & 0 & -0.444 & 6.67 \\ & 133.3 & 0 & 0 & -6.67 & 66.67 \\ \text{Sym.} & & 80 & 0 & 0 & 0 \\ & & & 0.444 & -6.67 & 133.3 \end{bmatrix}$$

$$[T_1] = [T_3] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

	1	2	3	4	5	6	7	8
Member 3	0	0	0	4	5	6		
Member 1	0	0	0	1	2	3		

$$[K_1] = [K_3] = EI \times 10^{-3} \begin{bmatrix} 0.444 & 0 & -6.67 & -0.444 & 0 & -6.67 & 0 & 0 \\ & 80 & 0 & 0 & -80 & 0 & 0 & 0 \\ & & 133.3 & 6.67 & 0 & 66.67 & 0 & 0 \\ & & & 0.444 & 0 & 6.67 & 1 & 4 \\ & & & & 80 & 0 & 2 & 5 \\ & & & & & 133.3 & 3 & 6 \end{bmatrix}$$

Member 2: $L = 40 \text{ ft}$; $\cos \theta = 1$; $\sin \theta = 0$

$$[k_2] = [K_2] = EI \times 10^{-3} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 120 & 0 & 0 & -120 & 0 & 0 \\ & 0.375 & 7.5 & 0 & -0.375 & 7.5 \\ & & 200 & 0 & -7.5 & 100 \\ & & & 120 & 0 & 0 \\ \text{Sym.} & & & & 0.375 & -7.5 \\ & & & & & 200 \end{bmatrix}$$

18.9 (Contd.)

$$\{F_{f2}\} = \{Q_{f2}\} = \begin{bmatrix} 0 \\ 30 \\ 200 \\ 0 \\ 30 \\ -200 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

Structure matrices:

$$[S] = EI \times 10^{-3} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 120.444 & 0 & 6.67 & -120 & 0 & 0 \\ & 80.375 & 7.5 & 0 & -0.375 & 7.5 \\ & & 333.3 & 0 & -7.5 & 100 \\ & & & 120.444 & 0 & 6.67 \\ \text{Sym.} & & & & 80.375 & -7.5 \\ & & & & & 333.33 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

$$\{P\} = \begin{bmatrix} 20 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \quad \{P_f\} = \{F_{f2}\}; \quad \{P\} - \{P_f\} = \begin{bmatrix} 20 \\ -30 \\ -200 \\ 0 \\ -30 \\ 200 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

Joint displacements: By solving the equations

$\{P\} - \{P_f\} = [S]\{d\}$, we obtain:

$$\{d\} = \frac{1}{EI \times 10^{-3}} \begin{bmatrix} 29.4204 \\ -0.2906 \\ -1.314 \\ 29.289 \\ -0.4594 \\ 0.4044 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

Member forces:

$$\{V_1\} = \frac{1}{EI \times 10^{-3}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 29.4204 \\ -0.2906 \\ -1.314 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{matrix}$$

$$\{V_4\} = \frac{1}{EI \times 10^{-3}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.2906 \\ -29.4204 \\ -1.314 \end{bmatrix}$$

18.9 (contd.)

$$\{Q_1\} =$$

$$\begin{bmatrix} 23.26 \text{ k} \\ 4.3 \text{ k} \\ 108 \text{ k-ft} \\ -23.26 \text{ k} \\ -4.3 \text{ k} \\ 21 \text{ k-ft} \end{bmatrix}$$

$$\{u_2\} = \{v_2\} = \{d\};$$

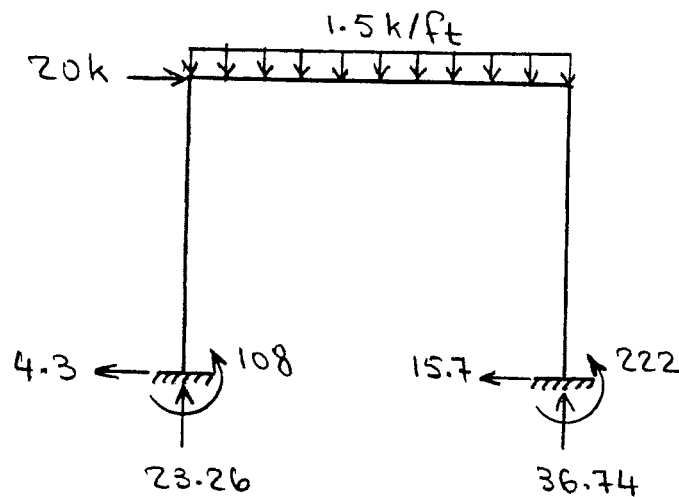
$$\{Q_2\} =$$

$$\begin{bmatrix} 15.7 \text{ k} \\ 23.26 \text{ k} \\ -21 \text{ k-ft} \\ -15.7 \text{ k} \\ 36.74 \text{ k} \\ -249 \text{ k-ft} \end{bmatrix}$$

$$\{v_3\} = \frac{1}{EI \times 10^{-3}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 29.289 \\ -0.4594 \\ 0.4044 \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 4 \\ 5 \\ 6 \end{matrix}$$

$$\{u_3\} = \frac{1}{EI \times 10^{-3}} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.4594 \\ -29.289 \\ 0.4044 \end{bmatrix}$$

$$\{Q_3\} = \begin{bmatrix} 36.74 \text{ k} \\ 15.7 \text{ k} \\ 222 \text{ k-ft} \\ -36.74 \text{ k} \\ -15.7 \text{ k} \\ 249 \text{ k-ft} \end{bmatrix}$$



Reactions

Appendix B
Review of Matrix Algebra
&
Appendix C
Computer Software

APPENDIX B

B.1

$$[C] = [A] + 3[B] = \begin{bmatrix} 18 & -11 & 18 \\ -11 & 19 & 28 \\ 18 & 28 & 4 \end{bmatrix}$$

B.2

$$[C] = 2[A] - [B] = \begin{bmatrix} 7 & 8 \\ 11 & 7 \\ 1 & 0 \end{bmatrix}$$

B.3

$$[C] = \begin{bmatrix} -6 & 4 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} = -12 - 4 - 10 = \underline{-26}$$

$$[D] = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} \begin{bmatrix} -6 & 4 & -2 \end{bmatrix} = \begin{bmatrix} -12 & 8 & -4 \\ 6 & -4 & 2 \\ -30 & 20 & -10 \end{bmatrix}$$

B.4

$$[C] = \begin{bmatrix} 2 & -5 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} -3 & 4 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} -26 & 3 \\ 27 & -17 \end{bmatrix}$$

$$[D] = \begin{bmatrix} -3 & 4 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & -5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} -26 & 27 \\ 3 & -17 \end{bmatrix}$$

B.5

$$\begin{aligned}[A][B] &= \begin{bmatrix} 8 & -2 & 5 \\ 1 & -4 & 3 \\ 2 & 0 & 6 \end{bmatrix} \begin{bmatrix} 1 & -5 \\ 7 & 0 \\ 0 & -3 \end{bmatrix} \\ &= \begin{bmatrix} (8-14) & (-40-15) \\ (1-28) & (-5-9) \\ (2) & (-10-18) \end{bmatrix} = \begin{bmatrix} -6 & -55 \\ -27 & -14 \\ 2 & -28 \end{bmatrix} \\ [AB]^T &= \begin{bmatrix} -6 & -27 & 2 \\ -55 & -14 & -28 \end{bmatrix} \quad (1)\end{aligned}$$

$$\begin{aligned}[B]^T[A]^T &= \begin{bmatrix} 1 & 7 & 0 \\ -5 & 0 & -3 \end{bmatrix} \begin{bmatrix} 8 & 1 & 2 \\ -2 & -4 & 0 \\ 5 & 3 & 6 \end{bmatrix} \\ &= \begin{bmatrix} (8-14) & (1-28) & (2) \\ (-40-15) & (-5-9) & (-10-18) \end{bmatrix} \\ &= \begin{bmatrix} -6 & -27 & 2 \\ -55 & -14 & -28 \end{bmatrix} \quad (2)\end{aligned}$$

From Eqs. (1) and (2) we can see that:

$$\underline{[AB]^T = [B]^T[A]^T}$$

B-6

$$\left[\begin{array}{ccc|c} 2 & 5 & -1 & 15 \\ 5 & -1 & 3 & 27 \\ -1 & 3 & 4 & 14 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 2.5 & -0.5 & 7.5 \\ 0 & -13.5 & 5.5 & -10.5 \\ 0 & 5.5 & 3.5 & 21.5 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0.518 & 5.555 \\ 0 & 1 & -0.407 & 0.778 \\ 0 & 0 & 5.739 & 17.221 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

Thus,

$$\underline{x_1 = 4 \quad x_2 = 2 \quad x_3 = 3}$$

B.7

$$\left[\begin{array}{ccc|c} -12 & -3 & 6 & 45 \\ 5 & 2 & -4 & -9 \\ 10 & 1 & -7 & -32 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0.25 & -0.5 & -3.75 \\ 0 & 0.75 & -1.5 & 9.75 \\ 0 & -1.5 & -2 & 5.5 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -7 \\ 0 & 1 & -2 & 13 \\ 0 & 0 & -5 & 25 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -7 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \end{array} \right]$$

Thus,

$$\underline{x_1 = -7 \quad x_2 = 3 \quad x_3 = -5}$$

B.8

$$\left[\begin{array}{cccc|c} 5 & -2 & 6 & 0 & 0 \\ -2 & 4 & 1 & 3 & 18 \\ 6 & 1 & 6 & 8 & -29 \\ 0 & 3 & 8 & 7 & 11 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & -0.4 & 1.2 & 0 & 0 \\ 0 & 3.2 & 3.4 & 3 & 18 \\ 0 & 3.4 & -1.2 & 8 & -29 \\ 0 & 3 & 8 & 7 & 11 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1.625 & 0.375 & 2.25 \\ 0 & 1 & 1.063 & 0.938 & 5.625 \\ 0 & 0 & -4.814 & 4.811 & -48.125 \\ 0 & 0 & 4.811 & 4.186 & -5.875 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & -14 \\ 0 & 1 & 0 & 2 & -5 \\ 0 & 0 & 1 & -1 & 10 \\ 0 & 0 & 0 & 9 & -54 \end{array} \right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & -6 \end{array} \right]$$

Thus,

$$\underline{x_1 = -2 \quad x_2 = 7 \quad x_3 = 4 \quad x_4 = -6}$$

B.9

$$\left[\begin{array}{ccc|ccc} 4 & -3 & -1 & 1 & 0 & 0 \\ -2 & 5 & 1 & 0 & 1 & 0 \\ 6 & -4 & -5 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & -0.75 & -0.25 & 0.25 & 0 & 0 \\ 0 & 3.5 & 0.5 & 0.5 & 1 & 0 \\ 0 & 0.5 & -3.5 & -1.5 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -0.143 & 0.357 & 0.215 & 0 \\ 0 & 1 & 0.143 & 0.143 & 0.286 & 0 \\ 0 & 0 & -3.572 & -1.572 & -0.143 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0.42 & 0.22 & -0.04 \\ 0 & 1 & 0 & 0.08 & 0.28 & 0.04 \\ 0 & 0 & 1 & 0.44 & 0.04 & -0.28 \end{array} \right]$$

Thus,

$$[A]^{-1} = \begin{bmatrix} 0.42 & 0.22 & -0.04 \\ 0.08 & 0.28 & 0.04 \\ 0.44 & 0.04 & -0.28 \end{bmatrix}$$

B.10

$$\left[\begin{array}{cccc|cccc} 4 & 2 & 0 & -3 & 1 & 0 & 0 & 0 \\ 2 & 3 & -4 & 0 & 0 & 1 & 0 & 0 \\ 0 & -4 & 2 & -1 & 0 & 0 & 1 & 0 \\ -3 & 0 & -1 & 5 & 0 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0.5 & 0 & -0.75 & 0.25 & 0 & 0 & 0 \\ 0 & 2 & -4 & 1.5 & -0.5 & 1 & 0 & 0 \\ 0 & -4 & 2 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1.5 & -1 & 2.75 & 0.75 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 1 & -1.125 & 0.375 & -0.25 & 0 & 0 \\ 0 & 1 & -2 & 0.75 & -0.25 & 0.5 & 0 & 0 \\ 0 & 0 & -6 & 2 & -1 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1.625 & 1.125 & -0.75 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -0.7917 & 0.2083 & 0.0833 & 0.1667 & 0 \\ 0 & 1 & 0 & 0.0833 & 0.0833 & -0.1667 & -0.3333 & 0 \\ 0 & 0 & 1 & -0.3333 & 0.1667 & -0.3333 & -0.1667 & 0 \\ 0 & 0 & 0 & 2.2917 & 0.7917 & -0.0833 & 0.3333 & 1 \end{array} \right]$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0.4818 & 0.0545 & 0.2818 & 0.3455 \\ 0 & 1 & 0 & 0 & 0.0545 & -0.1636 & -0.3455 & -0.0364 \\ 0 & 0 & 1 & 0 & 0.2818 & -0.3455 & -0.1182 & 0.1455 \\ 0 & 0 & 0 & 1 & 0.3455 & -0.0364 & 0.1455 & 0.4364 \end{array} \right]$$

Thus,

$$[A]^{-1} = \begin{bmatrix} 0.4818 & 0.0545 & 0.2818 & 0.3455 \\ 0.0545 & -0.1636 & -0.3455 & -0.0364 \\ 0.2818 & -0.3455 & -0.1182 & 0.1455 \\ 0.3455 & -0.0364 & 0.1455 & 0.4364 \end{bmatrix}$$

APPENDIX C

$$\boxed{C.1} \quad \frac{L}{360} = \frac{80}{360} = 0.222 \text{ ft} = 2.67 \text{ in.}$$

$$(a) \quad \underline{A = 9.12 \text{ in}^2}$$

$$(b) \quad \underline{A = 6.33 \text{ in}^2}$$

$$(c) \quad \underline{A = 8.66 \text{ in}^2}$$

$$\boxed{C.2} \quad \frac{L}{360} = \frac{24}{360} = 0.0667 \text{ m}$$

$$(a) \quad \underline{A = 8470 \text{ mm}^2}$$

$$(b) \quad \underline{A = 7620 \text{ mm}^2}$$

$$(c) \quad \underline{A = 6860 \text{ mm}^2}$$

C.3

$$\underline{I = 1257 \text{ in}^4}$$